Engineering Mechanics

References:

1. Mechanics of Materials I - E. J. Hearn

2. Strength of Materials - Ferdinand L. Singer and Andrew Pytel

Simple Stress and Strain

1.1 Load

In any engineering structure or mechanism, the individual components will be subjected to external forces arising from the service conditions or environment in which the component works.

1.2 Type of Load

(a) Static or **dead load**, generally caused by gravity effects.

(b) Live load, produced by cars crossing a bridge.

(c) Impact or Shock loads, caused by sudden blows.

(d) Fatigue, fluctuating or alternating loads, the magnitude and sign of the load changing with time.

1.3 Direct or Normal Stress (σ)

Is the normal force applied to the cross-sectional area.

$$\textbf{\textit{Stress}}\ (\sigma) = \frac{\textbf{\textit{Load}}}{\textbf{\textit{Area}}} = \frac{\boldsymbol{P}}{\boldsymbol{A}}$$

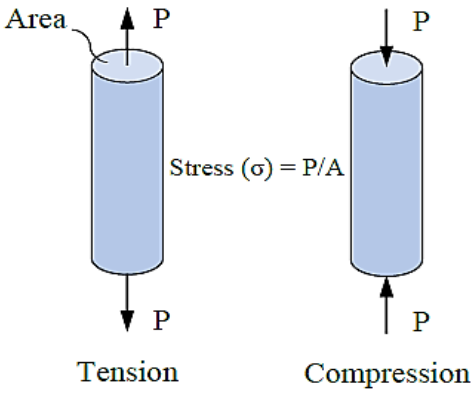


Fig. 1.1 Types of direct stress.

1.4 Direct Strain (Є)

If a bar is subjected to a direct load, and hence stress, the bar will change in length. If the bar has an original length L and changes in length by an amount δL, the strain produced is defined as follows.

$$\textbf{Strain } (\epsilon) = \frac{\textit{Change in length}}{\textit{Original length}} = \frac{\delta L}{L}$$

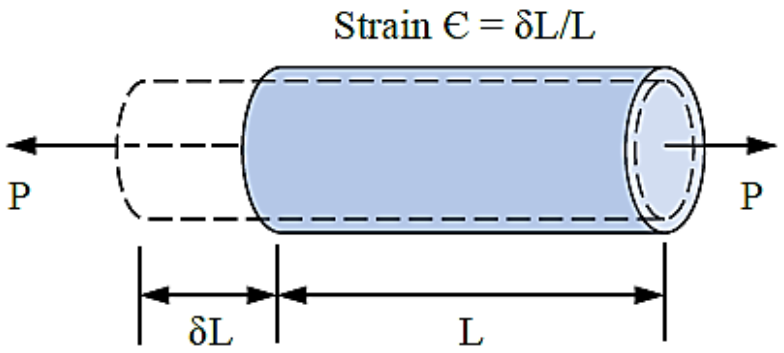


Fig. 1.2 Deformation of the material.

1.5 Elastic materials-Hooke's law

A material is said to be elastic if it returns to its original dimensions when the load is removed. Since loads are proportional to the stresses they produce, and the deformations are proportional to the strains, stress is proportional to strain. Hooke's law, in its simplest form:

$$\textit{Stress } (\sigma) \propto \textit{Strain } (Є)$$

$$\frac{\textit{stress}}{\textit{strain}} = \textit{constant}$$

1.6 Modulus of Elasticity-Young's modulus

Within the elastic limits of materials, the limits in which Hooke's law applies it has a constant, this constant is termed by the symbol ***E*** and called the modulus of elasticity or Young's modulus.

$$\boldsymbol{E} = \frac{\boldsymbol{stress}}{\boldsymbol{strain}} = \frac{(\boldsymbol{\sigma})}{(\epsilon)}$$

1.7 Examples

1. A prismatic bar with a circular cross section with diameter d= 50mm and axial tensile load P = 10 kN. Find the normal stress.

Solution:

$$\sigma = \frac{P}{A} = \frac{P}{\pi d^2/4} = \frac{4(10 \times 10^3)}{\pi(50 \times 10^{-3})^2} \frac{N}{m^2} = 5.0929 \times 10^6 \text{ Pa}$$

$$\sigma = 5.0929 \text{ MPa}$$

2. Prismatic bar has length L= 2.0 m. A tensile load is applied which causes the bar to extend by δ=1.4 mm. Find the normal strain.

Solution:

$$\epsilon = \frac{\delta L}{L} = \frac{1.4 \times 10^{-3}}{2}\,\frac{m}{m} = 0.0007$$

Units: none, although sometime quoted as $\mu\epsilon$ (micro strain $\epsilon \times 10^{-6}$) or % strain.

$$\epsilon = 0.0007 = 7 \times 10^{-4} = (7 \times 10^{2})(10^{-4} \times 10^{-2}) = 700 \times 10^{-6}$$

$$\epsilon = 700\ \mu\epsilon$$

$$\epsilon = 0.0007 = 0.07\%\ strain$$

3. A Steel rod (E=200 GPa) has a circular cross section and is 10m long. Determine the minimum diameter if the rod must hold a 30 kN tensile force without deforming more than 5mm. Assume the steel stays in the elastic region. Note, 1 GPa = 10^9 Pa.

Solution:

$\delta L = 5mm = 5 \times 10^{-3}m$, $L = 10\ m$, $P = 30 \times 10^{3}N$, $E = 200 \times 10^{9}Pa$

$$\varepsilon = \frac{\delta L}{L} = \frac{5 \times 10^{-3}}{10} = 0.0005$$

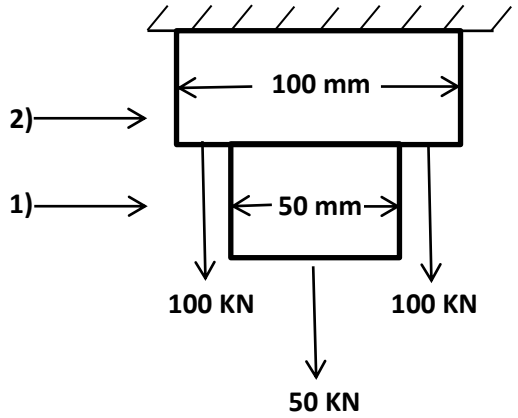
$$\sigma = E\epsilon = 200 \times 10^{9} \times 0.0005 = 0.1 \times 10^{9}\ N/m^{2}$$

$$A = \frac{P}{\sigma} = \frac{30 \times 10^{3}}{0.1 \times 10^{9}} = 0.0003m^{2} = 300mm^{2}$$

$$A = \frac{\pi d^{2}}{4}$$

$$d = \sqrt[2]{\frac{300 \times 4}{\pi}} = 19.5mm$$

4. A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self-weight, the maximum tensile stress in N/mm² anywhere is



Solution:

$\sigma = \frac{P}{A}$

$P_1 = 50 \times 10^3 N$

$P_2 = 100 + 100 + 50 = 250 \times 10^3\ N$

$A_1 = 50 \times 50 = 2500\ mm^2$

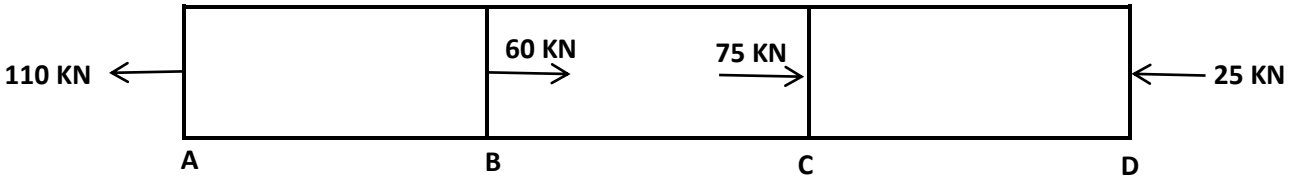
$A_2 = 100 \times 100 = 1 \times 10^4\ mm^2$

$\sigma_1 = \frac{P_1}{A_1} = \frac{50 \times 10^3}{2500} = 20\ N/\ mm^2$

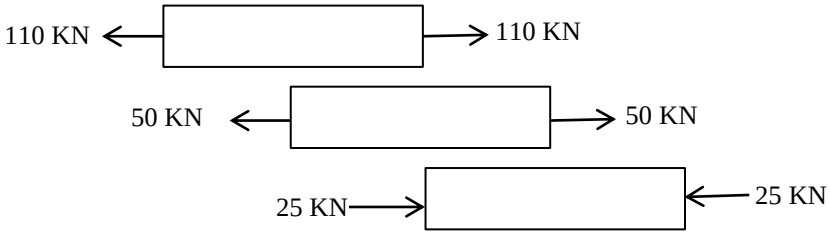
$\sigma_2 = \frac{P_2}{A_2} = \frac{250 \times 10^3}{1 \times 10^4} = 25\ N/\ mm^2$,,

Maximum tensile stress= $\sigma_2 = 25\ \mathrm{N}/\ \mathrm{mm}^2$

5. A bar having a cross sectional area of (2500 $\boldsymbol{mm^2}$) is subjected to axial loads at the position indicated. Determine the value of stress in the segment AB, BC, CD, and total stress?



Solution:



$\sigma_{AB} = \frac{P_{AB}}{A} = \frac{110\times10^3}{2500} = 44\,\frac{N}{mm^2}$ (tensile)

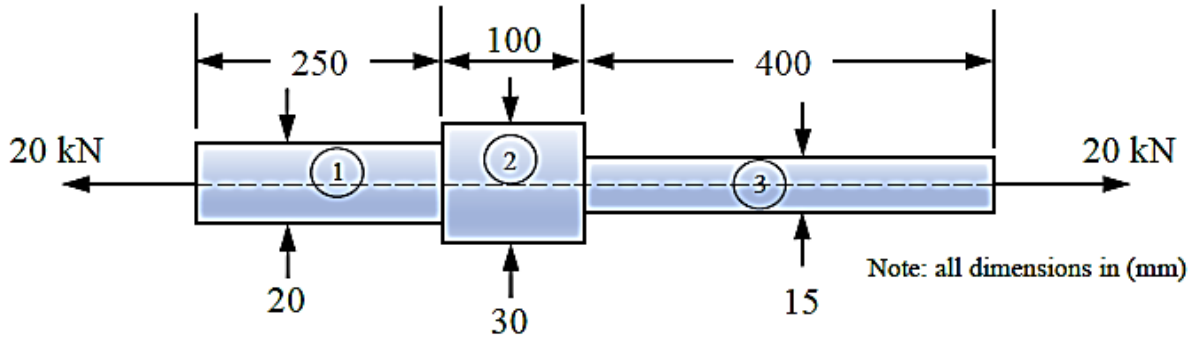
$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{50\times10^3}{2500} = 20\,\frac{N}{mm^2}$ (tensile)

$\sigma_{CD} = \frac{P_{CD}}{A} = \frac{25\times10^3}{2500} = 10\,\frac{N}{mm^2}$ (compression)

$total\ stress\ (\sigma_t) = \sigma_{AB} + \sigma_{BC} + \sigma_{CD}$

$\sigma_t = 44 + 20 - 10 = 54\,\frac{N}{mm^2}$

6. Determine the stress in each section of the bar shown in Figure when subjected to an axial tensile load of 20 kN. The central section is 30 mm square cross-section; the other portions are of circular section, their diameters being indicated. What will be the total extension of the bar? For the bar material E = 210 GN/m^2.



Solution:

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{P}{A}$$

$$\text{Stress in section (1)} = \frac{20\times10^{3}}{\frac{\pi(20\times10^{-3})^{2}}{4}} = \frac{80\times10^{3}}{\pi\times400\times10^{-6}} = 63.66\ \text{MN/m}^2$$

$$\text{Stress in section (2)} = \frac{20\times10^{3}}{30\times30\times10^{-6}} = 22.2\ \text{MN/m}^2$$

$$\text{Stress in section (3)} = \frac{20\times10^{3}}{\frac{\pi(15\times10^{-3})^{2}}{4}} = \frac{80\times10^{3}}{\pi\times225\times10^{-6}} = 113.2\ \text{MN/m}^2$$

Now the extension of a bar can always be written in terms of the stress in the bar since,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\delta / L}$$

$$\delta = \frac{\sigma L}{E}$$

$$\text{Extension of section (1)} = 63.66\times10^{6}\times\frac{250\times10^{-3}}{210\times10^{9}} = 75.8\times10^{-6}\ \text{m}$$

$$\text{Extension of section (2)} = 22.2 \times 10^6 \times \frac{100 \times 10^{-3}}{210 \times 10^9} = 10.6 \times 10^{-6} \text{ m}$$

$$\text{Extension of section (3)} = 113.2 \times 10^6 \times \frac{400 \times 10^{-3}}{210 \times 10^9} = 215.6 \times 10^{-6} \text{ m}$$

$$\text{Total extension} = (75.8 + 10.6 + 215.6)10^{-6}$$

$$= 302 \times 10^{-6} \ m$$

$$= \mathbf{0.302 \ mm}$$

7. A 25 mm diameter bar is subjected to an axial tensile load of 100 kN. Under the action of this load, a 200 mm gauge length is found to extend 0.19×10^{-3}mm. Determine the modulus of elasticity for the bar material.

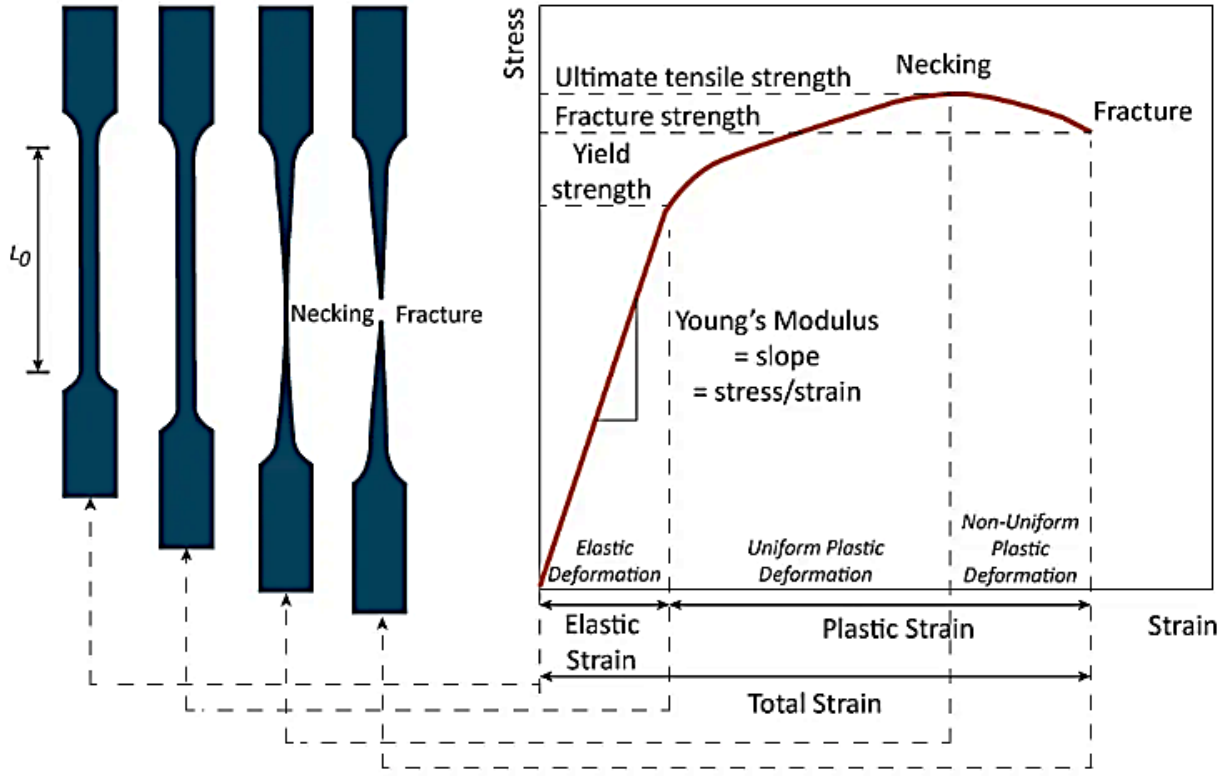
Solution :

$$\text{Young's modulus } E = \frac{PL}{A\,\delta L}$$

$$= \frac{100 \times 10^3 \times 200 \times 10^{-3}}{\frac{\pi(25 \times 10^{-3})^2}{4} \times 0.19 \times 10^{-3}} = \mathbf{214 \ GN/m^2}$$

1.8 Tensile Test

In order to compare the strengths of various materials, it is necessary to carry out some standard forms of the test to establish their relative properties. One such test is the standard tensile test in which a circular bar of the uniform cross-section is subjected to a gradually increasing tensile load until failure occurs. Measurements of the change in length of a selected gauge length of the bar are recorded throughout the loading operation by means of extensometers and a graph of load against extension or stress against strain is produced, as shown in Fig. 1.3. This shows a typical result for a test on a mild (low carbon) steel bar. Other materials will exhibit different graphs but of a similar general form



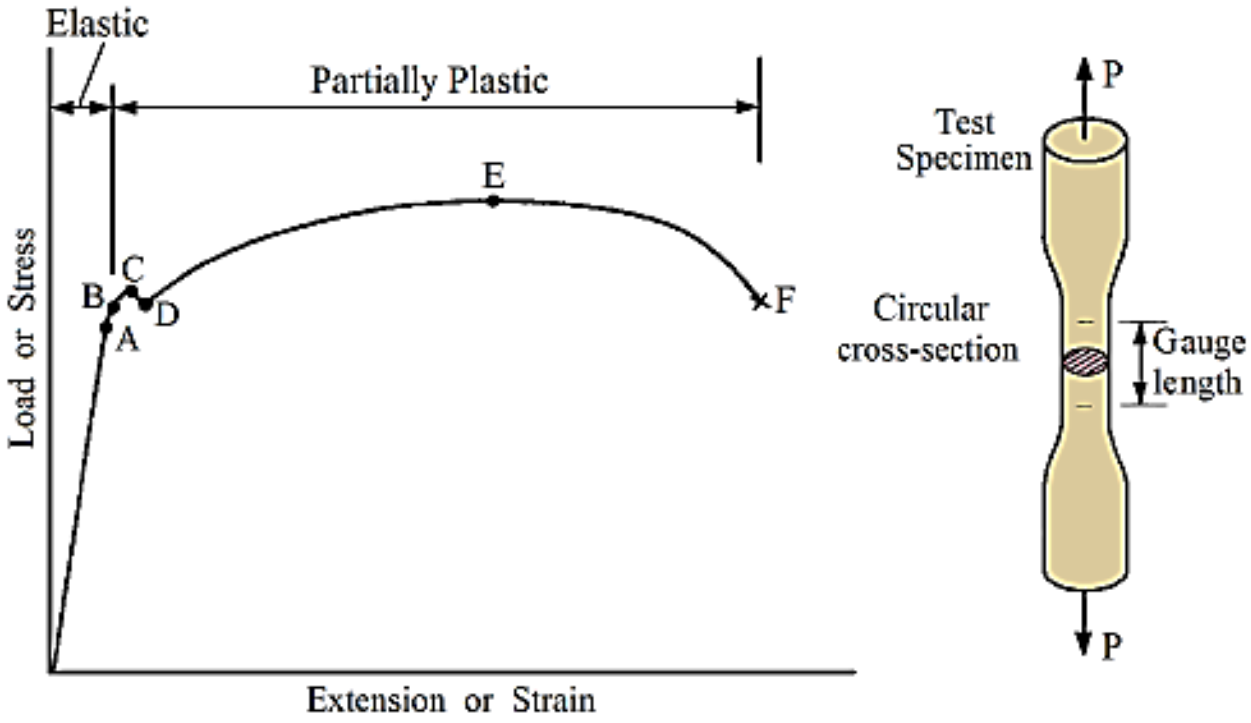


Fig. 1.3 Typical tensile test curve for mild steel.

For the first part of the test, it will be observed that Hooke's law is obeyed, i.e. the material behaves elastically and stress is proportional to strain, giving the straight-line graph indicated. Some point A is eventually reached, however, when the linear nature of the graph ceases and this point is termed the *limit of proportionality*.

For a short period beyond this point the material may still be elastic in the sense that deformations are completely recovered when the load is removed (i.e. strain returns to zero) but Hooke's law does not apply. The limiting point B for this condition is termed the *elastic limit*. For most practical purposes, it can often be assumed that points A and B are coincident.

Beyond the elastic limit, plastic deformation occurs and strains are not recoverable. There will thus be some permanent deformation or permanent set when the load is removed. After the point C termed the upper yield point and D the lower yield point.

For example, the certain materials of high carbon steels and non-ferrous metals, it is not possible to detect any difference between the upper and lower yield points and in some cases, no yield point exists at all. In such cases, proof stress is used to indicate the onset of plastic strain or as a comparison of the relative properties with

another similar material. This involves a measure of the permanent deformation produced by a loading cycle. For example, the 0.1 % proof stress is that stress which, when removed, produces a permanent strain or "set" of 0.1 % of the original gauge length, as shown in Fig. l.4.

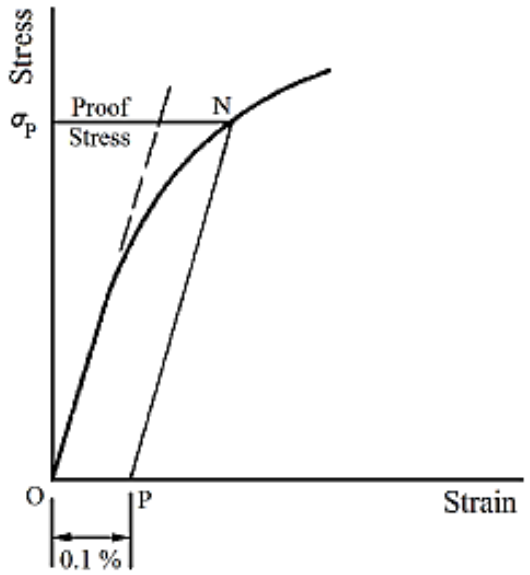


Fig. 1.4 Determination of 0.1 % proof stress.

1.9 Ductile and Brittle Materials

Ductility is the capacity of a material to allow large extensions or the ability to be drawn out plastically. While a brittle material is one which exhibits relatively small extensions to fracture so that the partially plastic region of the tensile test graph is much reduced.

$$\text{Percentage elongation} = \frac{\text{increase in gauge length to fracture}}{\text{original gauge length}} \times 100\%$$

$$\text{Percentage reduction in area} = \frac{\text{reduction in cross} - \text{sectional area of necked portion}}{\text{original area}} \times 100\%$$

1.10 Example for Tensile Test

1. The following observations were made during the tensile test on a 25 mm diameter and 175 mm long metallic bar: Elastic elongation is 0.05 mm at a load of 65 KN, load at yield point is 140 KN, maximum load is 225 KN, length of specimen at fracture is 180 mm. Determine:

1. Young's modulus 2. yield strength 3. ultimate tensile strength 4. Percent elongation.

Solution:

$d = 25mm, L = 175mm, \delta L = 0.05mm, P_E = 65\ KN, P_Y = 140\ KN,$

$P_{Max} = 225\ KN,\ L_f = 180mm$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (25^2)}{4} = 490.87mm^2$$

$$\varepsilon = \frac{\delta L}{L} = \frac{0.05}{175} = 2.85 \times 10^{-4}$$

$$\sigma_E = \frac{P_E}{A} = \frac{65}{490.87} = 0.1324\ \frac{KN}{mm^2}$$

1. At Elastic region: (Hook's low)

$$E = \frac{\sigma_E}{\varepsilon} = \frac{0.1324}{2.85 \times 10^{-4}} = 464.561 \frac{KN}{mm^2}$$

2. *At yield point* :

$$\sigma_Y = \frac{P_Y}{A} = \frac{140}{490.87} = 0.285\ \frac{KN}{mm^2}$$

3. *At maximum load* :

$$\sigma_{Max} = \frac{P_{Max}}{A} = \frac{225}{490.87} = 0.458\ \frac{KN}{mm^2}$$

4. Percent elongation.

$$\%elongation = \left(\frac{L_f - L}{L}\right) \times 100\% = \left(\frac{180 - 175}{175}\right) \times 100\% = 2.85\%$$

1.11 Poisson's ratio

Consider the rectangular bar as shown in Fig. 1.5 subjected to a tensile load. Under the action of this load, the bar will increase in length by an amount δL giving a longitudinal strain in the bar of

$$\boldsymbol{\epsilon_L = \frac{\delta L}{L}}$$

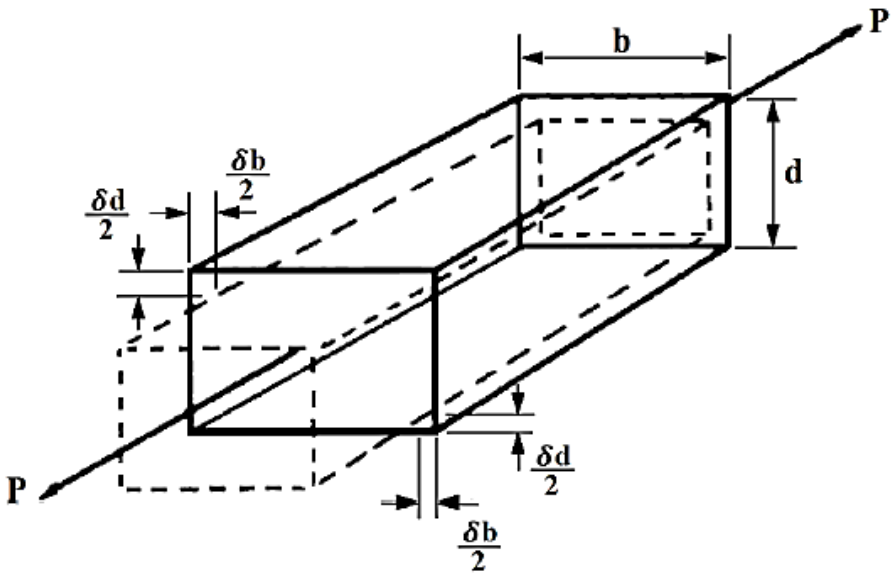


Fig. 1.5 Rectangular bar.

The bar will also exhibit, a reduction in dimensions laterally, i.e. its breadth and depth will both reduce. The associated lateral strains will both be equal, will be of opposite sense to the longitudinal strain, and given by

$$\epsilon_{lat} = -\frac{\delta b}{b} = -\frac{\delta d}{d}$$

$$\boldsymbol{Poissosn's\ ratio}(v) = \frac{\boldsymbol{lateral\ strain}}{\boldsymbol{longitudinal\ strain}} = \frac{(-\boldsymbol{\delta d}/\boldsymbol{d})}{\boldsymbol{\delta L}/\boldsymbol{L}}$$

1.12 Example for Poisson's ratio

1. For the two-dimensional stress case:

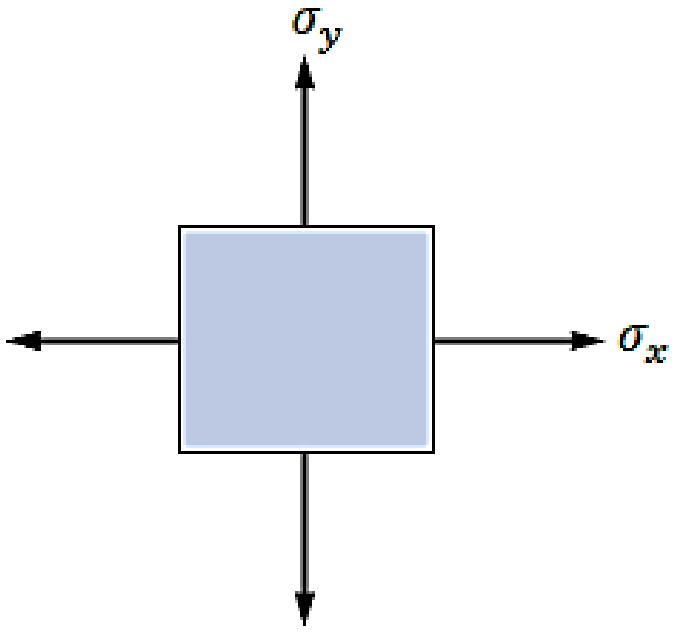


Fig. 1.6 Simple two-dimensional system of direct stresses.

The following strains will be produced
(a) In the X direction resulting from $\sigma_x, \epsilon_x = \sigma_x/E$.
(b) In the Y direction resulting from $\sigma_y, \epsilon_y = \sigma_y\ /E$.
(c) In the X direction resulting from $\sigma_y, \epsilon_x = -v(\sigma_y,/E)$.
(d) In the Y direction resulting from $\sigma_x, \epsilon_y = -v(\sigma_x/E)$.
Strains (c) and (d) being the so-called ***Poisson's ratio strain***, opposite in sign to the applied strains, i.e. compressive.

The total strain in the X direction will therefore be given by:

$$\epsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E} = \frac{1}{E}(\sigma_x - v\sigma_y)$$

and the total strain in the Y direction will be:

$$\epsilon_y = \frac{\sigma_y}{E} - v\frac{\sigma_x}{E} = \frac{1}{E}(\sigma_y - v\sigma_x)$$

2. Consider a steel bar of 100 mm length and 50 mm width. If after the application of 50N, steel bar length increases to 102 mm. What will be the change in width?., Note that positon′s ratio is 0.3.

Solution :

$\mathrm{L} = 100\mathrm{mm}, \quad \mathrm{W} = 50\mathrm{mm}, \quad \mathrm{P} = 50\mathrm{KN}, \ \text{increase in length} = 102\mathrm{mm}$

$$\epsilon_l = \frac{\delta L}{L} = \frac{\text{increase in length} - \text{original length}}{\text{original length}} = \frac{l - l_\circ}{l_\circ} = \frac{102 - 100}{100} = 0.02$$

$$\text{positon's ratio (v)} = \frac{\epsilon_{lat}}{\epsilon_l} = \frac{\delta w/w}{\delta L/L}$$

$$0.3 = \frac{\delta w/50}{0.02} \quad => \delta w = 0.3$$

3. When a brass rod of diameter 6 mm is subjected to a tension of 5×10^3 N, the diameter changes by 3.6×10^{-4} cm. Calculate the longitudinal strain and Poisson's ratio for brass given that E for the brass is 9×10^{10} N/m².

Solution :

Given: Diameter of rod = d = 6 mm , Load P = 5×10^3 N, $\delta d = 3.6 \times 10^{-4}$ cm = 3.6×10^{-6} m, E =9×10^{10} N/m².

$$\sigma = \frac{P}{A} = \frac{5 \times 10^3}{\frac{\pi(6 \times 10^{-3})^2}{4}} = 176838825.7 \text{ pa}$$

$$E = \frac{\sigma}{\epsilon_l} => \epsilon_l = \frac{\sigma}{E} = \frac{176838825.7}{9\times10^{10}} \quad => \epsilon_l = 1.96 \times 10^{-3}$$

$$\epsilon_{lat} = \frac{\delta d}{d} = \frac{3.6 \times 10^{-6}}{6 \times 10^{-3}} = 6 \times 10^{-4}$$

$$v = \frac{\epsilon_{lat}}{\epsilon_l} = \frac{6 \times 10^{-4}}{1.96 \times 10^{-3}} \quad => v = 31$$

1.13 Shear stress

Shear stress always tangential to the area on which it acts, and the direct stresses are always normal to the area on which they act.

$$\boldsymbol{Shear\ Strass\ (\tau) = \frac{shear\ load}{area\ resisting\ shear} = \frac{Q}{A}}$$

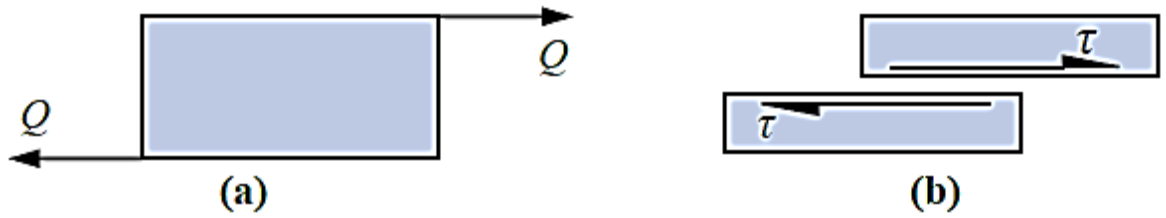
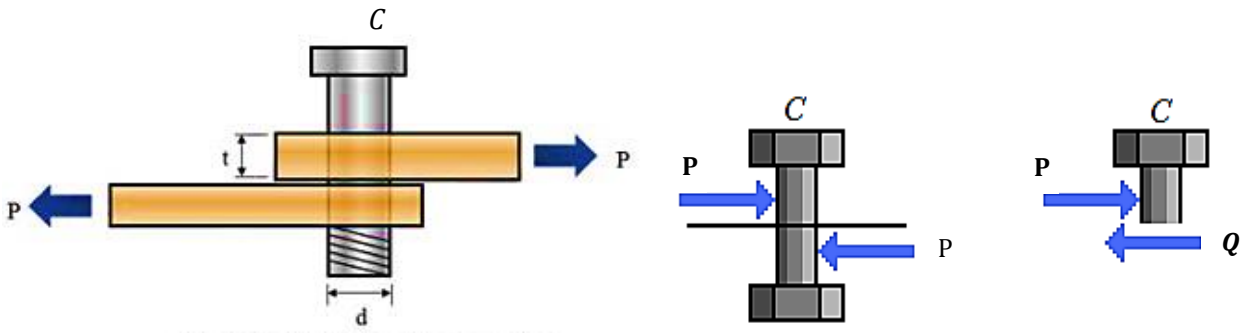


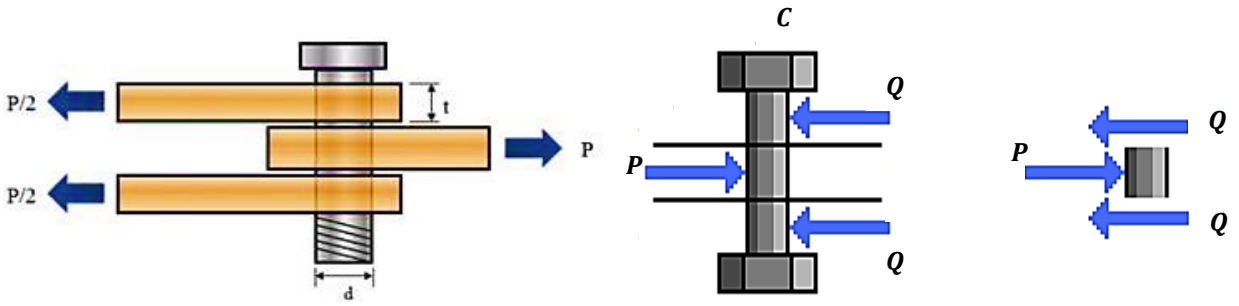
Fig. 1.7 Shear force and resulting shear stress system showing a typical form of failure by relative sliding of planes.

- Single shear stress



$$\tau = \frac{Q}{A}$$

- Double shear stress



$$\tau = \frac{Q}{2A}$$

1.14 Shear strain

The angle of deformation γ is the shear strain. Shear strain is dimensionless; it has no units and is measured in radians.

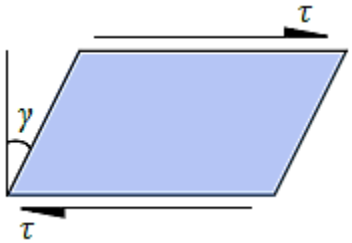


Fig. 1.8 Deformation (shear strain) produced by shear stresses.

For materials within the elastic range, the shear strain is proportional to the shear stress producing it,

$$\frac{shear\ stress}{shear\ strain} = \frac{\tau}{\gamma} = constant = G$$

The constant **G** is termed the modulus of rigidity or shear modulus and is directly comparable to the modulus of elasticity used in the direct stress application.

1.15 Example for shear stress

1. The coupling shown in figure (1.9)is constructed from steel of rectangular cross-section and is designed to transmit a tensile force of (50 kN). If the bolt is (15 mm) diameter calculate:
   a) The shear stress in the bolt.
   b) The direct stress in the plate.
   c) The direct stress in the forked end of the coupling.

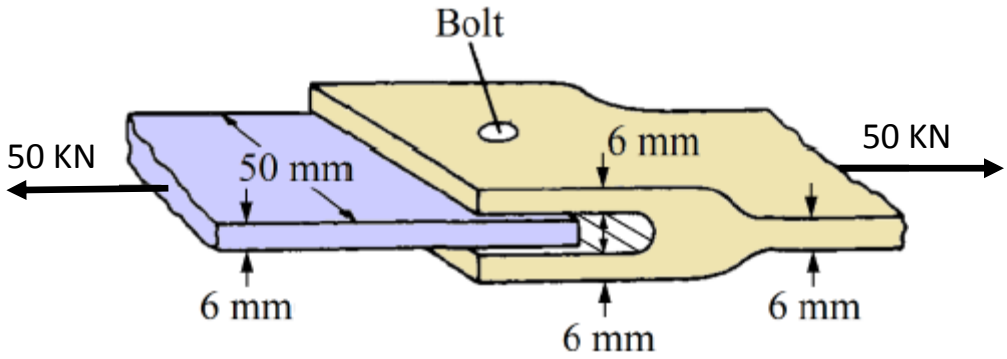


Fig 1.9 Steel rectangular coupling.

Solution :

(a) The bolt is subjected to double shear, tending to shear. There is thus twice the area of the bolt resisting the shear.

$$shear\ stress\ in\ bolt\ (\tau) = \frac{Q}{2A} = \frac{\mathbf{50 \times 10^3}}{\mathbf{2 \times \frac{\pi(15 \times 10^{-3})}{4}}} = \mathbf{141.5\ MPa}$$

(b) The plate will be subjected to a direct tensile stress given by.

$$tensile\ stress\ (\sigma) = \frac{P}{A} = \frac{\mathbf{50 \times 10^3}}{\mathbf{50 \times 6 \times 10^{-6}}} = \mathbf{166.7\ MPa}$$

(c) The force in the coupling is shared by the forked end pieces, each being subjected to direct stress.

$$direct\ stress\ (\sigma) = \frac{P}{A} = \frac{\mathbf{25 \times 10^3}}{\mathbf{50 \times 6 \times 10^{-6}}} = \mathbf{83.3\ MPa}$$

2. Find the smallest diameter bolt that can be used in the clevis shown in Fig(1.10) if P =400 kN. The shearing strength of the bolt is 300 MPa.

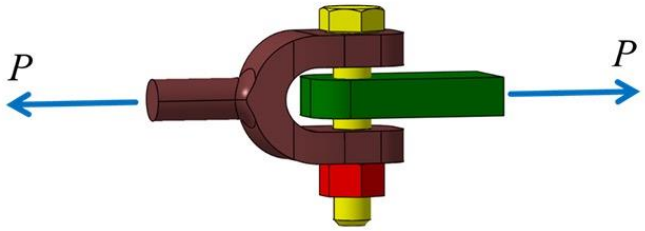


Fig 1.10

Solution :

$$\tau = \frac{Q}{2A} => A = \frac{Q}{2\tau} = \frac{400 \times 10^3}{2 \times 300} = 666.67$$

$$A = \frac{\pi d^2}{4} => d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 666.67}{\pi}} = 29.13\ mm$$

3. What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m2.

Solution :

$$\tau = \frac{Q}{A} => Q = A\tau = (\pi Dt)\tau$$

$$Q = \pi \times 20 \times 25 \times 350 = 549778\ N$$

$$Q = 549.8\ KN$$

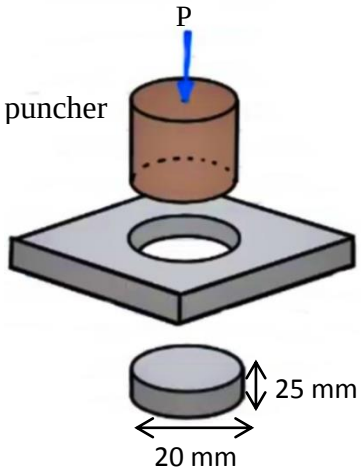


Fig 1.11

4. As in Figure (1.12) hole is to be punched out of a plate having a shearing strength of 40ksi. The compressive stress in the punch is limited to 50 ksi. **(a)** Compute the maximum thickness of plate in which a hole 2.5 inches in diameter can be punched. **(b)** If the plate is 0.25 inch thick, determine the diameter of the smallest hole that can be punched.

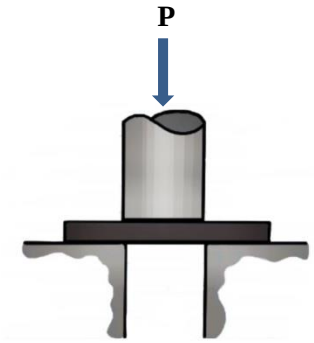


Fig 1.12

Solution :

a) Maximum thicknesses of plate.

Based on puncher strength:

$$P = \sigma A = 50\left(\frac{\pi \times 2.5^2}{4}\right) = 78.125\pi\ Kips$$

Based on shear strength:

$$Q = \tau A$$

$$78.125\pi = 40(\pi \times 2.5 \times t)$$

$$t = 0.781\ inch$$

b) Diameter of smallest hole

Based on compression of puncher :

$$P = \sigma A = \sigma(\frac{\pi d^2}{4})$$

$$P = 50\left(\frac{\pi d^2}{4}\right) => P = 12.5\pi d^2$$

Based on shearing of plate:

$$Q = \tau A$$

$$12.5\pi d^2 = 40[\pi d(0.25)]$$

$$d = 0.8\ inch$$

Fluid Mechanics

Fluid Mechanics

A **Fluid** *is a substance which deforms continuously, or flows, when subjected to shearing forces.*

What is **fluid mechanics**? As its name *suggests it is the branch of applied mechanics concerned with the statics and dynamics of fluids - both liquids and gases*. The analysis of the behavior of fluids is based on the fundamental laws of mechanics which relate continuity of mass and energy with force and momentum together with the familiar solid mechanics properties. Fluid mechanics may be divided into three branches:

1- Fluid statics: is the study of the mechanics of fluid at rest.
2- Kinematics: deals with velocity and streamlines without considering forces and energy.
3- Hydrodynamics: is concerned with the relations between velocities and acceleration and the forces exerted by or upon fluids in motion.

Newtonian / Non-Newtonian/Ideal/Perfect Fluids

Even among fluids which are accepted as fluids there can be wide differences in behavior under stress. *Fluids obeying Newton's law where the value of* μ *is constant are known as* **Newtonian** fluids. If μ is constant the shear stress is linearly dependent on velocity gradient. This is true for most common fluids.

Fluids in which the value of μ *is not constant are known* as **non-Newtonian** fluids. These categories are based on the relationship between shear stress and the velocity gradient (rate of shear strain) in the fluid.

There is also one more - which is not real, it does not exist - known as the **ideal fluid**. *This is a fluid which is assumed to have no viscosity*. This is a useful concept when theoretical solutions are being considered - it does help achieve some practically useful solutions.

In some cases thermodynamics relations and compressible-fluid flow have been limited, so the fluid is said to be **perfect gas** and it obeys the perfect gas law (the equation of state: $p v = mRT$)

Liquids vs. Gasses

Although liquids and gasses behave in much the same way and share many similar characteristics, they also possess distinct characteristics of their own. Specifically

- A liquid is difficult to compress and often regarded as being incompressible.
  A gas is easily to compress and usually treated as such - it changes volume with pressure.
- A given mass of liquid occupies a given volume and will occupy the container it is in and form a free surface (if the container is of a larger volume).
  A gas has no fixed volume; it changes volume to expand to fill the containing vessel. It will completely fill the vessel so no free surface is formed.

Fluid Properties

1-Density

Density is defined as an object mass per unit volume. Mass is a property.

The density can be expressed as: $\rho = m / V = 1 / v_g$

where: ρ = *density* (kg/m^3)
m = *mass* (kg)
V = *volume* (m^3)
v_g = *specific volume* (m^3/kg)

The SI units for density are kg/m^3. The imperial (BG) units are lb/ft^3 ($slugs/ft^3$). While people often use pounds per cubic foot as a measure of density in the U.S., pounds are really measures of force, not mass. Slugs are the correct measure of mass. You can multiply slugs by 32.2 for a rough value in pounds.

The higher the density, the tighter the particles are packed inside the substance. Density is a physical property constant at a given temperature and density can help to identify a substance.

Example 1 - Use the Density to Identify the Material:

An unknown liquid substance has a mass of **18.5 g** and occupies a volume of **23.4 ml**. (milliliter). The density can be calculated as

$\rho = [18.5\ (g) / 1000\ (g/kg)] / [23.4\ (ml) / 1000\ (ml/l)\ 1000\ (l/m^3)\ ]$

$= 18.5\ 10^{-3}\ (kg) / 23.4\ 10^{-6}\ (m^3)$

$= \underline{790}\ kg/m^3$

If we look up densities of some common substances, we can find that ethyl alcohol, or ethanol, has a density of 790 kg/m^3. Our unknown liquid may likely be ethyl alcohol!

Example 2 - Use Density to Calculate the Mass of a Volume

The density of titanium is **4507 kg/m^3** . Calculate the mass of **0.17 m^3** titanium!

$m = 0.17\ (m^3)\ 4507\ (kg/m^3)$

$= \underline{766.2}\ kg$

2-Specific Gravity

The Specific Gravity - *SG* - is a dimensionless unit defined *as the ratio of density of the material to the density of water at a specified temperature*. Specific Gravity can be expressed as

$$SG = = \rho / \rho_{H2O}$$

Where :

SG = specific gravity
ρ = *density of fluid or substance* (kg/m^3)
ρ_{H2O} = *density of water* (kg/m^3)

It is common to use the density of water at 4 °C (39°F) as reference - at this point the density of water is at the highest - 1000 kg/m^3 or 62.4 lb/ft^3.

Typical values: Water = 1, Mercury = 13.6, Paraffin Oil =0.8.

Typical values:

Water $=1.14\times10^{-3}$ kg m^{-1} s^{-1}, Air $=1.78\times10^{-5}$ kg m^{-1} s^{-1},
Mercury $=1.552$ kg m^{-1} s^{-1}, Paraffin Oil $=1.9$ kg m^{-1} s^{-1}.

- Kinematics Viscosity

Kinematics Viscosity, ν , *is defined as the ratio of dynamic viscosity to mass density.*

$$\nu = \frac{\mu}{\rho}$$

. Units:m^2 s^{-1} square meters per second,
(Although note that **ν** is often expressed in Stokes, St, where10^4 St = 1 m^2 s^{-1})
Typical values:

Water $=1.14 \times 10^{-6}$ m^2 s^{-1} , Air $=1.46 \times 10^{-5}$ m^2 s^{-1} ,
Mercury $=1.145 \times 10^{-4}$ m^2 s^{-1} , Paraffin Oil $=2.375 \times 10^{-3}$ m^2 s^{-1} .

Viscosity and Reference Temperatures

The viscosity of a fluid is highly temperature dependent and for either dynamic or kinematic viscosity to be meaningful, the **reference temperature** must be quoted. In ISO 8217 the reference temperature for a residual fluid is 100°C. For a distillate fluid the reference temperature is 40°C.

For a liquid - the kinematic viscosity will **decrease** with higher temperature
For a gas - the kinematic viscosity will **increase** with higher temperature

Example 5 - Converting between Kinetic and Absolute Viscosity for Air

Kinematic viscosity of air at 1 bar and 40°C is 16.97 cSt ($16.97\ 10^{-6}$ m^2/s).
The density of air may be estimated by the Ideal Gas Law:

$\rho = 10^5 / (287 \times 313) = 1.113\ kg/m^3$

The absolute viscosity can be expressed as

$\mu = 1.113 \times 16.97\ 10^{-6}$
$= 1.88\ 10^{-5}\ kg/ms\ (Ns/m^2)\ (P)$

Example 6:

The density of an oil is **850 kg/m^3**. Find its relative density and Kinematic viscosity if the dynamic viscosity is **5 x 10^{-3} kg/ms**.

$\rho_{oil} = 850$ kg/m^3 , $\rho_{water} = 1000$ kg/m^3
$S.G_{oil} = 850 / 1000 = 0.85$
Dynamic viscosity $= \mu = 5\text{x}\ 10^{-3}$ kg/ms
Kinematic viscosity $= \nu = \mu / \rho$

$$\nu = \frac{5\text{x}\ 10-3}{1000} = 5\text{x}10^{-6}\ m^2\ s^{-1}$$

Example 7:
5.6m^3 of oil weighs **46 800 N**. Find its mass density, and relative density.

Weight 46 800 = mg
Mass m = 46 800 / 9.81 = 4770.6 kg .
Mass density ρ = Mass / volume = 4770.6 / 5.6 = 852 kg/m^3

Relative density $\gamma = \frac{\rho}{\rho w} = \frac{852}{1000} = 0.852$

Example 8:

From table of fluid properties the viscosity of water is given as **0.01008 poises**. What is this value in Ns/m^2 and Pa s units?

$\mu = 0.01008$ poise

1 poise = 0.1 Pa s = 0.1 Ns/m^2

$\mu = 0.001008$ Pa s = 0.001008 Ns/m^2

Example 9:

In a fluid the velocity measured at a distance of **75mm** from the boundary is **1.125m/s**. The fluid has absolute viscosity **0.048 Pa s** and relative density **0.913**. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution.

$\mu = 0.048$ Pa s

S.G= 0.913

$$\frac{\partial u}{\partial y} = \frac{1.125}{0.075} = 15 s^{-1}$$

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$= 0.048 \times 15 = 0.720 Pa s$$

Table(1): Approximate Physical Properties of some Common Liquids

Liquid	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (kN/m^3)	Dynamic Viscosity, μ ($N \cdot s/m^2$)	Kinematic Viscosity, ν (m^2/s)	Surface Tension,[a] σ (N/m)	Vapor Pressure, p_v [N/m^2 (abs)]	Bulk Modulus,[b] E_v (N/m^2)
Carbon tetrachloride	20	1,590	15.6	9.58 E − 4	6.03 E − 7	2.69 E − 2	1.3 E + 4	1.31 E + 9
Ethyl alcohol	20	789	7.74	1.19 E − 3	1.51 E − 6	2.28 E − 2	5.9 E + 3	1.06 E + 9
Gasoline[c]	15.6	680	6.67	3.1 E − 4	4.6 E − 7	2.2 E − 2	5.5 E + 4	1.3 E + 9
Glycerin	20	1,260	12.4	1.50 E + 0	1.19 E − 3	6.33 E − 2	1.4 E − 2	4.52 E + 9
Mercury	20	13,600	133	1.57 E − 3	1.15 E − 7	4.66 E − 1	1.6 E − 1	2.85 E + 10
SAE 30 oil[c]	15.6	912	8.95	3.8 E − 1	4.2 E − 4	3.6 E − 2	—	1.5 E + 9
Seawater	15.6	1,030	10.1	1.20 E − 3	1.17 E − 6	7.34 E − 2	1.77 E + 3	2.34 E + 9
Water	15.6	999	9.80	1.12 E − 3	1.12 E − 6	7.34 E − 2	1.77 E + 3	2.15 E + 9

[a] In contact with air.
[b] Isentropic bulk modulus calculated from speed of sound.
[c] Typical values. Properties of petroleum products vary.

Pressure

The pressure in a fluid is defined as: *"The normal force per unit area exerted on a imaginary or real plane surface in a fluid or a gas"*
Fluid pressure is transmitted with equal intensity in all directions and acts normal to any plane. In the same horizontal plane the pressure intensities in a liquid are equal.

The equation for pressure can expressed as:

$$p = F / A$$

where

p = *pressure [lb/in^2 (psi) or lb/ft^2 (psf), N/m^2 or kg/ms^2 (Pa)]*
F = *force [N]*
A = *area [in^2 or ft^2, m^2]*

Absolute Pressure

The **absolute** pressure - p_a - is measured relative to the *absolute zero pressure* - the pressure that would occur at absolute vacuum. In all calculation involving the gas laws requires pressure (and temperature) to be in absolute units.

Gauge Pressure

A **gauge** is often used to measure the pressure difference between a system and the surrounding atmosphere. This pressure is often called the **gauge pressure**.

Atmospheric Pressure

The **atmospheric** pressure is the pressure in the surrounding air at or "close" to the surface of the earth. The atmospheric pressure varies with temperature and altitude above sea level.

$$p_{abs} = p_{atm} + p_g$$

where

p_g = *gauge pressure*
p_{atm} = *atmospheric pressure*
p_{abs} = *absolute pressure*

Standard Atmospheric Pressure

The **Standard Atmospheric Pressure** (atm) is used as a reference for gas densities and volumes. The Standard Atmospheric Pressure is defined at sea-level at 273°K (0°C) and is **1.01325 bar** or 101325 Pa (absolute). The temperature of 293°K (20°C) is also used.

In imperial units the Standard Atmospheric Pressure is 14.696 psi.

- *1 atm = 1.01325 bar = 101.3 kPa = 14.696 psi (lb$_f$/in^2)= 760 mmHg =10.33 mH$_2$O = 760 torr = 29.92 inHg = 1013 mbar = 1.0332 kg$_f$/cm^2 = 33.90 ftH$_2$O*

Pressure Units

Since 1 Pa is a small pressure unit, the unit hectopascal (hPa) is widely used, especially in meteorology. The unit kilopascal (kPa) is commonly used designing technical applications like HVAC systems, piping systems and similar.

- *1 hectoPascal = 100 Pascal = 1 millibar*
- *1 kiloPascal = 1000 Pascal*

Some Alternative Units of Pressure

- *1 bar - 100,000 Pa*
- *1 millibar - 100 Pa*
- *1 atmosphere - 101,325 Pa*
- *1 mm Hg - 133 Pa*
- *1 inch Hg - 3,386 Pa*

A **torr** (torr) is named after Torricelli and is the pressure produced by a column of mercury 1 mm high equals to 1/760th of an atmosphere. 1 atm = 760 torr = 14.696 psi

Pounds per square inch (psi) was common in U.K. but has now been replaced in almost every country except in the U.S. by the SI units. The Normal atmospheric pressure is 14.696 psi, meaning that a column of air on one square inch in area rising from the Earth's atmosphere to space weights 14.696 pounds.
The **bar** (bar) is common in the industry. One bar is 100,000 Pa, and for most practical purposes can be approximated to one atmosphere even if

1 Bar = 0.9869 atm

There are 1,000 **millibar** (mbar) in one bar, a unit common in meteorology.

1 millibar = 0.001 bar = 0.750 torr = 100 Pa

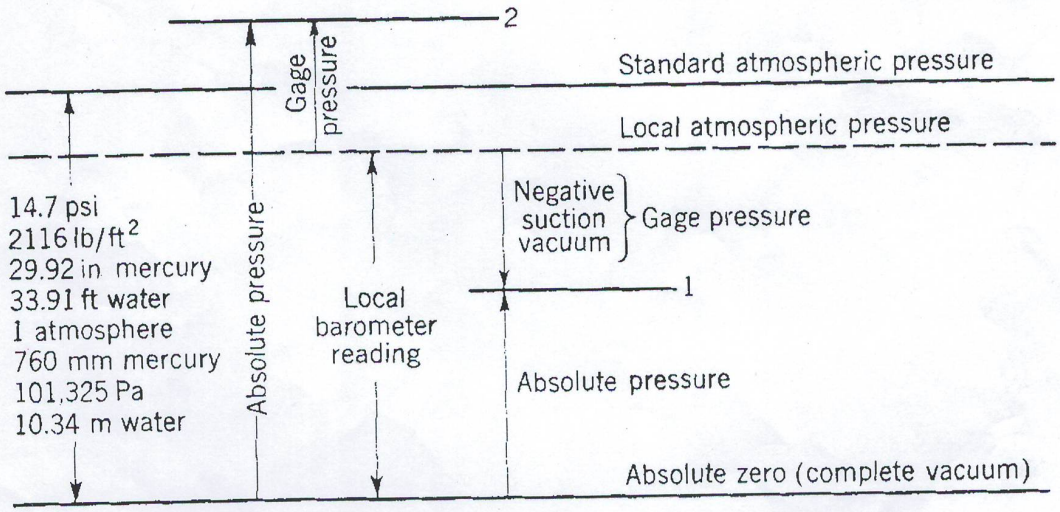


Fig 1:Units and Scales for pressure measurement

Static Pressure and Pressure Head in Fluids

Fluids statics

The pressure indicates the normal force per unit area at a given point acting on a given plane. Since there is no shearing stresses present in a fluid at rest - the pressure in a fluid is independent of direction.
For fluids - liquids or gases - at rest the pressure gradient in the vertical direction depends only on the specific weight of the fluid.

How pressure changes with elevation can be expressed as

$$\frac{dp}{dz} = -\gamma \quad ,$$

Where: dp = *change in pressure*
dz = *change in height*
γ = *specific weight*

The pressure gradient in vertical direction is negative – since the pressure decrease upwards.
For incompressible fluid - as a liquid - the pressure difference between two elevations can be expressed as:

$$dp = -\gamma \, dz$$

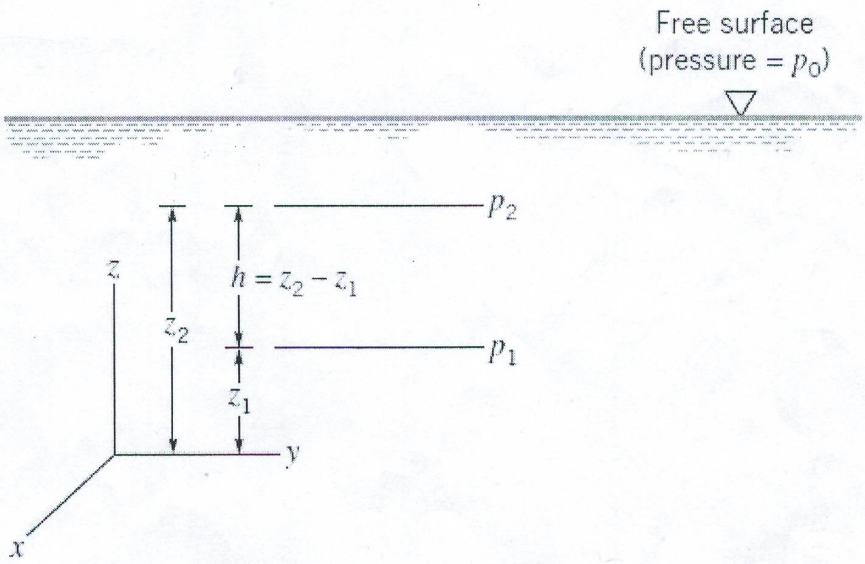


Fig 8: pressure variation in a fluid at rest with a free surface.

$$p_2 - p_1 = -\gamma \, (z_2 - z_1)$$

Where: p_2 = *pressure at level 2*
p_1 = *pressure at level 1*
z_2 = *level 2*
z_1 = *level 1*

It can be transformed to:

$$p_1 - p_2 = \gamma \, (z_2 - z_1)$$

or $p_1 - p_2 = \gamma h$

where: $h = z_2 - z_1$ *difference in elevation.*

$$p_1 = \gamma h + p_2$$

Then it can be transformed to:

$$h = \frac{(P2 - P1)}{\gamma}$$

where **h** express **the pressure head** (the height of a column of fluid of specific weight **γ**) required to give a pressure difference of $(p_2 - p_1)$.

Hydrostatic pressure

Hydrostatic pressure is the pressure due to the weight of a fluid.

$$\mathbf{P = \rho g h}$$

where: ρ (rho) is the density of the fluid (i.e., the practical density of fresh water is 1000 kg/m^3);
g is the acceleration due to gravity (approximately 9.81 m/s^2 on earth's surface);
h is the height of the fluid column (in metres).

Example 9:

We can quote a pressure of **500 kN/m^2** in terms of the height of a column of water and mercury.,

Using P = ρ g h
In terms of water with density 1000 kg/m^3

$$h = \frac{p}{\rho g} = \frac{500 \times 10^3}{1000 \times 9.81} = 50.95m \text{ of water}$$

. And in terms of Mercury with density 13600 kg/m^3

$$h = \frac{500 \times 10^3}{13.6 \times 10^3 \times 9.81} = 3.75m \text{ of Mercury}$$

Example 10:

A storage tank contains petrol to a height of **4.7 m**. If the pressure at the base of the tank is **32.3 kPa**, determine the density of the petrol. Take the gravitational field force as **9.8 m/s^2**.

ρ = p/gh

= 32.2x10^3/ 9.8x4.7

= 699 kg/m^3

Pressure Measurement

Pressure Measurement By Bourdon Gauge

A **gauge** is often used to measure the pressure difference between a system and the surrounding atmosphere. This pressure is often called the **gauge pressure** .The pressure element is a hollow, curved, flat metallic tube closed at one end; the other end is connected to the pressure to be measured. When the internal pressure is increased, the tube tends to straighten, pulling on a linkage to which is attached a pointer and causing the pointer to move.

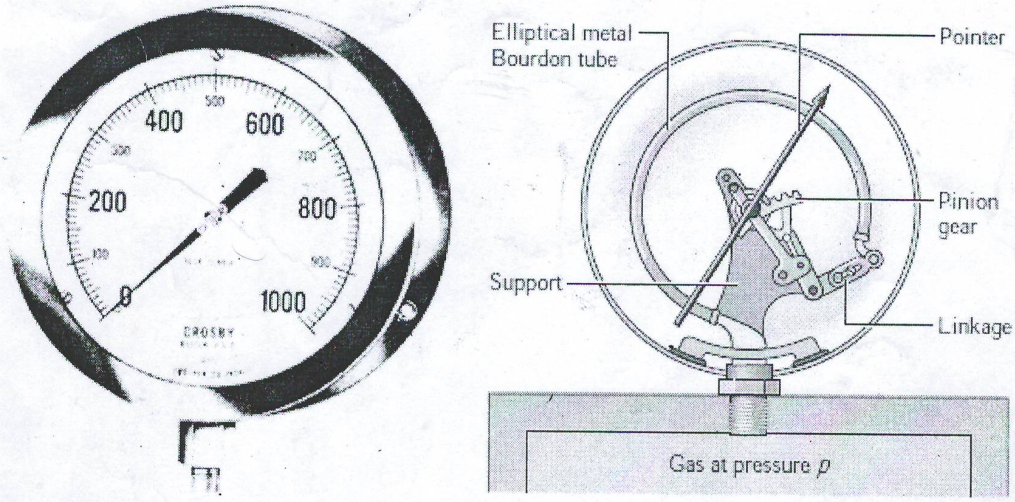


Fig 2: Bourdon Gauge

Barometers

A barometer is an instrument for measuring atmospheric pressure. It is affected by seasonal changes of temperature. Barometers are therefore also used for the measurement of altitude and also as one of the aids in weather forecasting. The value of atmospheric pressure will thus vary with climatic conditions, although not usually by more than about 10% of standard atmospheric pressure.

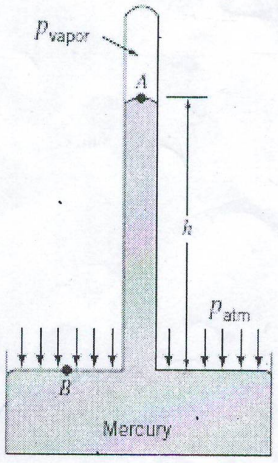


Fig 3 : A barometer

Barometer in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in Fig.3 . The tube is initially filled with mercury (inverted with its open end up) and then turned upside down (open end down) with the open end in the container of mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor

pressure (which develops in the space above the column) balances the force due to the atmospheric pressure. For most practical purposes the contribution of the vapor pressure can be neglected since it is very small.

$$Patm = \gamma h - Pvapor$$
$$\therefore Patm = \gamma h$$

Pressure Measurement By Manometer

The relationship between pressure and head is used to measure pressure with a manometer (also know as a liquid gauge). It is necessary to demonstrate the analysis and use of various types of manometers for pressure measurement.

1. The Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured. An example can be seen in the figure below. This simple device is known as a *Piezometer tube*. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is **gauge pressure**.

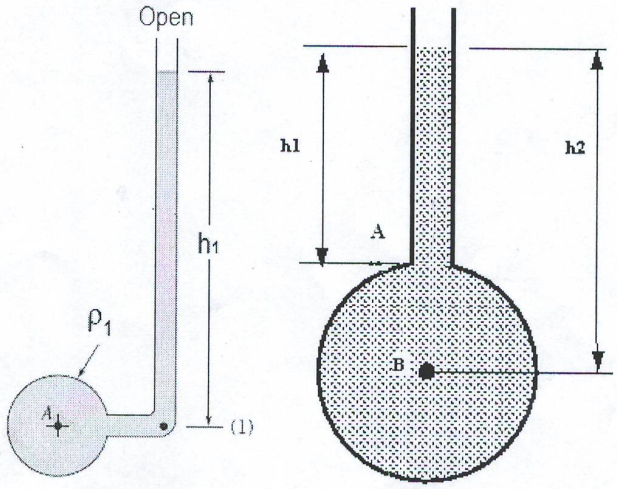


Fig 4: A simple piezometer tube manometer

Pressure at A = pressure due to column of liquid above A

$$P_A = \rho g h_1$$

Pressure at B = pressure due to column of liquid above B

$$P_B = \rho g h_2$$

Although the piezometer tube is a very simple and accurate pressure-measuring device, it has several disadvantages. It is only suitable if the pressure in the container is greater than atmospheric pressure (otherwise air would be sucked into the system), and the pressure to be measured must not be too small or too large so the required height of the column is reasonable. Also the fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.

2.The "U"-Tube Manometer

Using a "U"-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The "U" is connected as in the figure below and filled with a fluid called the *manometric fluid.*

The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.

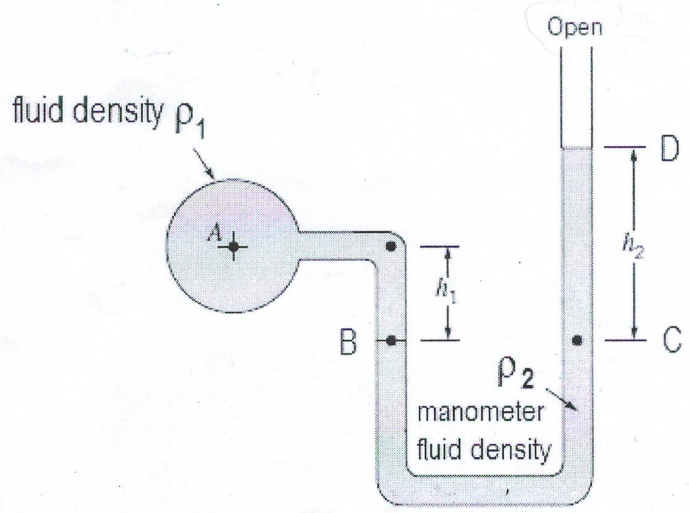


Fig 5: A "U"-Tube manometer

Note that ρ_1 is the density of fluid inside the tank, where ρ_2 (ρ_{man}) is manometer fluid density.
Pressure in a continuous static fluid is the same at any horizontal level so,

Pressure at B = Pressure at C

$$P_B = P_C$$

For the **left hand** arm

Pressure at B = Pressure at A + Pressure due to height h_1 of fluid being measured

$$P_B = P_A + \rho g h_1$$

For the **right hand** arm

Pressure at C = Pressure at D+ Pressure due to height h_2 of manometric fluid

$$P_C = P_{atm} + \rho_2 g h_2$$

As we are measuring *gauge pressure* we can subtract P_{atm} giving

$$P_B = P_C$$

$$P_A = \rho_2 g h_2 + \rho_1 g h_1$$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid i.e $\rho_{man} >> \rho_1$.

. In this case the term $\rho_1 g h_1$ can be neglected, and the gauge pressure is given by:

$$P_A = \rho_2 g h_2$$

3. Measurement Of Pressure Difference Using a "U"-Tube Manometer.

The U-tube manometer is also widely used to measure the *difference* in pressure between two containers or two points in a given system. Consider a manometer connected between containers *A* and *B* as is shown in Fig.**6**. The difference in pressure between *A* and *B* can be found by again starting at one end of the system and working around to the other end. For example, at *A* the pressure is P_A, which is equal to P_1, and as we move to point (2) the pressure increases by($\gamma_1 h_1$).The pressure at P_2 is equal to P_3, and as we move upward to point (4) the pressure decreases by ($\gamma_2 h_2$). Similarly, as we continue to move upward from point (4) to (5) the pressure decreases by ($\gamma_3 h_3$). Finally, $P_5 = P_B$, as they are at equal elevations. Thus,

$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

Or, as indicated in the figure in the margin on the previous page, we could start at *B* and work our way around to *A* to obtain the same result. In either case, the pressure difference is

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

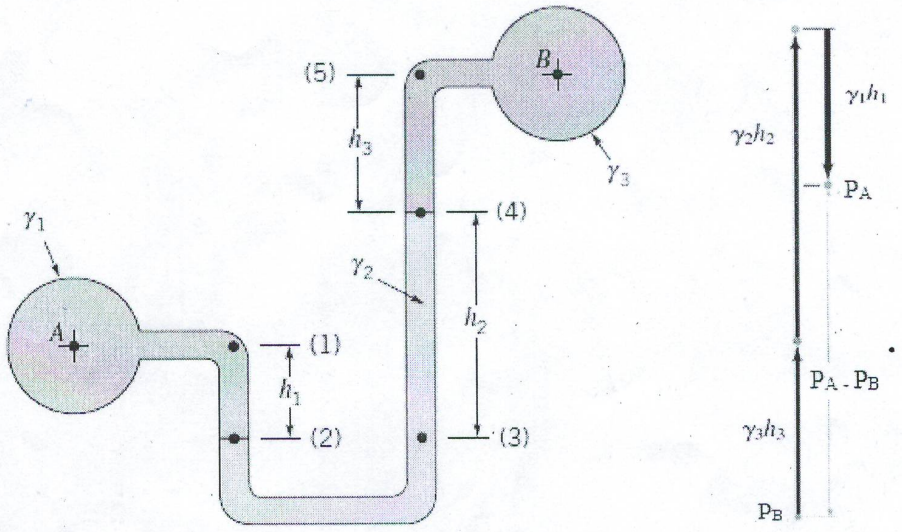


Fig 6: Pressure difference measurement by the "U"-Tube manometer

4. Tilted manometer

To measure small pressure changes, a manometer of the type shown in Fig.7 is frequently used. One leg of the manometer is inclined at an angle θ, and the differential reading L_2 is measured along the inclined tube. The difference in pressure $P_A - P_B$ can be expressed as:

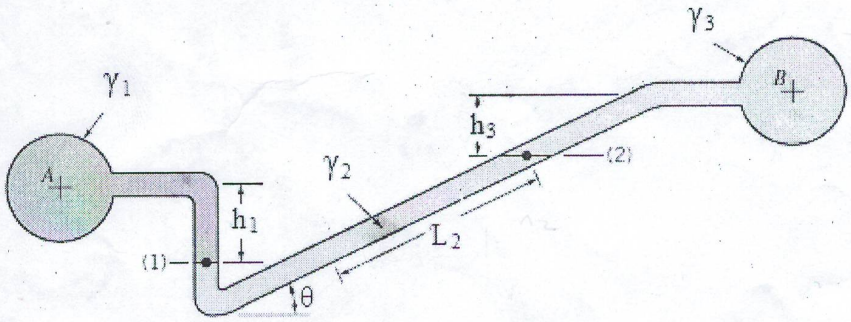


Fig 7: Tilted manometer.

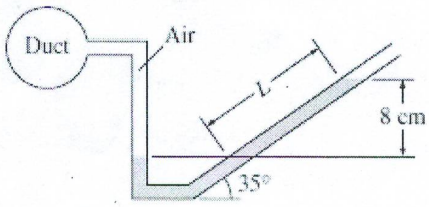
$$P_A + \gamma_1 h_1 - \gamma_2 L_2 \sin\theta - \gamma_3 h_3 = P_B$$
$$P_A - P_B = \gamma_2 L_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

where it is to be noted the pressure difference between points (1) and (2) is due to the vertical distance between the points, which can be expressed as $L_2 \sin\theta$ .Thus, for relatively small angles the differential reading along the inclined tube can be made large even for small pressure differences. The inclined-tube is often used to measure small differences in gas pressures so that if pipes A and B contain a gas, then

$$P_A - P_B = \gamma_2 L_2 \sin\theta$$

Example 11:

The air pressure in a duct is measured by an inclined manometer whose open arm inclined by **35 degrees** from the horizontal, as shown in Fig. The density of the liquid in the manometer is **0.81 kg/L**, and the vertical distance between the fluid levels in the two arms of the manometer is **8 cm**. Determine the gage pressure of air in the duct and the length of the fluid column in the inclined arm above the fluid level in the vertical arm.



$P_{gage} = \rho g h$

$= 810\ kg/m^3 \times 9.81\ m/s^2 \times 0.08\ m = 635.688\ Pa$

The length of the differential fluid column is

$L = h / \sin\theta = (8\ cm) / \sin 35° =$ **13.94 cm**

Choice of Manometer

Care must be taken when attaching the manometer to vessel, no burrs must be present around this joint. Burrs would alter the flow causing local pressure variations to affect the measurement.

Some disadvantages of manometers:

- Slow response - only really useful for very slowly varying pressures - no use at all for fluctuating pressur
- For the "U" tube manometer two measurements must be taken simultaneously to get the h value. This may be avoided by using a tube with a much larger cross-sectional area on one side of the manometer than the other;
- It is often difficult to measure small variations in pressure - a different manometric fluid may be required - alternatively a sloping manometer may be employed; It cannot be used for very large pressures unless several manometers are connected in series;
- For very accurate work the temperature and relationship between temperature and ρ must be known;

Some advantages of manometers:

- They are very simple.
- No calibration is required - the pressure can be calculated from first principles

Continuity Equation and Conservation of Matter

Mass and Volumetric Flow Rate

To characterize the fluid flow, we have to specify the quantity of the fluid flowing through a given location in the system per unit of time. This is called the *flow rate*. Since fluid quantity can be expressed as volume or mass, there are two types of flow rates: volumetric and mass.

Volumetric flow rate ($\dot{V}$) is the volume of the fluid passing through a given cross section in a unit of time. The basic unit is m^3/s. If m^3/s is too large, litre/second (l/s) is used.

Volumetric flow rate is a function of the fluid velocity (v) and the cross section of the channel through which the fluid is flowing, e.g., a pipe, (A). Let's mark two cross sections 1 and 2 on the pipe. The distance between them is L. Follow a layer of fluid passing through the cross section 1 and, a moment later, through cross section 2 over the distance L. Assuming all fluid particles move at the same velocity, we conclude that the volume of fluid that passed through cross section 1 was:

$$V = A \cdot L$$

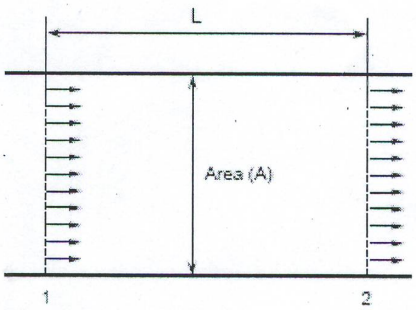


Fig 13: A simple case of flow through a straight pipe.

Per unit of time, it will be:

$$\dot{V} = \frac{V}{t} = \frac{A \cdot L}{t}$$

Since L/t is the average velocity of the fluid (v), the volumetric flow rate can be expressed as:

$$\dot{V} = A\ v$$

Mass flow rate (ṁ) is the mass of fluid passing through a given cross section in a unit of time. Its basic unit is kg/s. We can derive the formula for the mass flow rate using the formula for the volumetric flow rate and by converting volume to mass using density (ρ).

$$\text{Since } \rho = \frac{m}{V}, \text{ then } m = V \cdot \rho$$

If we divide the both sides of the above equation by time, we get the flow rates. The final formula for the mass flow rate is:

$$\dot{m} = \rho \dot{V} = \rho\ A\ v$$

Where: *m = mass flow rate (kg/s),ρ = density (kg/m^3),v = speed (m/s),A = area (m^2)*

Although we have used a pipe to discuss the flow rates, the concepts apply equally well to channels with cross sections of any shape, e.g., rectangular ducts.

Both types of flow rates are commonly used to characterize system conditions in our plants. This knowledge will help you, in your daily work, foresee what happens to these flow rates when a system parameter changes or diagnose the cause of their change.

Discharge and mean velocity

If we know the size of a pipe, and we know the discharge, we can deduce the mean velocity

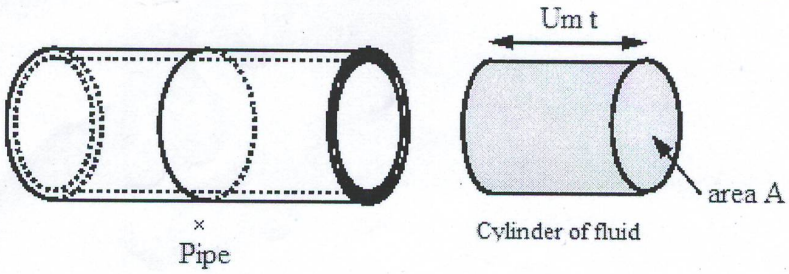


Fig 14: Discharge in a pipe

If the area of cross section of the pipe at point X is A, and the mean velocity here is u_m (or average u_a). During a time t, a cylinder of fluid will pass point X with a volume A u_m t. The volume per unit time (the discharge) will thus be:

$$Q = \frac{\text{volume}}{\text{time}} = \frac{A \times Um \times t}{t} = A\ Um$$

Note how carefully we have called this the *mean velocity*. This is because the velocity in the pipe is not constant across the cross section. Crossing the centerline of the pipe, the velocity is zero at the walls increasing to a maximum at the centre then decreasing symmetrically to the other wall. This variation across the section is known as the velocity profile or distribution. A typical one is shown in the figure below.

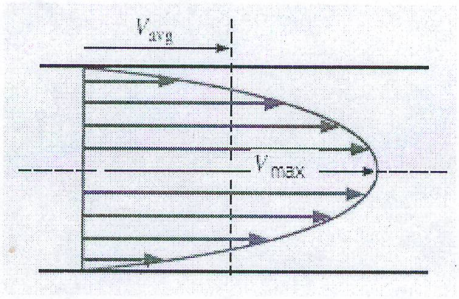


Fig 15: A typical velocity profile across a pipe

The Law of Conservation of Mass

It states that mass can be neither created nor destroyed. Using the Mass Conservation Law on a **steady flow** process - flow where the flow rate does not change:

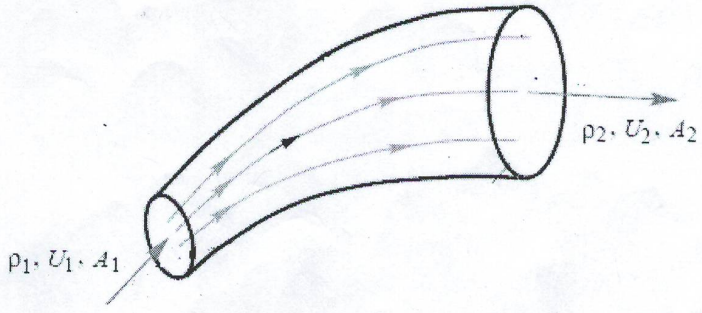


Fig 16: Flowing fluid with mass conservation

The equation of continuity results from the principle of conservation of mass. For steady flow, the mass of fluid passing all sections in a stream of fluid per unit of time is the same. This can be evaluated as:

$$\text{Mass Inflow} = \text{Mass Outflow}$$

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

For incompressible fluids where the density remains constant, the equation becomes:

$$u_1 A_1 = u_2 A_2$$

This statement is called **the Equation of Continuity.** Common application of **the Equation of Continuity** are pipes, tubes and ducts with flowing liquid or gases, rivers.

Example 12:

10 m^3/h of water flows through a pipe with *100 mm* inside diameter. The pipe is reduced to an inside dimension of *80 mm*. Find the velocity in each section.

Solution:

the velocity in the 100 mm pipe can be calculated as

$V_{100} = (10\ m^3/h) = 10/3600\ m^3/s.$

$= u\ (3.14\ (0.1\ m)^2 / 4)$

$\therefore\ u = 0.35\ m/s$

the velocity in the 80 mm pipe can be calculated

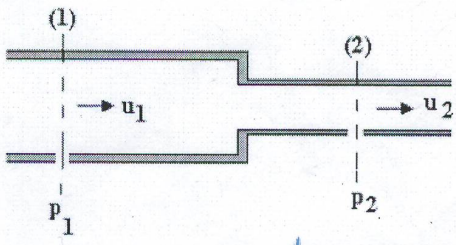
$V_{80} = V_{100} = (10\ m^3/h) = 10/3600\ m^3/s$

$= u\ (3.14\ (0.08\ m)^2 / 4)$

$\therefore\ u = 0.55\ m/s$

Example 13:

A pipe carrying water as shown in the figure. The area at point 1 is 0.002 m^2 and at point 2 is 0.001 m^2. The velocity at point 2 is 8 m/s. At point 1, calculate the velocity, the mass flow rate and volume flow rate.



Solution:

With continuity equation: $A_1 u_1 = A_2 u_2$

$0.002\ m^2 \times u_1 = 0.001\ m^2 \times 8\ m/s$

$\therefore\ u_1 = 4\ m/s$

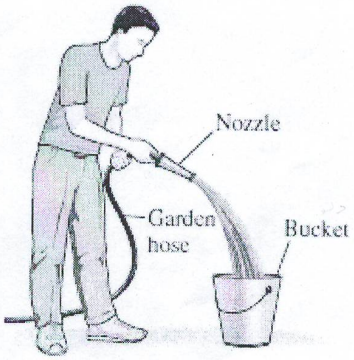
$\dot{m} = \rho A_1 u_1 = 1000\ kg/m^3 \times 0.002\ m^2 \times 4\ m/s = 8\ kg/s$

$V = A_1 u_1 = 0.002\ m^2 \times 4\ m/s = 0.008\ m^3/s$,

or $= \dot{m}/\rho = (8\ kg/s) / (1000\ kg/m^3) = 0.008\ m^3/s$

Example 14:

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine (*a*) the volume and mass flow rates of water through the hose, and (*b*) the average velocity of water at the nozzle exit.



Solution:

We take the density of water to be 1000 kg/m^3 = 1 kg/L.
Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{t}$$

$$= (10 \times 3.785 \text{ litre}) / 50 \text{ s} = 0.757 \text{ L/s} = 757 \text{ cm}^3/\text{s} = 757 \times 10^{-6} \text{ m}^3/\text{s}$$

$$\dot{m} = \dot{V} \cdot \rho$$

$$= 0.757 \text{ L/s} \times 1 \text{ kg/L} = 0.757 \text{ kg/s}$$

The cross-sectional area of the nozzle exit is:

$$A_e = \pi r^2 = \pi (0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

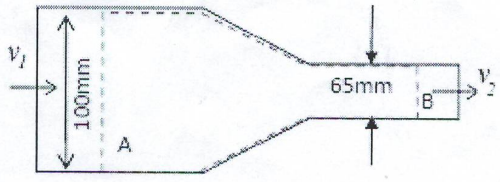
The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$Ve = \dot{V} / A_e$$

$$= 757 \times 10^{-6} \text{ m}^3/\text{s} / 0.5027 \times 10^{-4} \text{ m}^2 = 15.1 \text{ m/s}$$

Example 15:

A pipe **A** with ID 100 mm is connected to a pipe **B** with ID 65 mm through converging cone. Kerosene (ρ = 800 kg/m^3) is pumped through the pipe> If the average velocity in the pipe **A** is 0.75 m/s, Calculate: a- velocity in pipe B , b- mass flow rate in A and c- mass flow rate in B



Solution:

a) Velocity in pipe B

From continuity of mass flow suggest that in the absence of accumulation

Mass flow in A = mass flow in B

Mass flow in A = density × volumetric flowrate

$$= \rho \times v_1 \times \frac{\pi D^2}{4}$$

Similarly mass flow in B

$$= \rho \times v_2 \times \frac{\pi d^2}{4}$$

Substituting (2) and (3) in (1)

$$\rho \times v_1 \times \frac{\pi D^2}{4} = \rho \times v_2 \times \frac{\pi d^2}{4}$$

$$v_1 \times D^2 = v_2 \times d^2$$

$$\therefore v_2 = v_1 \times \frac{D^2}{d^2}$$

$$v_2 = 0.75 \times \left[\frac{0.1}{0.065}\right]^2 = 1.775\ m/s$$

b) Mass flow rate in A

From equation 3 the mass flowrate in A

$$A = \rho \times v_1 \times \frac{\pi D^2}{4}$$

$$= 800 \times 0.75 \times \frac{\pi (0.1)^2}{4}$$

$$= 4.71 kg/s$$

c) Mass flowrate in B = mass flow in A

BERNOULLI PRINCIPLE

Bernoulli's principle states that for an inviscid flow, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's energy. Bernoulli's principle is named after the Dutch-Swiss mathematician Daniel Bernoulli who published his principle in his book *"Hydrodynamica "* in 1738.

"The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible".

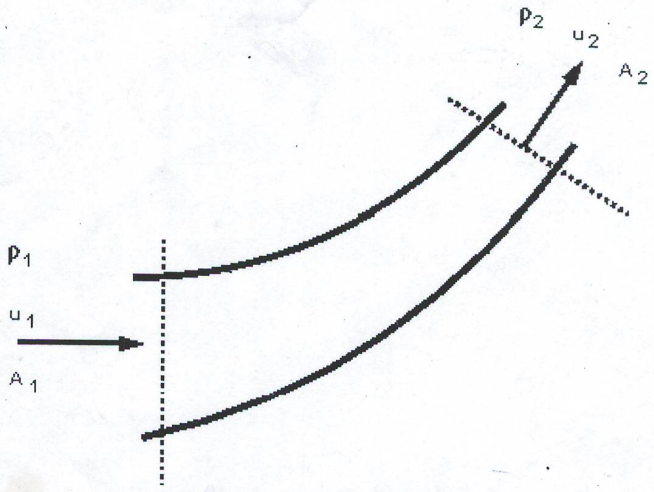


Fig 17: Flowing fluid through stream lines

In mathematical relation:

$$(KE + PE + PV)_1 = (KE + PE + PV)_2$$

In most flows of liquids, and of gases at low Mach number, the mass density of a fluid parcel can be considered to be constant, regardless of pressure variations in the flow. For this reason the fluid in such flows can be considered to be incompressible and these flows can be described as incompressible flow. Bernoulli performed his experiments on liquids and his equation in its original form is valid only for incompressible flow. A common form of Bernoulli's equation, valid at any arbitrary point along a streamline where gravity is constant, is:

$$\frac{p}{\rho} - \frac{v^2}{2} - gz = \text{constant}$$

where:

v is the fluid flow speed at a point on a streamline,

g is the acceleration due to gravity,

z is the elevation of the point above a reference plane, with the positive z-direction pointing upward – so in the direction opposite to the gravitational acceleration,

p is the pressure at the chosen point, and

ρ is the density of the fluid at all points in the fluid.

or in head form:

$$\frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{constant}$$

Where:

$\frac{P}{\rho g}$ = Pressure head (due to static pressure)

$\frac{v^2}{2g}$ = Velocity head(due to dynamic pressure)

Z = Elevtion head

Head is a measure of energy per unit weight of the fluid. For example, a fluid volume with mass *m* elevated to a height of *h* above the ground has a potential energy *mgh* (units: Joules). In this case, ground is considered as the reference level with zero potential energy. If we compute the energy per unit weight , it gives *h*. The elevation head in equation can be considered as the height above some reference level which is often called the "Datum level".

Similarly pressure head gives the energy due to pressure force and velocity head gives the kinetic energy of a unit weight of the fluid. In this sense, Bernoulli equation gives a simpler form of energy conservation.Inviscid fluids are absent of energy dissipating mechanism called viscous dissipation. Since there is no energy dissipation, equation suggests that the total head of a fluid element remains a constant as it flows along the streamline.

Applications of the Bernoulli Equation

In modern everyday life there are many observations that can be successfully explained by application of Bernoulli's principle, even though no real fluid is entirely inviscid and a small viscosity often has a large effect on the flow.

- Bernoulli's Principle can be used to calculate the lift force on an airfoil if you know the behavior of the fluid flow in the vicinity of the foil. For example, if the air flowing past the top surface of an aircraft wing is moving faster than the air flowing past the bottom surface then Bernoulli's principle implies that the pressure on the surfaces of the wing will be lower above than below. This pressure difference results in an upwards force. Whenever the distribution of speed past the top and bottom surfaces of a wing is known, the lift forces can be calculated (to a good approximation) using Bernoulli's equations established by Bernoulli over a century before the first man-made wings were used for the purpose of flight. Bernoulli's principle does not explain why the air flows faster past the top of the wing and slower past the underside.
- The carburetor used in many reciprocating engines contains a venturi to create a region of low pressure to draw fuel into the carburetor and mix it thoroughly with the incoming air. The low pressure in the throat of a venturi can be explained by Bernoulli's principle; in the narrow throat, the air is moving at its fastest speed and therefore it is at its lowest pressure.

- The Pitot tube and static port on an aircraft are used to determine the airspeed of the aircraft. These two devices are connected to the airspeed indicator which determines the dynamic pressure of the airflow past the aircraft. Dynamic pressure is the difference between stagnation pressure and static pressure. Bernoulli's principle is used to calibrate the airspeed indicator so that it displays the indicated airspeed appropriate to the dynamic pressure.

- The flow speed of a fluid can be measured using a device such as a Venturi meter or an orifice plate, which can be placed into a pipeline to reduce the diameter of the flow. For a horizontal device, the continuity equation shows that for an incompressible fluid, the reduction in diameter will cause an increase in the fluid flow speed. Subsequently Bernoulli's principle then shows that there must be a decrease in the pressure in the reduced diameter region. This phenomenon is known as the Venturi effect.

- The maximum possible drain rate for a tank with a hole or tap at the base can be calculated directly from Bernoulli's equation, and is found to be proportional to the square root of the height of the fluid in the tank. This is Torricelli's law, showing that Torricelli's law is compatible with Bernoulli's principle. Viscosity lowers this drain rate. This is reflected in the discharge coefficient which is a function of the Reynold's number and the shape of the orifice.

We know that Bernoulli equation can be applied to a great many situations not just the pipe flow we have been considering up to now. In the following sections we will see some examples of its application to flow measurement from tanks, within pipes as well as in open channels.

1. Free jets

A free jet forms when a liquid accelerate through a nozzle. Consider a nozzle at the bottom of a reservoir as shown in fig below. The smooth and contoured nozzle has a diameter **d**.

Consider a fluid element moving from (1) at the surface to (2) at the nozzle exit along the streamline. The free surface of the liquid is h distance above the nozzle.

Assuming the datum is at the nozzle, applying Bernoulli equation between (1) and (2) gives

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

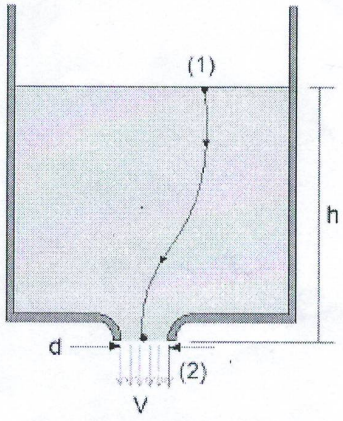


Fig 18 : Free jet at bottom

Hydrostatic Pressure امثلة على

Example/ **Calculate the total force acting on the surface of a tank with dimensions (300cm length, 250cm width, and 450cm height) if it is filled with water with a density of (1000 kg/m^3). Also calculate the pressure at a point of the base.**

Solution/

$$F = m \times g$$

$$F = (\rho \times V) \times g$$

$$F = \rho \times (A \times h) \times g$$

$$F = 1000 \times (3 \times 2.5 \times 4.5) \times 9.81$$

$$F = 331087.5\ N$$

الضغط المؤثر على القاعدة...

$$P = \frac{F}{A} = \frac{331087.5}{3 \times 2.5} = 44145\ \frac{N}{m^2}$$

او بطريقة اخرى

$$P = \rho \times g \times h$$

$$P = 1000 \times 9.81 \times 4.5 = 4414 \frac{N}{m^2}$$

Example/ **Two points A and B are at a depth of 300m and 200m on the sea floor. Find the difference in pressure between the two points if you know that the density of sea water is** $1023 kg/m^3$

Solution/

(A)من معادلة الضغط الهيدروستاتيكي نحسب الضغط عند النقطة

$$P_A = \rho \times g \times h_A$$

$$P_A = 1023 \times 9.81 \times 300 = 3010689 \ N/m^2$$

(B)من معادلة الضغط الهيدروستاتيكي نحسب الضغط عند النقطة

$$P_B = \rho \times g \times h_B$$

$$P_B = 1023 \times 9.81 \times 200 = 2007126 \ N/m^2$$

ثم نحسب الفرق في الضغط بين النقطتين

$$P_A - P_B = 3010689 - 2007126 = 1003563 \ Pa$$

ملاحظة/ يمكن حساب الضغط الهيدروستاتيكي مباشرة من القانون

$$\Delta P = \rho \times g \times \Delta h$$

$$\Delta P = 1023 \times 9.81 \times (300 - 200) = 1003563 \ Pa$$

U-tube manometer امثلة على

Example 1/ A simple manometer is used to measure the pressure of oil S.G. 0.8 flowing in a pipeline shown in Figure (1). Its right limb is open to atmosphere and the left limb is connected to the pipe. The center of the pipe is 9.0 cm below the level of the mercury in the right limb. If the difference of the mercury level in the two limbs is 15 cm, determine the absolute and the gauge pressures of the oil in the pipe.

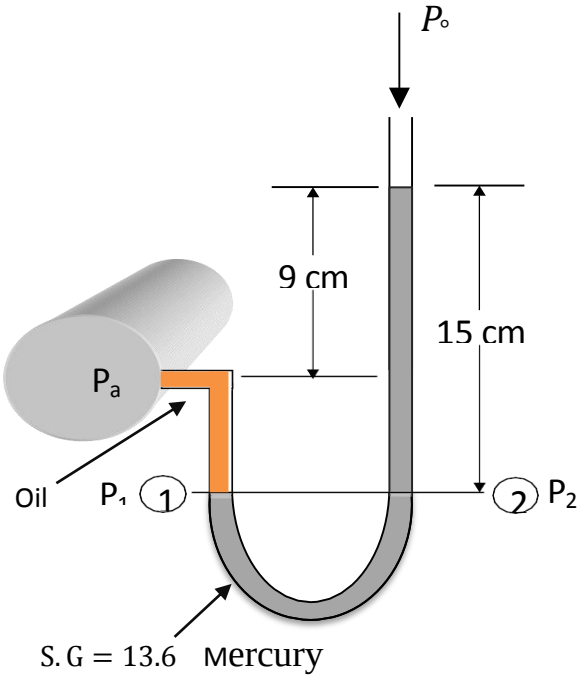


Figure (1)

solution/

$\rho oil = 0.8\ (1000) = 800\ Kg/m^3$,

$\rho_m = 13.6 \times 1000 = 13600\ Kg/m^3$

$P1 = Pa + Poil$

$P2 = Po + P_m$

$P_1 = P_2$

$P_a + P_{oil} = P_o + P_m$

$P_a = P_o + P_m - P_{oil}$ ..... (1)

$P_a = 1.0325 \times 10^5 + 13600 \times 9.81 \times 0.15 - [800 \times 9.81 \times (0.15 - 0.0\ 9)]$

$P_a = 122791.5\ Pa$

Example 2/ The following Figure (2) shows a manometer connected to the pipeline containing oil of S.G= 0.8. Determine the absolute pressure of the oil in the pipe, and the gauge pressure.

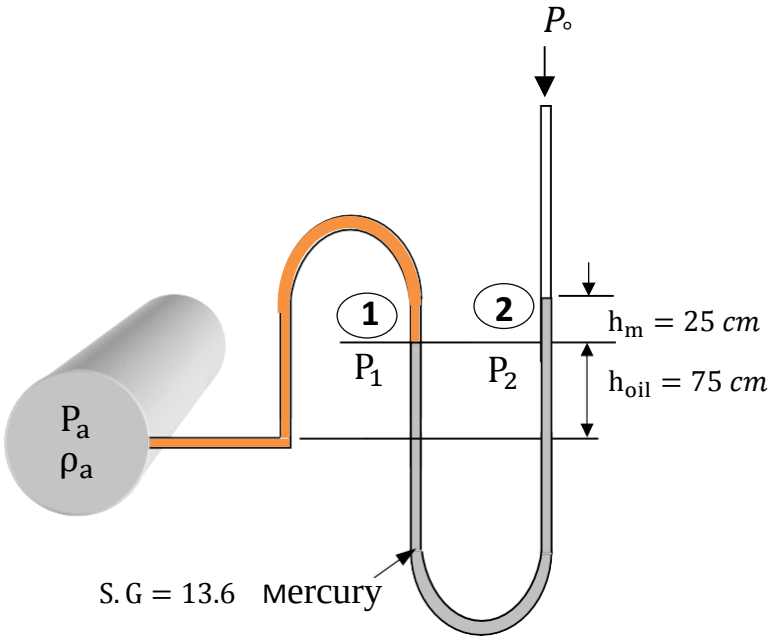


Figure (2)

Solution/

$\rho_a = 0.8\ (1000) = 800\ kg/m^3$

$\rho_m = 13.6 \times 1000 = 13600\ Kg/m^3$

$P_1 = P_2$

$P_1 = P_a - P_{oil}$

$P_2 = P_o + P_m$

$\Rightarrow P_a = P_o + h_m \rho_m g + h_{oil} \rho_{oil} g$

$= 1.0325 \times 10^5 + (0.25)\ m\ (13600\ kg/m^3)\ 9.81\ m/s^2 + (0.75)\ m\ (800\ kg/m^3)\ 9.81\ m/s^2$

$= 1.40565 \times 10^5\ Pa$

Example 3/ A manometer containing oil ($\rho = 850\ kg/m^3$) is attached to a tank filled with air, as shown in Figure (3). If the oil-level difference between the two columns is (1.5 m) and the atmospheric pressure is (98 kPa) , determine the absolute pressure of the air in the tank.

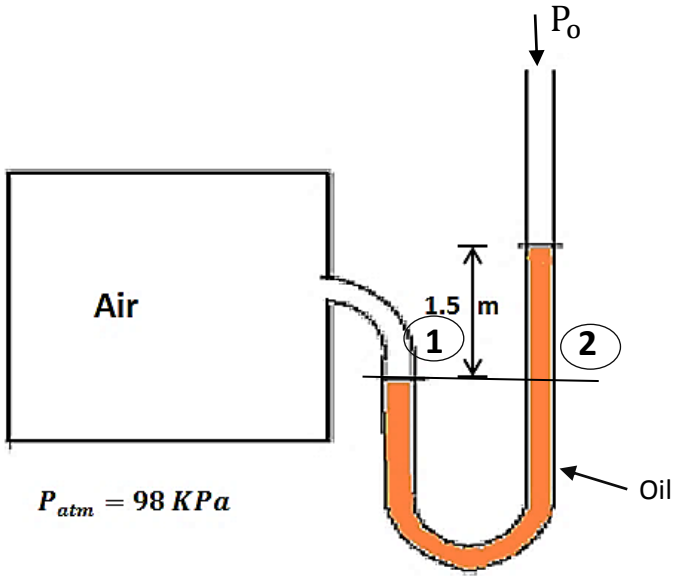


Figure (3)

Solution/

$P_1 = P_{air}$

$P_2 = P_o + P_{oil}$

$P_1 = P_2$

$P_{air} = P_o + \rho_{oil}\ g\ h_{oil}$

$P_{air} = 98 \times 10^3 + (850 \times 9.81 \times 1.5)$

$P_{air} = 110507.75\ pa$

Example 4/ The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Figure (4). The tank is located on a mountain at an altitude of 1400m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if ($h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35\ m$).Take the densities of water, oil, and mercury to be 1000 Kg/m^3, 850 Kg/m^3, and 13,600 Kg/m^3, respectively.

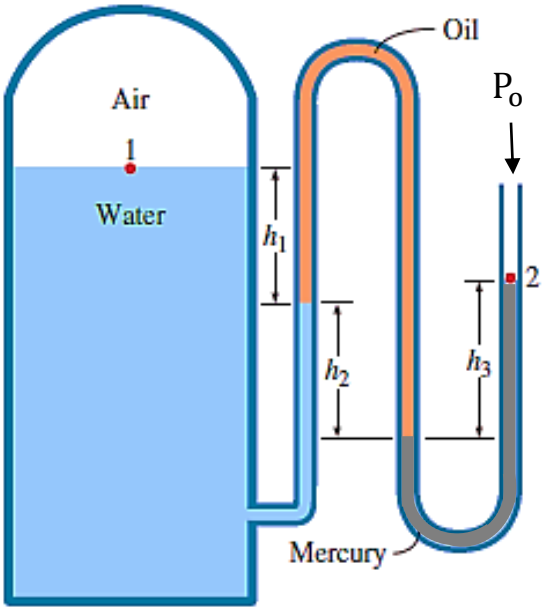


Figure (4)

Solution/

$P_{air} + P_{water} + P_{oil} - P_{mercury} - P_o = 0$

$P_{air} = P_o + P_{mercury} - P_{water} - P_{oil}$

$P_{air} = P_o + \rho_{mercury} g h_3 - \rho_{water} g h_1 - \rho_{oil} g h_2$

$P_{air} = 85.6 \times 10^3 + (13600 \times 9.81 \times 0.35) - (1000 \times 9.81 \times 0.1) - (850 \times 9.81 \times 0.2)$

$P_{air} = 130\ 000$ Pa

Example 5/ The gage pressure of the air in the tank shown in Figure (5) is measured to be 65 kPa. Determine the differential height **h** of the mercury column.

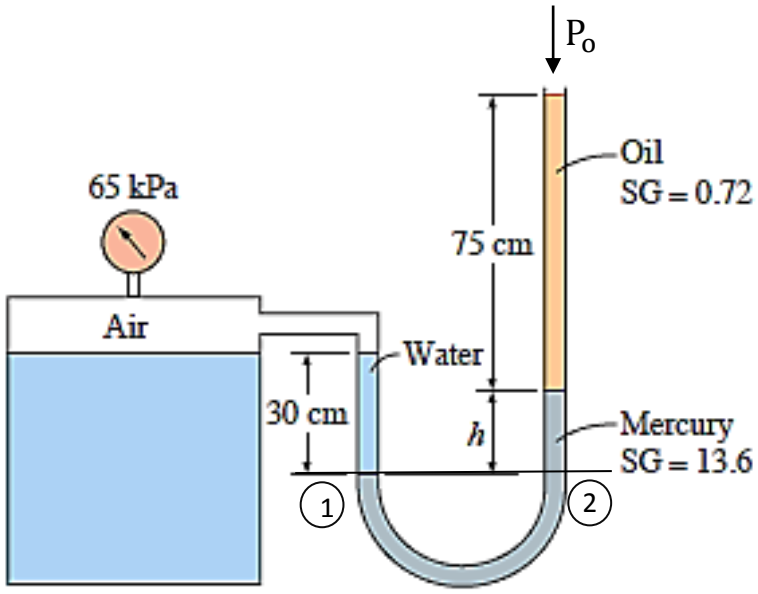


Figure (5)

Solution/

$\rho = \text{S.G} \times \rho_{H2O}$

$\rho_{oil} = 0.72 \times 1000 = 720\ \text{Kg/m}^3$

$\rho_{mercury} = 13.6 \times 1000 = 13600\ \text{Kg/m}^3$

$P_1 = P_{air} + P_{water}$

$P_2 = P_o + P_{oil} + P_{mercury}$

$P_1 = P_2$

$P_{air} + P_{water} = P_o + P_{oil} + P_{mercury}$

$P_{mercury} = P_{air} + P_{water} - P_o - P_{oil}$

$P_{mercury} = P_{air} - P_o + \rho_{water} g h_w - \rho_{oil} g h_{oil}$

$P_{mercury} = 65 \times 10^3 + (1000 \times 9.81 \times 0.3) - (720 \times 9.81 \times 0.75)$

$P_{mercury} = 62645.6\ Pa = \rho_m g h_m$

$\Rightarrow h_m = \frac{P_{mercury}}{\rho_m g} = \frac{62645.6}{13600 \times 9.81} = 0.47\ m$

Example 6/ A differential manometer is connected to two pipes as shown in Figure(6). The pipe A is containing carbon tetrachloride S.G. = 1.594 and the pipe B is contain an oil of S.G. = 0.8. Find the difference of mercury level if the pressure difference in the two pipes be $7.848 \times 10^4\ Pa$. Note; $\rho_{mercury} = 13600\ Kg/m^3$

Solution/

ملاحظة:

$h_c = 2.5 + 1 = 3.5\ m$

$h_m = h$

$h_{oil} = 1 + h$

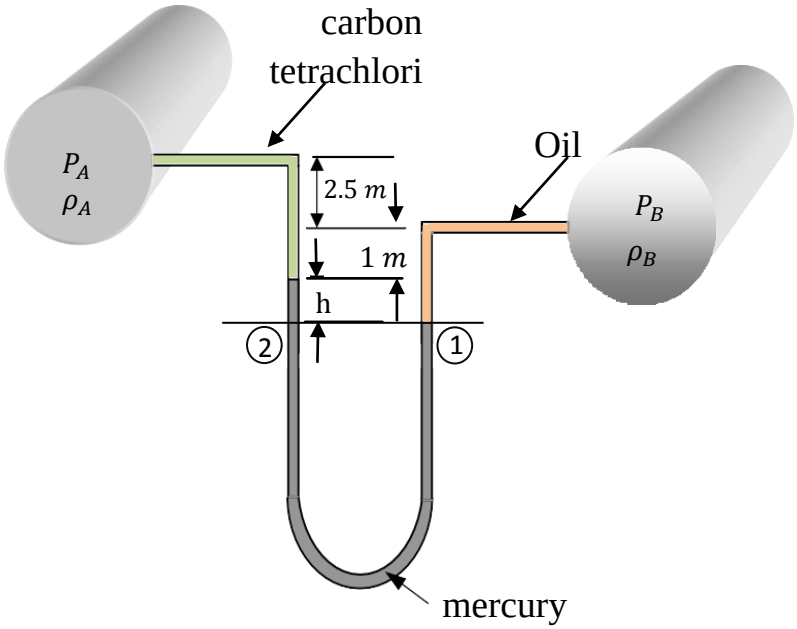


Figure (6)

$\rho = S.G \times \rho_{H2O}$

$\rho_c = 1.594 \times 1000 = 1594\ Kg/m^3$

$\rho_{oil} = 0.8 \times 1000 = 800\ Kg/m^3$

$P_1 = P_B + P_{oil}$

$P_2 = P_A + P_c + P_m$

$P_1 = P_2$

$P_B + P_{oil} = P_A + P_c + P_m$

$P_B - P_A = P_c + P_m - P_{oil}$

$P_B - P_A = \rho_{oil}\, g h_{oil} - \rho_c\, g h_c - \rho_m\, g h_m$

$7.848 \times 10^4 = [1594 \times 9.81 \times 3.5] + [13600 \times 9.81 \times h] - [800 \times 9.81 \times (1 + h)]$

$h = 25.16$ cm.

Example 7/ An inverted manometer, when connected to two pipes A and B, gives the readings as shown in Figure (7). Determine the pressure in tube B, if the pressure in pipe A $9.81 \times 10^4\ Pa$.

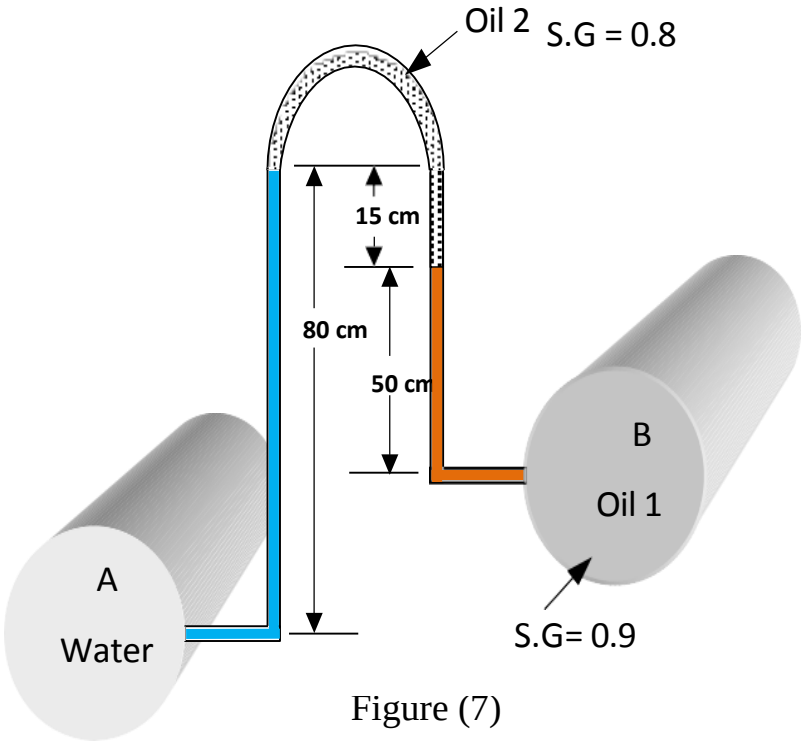


Figure (7)

Solution/

$\rho = \text{S.G} \times \rho_{H2O}$

$\rho_{oil\ 1} = 0.9 \times 1000 = 900\ \text{Kg/m}^3$

$\rho_{oil\ 2} = 0.8 \times 1000 = 800\ \text{Kg/m}^3$

$P_1 = P_A - P_w$

$P_2 = P_B + P_{oil\ 1} + P_{oil\ 2}$

$P_1 = P_2$

$P_A - P_w = P_B + P_{oil\ 1} + P_{oil\ 2}$

$P_B = P_A - P_w - P_{oil\ 1} - P_{oil\ 2}$

$P_B = P_A - \rho_w\, g h_w - \rho_{oil\ 1}\, g h_{oil\ 1} - \rho_{oil\ 2}\, g h_{oil\ 2}$

$P_B = 9.81 \times 10^4 - (1000 \times 9.81 \times 0.8) - (900 \times 9.81 \times 0.5) - (800 \times 9.81 \times 0.15)$

$P_B = 84660.3\ \text{Pa}$

Problems:

1. A Micro manometer, having ratio of basin to limb areas as 40, was used to determine the pressure in a pipe containing water. Determine the pressure in the pipe for the manometer

reading shown in Figure (8). Ans: $P_A = 1.11507 \text{ x}10^5$ Pa

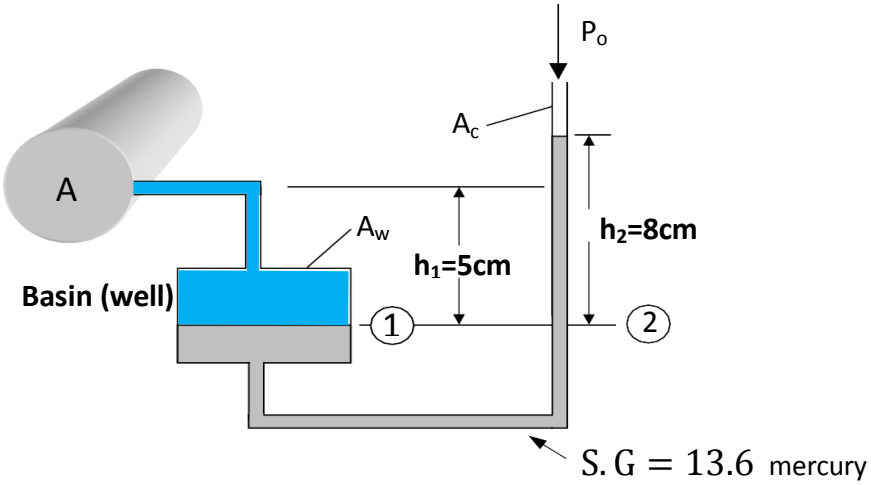


Figure (8)

2. A differential manometer is connected to two pipes as shown in Figure (9). At B the air pressure is $9.81 \times 10^4\ Pa$, find the absolute pressure at A.

Ans: P_{air} = 9.54513 x104 Pa

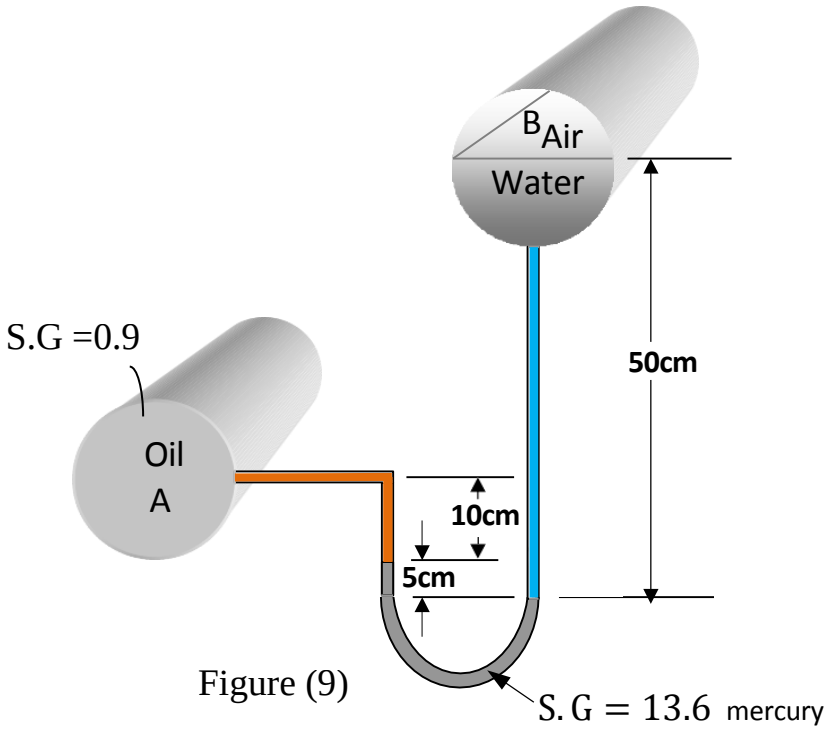
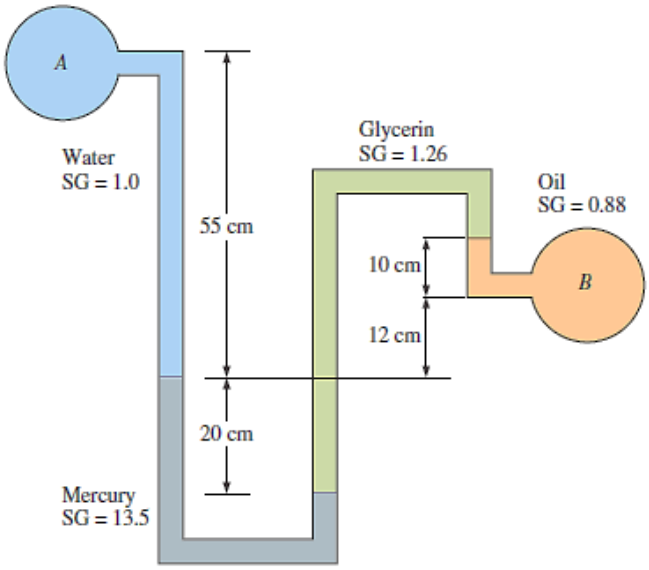


Figure (9)

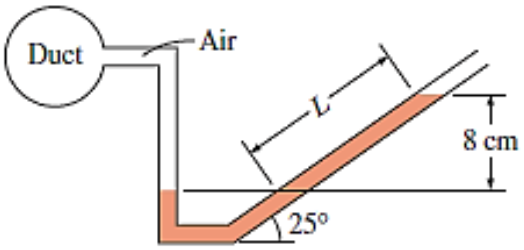
3. The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown in Figure (10). For the given fluid heights and specific gravities, calculate the pressure difference $\Delta P = P_B - P_A$. Ans: ΔP = 27.6 KPa



Figure(10)

4. The air pressure in a circular duct is to be measured using a manometer whose open arm is inclined 25° from the horizontal, as shown in Figure (11). The density of the liquid in the manometer is 0.81 kg/L, and the vertical distance between the fluid levels in the two arms of the manometer is 8 cm. Determine the gage pressure of air in the duct and the length of the fluid column in the inclined arm above the fluid level in the vertical arm.

Ans: $P_{gage} = 636$ pa, $L = 18.9\ cm$



Figure(11)

5. A gasoline line is connected to a pressure gage through a double-U manometer, as shown in Figure (12). If the reading of the pressure gage is 260 kPa, determine the gage pressure of the gasoline line. Ans: $P_{gasoline} = 245$ kPa

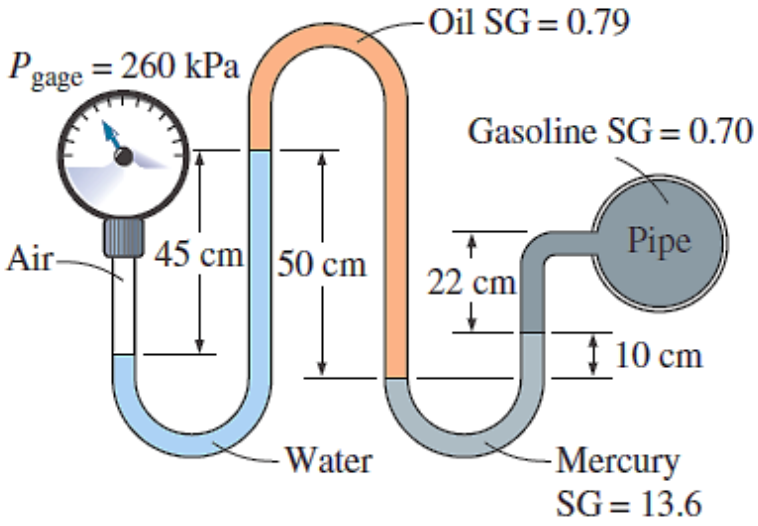


Figure (12)

6. A water pipe is connected to a double-U manometer as shown in Figure (13), at a location where the local atmospheric pressure is 14.2 psia. Determine the absolute pressure at the center of the pipe. Ans: $P_{water\ pipe} = 22.3$ psia

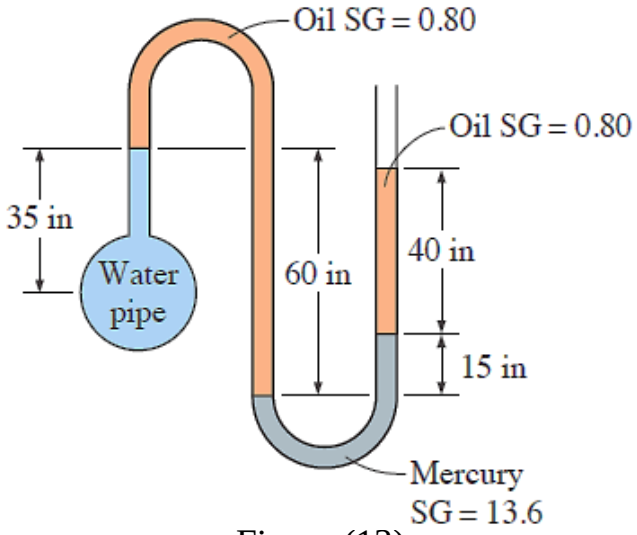
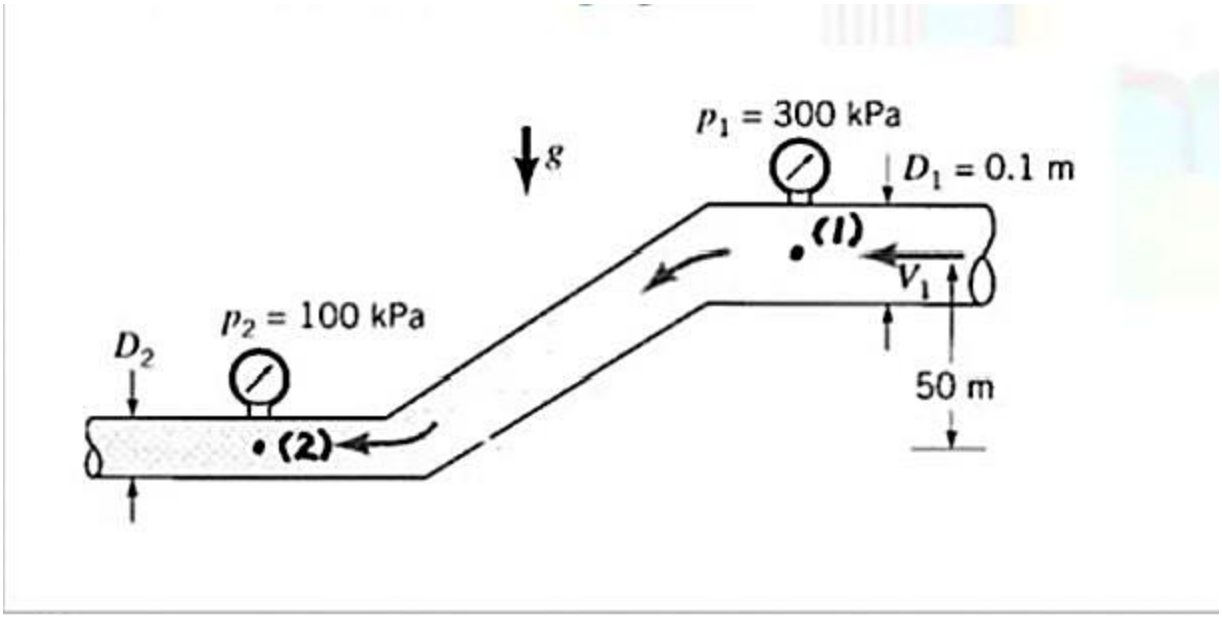


Figure (13)

Example for Bernoulli Equation

Water flows steadily through the pipe as shown below, such that the pressure at section (1) and at section (2) are (300 kPa and 100 kPa) respectively. Determine the diameter of the pipe at section (2), if the velocity at section (1) is (20 m/sec) and viscous effects are negligible.



Solution:

$P_1 = 300\ KPa\ , D_1 = 0.1\ m,\ V_1 = 20\ m/s, Z_1 = 50\ m$

$P_2 = 100\ KPa\ , D_2 = ?\ ,\ V_2 = ?\ , Z_2 = 0\ m$

$$A_1 = \frac{\pi\, d_1^2}{4} = \frac{\pi \times 0.1^2}{4} = 0.008\ m^2$$

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + Z_2 + h_f$$

$$\frac{300 \times 10^3}{1000 \times 9.81} + \frac{20^2}{2 \times 9.81} + 50 = \frac{100 \times 10^3}{1000 \times 9.81} + \frac{{v_2}^2}{2 \times 9.81} + 0$$

$$30.612 + 20.387 + 50 = 10.204 + \frac{{v_2}^2}{19.62}$$

$$100.999 = 10.204 + \frac{{v_2}^2}{19.62} \rightarrow \quad {v_2}^2 = \sqrt{90.795 \times 19.62}$$

$${v_2}^2 = 42.21\ m/s$$

$$\dot{V}_1 = \dot{V}_2$$

$$A_1 v_1 = A_2 v_2$$

$$0.008 \times 20 = A_2 \times 42.21$$

$$A_2 = 0.004\ m^2$$

$$A_2 = \frac{\pi\, d_2^2}{4}$$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 \times 0.004}{\pi}} = 0.071\ m$$

H.w/

Water flows in a rectangular channel that is (0.5 m) wide as shown. The upstream depth is (70 mm). The water surface rises (40 mm) as it passes over a portion where the channel bottom rises (10 mm). If viscous effects are negligible. What is the flow rate?

