

CHAPTER ONE

Some Concepts and Definitions

“**Thermodynamics**” is derived from the Greek words *threme*, meaning “ heat/ thermal” and *dynamics* meaning “ movement/ power/ flow ”.

Thus; thermodynamics is the science that deals with energy transformation; the conversion of heat into work, or chemical energy into electrical energy, both of these are energy transformations, and thermodynamics is the science that provides the tools to analyze them.

Application of Thermodynamics:

Any application that deals with heat and work transformation such as:

- 1- Internal combustion engines (IC engine).
- 2- Power plants.
- 3- Car radiators.
- 4- Air conditioning systems.



Refrigerator



Boats



Aircraft and spacecraft



Power plants



Human body



Cars

Dimensions and Units:

Any physical quantity can be characterized by **dimensions**. The magnitudes assigned to the dimensions are called **units**. Some basic dimensions such as mass m , length L , time t , and temperature T are selected as **primary** or **fundamental dimensions**,

while others such as velocity V , energy E , and volume V are expressed in terms of the primary dimensions and are called **secondary dimensions**, or **derived dimensions**.

Quantity	Symbol	SI Units	English Units	To Convert from English to SI Units Multiply by
Length	L	m	ft	0.3048
Mass	m	kg	lbm	0.4536
Time	t	s	sec	1
Area	A	m ²	ft ²	0.09290
Volume	V	m ³	ft ³	0.02832
Velocity	V	m/s	ft/sec	0.3048
Acceleration	a	m/s ²	ft/sec ²	0.3048

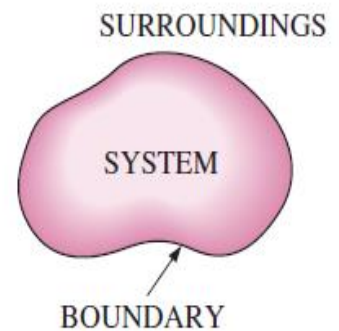
Force, Weight	F, W	N	lbf	4.448
Density	ρ	kg/m ³	lbm/ft ³	16.02
Specific weight	γ	N/m ³	lbf/ft ³	157.1
Pressure	P	kPa	psi	6.895
Work, Energy	W, E, U	J	ft · lbf	1.356
Heat transfer	Q	J	Btu	1055
Power	$\dot{W}$	W	ft · lbf/sec	1.356
		W	hp	746

Prefixes for SI Units

Multiplication Factor	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi*	c
10^{-3}	mili	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Thermodynamics definitions:

- 1- **System:** is a quantity of matter or a region in space chosen for study.
- 2- **Working substance:** is a matter that transfers the energy through the system (steam, gas, liquid,.....etc).
- 3- **Boundary:** is the real or imaginary surface that separates the system from its surroundings.
- 4- **Surroundings:** the mass or region outside the system.

**FIGURE**

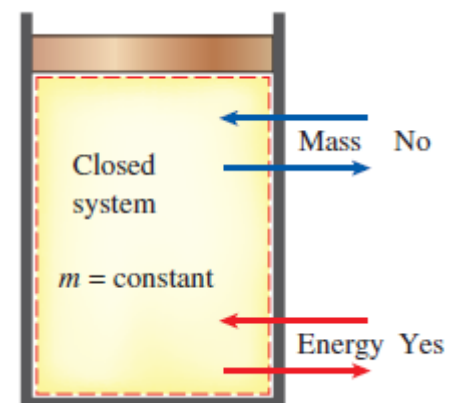
System, surroundings, and boundary.

Classification of Thermodynamics Systems:

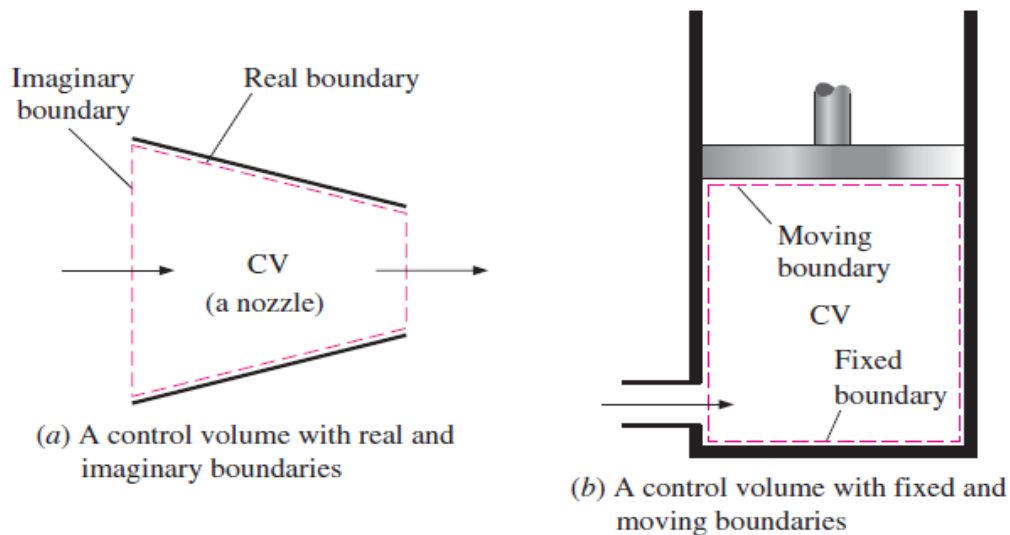
The system may be divided into the following:

1- Closed system (also known as a control mass):

- consists of a fixed amount of mass.
- no mass can cross its boundary (no mass can enter or leave a closed system). Only energy, in the form of heat or work, can cross the boundary.

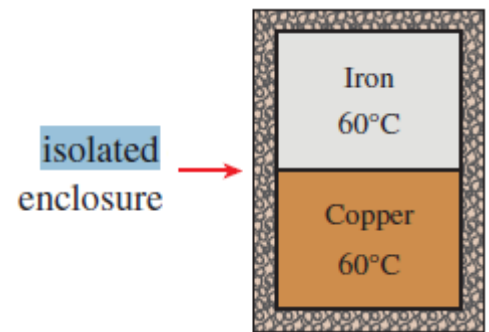
**2- Open system (or a control volume):**

- Both mass and energy can cross the boundary such as a compressor, turbine, or nozzle.
- The boundaries of a control volume are called a **control surface**, and they can be real or imaginary, moving or fixed. In the case of a nozzle, the inner surface of the nozzle forms the real part of the boundary, and the entrance and exit areas form the imaginary part. A control volume can be fixed in size and shape, as in the case of a nozzle, or it may involve a moving boundary.



3- isolated system :

- Special case of closed system
- NO mass and NO energy is allowed to cross the boundary.



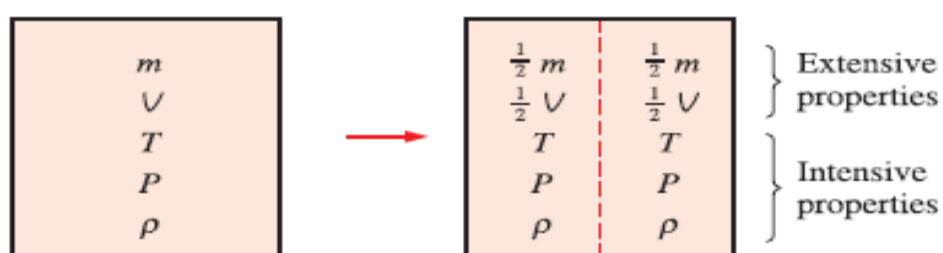
PROPERTIES OF A SYSTEM:

Any characteristic that describes or identifies a system is called a **property**. Some familiar properties are **pressure P , temperature T , volume V , and mass m** .

Properties are divided into two groups (*Intensive and extensive*):

- 1- **Intensive properties** are those that are independent of the mass of a system, such as temperature, pressure, and density.
- 2- **Extensive properties** are those whose values depend on the mass of the system.

An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition, as shown in Fig.



Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

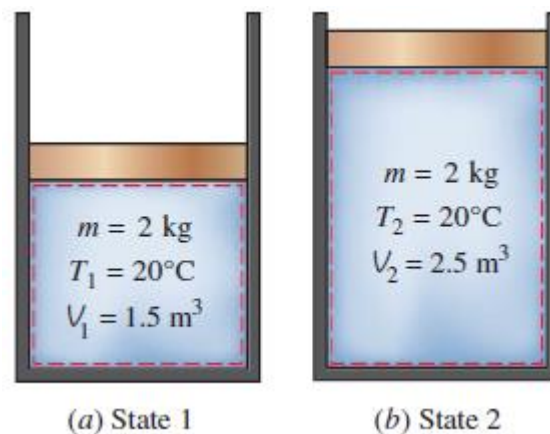
- Extensive properties per unit mass are called **specific properties**. Some examples of specific properties are specific volume ($v = V/m$) and specific energy ($e = E/m$).

Thermodynamics terms

1- STATE:

Consider a system not undergoing any change. At this point, all the properties can be measured or calculated throughout the entire system, which gives us a set of properties that completely describes the condition, or the **state**, of the system. At a given state, all the properties of a system have fixed values. Thus; **state of a system is its condition as described by giving values to its properties at a particular instant.**

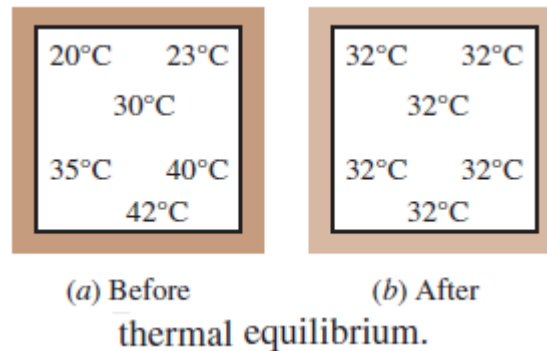
If the value of even one property changes, the state will change to a different one.



2- EQUILIBRIUM

Thermodynamics deals with *equilibrium* states. The word **equilibrium** implies a state of balance. There are many types of equilibrium, For example, a system is in **thermal equilibrium** if the temperature is the same throughout the entire system, and a system is in **mechanical equilibrium** if there is no change in pressure at any point of the system with time. If a system involves two phases, it is in **phase equilibrium** when the mass of each phase reaches an equilibrium level and stays there. A system

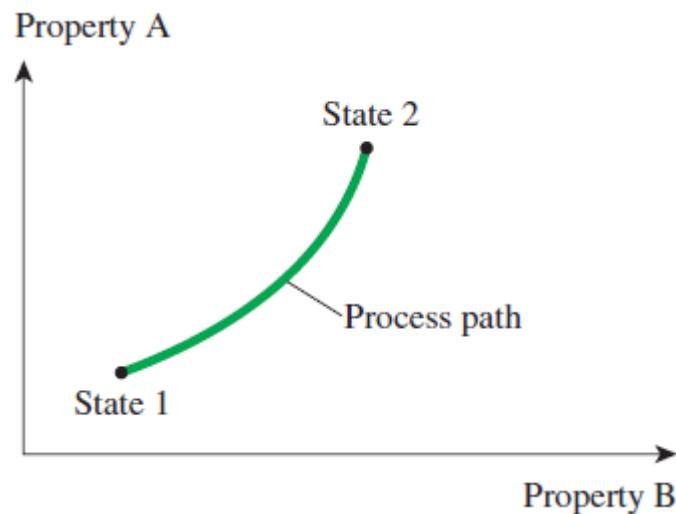
is in **chemical equilibrium** if its chemical composition does not change with time, that is, no chemical reactions occur.



3- PROCESSES:

Any change that a system undergoes from one equilibrium state to another is called a **process**.

The series of states through which a system passes during a process is called the **path** of the process.



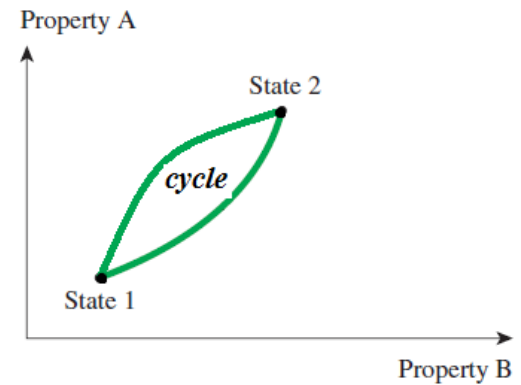
An **isothermal process**, is a process during which the temperature T remains constant.

An **isobaric process** is a process during which the pressure P remains constant.

An **isochoric** (or **isometric**) **process** is a process during which the specific volume remains constant.

4- CYCLE:

A system is said to have undergone a **cycle** if it returns to its initial state at the end of the process. That is, for a cycle the initial and final states are identical.

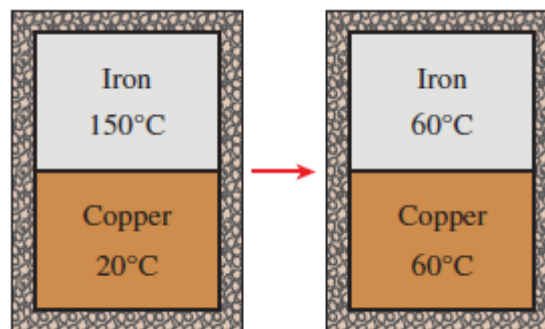


TEMPERATURE:

It is a measure of relative "hotness" or "coldness" of an object.

Thermal equilibrium

It is a common experience that a cup of hot coffee left on the table eventually cools off and a cold drink eventually warms up. That is, when a body is brought into contact with another body that is at a different temperature, heat is transferred from the body at higher temperature to the one at lower temperature until both bodies attain the same temperature. At that point, the heat transfer stops, and the two bodies are said to have reached **thermal equilibrium**.



The zeroth law of thermodynamics:

It states that if two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other.

By replacing the third body with a thermometer, the zeroth law can be restated as *two bodies are in thermal equilibrium if both have the same temperature reading even if they are not in contact*.

Temperature Scales:

Two scales are commonly used for measuring temperature:

- 1- **Celsius scale** (formerly called the *centigrade scale*)
- 2- **Fahrenheit scale**

On the Celsius scale, the ice and steam points were originally assigned the values of 0 and 100°C, respectively. The corresponding values on the Fahrenheit scale are 32 and 212°F.

- The absolute scale relate to the Celsius or centigrade is Kelvin (K):

$$T(K) = T(^{\circ}C) + 273.15$$

- The absolute scale relate to the Fahrenheit Rankine (R):

$$T(R) = T(^{\circ}F) + 459.67$$

The temperature scales in the two unit systems are related by

$$T(R) = 1.8T(K)$$

$$T(^{\circ}F) = 1.8T(^{\circ}C) + 32$$

DENSITY AND SPECIFIC GRAVITY:

Density is defined as mass per unit volume.

$$\rho = \frac{m}{V} \quad (\text{kg/m}^3)$$

The reciprocal of density is the **specific volume** v , which is defined as volume per unit mass. That is,

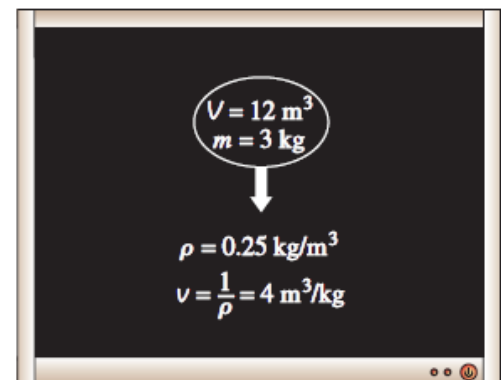
$$v = \frac{V}{m} = \frac{1}{\rho}$$

specific gravity, or **relative density**, and is defined as the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C, for which $\rho_{\text{(H}_2\text{O)}} = 1000 \text{ kg/m}^3$). That is

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}}}$$

Note that the specific gravity of a substance is a dimensionless quantity. The weight of a unit volume of a substance is called **specific weight** and is expressed as:

$$\gamma_s = \rho g \quad (\text{N/m}^3)$$



PRESSURE:

Pressure is defined as *a normal force exerted by a fluid per unit area*. it has the unit of Newtons per square meter (N/m^2), which is called a **Pascal (Pa)**. That is,

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

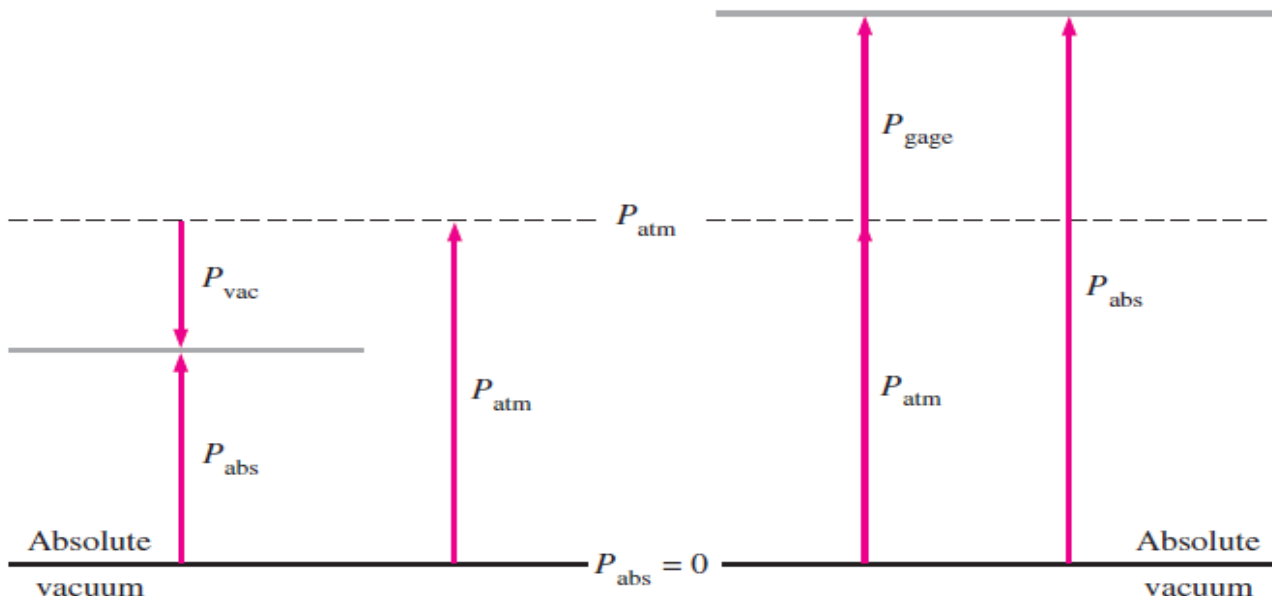
$$1 \text{ atm} = 101,325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$1 \text{ atm} = 14.696 \text{ psi.}$$

The actual pressure at a given position is called the **absolute pressure**, and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere. and so they indicate the difference between the absolute pressure and the local atmospheric pressure. difference is called the **gage pressure**. Pressures below atmospheric pressure are called **vacuum pressures** and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are all positive quantities.

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}}$$



Like other pressure gages, the gage used to measure the air pressure in an automobile tire reads the gage pressure. Therefore, the common reading of 32 psi (2.25 kgf/cm²) indicates a pressure of 32 psi above the atmospheric pressure. At a location where the atmospheric pressure is 14.3 psi, for example, the absolute pressure in the tire is $32 + 14.3 = 46.3$ psi.

the gage used to measure the air pressure in an automobile tire reads the gage pressure. Therefore, the common reading of 32.0 psi indicates a pressure of 32.0 psi above the atmospheric pressure. At a location where the atmospheric pressure is 14.3 psi, for example, the absolute pressure in the tire is:

$$32.0 + 14.3 = 46.3 \text{ psi.}$$

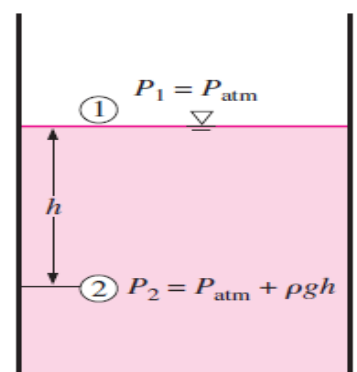
EX.:

A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. determine the absolute pressure in the chamber.

$$P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 14.5 - 5.8 = \mathbf{8.7 \text{ psi}}$$

If we take point 1 to be at the free surface of a liquid open to the atmosphere where the pressure is the atmospheric pressure P_{atm} , then the pressure at a depth h from the free surface becomes:

$$P = P_{\text{atm}} + \rho gh \quad \text{or} \quad P_{\text{gage}} = \rho gh$$



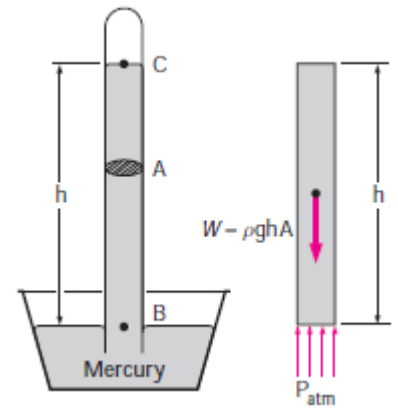
PRESSURE MEASUREMENT DEVICES

Tow commonly used devices for measuring pressure:

- 1- **Barometer.**
- 2- **Manometer.**
- 3- **Bourdon gage.**

- 1- **Barometer:** Atmospheric pressure is measured by a device called a **barometer**; thus, the atmospheric pressure is often referred to as the *barometric pressure*.

$$P_{\text{atm}} = \rho gh$$



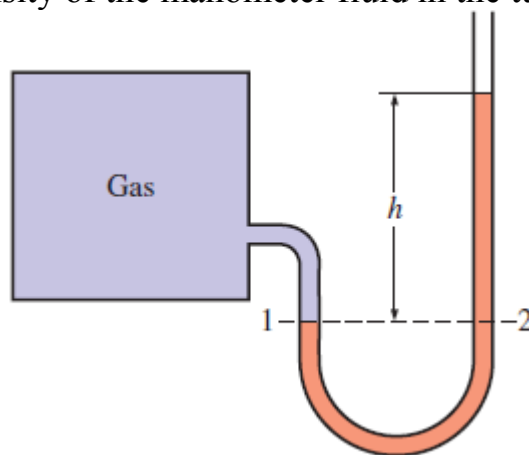
- 2- **Manometer:** A manometer consists of a glass or plastic U-tube containing one or more fluids such as mercury, water, alcohol, or oil. It can be used to measure pressure differences.

Consider the manometer shown in Figure that is used to measure the pressure in the tank. Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value. Since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, thus;

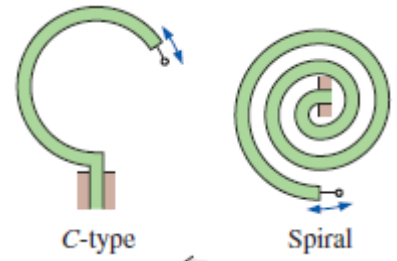
$$P_2 = P_1.$$

$$P_2 = P_{\text{atm}} + \rho gh$$

where ρ is the density of the manometer fluid in the tube.



3- **Bourdon tube:** which consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle. When the tube is open to the atmosphere, the tube is undeflected, and the needle on the dial at this state is calibrated to read zero (gage pressure). When the fluid inside the tube is pressurized, the tube stretches and moves the needle in proportion to the pressure applied.



4- **Pressure transducers:** Modern pressure sensors, use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance.

5- **Strain-gage pressure transducers.**

6- **Piezoelectric transducers.**

7- **Deadweight tester.**

ENERGY

Energy (E):

Energy exists in numerous forms such as thermal, mechanical, electric, chemical, and nuclear. Even mass can be considered a form of energy. Energy can be transferred to or from a closed system (a fixed mass) in two distinct forms: *heat* and *work*. For control volumes, energy can also be transferred by mass flow. An energy transfers to or from a closed system is *heat* if it is caused by a temperature difference. Otherwise it is *work*, and it is caused by a force acting through a distance.

FORMS OF ENERGY:

A system may possess several different forms of energy such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear and their sum constitutes the **total energy (E)** of a system. The total energy of a system on a *unit mass* basis is denoted by (**e**) and is expressed as:

$$e = \frac{E}{m} \quad (\text{kJ/kg})$$

TYPES OF ENERGY:

- a- **Potential energy (P.E):** The energy that a system possesses as a result of its elevation in a gravitational field.

$$PE = mgz \quad (\text{kJ})$$

or, on a unit mass basis,

$$pe = gz \quad (\text{kJ/kg})$$

Where;

g = is the gravitational acceleration

z = the elevation of the center of gravity of a system.

- b- **Kinetic energy (K.E):** The energy that a system possesses as a result of its motion.

$$KE = m \frac{V^2}{2} \quad (\text{kJ})$$

or, on a unit mass basis,

$$ke = \frac{V^2}{2} \quad (\text{kJ/kg})$$

Where;

V = the velocity of the system.

c- **Internal energy (U):** A form of energy related to the molecular structure of a system and the degree of the molecular activity. The energy stored in a battery, energy stored in an electrical condenser, and vibration of the molecules, electrons, protons, and neutrons, and the chemical energy due to bonding between atoms and between subatomic particles. All of these forms of energy will be referred to as **internal energy** and designated by the letter **(U)**. A substance always has internal energy; if there is molecular activity there is internal energy.

☒ The total energy of a system consists of the kinetic, potential, and internal energies and is expressed as:

$$E = U + KE + PE = U + m \frac{V^2}{2} + mgz \quad (\text{kJ})$$

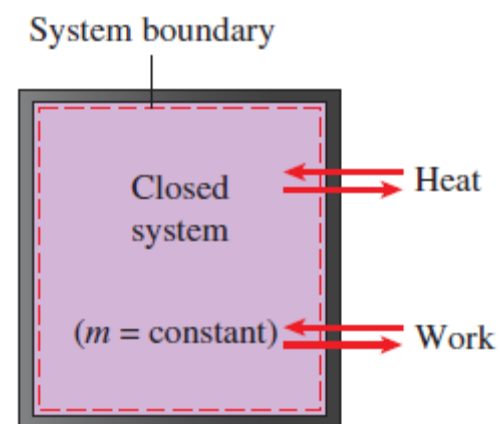
or, on a unit mass basis,

$$e = u + ke + pe = u + \frac{V^2}{2} + gz \quad (\text{kJ/kg})$$

ENERGY TRANSFER BY HEAT

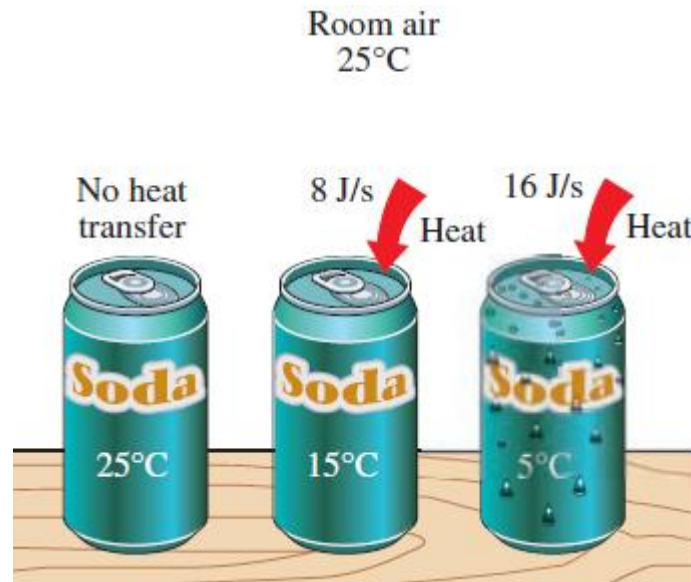
Energy can cross the boundary of a closed system in two distinct forms: *heat* and *work*.

We know from experience that a can of cold soda left on a table eventually warms up and that a hot baked potato on the same table cools down. When a body is left in a medium that is at a different temperature, energy transfer takes place between the body and the surrounding medium until thermal equilibrium is established, that is, the body and the medium reach the same temperature. The direction of energy transfer is always from the higher temperature body to the lower temperature one. Once the temperature equality is established, energy transfer stops. In the processes described



above, energy is said to be transferred in the form of heat.

Heat is defined as the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference.



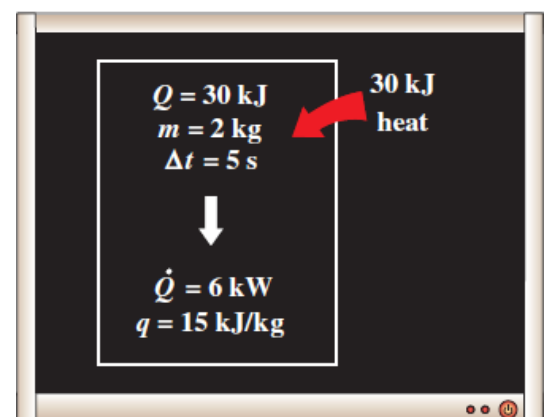
As a form of energy, heat has energy units, kJ (or Btu).

The amount of heat transferred during the process between two states (states 1 and 2) is denoted by (Q_{12}) , or just (Q) . Heat transfer per unit mass of a system is denoted q and is determined from:

$$q = \frac{Q}{m} \quad (\text{kJ/kg})$$

Sometimes it is desirable to know the *rate of heat transfer* (the amount of heat transferred per unit time) instead of the total heat transferred over some time interval.

The heat transfer rate is denoted $(\dot{Q})$, where the overdot stands for the time derivative, or “per unit time.” The heat transfer rate $(\dot{Q})$ has the unit (kJ/s) , which is equivalent to (kW) .



Heat is transferred by three mechanisms: conduction, convection, and radiation.

- **Conduction** is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interaction between particles.
- **Convection** is the transfer of energy between a solid surface and the adjacent fluid that is in motion, and it involves the combined effects of conduction and fluid motion.
- **Radiation** is the transfer of energy due to the emission of electromagnetic waves (or photons).

ENERGY TRANSFER BY WORK

Work like heat, is an energy interaction between a system and its surroundings. As mentioned earlier, energy can cross the boundary of a closed system in the form of heat or work. Therefore, ***if the energy crossing the boundary of a closed system is not heat, it must be work.***

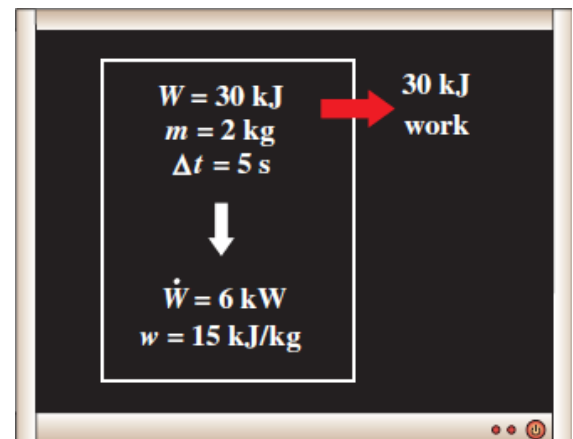
Work is the energy transfer associated with a force acting through a distance. Or it is the product of a force and the distance moved in the direction of the force.

Work is also a form of energy transferred like heat and, therefore, has energy units such as **(N.m = J) or (kJ)**.

The work done per unit mass of a system is denoted by w and is expressed as;

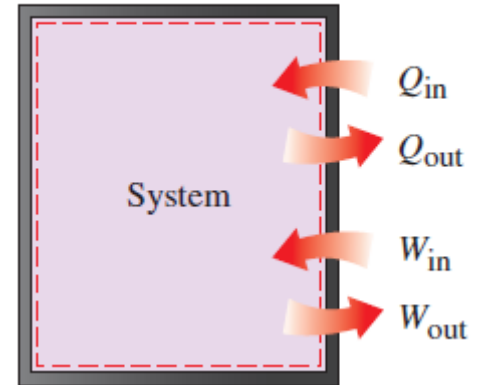
$$w = \frac{W}{m} \quad (\text{kJ/kg})$$

The work done per unit time is called **POWER**. The unit of power is **(kJ/s), or (Kw)**.



Surroundings

Heat and work are directional quantities, require the specification of both the magnitude and direction. Heat transfer to a system and work done by a system are **positive**; heat transfer from a system and work done on a system are **negative**.



ANOTHER FORMS OF WORK:

Electrical Work:

The electrical work done during a time interval is expressed as:

$$W_e = VI \Delta t \quad (\text{kJ})$$

V = a potential difference

I = the current

Δt = a time interval

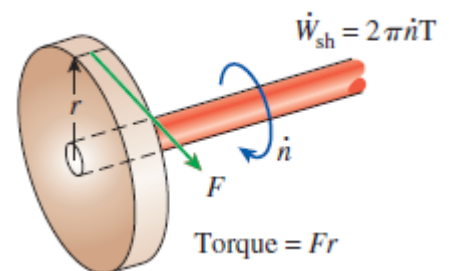
MECHANICAL WORK:

There are several different ways of doing work, each in some way related to a force acting through a distance.

$$W = Fs$$

Shaft Work:

Energy transmission with a rotating shaft is very common in engineering practice. Often the torque T applied to the shaft is constant, which means that the force F applied is also constant. For a specified constant torque, the work done during n revolutions is determined as follows: A force F acting through a moment arm r generates a torque T of:



$$T = Fr \rightarrow F = \frac{T}{r}$$

This force acts through a distance s , which is related to the radius r by:

$$s = (2\pi r)n$$

Then the shaft work is determined from:

$$W_{sh} = Fs = \left(\frac{T}{r}\right)(2\pi rn) = 2\pi nT$$

EXAMPLE Power Transmission by the Shaft of a Car

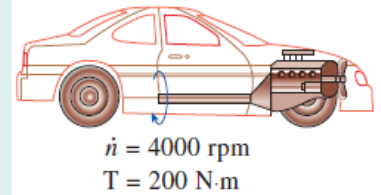
Determine the power transmitted through the shaft of a car when the torque applied is 200 N·m and the shaft rotates at a rate of 4000 revolutions per minute (rpm).

SOLUTION The torque and the rpm for a car engine are given. The power transmitted is to be determined.

Analysis A sketch of the car is given in Fig. 2–31. The shaft power is determined directly from

$$\dot{W}_{\text{sh}} = 2\pi nT = (2\pi)\left(4000 \frac{1}{\text{min}}\right)(200 \text{ N}\cdot\text{m})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ kJ}}{1000 \text{ N}\cdot\text{m}}\right)$$

$$= 83.8 \text{ kW}$$

**Spring Work**

It is common knowledge that when a force is applied on a spring, the length of the spring changes.

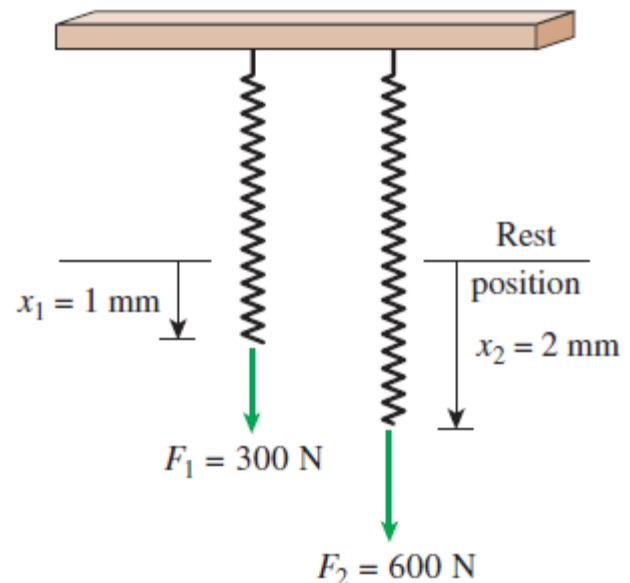
To determine the total spring work, we need to know a functional relationship between F and x . For linear elastic springs, the displacement x is proportional to the force applied:

$$F = kx$$

where k is the spring constant and has the unit kN/m.

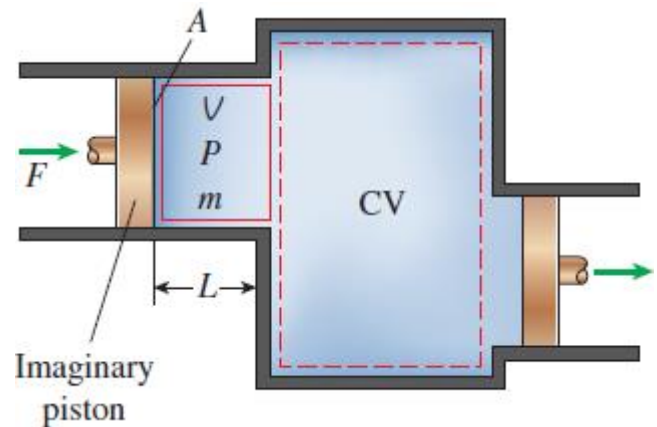
$$W_{\text{spring}} = \frac{1}{2}k(x_2^2 - x_1^2) \quad (\text{kJ})$$

where x_1 and x_2 are the initial and the final displacements of the spring, respectively, measured from the undisturbed position of the spring.



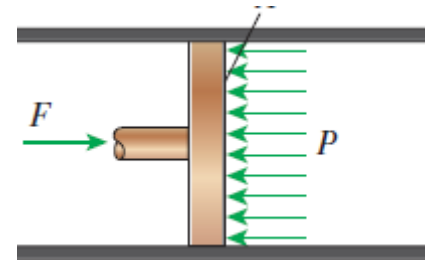
FLOW WORK THE ENERGY OF A FLOWING FLUID

Unlike closed systems, control volumes involve mass flow across their boundaries, and some work is required to push the mass into or out of the control volume. This work is known as the **flow work**, or **flow energy**, and is necessary for maintaining a continuous flow through a control volume.



To obtain a relation for flow work, consider a fluid element of volume V as shown in Figure. The fluid immediately upstream forces this fluid element to enter the control volume; thus, it can be regarded as an imaginary piston. If the fluid pressure is P and the cross-sectional area of the fluid element is A , the force applied on the fluid element by the imaginary piston is:

$$F = PA$$



To push the entire fluid element into the control volume, this force must act through a distance L . Thus, the work done in pushing the fluid element across the boundary (i.e., the flow work) is:

$$W_{\text{flow}} = FL = PAL = PV$$

The flow work per unit mass is obtained by dividing both sides of this equation by the mass of the fluid element:

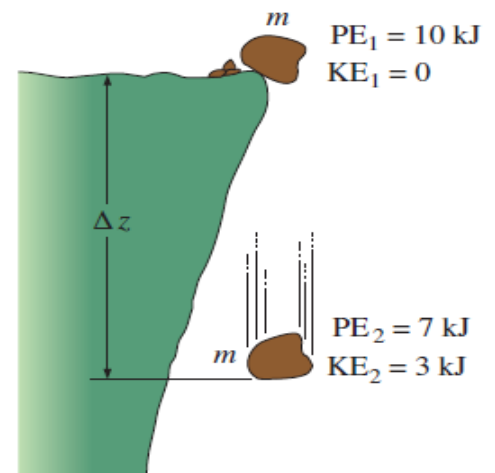
$$w_{\text{flow}} = Pv \quad (\text{kJ/kg})$$

It is interesting that unlike other work quantities, flow work is expressed in terms of properties and sometimes refer to it as **flow energy** (energy of a flowing fluid).

THE FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics, also known as the conservation of energy principle, it states that **energy can be neither created nor destroyed during a process; it can only change forms.**

We all know that a rock at some elevation possesses some potential energy, and part of this potential energy is converted to kinetic energy as the rock falls. Experimental data show that the decrease in potential energy ($mg \Delta z$) exactly equals the increase in kinetic energy [$m (V_2^2 - V_1^2)/2$] when the air resistance is negligible.



the conservation of energy principle can be expressed as follows: The net change (increase or decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process.

$$\left(\begin{array}{c} \text{Total energy} \\ \text{entering the system} \end{array} \right) - \left(\begin{array}{c} \text{Total energy} \\ \text{leaving the system} \end{array} \right) = \left(\begin{array}{c} \text{Change in the total} \\ \text{energy of the system} \end{array} \right)$$

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

This relation is often referred to as the **energy balance** and is applicable to any kind of system undergoing any kind of process.

Energy Change of a System, ΔE_{system}

The change in the total energy of a system during a process is the sum of the changes in its internal, kinetic, and potential energies and can be expressed as:

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

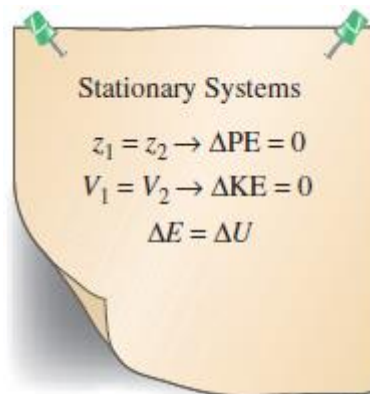
Where;

$$\Delta U = m(u_2 - u_1)$$

$$\Delta KE = \frac{1}{2} m(V_2^2 - V_1^2)$$

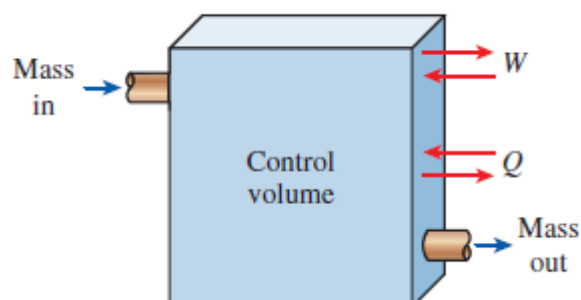
$$\Delta PE = mg(z_2 - z_1)$$

☒ Most systems encountered in practice are stationary. Thus;

**Mechanisms of Energy Transfer, E_{in} and E_{out}**

Energy can be transferred to or from a system in three forms: heat, work, and mass flow.

The only two forms of energy interactions associated with a fixed mass or closed system are *heat transfer* and *work*.



$$E_{\text{in}} - E_{\text{out}} = (Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) + (E_{\text{mass,in}} - E_{\text{mass,out}})$$

1. Heat Transfer, Q Heat transfer to a system (heat gain) increases the energy of the molecules and thus the internal energy of the system, and heat transfer from a system (heat loss) decreases it since the energy transferred out as heat comes from the energy of the molecules of the system.

2. Work Transfer, W An energy interaction that is not caused by a temperature difference between a system and its surroundings is work. A rising piston, a rotating shaft, and an electrical wire crossing the system boundaries are all associated with work interactions. Work transfer to a system (i.e., work done on a system) increases the energy of the system, and work transfer from a system (i.e., work done by the system) decreases it since the energy transferred out as work comes from the energy contained in the system. Car engines and hydraulic, steam, or gas turbines produce work while compressors, pumps, and mixers consume work.

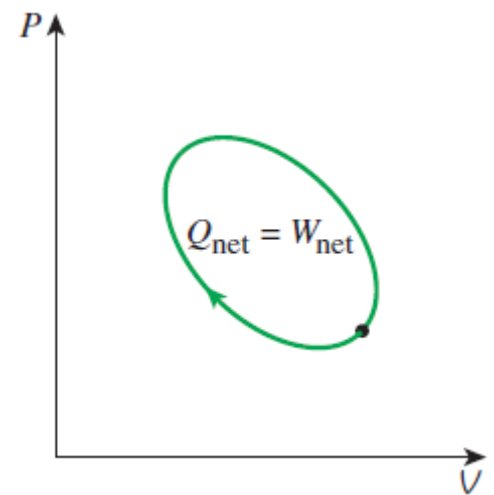
3. Mass Flow, m Mass flow in and out of the system serves as an additional mechanism of energy transfer. When mass enters a system, the energy of the system increases because mass carries energy with it (in fact, mass is energy). Likewise, when some mass leaves the system, the energy contained within the system decreases because the leaving mass takes out some energy with it. For example, when some hot water is taken out of a water heater and is replaced by the same amount of cold water,

the energy content of the hot-water tank (the control volume) decreases as a result of this mass interaction.

Noting that energy can be transferred in the forms of heat, work, and mass, and that the net transfer of a quantity is equal to the difference between the amounts transferred in and out, the energy balance can be written more explicitly as:

$$E_{\text{in}} - E_{\text{out}} = (Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) + (E_{\text{mass,in}} - E_{\text{mass,out}}) = \Delta E_{\text{system}}$$

where the subscripts “in” and “out” denote quantities that enter and leave the system, respectively. All six quantities on the right side of the equation represent “amounts,” and thus they are *positive* quantities. The direction of any energy transfer is described by the subscripts “in” and “out.”



The heat transfer Q is zero for adiabatic systems, the work transfer W is zero for systems that involve no work interactions, and the energy transport with mass E_{mass} is zero for systems that involve no mass flow across their boundaries (i.e., closed systems). Energy balance for any system undergoing any kind of process can be expressed more compactly as:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ})$$

or, in the **rate form**, as:

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \quad (\text{kW})$$

For a closed system undergoing a **cycle**, the initial and final states are identical, and thus:

$$\Delta E_{\text{system}} = E_2 - E_1 = 0.$$

Then the energy balance for a cycle simplifies to:

$$E_{\text{in}} - E_{\text{out}} = 0 \quad \text{or} \quad E_{\text{in}} = E_{\text{out}}.$$

Noting that a closed system does not involve any mass flow across its boundaries, the energy balance for a cycle can be expressed in terms of heat and work interactions as:

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \text{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}} \quad (\text{for a cycle})$$

That is, the net work output during a cycle is equal to net heat input.

Thus; the first law of thermodynamics then takes the form:

$$\begin{aligned} Q_{1-2} - W_{1-2} &= KE_2 - KE_1 + PE_2 - PE_1 + U_2 - U_1 \\ &= \frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) + U_2 - U_1 \end{aligned}$$

The energy balance can be expressed on a **per unit mass** basis as:

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \quad (\text{kJ/kg})$$

EXAMPLE

Cooling of a Hot Fluid in a Tank

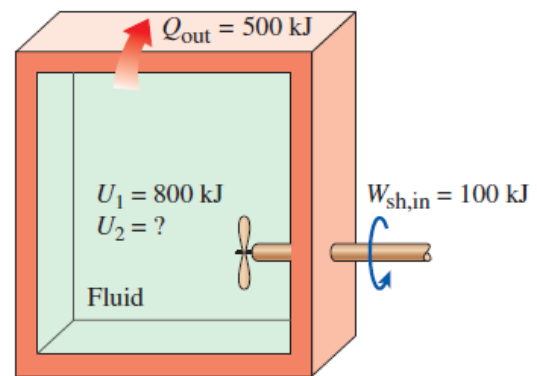
A rigid tank contains a hot fluid that is cooled while being stirred by a paddle wheel. Initially, the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat, and the paddle wheel does 100 kJ of work on the fluid. Determine the final internal energy of the fluid. Neglect the energy stored in the paddle wheel.

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$W_{\text{sh,in}} - Q_{\text{out}} = \Delta U = U_2 - U_1$$

$$100 \text{ kJ} - 500 \text{ kJ} = U_2 - 800 \text{ kJ}$$

$$U_2 = 400 \text{ kJ}$$



EXAMPLE Heating Effect of a Fan

A room is initially at the outdoor temperature of 25°C. Now a large fan that consumes 200 W of electricity when running is turned on (Fig. 2–51). The heat transfer rate between the room and the outdoor air is given as $\dot{Q} = UA(T_i - T_o)$ where $U = 6 \text{ W/m}^2 \cdot ^\circ\text{C}$ is the overall heat transfer coefficient, $A = 30 \text{ m}^2$ is the exposed surface area of the room, and T_i and T_o are the indoor and outdoor air temperatures, respectively. Determine the indoor air temperature when steady operating conditions are established.

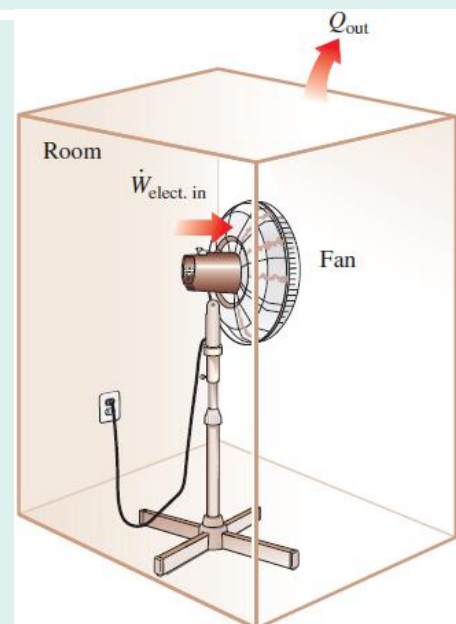
Analysis The electricity consumed by the fan is energy input for the room, and thus the room gains energy at a rate of 200 W. As a result, the room air temperature tends to rise. But as the room air temperature rises, the rate of heat loss from the room increases until the rate of heat loss equals the electric power consumption. At that point, the temperature of the room air, and thus the energy content of the room, remains constant, and the conservation of energy for the room becomes

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \xrightarrow{0(\text{steady})} 0 \rightarrow \dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{elect,in}} = \dot{Q}_{\text{out}} = UA(T_i - T_o)$$

$$200 \text{ W} = (6 \text{ W/m}^2 \cdot ^\circ\text{C})(30 \text{ m}^2)(T_i - 25^\circ\text{C})$$

$$T_i = 26.1^\circ\text{C}$$



EXAMPLE

A 5-hp fan is used in a large room to provide for air circulation. Assuming a well insulated, sealed room, determine the internal energy increase after 1 hour of operation.

Solution

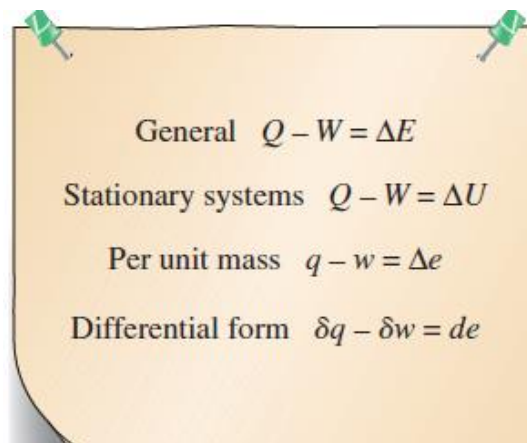
By assumption, $Q = 0$. With $\Delta PE = \Delta KE = 0$ the first law becomes $-W = \Delta U$. The work input is

$$W = (-5 \text{ hp})(1 \text{ hr})(746 \text{ W/hp})(3600 \text{ s/hr}) = -1.343 \times 10^7 \text{ J}$$

where $W = \text{J/s}$.

The negative sign results because the work is input to the system. Finally, the internal energy increase is

$$\Delta U = -(-1.343 \times 10^7) = 1.343 \times 10^7 \text{ J}$$

**ENERGY BALANCE FOR CLOSED SYSTEMS**

The energy balance relation (or the first-law) in that case for a closed system becomes:

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}} \quad \text{or} \quad Q - W = \Delta E$$

Where:

$Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$ is the *net heat input*

$W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$ is the *net work output*.

MASS AND ENERGY BALANCE FOR OPEN SYSTEMS

CONSERVATION OF MASS

The conservation of mass principle is one of the most fundamental principles in nature. For *closed systems*, the conservation of mass principle is implicitly used by requiring that the mass of the system remain constant during a process. For *control volumes*, however, mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

Mass and Volume Flow Rates:

The amount of mass flowing through a cross section per unit time is called the **mass flow rate** and is denoted by $\dot{m}$. The dot over a symbol is used to indicate *time rate of change*.

$$\dot{m} = \rho V_{\text{avg}} A_c \quad (\text{kg/s})$$

The volume of the fluid flowing through a cross section per unit time is called the **volume flow rate** $\dot{V}$

$$\dot{V} = V_{\text{avg}} A_c = V A_c \quad (\text{m}^3/\text{s})$$

The mass and volume flow rates are related by

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$

Conservation of Mass Principle

The **conservation of mass principle** for a control volume can be expressed as: *The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) of the total mass within the control volume during Δt . That is,*

$$\left(\begin{array}{c} \text{Total mass entering} \\ \text{the CV during } \Delta t \end{array} \right) - \left(\begin{array}{c} \text{Total mass leaving} \\ \text{the CV during } \Delta t \end{array} \right) = \left(\begin{array}{c} \text{Net change of mass} \\ \text{within the CV during } \Delta t \end{array} \right)$$

$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}} \quad (\text{kg})$$

where $\Delta m_{\text{CV}} = m_{\text{final}} - m_{\text{initial}}$ is the change in the mass of the control volume during the process. It can also be expressed in *rate form* as:

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt \quad (\text{kg/s})$$

where $\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ are the total rates of mass flow into and out of the control volume, and dm_{CV}/dt is the rate of change of mass within the control volume boundaries. The above equations are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ($m_{\text{CV}} = \text{constant}$). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

The conservation of mass principle for a general steady-flow system with multiple inlets and outlets is expressed in rate form as:

$$\text{Steady flow:} \quad \sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \quad (\text{kg/s})$$

It states that *the total rate of mass entering a control volume is equal to the total rate of mass leaving it.*

for *single-stream steady-flow systems*:

$$\text{Steady flow (single stream):} \quad \dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

A large number of engineering devices such as turbines, compressors, and nozzles are classified as *steady-flow devices*. Processes involving such devices can be represented reasonably well by a somewhat idealized process, called the **steady-flow process**, which was defined as *a process during which a fluid flows through a control volume steadily*. That is, the fluid properties can change from point to point within the control volume, but at any point, they remain constant during the entire process.

☒ (Remember, *steady means no change with time.*)

During a steady-flow process, no intensive or extensive properties *within the control volume* change with time. Thus, the volume V , the mass m , and the total energy content E of the control volume remain constant.

As a result, the boundary work is zero for steady-flow systems (since $V_{CV} = \text{constant}$), and the total mass or energy entering the control volume must be equal to the total mass or energy leaving it (since $m_{CV} = \text{constant}$ and $E_{CV} = \text{constant}$). These observations greatly simplify the analysis.

During a steady-flow process, the total energy content of a control volume remains constant ($E_{CV} = \text{constant}$), and thus the change in the total energy of the control volume is zero ($\Delta E_{CV} = 0$). Therefore, the amount of energy entering a control volume in all forms (by heat, work, and mass) must be equal to the amount of energy leaving it. Then the rate form of the general energy balance reduces for a steady-flow process to:

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

Energy balance:

$$\underbrace{\dot{E}_{\text{in}}}_{\text{Rate of net energy transfer in by heat, work, and mass}} = \underbrace{\dot{E}_{\text{out}}}_{\text{Rate of net energy transfer out by heat, work, and mass}} \quad (\text{kW})$$

The energy balance for a general steady-flow system can also be written more explicitly as:

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m}\theta = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m}\theta$$

or

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz \right)}_{\text{for each inlet}} = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz \right)}_{\text{for each exit}}$$

The first-law or energy balance relation in that case for a general steady-flow system becomes:

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz \right)}_{\text{for each exit}} - \sum_{\text{in}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz \right)}_{\text{for each inlet}}$$

For single-stream devices, the steady-flow energy balance equation becomes:

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

Dividing the equation by m^o gives the energy balance on a unit-mass basis as:

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

When the fluid experiences negligible changes in its kinetic and potential energies (that is, $\Delta ke = 0$, $\Delta pe = 0$), the energy balance equation is reduced further to:

$$q - w = h_2 - h_1$$

SOME STEADY-FLOW ENGINEERING DEVICES

1 Nozzles and Diffusers

2 Turbines and Compressors

3 Throttling Valves

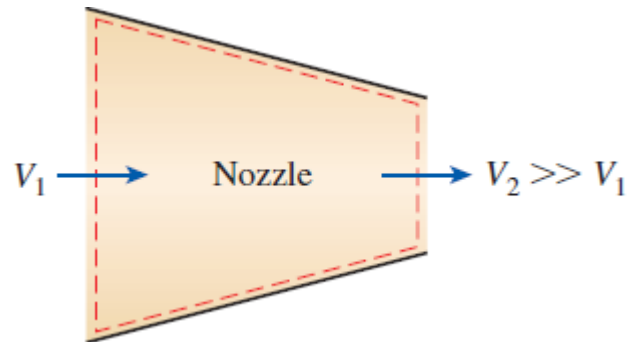
4 Mixing Chambers

5 Heat Exchangers

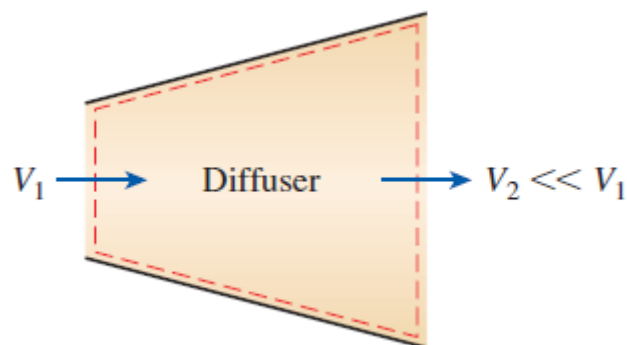
6 Pipe and Duct Flow

1 Nozzles and Diffusers

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.



A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.



- The rate of heat transfer between the fluid flowing through a nozzle or a diffuser and the surroundings is usually very small ($Q = 0$) since the fluid has high velocities, and thus it does not spend enough time in the device for any significant heat transfer to take place.
- Nozzles and diffusers typically involve no work ($W = 0$).
- Any change in potential energy is negligible ($\Delta p_e = 0$).
- But nozzles and diffusers usually involve very high velocities, and as a fluid passes through a nozzle or diffuser, it experiences large changes in its velocity. Therefore, the kinetic energy changes must be accounted for in analyzing the flow through these devices ($\Delta k_e \neq 0$).

2 Turbines and Compressors

In steam, gas, or hydroelectric power plants, the device that drives the electric generator is the turbine. As the fluid passes through the turbine, work is done against

the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as pumps and fans, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft. Therefore, compressors involve work inputs.

- Note that turbines produce power output whereas compressors, pumps, and fans require power input.
- Heat transfer from turbines is usually negligible ($Q = 0$) since they are typically well insulated.
- Heat transfer is also negligible for compressors unless there is intentional cooling. Potential energy changes are negligible for all of these devices ($\Delta pe = 0$).
- The velocities involved in these devices, with the exception of turbines and fans, are usually too low to cause any significant change in the kinetic energy ($\Delta ke = 0$).
- The fluid velocities encountered in most turbines are very high, and the fluid experiences a significant change in its kinetic energy. However, this change is usually very small relative to the change in enthalpy, and thus it is often disregarded.

3 Throttling Valves

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid. Some familiar examples are ordinary adjustable valves, capillary tubes.



(a) An adjustable valve



(b) A capillary tube

Throttling valves produce a pressure drop without involving any work. The pressure drop in the fluid is

often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

- Throttling valves are usually small devices, and the flow through them may be assumed to be adiabatic ($q = 0$) since there is neither sufficient time nor large enough area for any effective heat transfer to take place.
 - Also, there is no work done ($w = 0$).
 - The change in potential energy, is very small ($\Delta pe = 0$).
 - Even though the exit velocity is often considerably higher than the inlet velocity, in many cases, the increase in kinetic energy is insignificant ($\Delta ke = 0$).
- Then the conservation of energy equation for this single-stream steady-flow device reduces to:

$$h_2 \cong h_1 \quad (\text{kJ/kg})$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

or

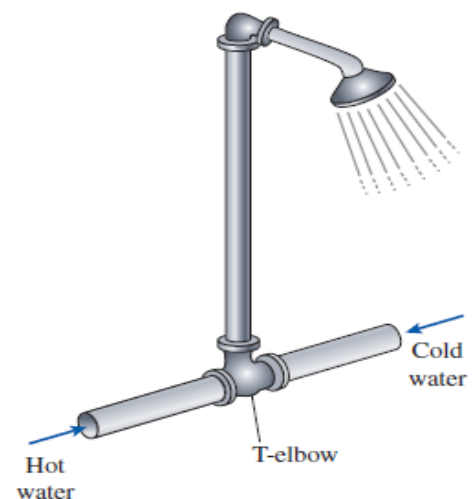
$$\text{Internal energy} + \text{Flow energy} = \text{Constant}$$

4a Mixing Chambers

The section where the mixing process takes place is commonly referred to as a **mixing chamber**. The conservation of mass principle for a mixing chamber requires that the sum of the incoming mass flow rates equal the mass flow rate of the outgoing mixture.

- Mixing chambers are usually well insulated ($q = 0$).
- do not involve any kind of work ($w = 0$).
- Also, the kinetic and potential energies of the fluid streams are usually negligible ($ke = 0$, $pe = 0$).

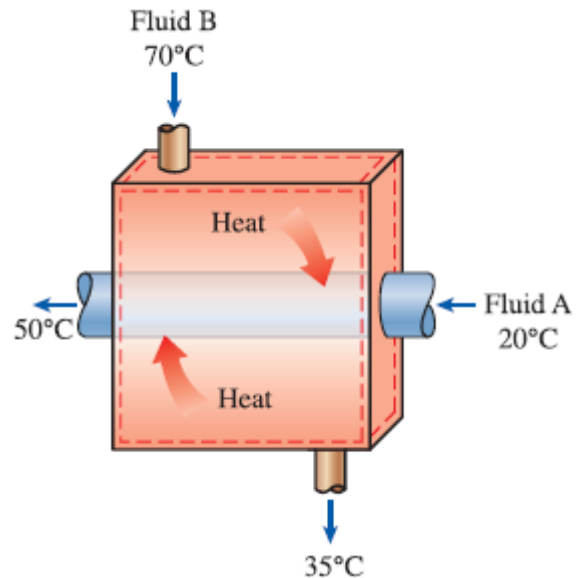
Then all there is left in the energy equation is the total energies of the incoming streams and the outgoing mixture.



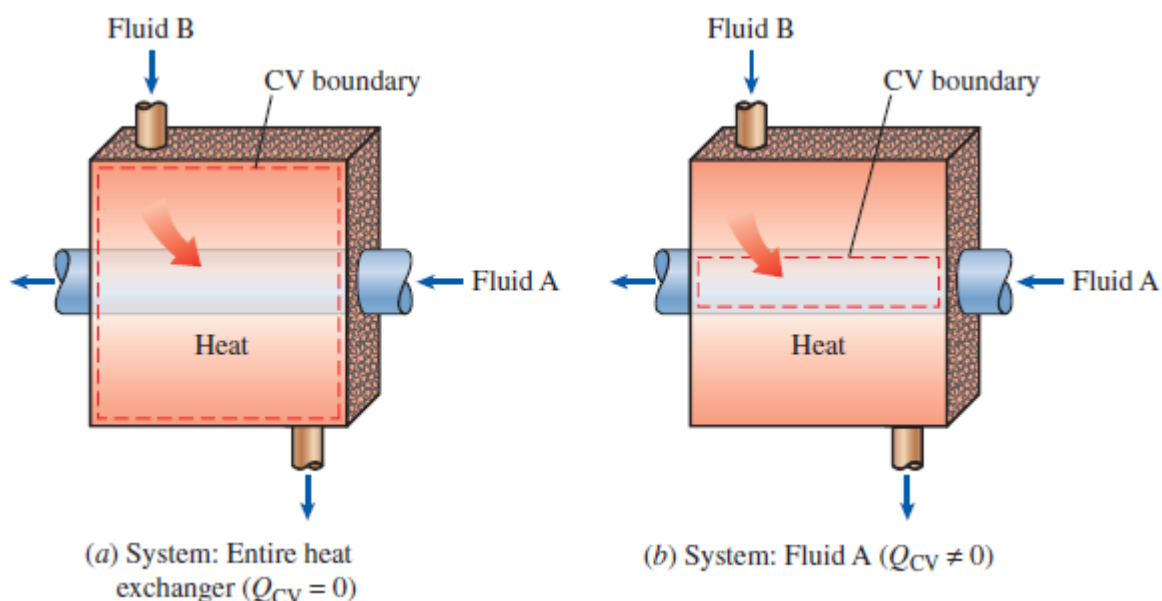
4b Heat Exchangers

Heat exchangers are devices where two moving fluid streams exchange heat without mixing. The simplest form of a heat exchanger is a *double-tube* (also called *tube-and-shell*) *heat exchanger*.

It is composed of two concentric pipes of different diameters. One fluid flows in the inner pipe, and the other in the annular space between the two pipes. Heat is transferred from the hot fluid to the cold one through the wall separating them.



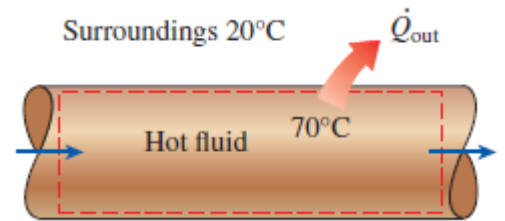
- Heat exchangers typically involve no work interactions ($w = 0$).
- Kinetic and potential energy changes are negligible ($\Delta ke = 0$, $\Delta pe = 0$) for each fluid stream.
- The heat transfer rate associated with heat exchangers depends on how the control volume is selected.
- When the entire heat exchanger is selected as the control volume, Q^0 becomes zero, since the boundary for this case lies just beneath the insulation and little or no heat crosses the boundary



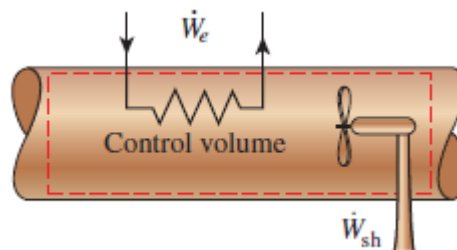
If, however, only one of the fluids is selected as the control volume, then heat will cross this boundary as it flows from one fluid to the other and Q^o will not be zero.

5 Pipe and Duct Flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. The control volume can be selected to coincide with the interior surfaces of the portion of the pipe or the duct that we are interested in analyzing.



- If the control volume involves a heating section (electric wires), a fan, or a pump (shaft), the work interactions should be considered.



- The velocities involved in pipe and duct flow are relatively low, and the kinetic energy changes are usually insignificant.
- The potential energy term may also be significant when the fluid undergoes a considerable elevation change as it flows in a pipe or duct.

Enthalpy:

In the solution of problems involving systems, certain products or sums of properties occur with regularity. One such combination of properties we define to be **enthalpy (H)** which is the sum of internal energy and flow work.

$$H = U + PV$$

The specific enthalpy h is found by dividing by the mass: $h = H/m$.

$$h = u + Pv$$

Example: Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m². The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (a) the mass flow rate of the air and (b) the enthalpy of the air leaving the diffuser. Take the enthalpy at inlet ($h_1 = 283.14$ kJ/kg).

SOLUTION:

1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$.

2 Air is an ideal gas.

3 The potential energy change is zero, $\Delta pe = 0$.

4 Heat transfer is negligible.

5 Kinetic energy at the diffuser exit is negligible.

6 There are no work interactions.

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(283 \text{ K})}{80 \text{ kPa}} = 1.015 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_1} V_1 A_1 = \frac{1}{1.015 \text{ m}^3/\text{kg}} (200 \text{ m/s})(0.4 \text{ m}^2) = 78.8 \text{ kg/s}$$

Since the flow is steady, the mass flow rate through the entire diffuser remains constant at this value.

$$\begin{aligned} \dot{E}_{\text{in}} - \dot{E}_{\text{out}} &= \frac{dE_{\text{system}}}{dt} \xrightarrow{0 \text{ (steady)}} = 0 \\ \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \end{aligned}$$

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ and } \Delta pe \cong 0)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

$$\begin{aligned} h_2 &= 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= 303.14 \text{ kJ/kg} \end{aligned}$$

EXAMPLE

Steam at 250 psia and 700°F steadily enters a nozzle whose inlet area is 0.2 ft². The mass flow rate of steam through the nozzle is 10 lbm/s. Steam leaves the nozzle at 200 psia with a velocity of 900 ft/s. Heat losses from the nozzle per unit mass of the steam are estimated to be 1.2 Btu/lbm. Determine (a) the inlet velocity and (b) the exit enthalpy of the steam.

The specific volume and enthalpy of steam at the nozzle inlet are

$$v_1 = 2.6883 \text{ ft}^3/\text{lbm}$$

$$h_1 = 1371.4 \text{ Btu/lbm}$$

SOLUTION:

1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$.

2 There are no work interactions.

3 The potential energy change is zero, $\Delta pe = 0$.

We observe that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

$$\dot{m} = \frac{1}{v_1} V_1 A_1$$

$$10 \text{ lbm/s} = \frac{1}{2.6883 \text{ ft}^3/\text{lbm}} (V_1)(0.2 \text{ ft}^2)$$

$$V_1 = 134.4 \text{ ft/s}$$

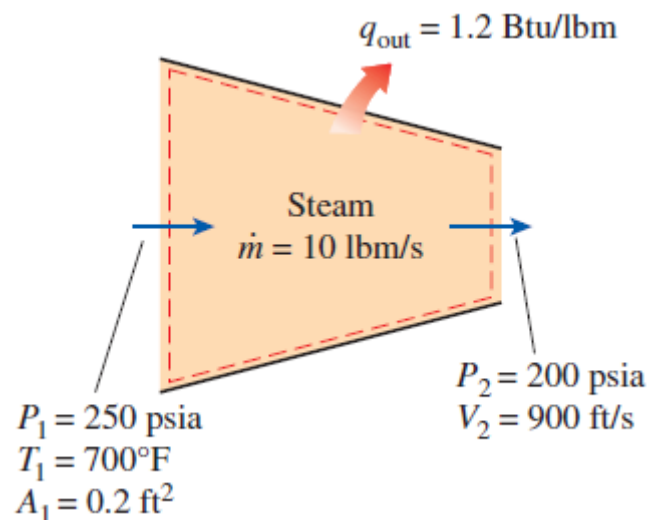
$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\dot{E}_{in} = \dot{E}_{out}} = \underbrace{\frac{dE_{system}}{dt}}_{0 \text{ (steady)}} = 0$$

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{Q}_{out} + \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) \quad (\text{since } \dot{W} = 0, \text{ and } \Delta pe \cong 0)$$

$$h_2 = h_1 - q_{out} - \frac{V_2^2 - V_1^2}{2}$$

$$= (1371.4 - 1.2) \text{ Btu/lbm} - \frac{(900 \text{ ft/s})^2 - (134.4 \text{ ft/s})^2}{2} \left(\frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/\text{s}^2} \right)$$

$$= 1354.4 \text{ Btu/lbm}$$



EXAMPLE

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

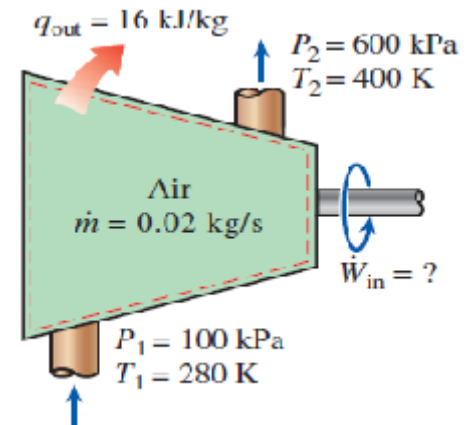
The enthalpy of air $h_1 = 280.13 \text{ kJ/kg}$ $h_2 = 400.98 \text{ kJ/kg}$

SOLUTION:

1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$.

2 Air is an ideal gas.

3 The kinetic and potential energy changes are zero, $\Delta ke = \Delta pe = 0$.



$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke = \Delta pe \cong 0)$$

$$\dot{W}_{in} = \dot{m}q_{out} + \dot{m}(h_2 - h_1)$$

$$\begin{aligned} \dot{W}_{in} &= (0.02 \text{ kg/s})(16 \text{ kJ/kg}) + (0.02 \text{ kg/s})(400.98 - 280.13) \text{ kJ/kg} \\ &= \mathbf{2.74 \text{ kW}} \end{aligned}$$

EXAMPLE

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are as indicated in Fig.

- Compare the magnitudes of Δh , Δke , and Δpe .
- Determine the work done per unit mass of the steam flowing through the turbine.
- Calculate the mass flow rate of the steam.

At the inlet, $h_1 = 3248.4 \text{ kJ/kg}$ At the turbine exit, $h_2 = 2361.01 \text{ kJ/kg}$

SOLUTION

1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$.

2 The system is adiabatic and thus there is no heat transfer.

$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4) \text{ kJ/kg} \\ = -887.39 \text{ kJ/kg}$$

$$\Delta ke = \frac{V_2^2 - V_1^2}{2} \\ = \frac{(180 \text{ m/s})^2 - (50 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ = 14.95 \text{ kJ/kg}$$

$$\Delta pe = g(z_2 - z_1) \\ = (9.81 \text{ m/s}^2)[(6 - 10) \text{ m}] \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ = -0.04 \text{ kJ/kg}$$

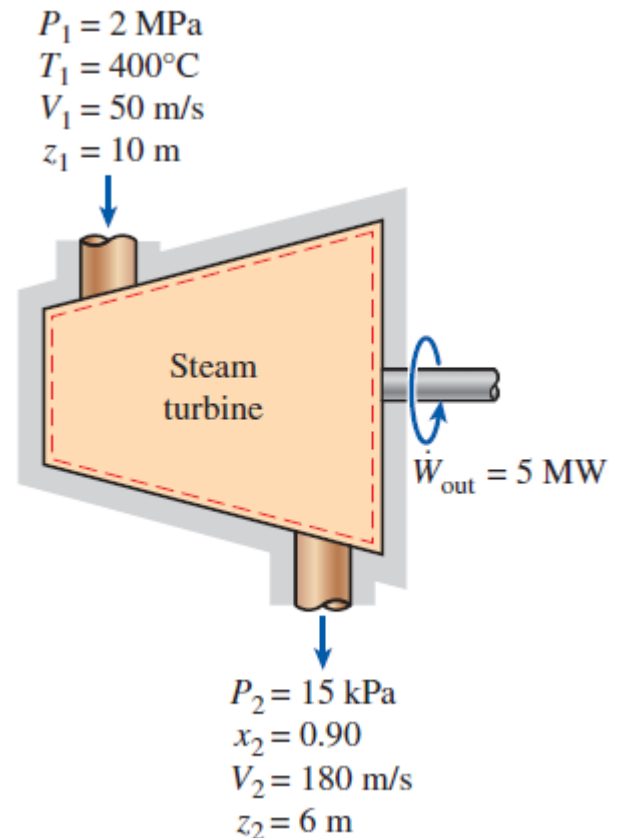
$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{system}}}{dt} \xrightarrow{0 \text{ (steady)}} = 0 \\ \dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m} \left(h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{W}_{\text{out}} + \dot{m} \left(h_2 + \frac{V_2^2}{2} + gz_2 \right) \quad (\text{since } \dot{Q} = 0)$$

$$w_{\text{out}} = - \left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = -(\Delta h + \Delta ke + \Delta pe) \\ = -[-887.39 + 14.95 - 0.04] \text{ kJ/kg} = 872.48 \text{ kJ/kg}$$

(c) The required mass flow rate for a 5-MW power output is

$$\dot{m} = \frac{\dot{W}_{\text{out}}}{w_{\text{out}}} = \frac{5000 \text{ kJ/s}}{872.48 \text{ kJ/kg}} = 5.73 \text{ kg/s}$$



EXAMPLE

Consider an ordinary shower where hot water at 140°F is mixed with cold water at 50°F. If it is desired that a steady stream of warm water at 110°F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia. $h_1 = 107.99$ Btu/lbm

$h_2 = 18.07$ Btu/lbm $h_3 = 78.02$ Btu/lbm

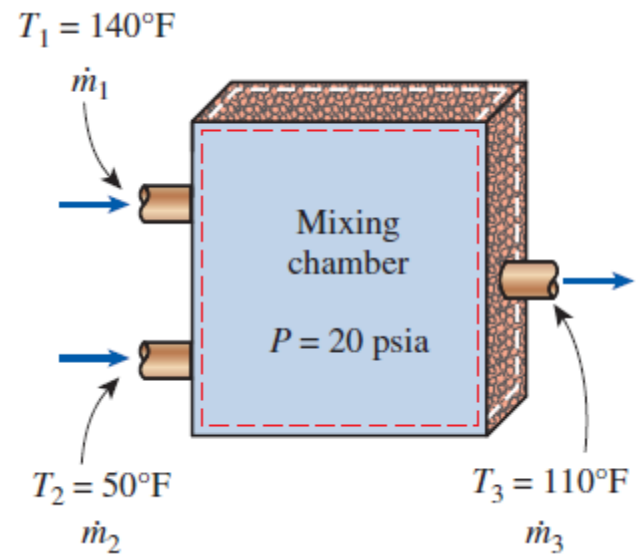
SOLUTION:

1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$.

2 The kinetic and potential energies are negligible, $ke = pe = 0$.

3 Heat losses from the system are negligible and thus $Q = 0$.

4 There is no work interaction involved.



$$\text{Mass balance: } \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{system}}}{dt} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\text{Energy balance: } \dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{system}}}{dt} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, ke \cong pe \cong 0)$$

Combining the mass and energy balances,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

Dividing this equation by $\dot{m}_2$ yields

$$y h_1 + h_2 = (y + 1) h_3$$

where $y = \dot{m}_1 / \dot{m}_2$ is the desired mass flow rate ratio.

$$y = \frac{h_3 - h_2}{h_1 - h_3} = \frac{78.02 - 18.07}{107.99 - 78.02} = \mathbf{2.0}$$

EXAMPLE

Refrigerant-134a is to be cooled by water in a condenser. The refrigerant enters the condenser with a mass flow rate of 6 kg/min at 1 MPa and 70°C and leaves at 35°C. The cooling water enters at 300 kPa and 15°C and leaves at 25°C. Neglecting any pressure drops, determine (a) the mass flow rate of the cooling water required and (b) the heat transfer rate from the refrigerant to water.

$$h_1 = 62.982 \text{ kJ/kg} \quad h_2 = 104.83 \text{ kJ/kg} \quad h_3 = 303.87 \text{ kJ/kg} \quad h_4 = 100.88 \text{ kJ/kg}$$

SOLUTION:

1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$.

2 The kinetic and potential energies are negligible, $ke = pe = 0$.

3 Heat losses from the system are negligible and thus $Q = 0$.

4 There is no work interaction involved.

Mass balance:

$$\dot{m}_{in} = \dot{m}_{out}$$

for each fluid stream since there is no mixing. Thus

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

Energy balance:

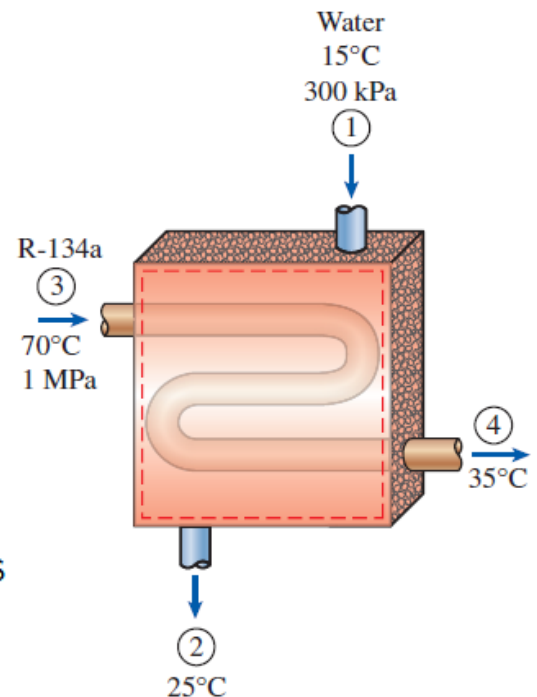
$$\begin{aligned} \dot{E}_{in} - \dot{E}_{out} &= \frac{dE_{system}}{dt} \rightarrow 0 \text{ (steady)} = 0 \\ \dot{E}_{in} &= \dot{E}_{out} \end{aligned}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, ke \cong pe \cong 0)$$

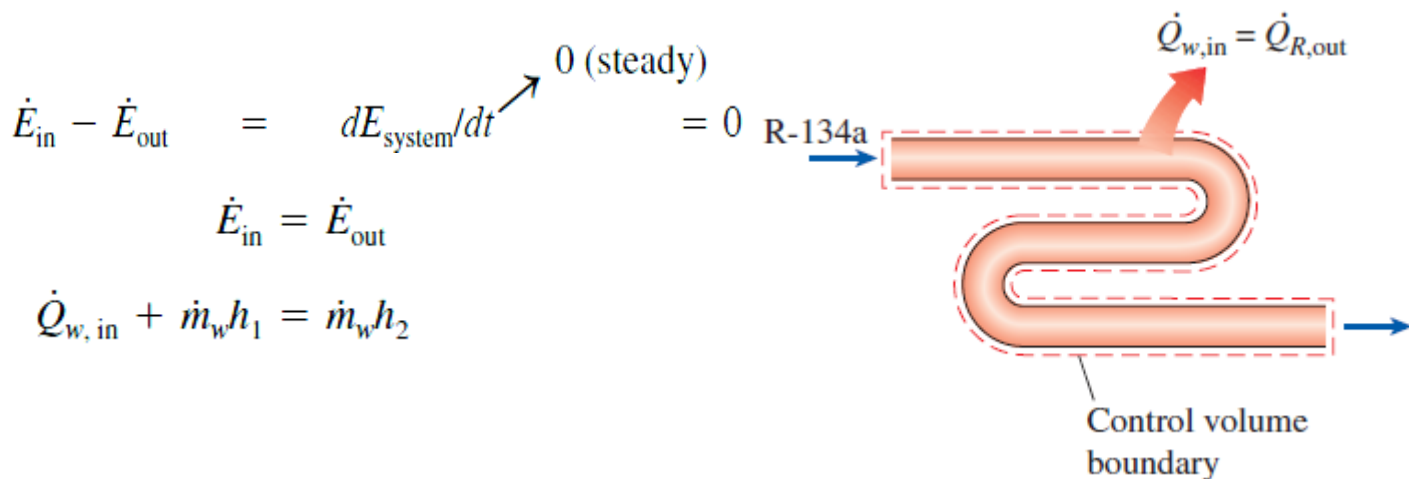
$$\dot{m}_w (h_1 - h_2) = \dot{m}_R (h_4 - h_3)$$

$$\dot{m}_w (62.982 - 104.83) \text{ kJ/kg} = (6 \text{ kg/min}) [(100.88 - 303.87) \text{ kJ/kg}]$$

$$\dot{m}_w = \mathbf{29.1 \text{ kg/min}}$$



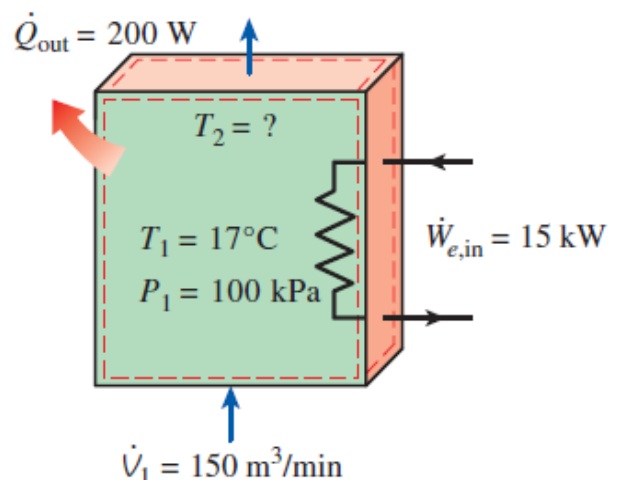
(b) To determine the heat transfer from the refrigerant to the water, we have to choose a control volume whose boundary lies on the path of heat transfer. All the assumptions stated earlier apply, except that the heat transfer is no longer zero.

**EXAMPLE**

The electric heating systems used in many houses consist of a simple duct with resistance heaters. Air is heated as it flows over resistance wires. Consider a 15-kW electric heating system. Air enters the heating section at 100 kPa and 17°C with a volume flow rate of 150 m³/min. If heat is lost from the air in the duct to the surroundings at a rate of 200 W, determine the exit temperature of air.

SOLUTION:

- 1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$.
- 2 Air is an ideal gas.
- 3 The kinetic and potential energy changes are negligible, $\Delta ke = \Delta pe = 0$.
- 4 Constant specific heats at room temperature can be used for air.



$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{system}}}{dt} \xrightarrow{0 \text{ (steady)}} = 0$$

At temperatures encountered in heating and air-conditioning applications, Δh can be replaced by $c_p \Delta T$ where $c_p = 1.005 \text{ kJ/kg}\cdot^\circ\text{C}$ –

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{e,\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{e,\text{in}} - \dot{Q}_{\text{out}} = \dot{m}c_p(T_2 - T_1)$$

From the ideal-gas relation, the specific volume of air at the inlet of the duct is

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290 \text{ K})}{100 \text{ kPa}} = 0.832 \text{ m}^3/\text{kg}$$

The mass flow rate of the air through the duct is determined from

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{150 \text{ m}^3/\text{min}}{0.832 \text{ m}^3/\text{kg}} \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 3.0 \text{ kg/s}$$

$$(15 \text{ kJ/s}) - (0.2 \text{ kJ/s}) = (3 \text{ kg/s})(1.005 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 17)^\circ\text{C}$$

$$T_2 = \mathbf{21.9^\circ\text{C}}$$

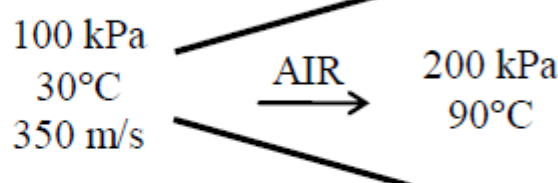
5–28 The diffuser in a jet engine is designed to decrease the kinetic energy of the air entering the engine compressor without any work or heat interactions. Calculate the velocity at the exit of a diffuser when air at 100 kPa and 30°C enters it with a velocity of 350 m/s and the exit state is 200 kPa and 90°C.

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} \stackrel{\text{no (steady)}}{=} 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_2 + V_2^2/2)$$

$$h_1 + V_1^2/2 = h_2 + V_2^2/2$$



Solving for exit velocity,

$$V_2 = \left[V_1^2 + 2(h_1 - h_2) \right]^{0.5} = \left[V_1^2 + 2c_p(T_1 - T_2) \right]^{0.5}$$

$$= \left[(350 \text{ m/s})^2 + 2(1.007 \text{ kJ/kg} \cdot \text{K})(30 - 90) \text{ K} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right) \right]^{0.5}$$

$$= \mathbf{40.7 \text{ m/s}}$$

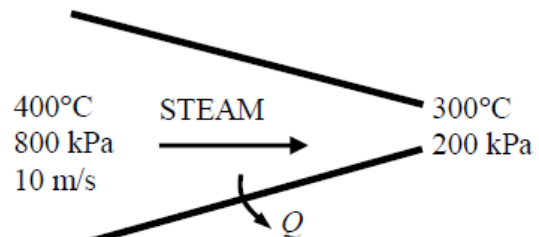
5-30 Steam enters a nozzle at 400°C and 800 kPa with a velocity of 10 m/s, and leaves at 300°C and 200 kPa while losing heat at a rate of 25 kW. For an inlet area of 800 cm², determine the velocity and the volume flow rate of the steam at the nozzle exit.

$$\begin{aligned} \nu_1 &= 0.38429 \text{ m}^3/\text{kg} & \nu_2 &= 1.31623 \text{ m}^3/\text{kg} \\ h_1 &= 3267.7 \text{ kJ/kg} & h_2 &= 3072.1 \text{ kJ/kg} \end{aligned}$$

Energy balance:

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} \stackrel{\text{no (steady)}}{=} 0$$

$$\begin{aligned} \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ \dot{m} \left(h_1 + \frac{V_1^2}{2} \right) &= \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) + \dot{Q}_{\text{out}} \quad \text{since } \dot{W} \cong \Delta p e \cong 0 \end{aligned}$$



or

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + \frac{\dot{Q}_{\text{out}}}{\dot{m}}$$

The mass flow rate of the steam is

$$\dot{m} = \frac{1}{\nu_1} A_1 V_1 = \frac{1}{0.38429 \text{ m}^3/\text{s}} (0.08 \text{ m}^2)(10 \text{ m/s}) = 2.082 \text{ kg/s}$$

Substituting,

$$3267.7 \text{ kJ/kg} + \frac{(10 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 3072.1 \text{ kJ/kg} + \frac{V_2^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) + \frac{25 \text{ kJ/s}}{2.082 \text{ kg/s}}$$

$$\longrightarrow V_2 = \mathbf{606 \text{ m/s}}$$

The volume flow rate at the exit of the nozzle is

$$\dot{V}_2 = \dot{m} \nu_2 = (2.082 \text{ kg/s})(1.31623 \text{ m}^3/\text{kg}) = \mathbf{2.74 \text{ m}^3/\text{s}}$$

5–44 Refrigerant-134a enters an adiabatic compressor as saturated vapor at -24°C and leaves at 0.8 MPa and 60°C . The mass flow rate of the refrigerant is 1.2 kg/s. Determine (a) the power input to the compressor and (b) the volume flow rate of the refrigerant at the compressor inlet.

$$\nu_1 = 0.17398 \text{ m}^3/\text{kg} \quad h_1 = 235.94 \text{ kJ/kg} \quad h_2 = 296.82 \text{ kJ/kg}$$

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}}^{\text{no (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \dot{Q} \cong \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$

Substituting,

$$\dot{W}_{\text{in}} = (1.2 \text{ kg/s})(296.82 - 235.94) \text{ kJ/kg} = \mathbf{73.06 \text{ kJ/s}}$$

(b) The volume flow rate of the refrigerant at the compressor inlet is

$$\dot{V}_1 = \dot{m}\nu_1 = (1.2 \text{ kg/s})(0.17398 \text{ m}^3/\text{kg}) = \mathbf{0.209 \text{ m}^3/\text{s}}$$

5–46 Steam flows steadily through an adiabatic turbine. The inlet conditions of the steam are 4 MPa, 500°C , and 80 m/s, and the exit conditions are 30 kPa, 92 percent quality, and 50 m/s. The mass flow rate of the steam is 12 kg/s. Determine (a) the change in kinetic energy, (b) the power output, and (c) the turbine inlet area.

$$\nu_1 = 0.086442 \text{ m}^3/\text{kg} \quad h_1 = 3446.0 \text{ kJ/kg} \quad h_2 = 2437.7 \text{ kJ/kg}$$

$$\Delta ke = \frac{V_2^2 - V_1^2}{2}$$

$$= \frac{(50 \text{ m/s})^2 - (80 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)$$

$$= \mathbf{-1.95 \text{ kJ/kg}}$$

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}}^{\text{no (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{W}_{\text{out}} + \dot{m}(h_2 + V_2^2/2) \quad (\text{since } \dot{Q} \equiv \Delta p_e \equiv 0)$$

$$\dot{W}_{\text{out}} = -\dot{m} \left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \right)$$

Then the power output of the turbine is determined by substitution to be

$$\dot{W}_{\text{out}} = -(12 \text{ kg/s})(2437.7 - 3446.0 - 1.95) \text{ kJ/kg} = 12,123 \text{ kW} = \mathbf{12.1 \text{ MW}}$$

(c) The inlet area of the turbine is determined from the mass flow rate relation,

$$\dot{m} = \frac{1}{v_1} A_1 V_1 \longrightarrow A_1 = \frac{\dot{m} v_1}{V_1} = \frac{(12 \text{ kg/s})(0.086442 \text{ m}^3/\text{kg})}{80 \text{ m/s}} = \mathbf{0.0130 \text{ m}^2}$$

5-71 Liquid water at 300 kPa and 20°C is heated in a chamber by mixing it with superheated steam at 300 kPa and 300°C. Cold water enters the chamber at a rate of 1.8 kg/s. If the mixture leaves the mixing chamber at 60°C, determine the mass flow rate of the superheated steam required.

$$h_1 = 83.91 \text{ kJ/kg} \quad h_2 = 3069.6 \text{ kJ/kg} \quad h_3 = 251.18 \text{ kJ/kg}$$

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \Delta \dot{m}_{\text{system}}^{\text{no (steady)}} = 0$$

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}}^{\text{no (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

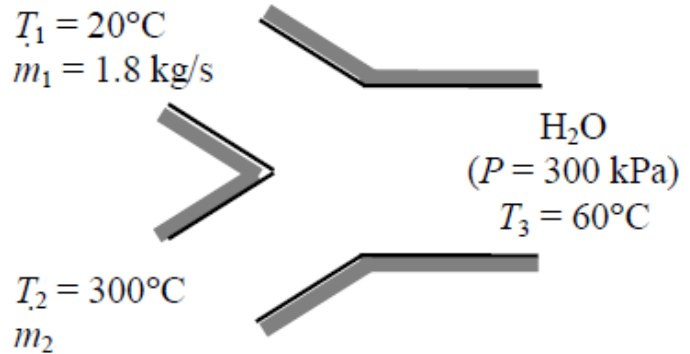
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} = \dot{W} = \Delta \text{ke} \equiv \Delta \text{pe} \equiv 0)$$

Combining the two,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$\dot{m}_2 = \frac{h_1 - h_3}{h_3 - h_2} \dot{m}_1$$

$$\dot{m}_2 = \frac{(83.91 - 251.18) \text{ kJ/kg}}{(251.18 - 3069.6) \text{ kJ/kg}} (1.8 \text{ kg/s}) = 0.107 \text{ kg/s}$$



Example

e: Water is flowing in a pipe that changes diameter from 20 to 40 mm. If the water in the 20-mm section has a velocity of 40 m/s, determine the velocity in the 40-mm section. Also calculate the mass flux.

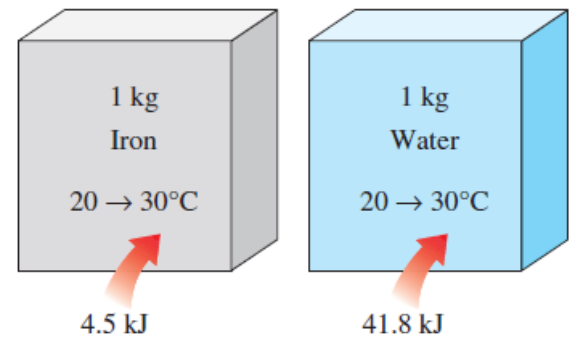
$$A_1 V_1 = A_2 V_2 \quad \left[\frac{\pi (0.02)^2}{4} \right] (40) = \frac{\pi (0.04)^2}{4} V_2 \quad \therefore V_2 = 10 \text{ m/s}$$

$$\dot{m} = \rho A_1 V_1 = (1000) \left(\frac{\pi (0.02)^2}{4} \right) (40) = 12.57 \text{ kg/s}$$

Specific Heats

We know from experience that it takes different amounts of energy to raise the temperature of identical masses of different substances by one degree.

For example, we need about 4.5 kJ of energy to raise the temperature of 1 kg of iron from 20 to 30°C, whereas it takes about 9 times this energy (41.8 kJ to be exact) to raise the temperature of 1 kg of liquid water by the same amount. Therefore, it is desirable to have a property that will enable us to compare the energy storage capabilities of various substances.



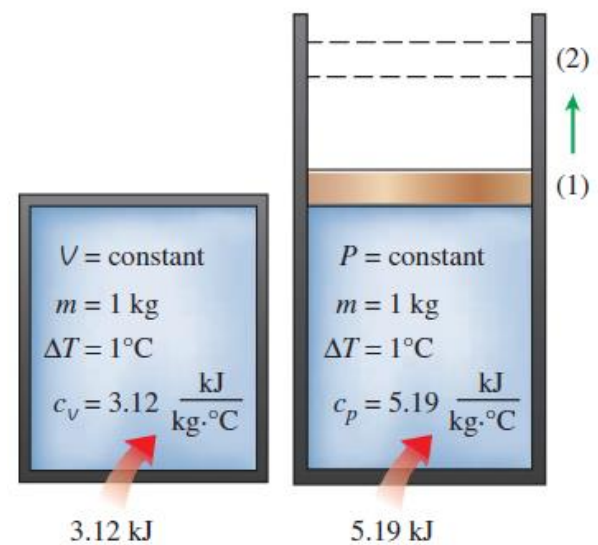
This property is the specific heat. The **specific heat** is defined as *the energy required to raise the temperature of a unit mass of a substance by one degree*.

In thermodynamics, we are interested in two kinds of specific heats: **specific heat at constant volume** c_v and **specific heat at constant pressure** c_p .

Physically, the specific heat at constant volume c_v can be viewed as *the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant*. The energy required to do

the same as the pressure is maintained constant is the specific heat at constant pressure c_p .

The specific heat at constant pressure c_p is always greater than c_v because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system.



$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \quad c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

= the change in enthalpy with temperature at constant pressure

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

= the change in internal energy with temperature at constant volume

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES:

We defined an ideal gas as a gas whose temperature, pressure, and specific volume are related by:

$$Pv = RT$$

Using the definition of enthalpy and the equation of state of an ideal gas, we have:

$$\left. \begin{array}{l} h = u + Pv \\ Pv = RT \end{array} \right\} h = u + RT$$

the internal energy and enthalpy of an ideal gas can be expressed as:

$$du = c_v dT \qquad u_2 - u_1 = c_{v,avg}(T_2 - T_1) \quad (\text{kJ/kg})$$

$$dh = c_p dT \qquad h_2 - h_1 = c_{p,avg}(T_2 - T_1) \quad (\text{kJ/kg})$$

Specific Heat Relations of Ideal Gases:

A special relationship between c_p and c_v for ideal gases can be obtained by differentiating the relation $h = u + RT$, which yields:

$$dh = du + R dT$$

Replacing dh by $c_p dT$ and du by $c_v dT$ and dividing the resulting expression by dT , we obtain

$$c_p = c_v + R \quad (\text{kJ/kg}\cdot\text{K})$$

At this point, we introduce another ideal-gas property called the **specific heat ratio** k , defined as:

$$k = \frac{c_p}{c_v}$$

$$C_p = R \frac{k}{k-1} \quad \text{and} \quad C_v = R \frac{1}{k-1}$$

EXAMPLE

Determine the enthalpy change for 1 kg of nitrogen which is heated from 300 to 1200 K by assuming constant specific heat.

Assuming constant specific heat (1.042 kJ/kg · °C) the enthalpy change is found to be

$$\Delta h = C_p \Delta T = 1.042 \times (1200 - 300) = 938 \text{ kJ/kg}$$

The First Law Applied to Various Processes:

- 1) THE CONSTANT-TEMPERATURE PROCESS
- 2) THE CONSTANT-VOLUME PROCESS
- 3) THE CONSTANT-PRESSURE PROCESS
- 4) THE ADIABATIC PROCESS
- 5) THE POLYTROPIC PROCESS

THE CONSTANT-TEMPERATURE PROCESS

For the isothermal process a gas that approximates an ideal gas, the internal energy depends only on the temperature and thus $\Delta U = 0$ for an isothermal process; for such a process the energy equation is:

$$Q = W$$

Using the ideal-gas equation $PV = mRT$, the work can be found to be:

$$\begin{aligned} W &= \int_{V_1}^{V_2} P dV \\ &= mRT \int_{V_1}^{V_2} \frac{dV}{V} = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2} \end{aligned}$$

where we used $P_1 V_1 = P_2 V_2$.

THE CONSTANT-VOLUME PROCESS

The work for a constant-volume process is zero, since dV is zero. For such a process the first law becomes:

$$Q = \Delta U$$

For a gas, approximated by an ideal gas with constant C_v , we would have:

$$Q = mC_v(T_2 - T_1) \quad \text{or} \quad q = C_v(T_2 - T_1)$$

THE CONSTANT-PRESSURE PROCESS

The first law, for a constant-pressure process is to be:

$$Q = \Delta H$$

For a process involving an ideal gas for which C_p is constant there results:

$$Q = mC_p(T_2 - T_1)$$

THE ADIABATIC PROCESS

the first law for the adiabatic process is:

$$-\Delta W = \Delta u$$

if an ideal gas can be assumed with constant C_v :

$$\frac{C_v}{R} \ln \frac{T_2}{T_1} = - \ln \frac{v_2}{v_1}$$

which can be put in the forms:

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{R/C_v} = \left(\frac{v_1}{v_2} \right)^{k-1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \quad \text{and} \quad \frac{P_2}{P_1} = \left(\frac{v_1}{v_2} \right)^k$$

EXAMPLE

A piston-cylinder arrangement contains 0.02 m³ of air at 50°C and 400 kPa. Heat is added in the amount of 50 kJ and work is done by a paddle wheel until the temperature reaches 700°C. If the pressure is held constant, how much paddle-wheel work must be added to the air? Assume constant specific heats.

Solution

$$Q - W = m(u_2 - u_1) \quad \text{or} \quad Q - W_{\text{paddle}} = m(h_2 - h_1) \\ = mC_p(T_2 - T_1)$$

To find m we use the ideal-gas equation. It gives us

$$m = \frac{PV}{RT} = \frac{400\,000 \times 0.02}{287 \times (50 + 273)} = 0.0863 \text{ kg}$$

From the first law the paddle-wheel work is found to be

$$W_{\text{paddle}} = Q - mC_p(T_2 - T_1) \\ = 50 - 0.0863 \times 1.0 \times (700 - 50) = -6.096 \text{ kJ}$$

EXAMPLE

Calculate the work necessary to compress air in an insulated cylinder from a volume of 2 m³ to a volume of 0.2 m³. The initial temperature and pressure are 20°C and 200 kPa, respectively.

Solution

process is adiabatic due to the presence of the insulation (we usually assume an adiabatic process anyhow since heat transfer is assumed to be negligible). The first law is then written as

$$-W = m(u_2 - u_1) = mC_v(T_2 - T_1)$$

The mass is found from the ideal-gas equation to

$$m = \frac{PV}{RT} = \frac{200 \times 2}{0.287(20 + 273)} = 4.757 \text{ kg}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1} = 293 \left(\frac{2}{0.2} \right)^{1.4-1} = 736 \text{ K}$$

Finally,

$$W = -4.757 \text{ kg} \left(0.717 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (736 - 293) \text{ K} = -1510 \text{ kJ}$$

EXAMPLE

Air flows through the supersonic nozzle shown. The inlet conditions are 7 kPa and 420°C. The nozzle exit diameter is adjusted such that the exiting velocity is 700 m/s. Calculate (a) the exit temperature, (b) the mass flow rate, and (c) the exit diameter. Assume an adiabatic quasiequilibrium flow.

Solution

- (a) To find the exit temperature the energy equation is used. It is, using $\Delta h = C_p \Delta T$,

$$\frac{V_1^2}{2} + C_p T_1 = \frac{V_2^2}{2} + C_p T_2$$

We then have, using $C_p = 1000 \text{ J/kg} \cdot \text{K}$,

$$T_2 = \frac{V_1^2 - V_2^2}{2C_p} + T_1 = \frac{400^2 - 700^2}{2 \times 1000} + 420 = 255^\circ\text{C}$$

Note: If T_1 is expressed in kelvins, then the answer would be in kelvins.

- (b) To find the mass flow rate we must find the density at the entrance. From the inlet conditions we have

$$\rho_1 = \frac{P_1}{RT_1} = \frac{7000}{287 \times 693} = 0.0352 \text{ kg/m}^3$$

The mass flow rate is then

$$\dot{m} = \rho_1 A_1 V_1 = 0.0352 \times \pi \times 0.1^2 \times 400 = 0.442 \text{ kg/s}$$

- (c) To find the exit diameter we would use $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$, the continuity equation. This requires the density at the exit. It is found by assuming adiabatic quasiequilibrium flow. with $\rho = 1/v$, we have

$$\rho_2 = \rho_1 \left(\frac{T_2}{T_1} \right)^{1/(k-1)} = 0.0352 \left(\frac{528}{693} \right)^{1/(1.4-1)} = 0.01784 \text{ kg/m}^3$$

Hence,

$$d_2^2 = \frac{\rho_1 d_1^2}{\rho_2 V_2} = \frac{0.0352 \times .2^2 \times 400}{0.01784 \times 700} = .0451. \quad \therefore d_2 = 0.212 \text{ m} \quad \text{or} \quad 212 \text{ mm}$$

EXAMPLE

Liquid sodium, flowing at 100 kg/s, enters a heat exchanger at 450°C and exits at 350°C. The C_p of sodium is 1.25 kJ/kg · °C. Liquid water enters at 5000 kPa and 20°C. Determine the minimum mass flow rate of the water so that the water just vaporizes. Also, calculate the rate of heat transfer.

Solution

The energy equation is used as

$$\dot{m}_s (h_{s1} - h_{s2}) = \dot{m}_w (h_{w2} - h_{w1}) \quad \text{or} \quad \dot{m}_s C_p (T_{s1} - T_{s2}) = \dot{m}_w (h_{w2} - h_{w1})$$

$$h_{w1} = 88.65 \text{ kJ/kg} \quad h_{w2} = 2794 \text{ kJ/kg}$$

$$100 \times 1.25 \times (450 - 350) = \dot{m}_w (2794 - 88.65). \quad \therefore \dot{m}_w = 4.62 \text{ kg/s}$$

The heat transfer is found using the energy equation applied to one of the separate control volumes:

$$\dot{Q} = \dot{m}_w (h_{w2} - h_{w1}) = 4.62 \times (2794 - 88.65) = 12\,500 \text{ kW} \quad \text{or} \quad 12.5 \text{ MW}$$

EXAMPLE

An insulated rigid tank initially contains 1.5 lbm of helium at 80°F and 50 psia. A paddle wheel with a power rating of 0.02 hp is operated within the tank for 30 min. Determine (a) the final temperature and (b) the final pressure of the helium gas.

$$c_v = 0.753 \text{ Btu/lbm} \cdot ^\circ\text{F}.$$

(a) The amount of paddle-wheel work done on the system is

$$W_{\text{sh}} = \dot{W}_{\text{sh}} \Delta t = (0.02 \text{ hp})(0.5 \text{ h}) \left(\frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right) = 25.45 \text{ Btu}$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$W_{\text{sh,in}} = \Delta U = m(u_2 - u_1) = mc_{v,\text{avg}}(T_2 - T_1)$$

$$25.45 \text{ Btu} = (1.5 \text{ lbm})(0.753 \text{ Btu/lbm} \cdot ^\circ\text{F})(T_2 - 80^\circ\text{F})$$

$$T_2 = \mathbf{102.5^\circ\text{F}}$$

(b) The final pressure is determined from the ideal-gas relation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

where V_1 and V_2 are identical and cancel out. Then the final pressure becomes

$$\frac{50 \text{ psia}}{(80 + 460) \text{ R}} = \frac{P_2}{(102.5 + 460) \text{ R}}$$

$$P_2 = \mathbf{52.1 \text{ psia}}$$

EXAMPLE

A piston–cylinder device initially contains 0.5 m^3 of nitrogen gas at 400 kPa and 27°C . An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120-V source. Nitrogen expands at constant pressure, and a heat loss of 2800 J occurs during the process. Determine the final temperature of nitrogen.

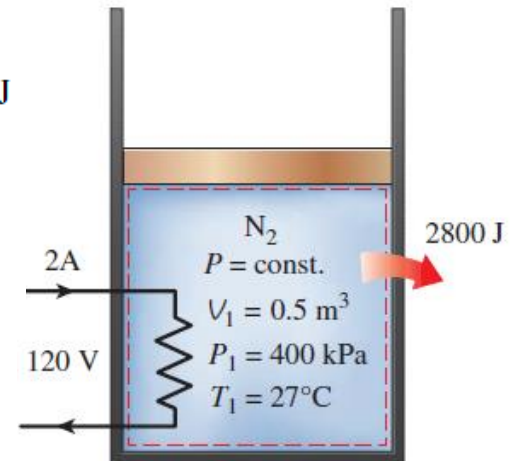
$c_p = 1.039 \text{ kJ/kg}\cdot\text{K}$ for nitrogen at room temperature.

$$W_e = VI \Delta t = (120 \text{ V})(2 \text{ A})(5 \times 60 \text{ s}) \left(\frac{1 \text{ kJ/s}}{1000 \text{ VA}} \right) = 72 \text{ kJ}$$

The mass of nitrogen is determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(400 \text{ kPa})(0.5 \text{ m}^3)}{(0.297 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300 \text{ K})} = 2.245 \text{ kg}$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$



$$W_{e,\text{in}} - Q_{\text{out}} - W_{b,\text{out}} = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) = mc_p(T_2 - T_1)$$

$$72 \text{ kJ} - 2.8 \text{ kJ} = (2.245 \text{ kg})(1.039 \text{ kJ/kg}\cdot\text{K})(T_2 - 27^\circ\text{C})$$

$$T_2 = \mathbf{56.7^\circ\text{C}}$$

EXAMPLE

A piston–cylinder device initially contains air at 150 kPa and 27°C . At this state, the piston is resting on a pair of stops, as shown in Fig. 4–32, and the enclosed volume is 400 L . The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (a) the final temperature, (b) the work done by the air, and (c) the total heat transferred to the air.

(a) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \longrightarrow \frac{(150 \text{ kPa})(V_1)}{300 \text{ K}} = \frac{(350 \text{ kPa})(2V_1)}{T_3}$$

$$T_3 = 1400 \text{ K}$$

(b) The work done could be

$$= (V_2 - V_1)P_2 = (0.4 \text{ m}^3)(350 \text{ kPa}) = 140 \text{ m}^3 \cdot \text{kPa}$$

$$W_{13} = 140 \text{ kJ}$$

(c) the energy balance on the system and observations, system between the initial and final states (process 1–3)

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U = m(u_3 - u_1)$$

The mass of the system can be determined from the ideal-gas relation:

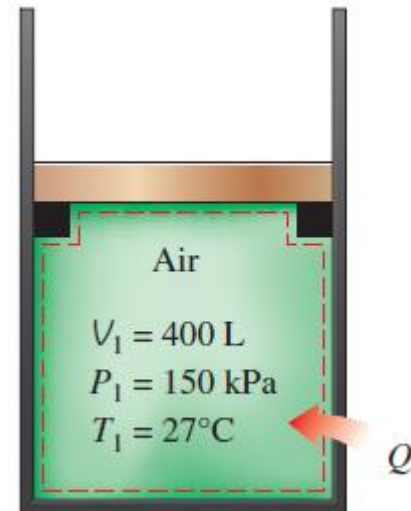
$$m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kPa})(0.4 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 0.697 \text{ kg}$$

$$u_1 = 214.07 \text{ kJ/kg} \quad u_3 = 1113.52 \text{ kJ/kg} \quad (\text{GIVEN})$$

Thus,

$$Q_{\text{in}} - 140 \text{ kJ} = (0.697 \text{ kg})[(1113.52 - 214.07) \text{ kJ/kg}]$$

$$Q_{\text{in}} = 767 \text{ kJ}$$



HOMEWORK

5–29 Air at 600 kPa and 500 K enters an adiabatic nozzle that has an inlet-to-exit area ratio of 2:1 with a velocity of 120 m/s and leaves with a velocity of 380 m/s. Determine (a) the exit temperature and (b) the exit pressure of the air. *Answers: (a) 437 K, (b) 331 kPa*

The enthalpy of air at the inlet temperature of 500 K is $h_1 = 503.02 \text{ kJ/kg}$

5-37 Refrigerant-134a enters a diffuser steadily as saturated vapor at 600 kPa with a velocity of 160 m/s, and it leaves at 700 kPa and 40°C. The refrigerant is gaining heat at a rate of 2 kJ/s as it passes through the diffuser. If the exit area is 80 percent greater than the inlet area, determine (a) the exit velocity and (b) the mass flow rate of the refrigerant.

Answers: (a) 82.1 m/s, (b) 0.298 kg/s

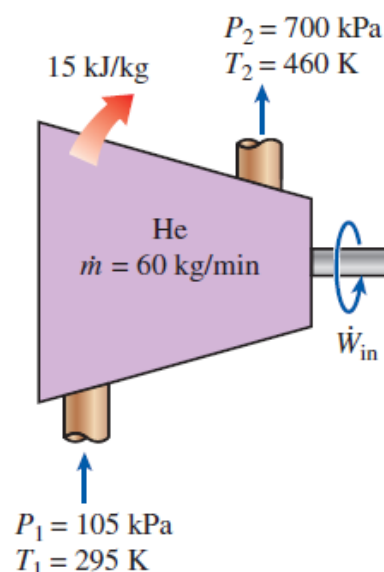
$$\begin{aligned} \nu_1 &= 0.034335 \text{ m}^3/\text{kg} & \nu_2 &= 0.031696 \text{ m}^3/\text{kg} \\ h_1 &= 262.46 \text{ kJ/kg} & h_2 &= 278.59 \text{ kJ/kg} \end{aligned}$$

5-48 Steam enters an adiabatic turbine at 10 MPa and 500°C and leaves at 10 kPa with a quality of 90 percent. Neglecting the changes in kinetic and potential energies, determine the mass flow rate required for a power output of 5 MW. Answer: 4.852 kg/s

$$h_1 = 3375.1 \text{ kJ/kg} \quad h_2 = 2344.7 \text{ kJ/kg}$$

5-50 Helium is to be compressed from 105 kPa and 295 K to 700 kPa and 460 K. A heat loss of 15 kJ/kg occurs during the compression process. Neglecting kinetic energy changes, determine the power input required for a mass flow rate of 60 kg/min.

$$c_p = 5.1926 \text{ kJ/kg} \cdot \text{K}$$

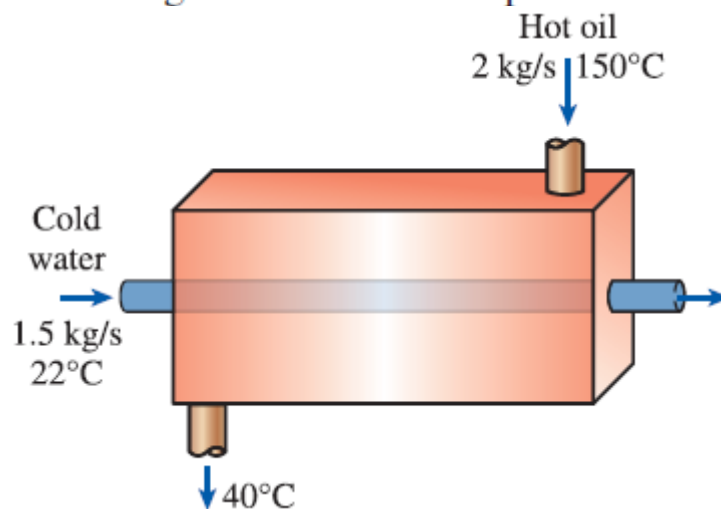


5-71 Liquid water at 300 kPa and 20°C is heated in a chamber by mixing it with superheated steam at 300 kPa and 300°C. Cold water enters the chamber at a rate of 1.8 kg/s. If the mixture leaves the mixing chamber at 60°C, determine the mass flow rate of the superheated steam required. *Answer: 0.107 kg/s*

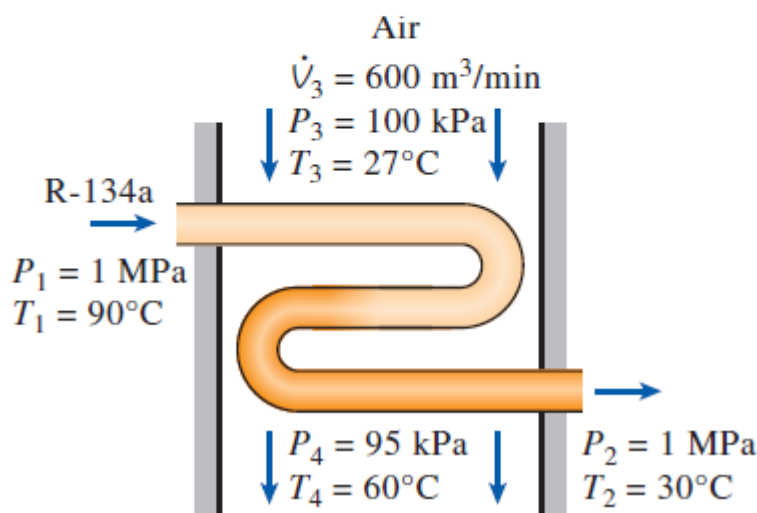
$$h_1 = 83.91 \text{ kJ/kg} \quad h_2 = 3069.6 \text{ kJ/kg} \quad h_3 = 251.18 \text{ kJ/kg}$$

5-76 A heat exchanger is to heat water ($c_p = 4.18 \text{ kJ/kg}\cdot^\circ\text{C}$) from 25 to 60°C at a rate of 0.2 kg/s. The heating is to be accomplished by geothermal water ($c_p = 4.31 \text{ kJ/kg}\cdot^\circ\text{C}$) available at 140°C at a mass flow rate of 0.3 kg/s. Determine the rate of heat transfer in the heat exchanger and the exit temperature of geothermal water.

5-78 A thin-walled double-pipe counter-flow heat exchanger is used to cool oil ($c_p = 2.20 \text{ kJ/kg}\cdot^\circ\text{C}$) from 150 to 40°C at a rate of 2 kg/s by water ($c_p = 4.18 \text{ kJ/kg}\cdot^\circ\text{C}$) that enters at 22°C at a rate of 1.5 kg/s. Determine the rate of heat transfer in the heat exchanger and the exit temperature of water.



5–81 Refrigerant-134a at 1 MPa and 90°C is to be cooled to 1 MPa and 30°C in a condenser by air. The air enters at 100 kPa and 27°C with a volume flow rate of 600 m³/min and leaves at 95 kPa and 60°C. Determine the mass flow rate of the refrigerant. *Answer: 100 kg/min*



The gas constant of air is 0.287 kPa·m³/kg·K

The constant pressure specific heat of air is $c_p = 1.005$ kJ/kg·°C

$$h_3 = 324.66 \text{ kJ/kg} \quad h_4 = 93.58 \text{ kJ/kg}$$

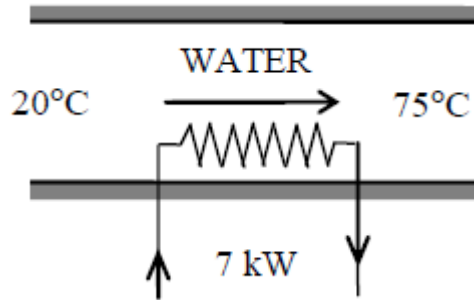
5–99 A 4-m × 5-m × 6-m room is to be heated by an electric resistance heater placed in a short duct in the room. Initially, the room is at 15°C, and the local atmospheric pressure is 98 kPa. The room is losing heat steadily to the outside at a rate of 150 kJ/min. A 200-W fan circulates the air steadily through the duct and the electric heater at an average mass flow rate of 40 kg/min. The duct can be assumed to be adiabatic, and there is no air leaking in or out of the room. If it takes 20 min for the room air to reach an average temperature of 25°C, find (a) the power rating of the electric heater and (b) the temperature rise that the air experiences each time it passes through the heater.

The gas constant of air is 0.287 kPa·m³/kg·K

The specific heats of air at room temperature are $c_p = 1.005$ and $c_v = 0.718$ kJ/kg·K

5–107 Water is heated in an insulated, constant-diameter tube by a 7-kW electric resistance heater. If the water enters the heater steadily at 20°C and leaves at 75°C , determine the mass flow rate of water.

The specific heat of water at room temperature is $c = 4.18 \text{ kJ/kg}\cdot^\circ\text{C}$

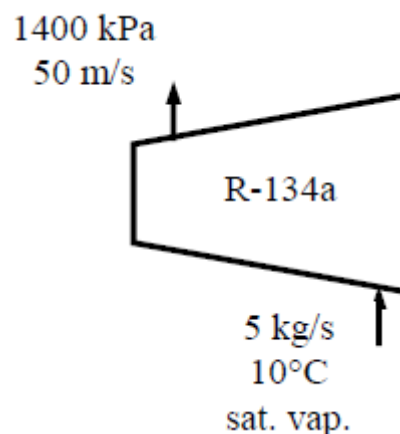


5–141E Nitrogen gas flows through a long, constant-diameter adiabatic pipe. It enters at 100 psia and 120°F and leaves at 50 psia and 70°F . Calculate the velocity of the nitrogen at the pipe's inlet and outlet.

The specific heat of nitrogen at the room temperature is $c_p = 0.248 \text{ Btu/lbm}\cdot\text{R}$

5–172 Refrigerant 134a enters a compressor with a mass flow rate of 5 kg/s and a negligible velocity. The refrigerant enters the compressor as a saturated vapor at 10°C and leaves the compressor at 1400 kPa with an enthalpy of 281.39 kJ/kg and a velocity of 50 m/s . The rate of work done on the refrigerant is measured to be 132.4 kW . If the elevation change between the compressor inlet and exit is negligible, determine the rate of heat transfer associated with this process, in kW.

$$h_1 = 256.22 \text{ kJ/kg} \quad ; \quad h_2 = 281.39 \text{ kJ/kg}$$



1.6 Heat Transfer

Heat is transferred between two objects by virtue of temperature difference. In practice, we note that after pouring hot water in a cup, the water cools down and the cup becomes warmer over time. This indicates that thermal energy is moving from the water to the cup, and the heat transfer process continues until both are at the same temperature. Similarly, if two solid objects at different temperatures brought into contact, heat will flow from the hotter body until both objects attain the same temperature. The above two examples indicate the existence of different mechanisms for heat transfer. Here we ask questions, how did heat flow from water to the cup? How did it flow from one solid object to another?

The examples also indicate that time is very important in the heat exchange process. Thus, the rate of heat transfer is actually the main controlling factor of the heat exchange process. Accordingly, we can state that heat transfer is *the science that deals with the determination of the rate of heat transfer*, which is the amount of heat flow per unit time. In addition, heat transfer analysis provides information about the **temperature gradient** within a system as well as the time required for heating or cooling.

1.6 انتقال الحرارة

تنتقل الحرارة من جسم الى آخر بسبب الفرق بدرجة الحرارة بينهما. ونلاحظ في مجريات الحياة العامة ان سكب الماء الحار في فنجان يؤدي الى رفع درجة حرارته, بينما تنخفض درجة حرارة الماء بمرور الزمن. وهذا يشير الى ان الطاقة الحرارية تنتقل من الماء الى الفنجان, وان عملية انتقال الحرارة تستمر حتى يصبح كلاهما في نفس درجة الحرارة. ولو تطرقنا الى مثال آخر يخص تلامس جسم صلب ساخن بجسم صلب بارد نسبياً, سنجد الحرارة تسري من الجسم الاول الى الثاني لحين بلوغهما نفس درجة الحرارة. يشير المثالان السابقان الى وجود طرائق مختلفة لانتقال الحرارة, وهنا نطرح الاسئلة, كيف انتقلت الحرارة من الماء الى الفنجان؟ وكيف انتقلت الحرارة من الجسم الصلب الساخن الى الجسم الآخر؟

يتضح من المثالين السابقين ايضاً ان الزمن مهم جداً في عملية التبادل الحراري, وبالتالي فإن معدل انتقال الحرارة (اوسرعة انتقال الحرارة) هو في الواقع عامل التحكم الاساسي بعملية التبادل الحراري. وهكذا يمكن القول بأن انتقال الحرارة هو العلم الذي يعنى بتقدير معدل انتقال الحرارة, او تخمين كمية الحرارة المنتقلة في وحدة الزمن. اضافة الى ذلك فإن تحليلات انتقال الحرارة توفر لنا معلومات تخص **التدرج بدرجة الحرارة** في المنظومة قيد الدراسة, اي تحديد درجات الحرارة في مواقع مختلفة, وكذلك معلومات تخص الزمن اللازم لتسخين او لتبريد اجزاء المنظومة.

1.7 Heat Transfer Modes

There are three methods of heat transfer, conduction, convection and radiation.

1.7.1 Conduction

It is the transfer of thermal energy from the more energetic particles to the adjacent less energetic ones of a substance. It may also occur between two solid objects in contact; particularly when the surfaces of the solid objects are in good physical contact. Fluids, that are liquids and gases, are much less conductive than solids. Conduction does not involve transfer of material; in fact, the energy transfer is caused by the collision of vibrating atoms as well as the free movement of electrons in the solid. Some materials are good conductors compared to others, for example **steel** is a good conductor whilst wood is a poor conductor. The ability of materials to conduct heat is governed by a property known as **thermal conductivity** denoted k in $W/m.K$ or $W/m.^{\circ}C$. The higher the value of the conductivity the more heat is conducted, for example, the thermal conductivity of copper is 0.4 kW/m.K and of water is 0.000613 kW/m.K , indicating that copper is a good conductor and water is poor.

In practice, the rate of heat transfer q in W depends on the temperature difference ΔT , the geometry of the object determined by the **characteristic length**, which is Δx for a flat plate, as well as the thermal conductivity k . In fact, temperature difference is the **driving force** for all heat transfer modes. **Fourier's Law** correlates the parameters by equation (1.12) which is applied to evaluate the rate of heat transfer for **one dimensional steady state conduction**.

1.7 طرائق انتقال الحرارة

هناك ثلاث طرق لانتقال الحرارة هي التوصيل والحمل والاشعاع.

1.7.1 التوصيل الحراري

هو انتقال الطاقة الحرارية من جزيئات المادة عالية الطاقة الى الجزيئات ذات الطاقة الاقل. ويحدث التوصيل ايضاً عند تلامس جسمين مصنوعين من مادة صلبة، حيث يكون انتقال الحرارة فعالاً اذا كان التماس بين اسطح الجسمين تماساً جيداً. اما الموائع التي تشمل السوائل والغازات فهي ضعيفة التوصيل مقارنة بالمواد الصلبة.

ان التوصيل الحراري لا ينطوي على اي نوع من انتقال المادة، لكن ما يحصل هو انتقال الطاقة نتيجة تصادم ذرات المادة التي تكون في حالة الاهتزاز، وكذلك بسبب حرية حركة الالكترونات في المواد الصلبة. ونجد بأن بعض المواد جيدة التوصيل مقارنة بمواد اخرى، ومثال ذلك جودة توصيل مادة **الصلب الحديدي** مقارنة بمادة الخشب. ويعود السبب الى قيمة **الموصلية الحرارية** التي يرمز لها (k) ، وحداتها $(W/m.K)$ او $(W/m.^{\circ}C)$ ، وهي خاصية تعنى بتحديد قدرة المواد على توصيل الحرارة. ولهذا تُصنّف المواد مرتفعة الموصلية على انها مواد جيدة التوصيل الحراري. فعلى سبيل المثال، موصلية مادة النحاس تساوي (0.4 kW/m.K) والماء $(0.000613 \text{ kW/m.K})$ مما يشير الى ان النحاس موصل حراري جيد، بينما الماء هو موصل حراري ضعيف.

وفي التطبيقات العملية، يعتمد معدل انتقال الحرارة (q) بالوحدات (W) على الفرق بدرجة الحرارة (ΔT) وعلى شكل الجسم الهندسي الذي يمثل **الطول المميز**، وهو سمك الصفيحة المستوية (Δx) مثلاً، كما ويعتمد ايضاً على قيمة الموصلية الحرارية (k) . وتجدر الاشارة الى ان الفرق بدرجة الحرارة هو **القوة الدافعة** لجميع انماط انتقال الحرارة. يربط **قانون فوريير** المقادير الثلاثة بالمعادلة (1.12) التي تستخدم لحساب معدل انتقال الحرارة في جميع حالات **التوصيل المستقر احادي البعد**.

$$q = k A (\Delta T / \Delta x) \quad (1.12)$$

Steady state means that temperature difference does not vary with time. However, if the temperature varies with time, **transient conduction** will prevail. It is also called **unsteady state conduction** and it engages complicated mathematical analysis. Additionally, in many industrial applications, heat may be transferred in more than one direction. In this case, the analytical solution to evaluate the temperature distribution becomes fairly complex. Note that solving transient-three dimensional conduction may require the employment of **numerical techniques** and computers to provide an approximate solution.

Conduction plays an essential part in heat transfer analysis for many engineering applications. Good examples are the design of heat exchangers, boilers, air-conditioning systems and thermal insulation.

Insulating materials exhibit low values of thermal conductivity; typical examples for buildings and industrial furnaces are: Glass fiber, Mineral wool, Cellulose, Polyurethane and Polystyrene (Styrofoam). Note that, it is quite common to encounter calculations of insulation thickness involving combined conduction with other modes of heat transfer such as convection or radiation.

1.7.2 Convection

is the transfer of heat by movement of the fluid from a hotter to a cooler spot. Hence, the bulk motion of a gas or liquid is a necessity for convective heat transfer to take place. In practice, convection is accomplished by forcing a fluid across objects of different geometrical shapes.

ان حالة الاستقرار هنا تعني ثبوت الفرق بدرجة الحرارة مع مرور الزمن. اما اذا تغير مقدار الفرق بدرجة الحرارة مع الزمن فينتحول انتقال الحرارة الى نمط **التوصيل العابر** الذي يعرف ايضاً **التوصيل غير المستقر**, ويتميز هذه النوع من التوصيل الحراري بتحليلات رياضية معقدة للغاية. بالإضافة الى ذلك, ففي بعض التطبيقات الصناعية, قد يحصل انتقال الحرارة باكثر من اتجاه متزامناً مع تغير الزمن, مما يؤدي الى صعوبة بالغة في حل معادلات التوصيل الحراري لتحديد قيم درجات الحرارة. وتجدر الإشارة الى ان حل معادلات التوصيل غير المستقر في ثلاثة اتجاهات قد يستلزم تطبيق **الطرق العددية** باستخدام الحاسبات الالكترونية للتوصل الى حلول تقريبية.

يلعب التوصيل دوراً أساسياً في تحليلات انتقال الحرارة للعديد من التطبيقات الهندسية, وان افضل الامثلة على ذلك هو, تصميم المبادلات الحرارية, والافران الصناعية, والمراجل البخارية ومنظومات التكيف, وكذلك حسابات تقدير سمك طبقة العزل الحراري.

تمتاز مواد العزل الحراري بقيم موصلية حرارية منخفضة, ومن اهم المواد المستخدمة لعزل البنايات السكنية وعزل الافران الصناعية هي الالياف الزجاجية, والصوف المعدني, والسيلولوز, والبوليبروتين, والبوليستايرين. وغالباً ما تتطلب حسابات تقدير سمك الطبقة العازلة حل معادلات التوصيل الحراري متزامناً مع طرق اخرى لانتقال الحرارة مثل الحمل الحراري او الاشعاع.

1.7.2 الحمل الحراري

هو انتقال الحرارة بفعل حركة المائع من نقطة حارة الى اخرى باردة, وعليه تعد حركة السائل او الغاز ضرورية لتوليد انتقال الحرارة بالحمل. وفي مجالات التطبيق العملي يحدث الحمل الحراري نتيجة دفع المائع قسرياً عبر اجسام ذات اشكال هندسية متنوعة.

Let us look at the cooling of a hot surface by a current of air. Heat is first conducted from the surface to the layer of air adjacent to the surface, and then it is carried away from the surface by convection. This mode of heat transfer depends on temperature difference as well as on the motion of the fluid.

Two common types of convection exist: one is **natural convection** or **free convection**, which is characterized by the movement of fluid caused by **buoyancy forces** due to the temperature difference. The second type is **forced convection** caused by forcing a fluid by a pump or a fan over the surface. The rate of convective heat transfer is determined by **Newton's cooling law** expressed by the equation

$$q = h A (T_s - T_f) \quad (1.13)$$

h is the **heat transfer coefficient** in $W/m^2.K$ or $W/m^2.^{\circ}C$, A is the surface area in m^2 , T_s is the temperature of the surface and T_f is the temperature of the fluid. It is worth noting here that the heat transfer coefficient h is not a property of the fluid, and its value should be determined experimentally for the setup under study.

Natural convection is demonstrated by heating the space inside a room. If a heater is placed in a corner inside a room, the air gets warmer and moves upward with the cooler air moving downward setting up warm currents around the space. Forced convection is more popular in industry, and pumping water into **heat exchanger** tubes is a typical example.

ولو تتبعنا عملية تبريد سطح ساخن بتيار من الهواء، سنلاحظ بأن الحرارة تنتقل من السطح الى طبقة الهواء الملاصقة لها بواسطة التوصيل، ثم تنتقل بواسطة الحمل الحراري مبتعدة عن السطح الصلب. ويعتمد هذا النوع من انتقال الحرارة على مقدار الفرق بدرجة الحرارة اضافة الى حركة المائع.

يوجد نوعان من الحمل الحراري، **الحمل الحراري الطبيعي** ويسمى ايضا **الحمل الحراري الحر** الذي يتميز بحركة المائع بسبب **قوى الطفو** الناجمة عن الاختلاف في درجة الحرارة، و**الحمل الحراري القسري** الذي يحدث نتيجة الجريان القسري للمائع بواسطة مضخة او مروحة عبر السطح. ويُحسب معدل انتقال الحرارة بالحمل بواسطة **قانون نيوتن للتبريد** وفقاً للمعادلة الآتية

يمثل الرمز (h) **معامل انتقال الحرارة** بالوحدات $(W/m^2.K)$ او $(W/m^2.^{\circ}C)$ ، والرمز (A) هو المساحة بالوحدات (m^2) ، و (T_s) درجة حرارة السطح و (T_f) درجة حرارة المائع. وتجدر الإشارة هنا الى ان معامل انتقال الحرارة هو ليس خاصية من خواص المادة، وانما ينبغي تحديد قيمته عملياً للمنظومة قيد الدراسة.

ان افضل مثال على ظاهرة الحمل الطبيعي هو تدفئة الغرفة، فعندما نضع مدفئة في احدى زواياها يرتفع الهواء الدافئ الى الاعلى ويحل محله هواء بارد مولداً تيارات دافئة فيها. اما الحمل القسري فهو شائع في المصانع حيث يُعد ضخ الماء في انابيب **المبادلات الحرارية** مثالا نموذجياً لذلك.

1.7.3 Thermal Radiation

It is the energy emitted by matter due to temperature. . All materials, solids, liquids and gases, radiate heat at a temperature above 0 K. Radiation is made up of **electromagnetic waves** with relatively long wave length compared to other forms of radiation such as ultraviolet or visible light. In contrast to convection and conduction, the transfer of radiation does not require an intervening transport medium; a good example is the travel of sun light to earth. Thermal radiation travels in **vacuum** at the speed of light without being subjected to any effect. An incident beam of radiation on a surface may undergo reflection, or absorption, or transmission, or the three phenomena simultaneously, see Figure 1.5. Radioactive heat transfer is the predominant mode of heat transfer at high temperatures. The amount of radiation emitted or absorbed by a solid object, at a given temperature, is determined by the value of the surface **emissivity** ϵ or **absorptivity** α . The value of both properties is in the range $0 \leq \epsilon \leq 1$ and $0 \leq \alpha \leq 1$. A body that have an emissivity $\epsilon = 1$ is called **blackbody**.

The maximum rate of radiation emitted by an object $q_{\max}$ in W is known as blackbody radiation; it is given by **Stefan-Boltzmann** law as

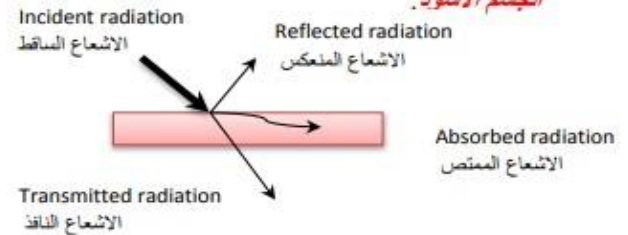
$$q_{\max} = \sigma A T_s^4$$

In fact, the **real surface** emits less radiation than the blackbody at the same temperature, and the rate of radiation q_{emit} is expressed as

$$q_{\text{emit}} = \epsilon \sigma A T_s^4$$

1.7.3 الاشعاع الحراري

هو الطاقة المنبعثة من المواد بسبب درجة الحرارة، حيث ان جميع المواد سواء اكانت صلبة او سائلة او غازية تبعث الاشعاع عند درجات حرارة تفوق الصفر المطلق (0 K). يتكون الاشعاع الحراري من **موجات كهرومغناطيسية** تتميز بموجاتها الطويلة نسبة الى انواع الاشعاع الأخرى كالاشعة فوق البنفسجية والضوء المرئي. وخلاف لما هو معروف عن الحمل الحراري والتوصيل فلا يتطلب انتقال الاشعاع بين نقطتين الى وسيلة نقل، ومثال ذلك هو وصول الضوء من الشمس الى الارض مروراً بالغلاف الجوي. وكذلك ينتقل الاشعاع الحراري خلال **الفراغ** بسرعة الضوء دون تعرضه الى اي تأثير. وقد تتعرض حزمة اشعاع ساقطة على سطح معين الى انعكاس او امتصاص او نفاذ، او جميع هذه الظواهر في آن معاً، كما هو مبين في الشكل 1.5. وتجدر الإشارة الى ان الاشعاع الحراري هو النمط السائد لانتقال الحرارة عند درجات الحرارة المرتفعة. تتحكم كل من **الانبعاثية** (ϵ) و**الامتصاصية** (α) بمقدار الاشعاع المنبعث او الممتص من الجسم الصلب عند درجة حرارة معينة، وتتخذ كل من الخاصيتين قيم تتراوح بين الصفر والواحد، اي: $(0 \leq \epsilon \leq 1)$ و $(0 \leq \alpha \leq 1)$ ، وعندما تكون انبعاثية الجسم مساوية الواحد ($\epsilon = 1$)، يطلق عليه **الجسم الاسود**.



الشكل 1.5 Figure

تُسمى كمية الاشعاع القصوى المنبعثة من جسم معين ($q_{\max}$) بوحدات (W) اشعاع الجسم الاسود، حيث يتم تقديره بموجب قانون **ستيفان - بولتزمان** كالآتي

$$(1.14)$$

وفي واقع الحال، ينبعث الاشعاع من **السطح الحقيقي** بكمية أقل من الجسم الاسود عند نفس درجة الحرارة، ويُكتب معدل الاشعاع الحراري الصادر (q_{emit}) منه بالصيغة الآتية

$$(1.15)$$

where ϵ is the *emissivity* of the surface, T_s is the absolute temperature of the surface expressed in K, A is the surface area in m^2 and σ is the **Stefan-Boltzmann constant** whose value is $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$.

A blackbody is also a perfect absorber of radiation and hence has an absorptivity equal to one $\alpha = 1$.

The value of emissivity and absorptivity depends on temperature and wavelength; however, **average values** are commonly used to evaluate radiation. According to **Kirchhoff's law**, the two average values are equal ($\epsilon = \alpha$). Evaluation of radiate heat transfer is based on the exchange of radiation between surfaces or between a surface and the surrounding atmosphere. This is common in the design of some industrial furnaces, where we find radiation exchange between the walls is essential. On the other hand, the design of **combustion chambers** involves radiative exchanges between surfaces and combustion gases. Thus, the emission and absorption of radiation by gases may affect the design calculations.

The **net rate of radiation heat transfer** q_{net} is given by the equation

$$q_{\text{net}} = \epsilon \sigma A (T_s^4 - T_{\infty}^4) \quad (1.16)$$

T_{∞} is the surrounding temperature in K.

There are many applications for radiation heat transfer, particularly in industry; examples are furnaces and heating systems. Solar energy system analysis and design involves radiation as well as other modes of heat transfer. The field of radiation is also important in space research.

ويمثل (ϵ) انبعاثية السطح، و (T_s) درجة حرارته المطلقة بوحدات (K)، و (A) مساحة السطح بالوحدات (m^2) و (σ) **ثابت ستيفان - بولتزمان** الذي يتخذ القيمة ($5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$).

ويُعد الجسم الاسود ممتصاً مثالياً للإشعاع، لذلك تكون امتصاصيته تساوي الواحد ($\alpha = 1$).

يعتمد مقدار كل من الانبعاثية والامتصاصية على درجة الحرارة وعلى طول الموجة، مع ذلك فإننا نستخدم عادة **قيماً وسطية** لغرض انجاز حسابات تقدير كمية الإشعاع الحراري. وبموجب **قانون كيرتشفوف** تصبح القيمتان الوسطيتان للانبعاثية والامتصاصية متساويتين، أي ان ($\epsilon = \alpha$).

تستند طريقة تقدير كمية الإشعاع الحراري الى ظاهرة التبادل الإشعاعي بين الأسطح او بين الأسطح وما يحيط بها. فمثلاً، يُعد تبادل الإشعاع الحراري بين الأسطح أساساً لتصميم الأفران الصناعية، بينما يستند تصميم **صناديق الاحتراق** الى تخمين تبادل الإشعاع بين الأسطح وغازات الاحتراق الموجودة داخلها، حيث يؤثر امتصاص وانبعاث الإشعاع بواسطة هذه الغازات في حسابات التصميم.

يعطى **المعدل الصافي لانتقال الحرارة بالإشعاع** (q_{net}) من المعادلة

(T_{∞}) هي درجة حرارة المحيط بوحدة (K).

هناك عدة تطبيقات لانتقال الحرارة بالإشعاع في المصانع، منها ما يتعلق بالمعدات الصناعية كالأفران ومنظومات التدفئة. كما يتضمن تصميم منظومات الطاقة الشمسية تحليلات انتقال الحرارة بالإشعاع مع طرائق انتقال الحرارة الأخرى. والمعروف أيضاً بأن الإشعاع الحراري يشكل جانباً مهماً في بحوث تكنولوجيا الفضاء.

1.8 انتقال الحرارة باكثر من طريقة

اصبح واضحاً ان الحرارة تنتقل باحدى طرائق الانتقال الثلاث، التوصيل او الاشعاع او الحمل الحراري. ومع ذلك نجد احياناً حدوث انتقال الحرارة باكثر من طريقة في آن معاً. فقد تنتقل الحرارة بنمط واحد او نمطين وذلك حسب نوع الوسط او المادة التي تمر من خلالها. فمثلاً يحصل الانتقال خلال الفراغ بالاشعاع فقط، بينما يُعد التوصيل الطريقة الوحيدة لانتقال الحرارة خلال الاجسام الممتعة كالقطع المعدنية. ويمكن ان يتزامن الاشعاع والتوصيل في المواد الصلبة شبه الشفافة مثل الزجاج. وكثيراً ما يحدث انتقال الحرارة من سطح مادة صلبة الى محيطها بالحمل والاشعاع في آن معاً، واذا شملنا انتقال الحرارة بالتوصيل خلال هذه المادة، سيكون هناك ثلاثة انماط متزامنة. وعموماً تنتقل الحرارة خلال الغازات الراكدة بواسطة التوصيل والاشعاع معاً، اما اذا كان الغاز متحركاً فيصبح الحمل النمط السائد اضافة الى قدر قليل من الاشعاع. ونلاحظ مما مر ذكره امكان انتقال الحرارة بنمطين خلال وسط مادي واحد، لكن تزامن الطرائق الثلاث يتطلب وجود وسط آخر، وافضل مثال على ذلك هو تبادل الحرارة بين قطعة معدنية والهواء المحيط بها.

1.8 Combined Heat Transfer Modes

Heat transfer may occur in one of three modes, conduction, radiation or convection. However, it is quite common to have more than one mode occurring simultaneously. Depending on the type of medium or material, we may experience one or two modes taking place at the same time. For example, heat transfer through vacuum takes place by radiation and through opaque solids by conduction. Nevertheless, conduction and radiation may occur simultaneously in semitransparent solids such as glass. Furthermore, heat transfer from the surface of a solid material to the surrounding may involve convection and radiation. However, if we include heat transfer through the material by conduction, we will have all three mechanisms involved. Likewise, heat transfer through stagnant gases is normally accomplished by both conduction and radiation. But, for moving gases, convection will be the dominant mode, plus some radiative exchange. So far, we noticed that only two modes of heat transfer can occur in a single medium. For the simultaneous occurrence of the three modes, another medium is required; a good example is heat exchanges between a metal plate and the ambient air.

1.9 تطبيقات انتقال الحرارة

تعد الاجهزة المنزلية كسخانات الماء والطباخات الكهربائية ومنظومات التكييف من وسائل التبريد والتسخين الشائعة، وكذلك جهاز تبريد محرك السيارة المعروف بالراديتور. وان تصميم هذه الاجهزة قد يتطلب طرائق انتقال الحرارة الثلاث.

1.9 Applications of Heat Transfer

Domestic equipment such as electric cookers, water heaters, fridges, air conditioning systems and car radiators are some of the popular means of heating and cooling. The three modes of heat transfer may well be involved in the design of these appliances.

2.1 Fourier's Law

Fourier's law states that the rate of conductive heat transfer through a material is proportional to the negative of the temperature gradient and to the area at right angle to the direction of heat flow. The law was developed from experimental data, by **Joseph Fourier**, in the nineteenth century; it is expressed for one-dimensional conduction as:

$$q_x \propto -A \frac{dT}{dx}$$

$$q_x = -k A \frac{dT}{dx}$$

q_x = Time rate of heat transfer in W

A = Area in m^2 perpendicular to the direction of heat flow

k = Thermal conductivity in W/m.K

$\frac{dT}{dx}$ = Temperature gradient in K/m

Fourier's law is applicable to all matter, Solids, liquids and gases, assuming that:

- One dimensional-steady conduction
- Constant thermal conductivity
- No internal heat generation
- Constant temperature gradient

Fourier's law can also be applied to conduction through solid of different shapes.

1.1 قانون فوريير

ينص **قانون فوريير** على أن معدل انتقال الحرارة بالتوصيل يتناسب طردياً مع سالب تدرج درجة الحرارة ومع المساحة المعامدة لاتجاه سريان الحرارة. وقد اشتق العالم **جوزيف فوريير** هذا القانون في القرن التاسع عشر مستنداً الى معطيات التجارب العملية. ويكتب القانون للتوصيل احادي البعد بالصيغة:

q_x = المعدل الزمني لانتقال الحرارة بوحدات (W)

A = المساحة المعامدة لاتجاه انتقال الحرارة (m^2)

k = الموصلية الحرارية (W/m.K)

$\frac{dT}{dx}$ = التدرج بدرجة الحرارة (K/m)

ينطبق قانون فوريير على جميع المواد سواء اكانت صلبة او سائلة او غازية, على افتراض:

- توصيل حراري مستقر احادي البعد
- ثبوت مقدار الموصلية الحرارية
- عدم وجود توليد حراري في المادة
- ثبوت تدرج ادرجة الحرارة خلال المادة

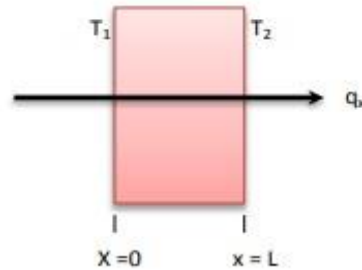
ويمكن ايضاً تطبيق قانون فوريير لانتقال الحرارة بالتوصيل خلال مواد صلبة مختلفة الاشكال.

2.2 Conduction through Plane Walls

Let us apply Fourier's law to a plane wall as shown in Fig 2.1

2.2 التوصيل خلال الجدران المستوية

دعنا نطبق قانون فورير على الجدار الموضح في الشكل 2.1



الشكل 2.1 Figure

التوصيل خلال الجدار المستوي Conduction through a plane wall

For steady one dimensional conduction and constant thermal conductivity equation (2.1) can be integrated to yield

وباعتبار التوصيل مستقراً ببعد واحد وعلى ان الموصلية الحرارية ثابتة المقدار, يمكننا مكاملة المعادلة (2.1) فنحصل على

$$q_x = -k A \frac{T_2 - T_1}{L} \quad (2.2)$$

Equation (2.2) is used to evaluate the rate of heat transfer through a plane wall. Also the equation provides the **heat flux** defined as the rate of heat transfer per unit area q_x'' in W/m^2 ,

تستخدم المعادلة (2.2) لتحديد مقدار معدل انتقال الحرارة خلال اي جدار مستوي. وتمنح هذه المعادلة ايضاً طريقة لحساب **الفيض الحراري**, وهو معدل انتقال الحرارة في وحدة المساحة, رمزه (q_x'') ووحداته (W/m^2).

$$q_x'' = \frac{q_x}{A} \quad (2.3)$$

المثال 2.1

Example 2.1

The temperature of the inner surface of an industrial furnace wall is 1100°C and that of the outer one is 850°C . The wall is made of bricks having thermal conductivity of 0.71 W/m.K . If the wall is $18 \text{ cm} \times 50 \text{ cm} \times 1.5 \text{ m}$, calculate the rate of heat loss through the wall, Evaluate the heat flux and the amount of heat lost in one hour?

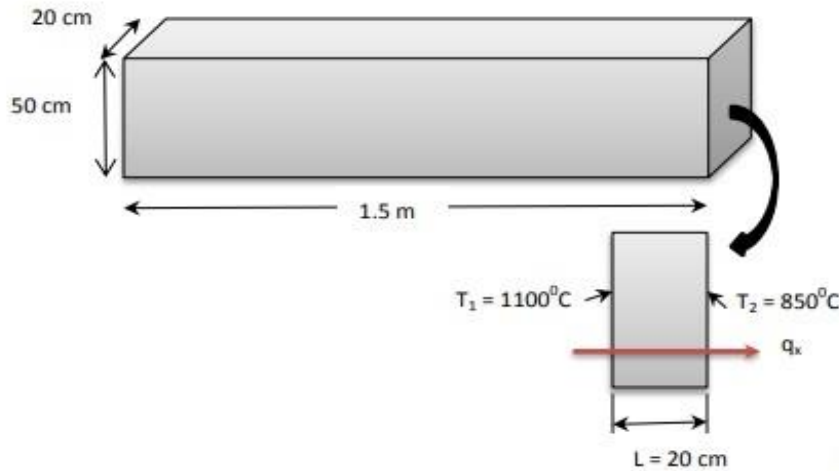
بلغت درجة حرارة سطح الجدار الداخلي لفرن صناعي (1100°C) ودرجة سطحه الخارجي (850°C)، وهو مبني من طابوق موصليته الحرارية تساوي (0.71 W/m.K)، فإذا كانت أبعاد الجدار ($18 \text{ cm} \times 50 \text{ cm} \times 1.5 \text{ m}$)، احسب معدل الفقدان الحراري عبر الجدار ثم اوجد مقدار الفيض الحراري ومقدار كمية الحرارة المفقودة خلال ساعة واحدة؟

Solution

الحل

Given:

المعلوم:



Find:

المطلوب ايجاده:

Heat loss q_x and heat flux q''

الفقدان الحراري (q_x) والفيض الحراري (q'')

Assumptions:

الافتراضات:

1. One dimensional-steady conduction.
2. Constant thermal conductivity.
3. No internal heat generation.
4. Constant temperature gradient.

1. التوصيل الحراري مستقر احادي البعد.
2. ثبوت مقدار الموصلية الحرارية.
3. عدم وجود توليد حراري في المادة.
4. ثبوت تدرج انتقال الحرارة خلال المادة.

Analysis:

التحليلات:

Step 1

أولاً

Convert the units of all known quantities to the SI system.

نحول وحدات المقادير المعروفة الى نظام (SI)

The height of the wall,

ارتفاع الجدار,

$$\frac{50}{100} = 0.5 \text{ m}$$

The width of the wall,

عرض الجدار,

$$\frac{20}{100} = 0.2 \text{ m}$$

The temperatures,

درجات الحرارة,

$$T_1 = 1100 + 273 = 1373 \text{ K}$$

$$T_2 = 850 + 273 = 1123 \text{ K}$$

Step 2

ثانياً

We apply Fourier's law to find the heat loss,

نطبق قانون فوريير لإيجاد الحرارة المفقودة,

$$q_x = -k A \frac{T_2 - T_1}{L} \quad \text{equation (2.2)}$$

$$q_x = -0.71 (1.5 \times 0.5) \frac{1123 - 1373}{0.2} = \underline{665.625 \text{ W}}$$

Step 3

ثالثاً

We calculate the flux using equation (2.3):

نحسب الفيض الحراري باستخدام المعادلة (2.3):

$$q_x'' = \frac{q_x}{A}$$

$$q_x'' = \frac{665.625}{1.5 \times 0.5} = \underline{887.5 \text{ W/m}^2}$$

رابعاً

Step 4

We determine the amount of heat lost in one hour which is 3600 s:

نقدر كمية الحرارة المفقودة في ساعة واحدة اي 3600 ثانية:

$$\text{Heat lost} = 665.625 \text{ W (J/s)}$$

$$\text{Heat lost in 1 hour} = 665.625 \times 3600 = 2.396 \times 10^6 \text{ J}$$

$$= \underline{2369 \text{ kJ}}$$

Comments:

الملاحظات:

1. Note of heat quantity is expressed in units of J or kJ, and the heat transfer rate in W or kW which is J/s or kJ/s. The units of the heat flux are W/m^2 or kW/m^2 .
2. The temperature difference in Kelvin has the same value in Celsius.

1. لاحظ ان كمية الحرارة تتخذ وحدات الجول (J) او الكيلوجول (kJ), ووحدات معدل انتقال الحرارة هي الواط (W) او الكيلوواط (kW), حيث انها تساوي جول/ثانية (J/s) او كيلوجول/ثانية (kJ/s). كما ان وحدات الفيض الحراري هي (W/m^2) او (kW/m^2).
2. ان قيمة الفرق بدرجة الحرارة بوحدات الكيلفن يساوي الفرق بوحدات السيليسيوس.

Exercise:

تمرين:

If the thickness of the wall in example 2.1 is increased to 23 cm, what will be the percentage saving in energy?

ما هو مقدار النسبة المئوية من الطاقة التي يمكن توفيرها عندما يكون سمك الجدار في المثال 2.1 يساوي (23 cm)؟

مسائل مع الحلول

Heat Conduction:

Problem:

A brick wall is **0.2 m** thick and has an area of **10 m²**. The temperature on the inner surface is **25°C** and on the outer surface is **5°C**. If the thermal conductivity of the brick is **k = 0.6 W/m °C**, calculate the rate of heat transfer through the wall.

Solution:

Using Fourier's Law of Conduction:

$$Q = \frac{kA(T_1 - T_2)}{L}$$

Where:

- $k = 0.6 \text{ W/m} \cdot ^\circ\text{C}$
- $A = 10 \text{ m}^2$
- $T_1 - T_2 = 25 - 5 = 20^\circ\text{C}$
- $L = 0.2 \text{ m}$

$$Q = \frac{0.6 \times 10 \times 20}{0.2} = \frac{120}{0.2} = 600 \text{ W}$$

Result: 600 W of heat is lost from the room to the outside.

Conduction + Composite Wall Resistance:

Problem:

A composite wall consists of two layers: a brick layer **0.15m** thick with **k₁ = 0.7 W/m °C**, and an insulation layer **0.05m** thick with **k₂ = 0.04 W/m °C**. The wall area is **20m²**. The inside temperature is **22°C** and the outside temperature is **-3°C**. Calculate the rate of heat transfer.

Solution:

Total thermal resistance for layers in series:

$$R_{total} = R_1 + R_2 = \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}$$

$$R_1 = \frac{0.15}{0.7 \times 20} = 0.0107 ^\circ\text{C/W}$$

$$R_2 = \frac{0.05}{0.04 \times 20} = 0.0625 ^\circ\text{C/W}$$

$$R_{total} = 0.0107 + 0.0625 = 0.0732 ^\circ\text{C/W}$$

$$Q = \frac{\Delta T}{R_{total}} = \frac{22 - (-3)}{0.0732} = \frac{25}{0.0732} \approx 341.5 \text{ W}$$

Heat Convection

Problem:

An electric heater has a surface area of **0.3 m²** and its surface temperature is **80°C**. It is placed in a room where the air temperature is **20°C**. The convective heat transfer coefficient is **h = 15 W/m².°C**. Calculate the heat lost by convection.

Solution:

Using Newton's Law of Cooling:

$$Q = hA(T_s - T_\infty)$$

$$Q = 15 \times 0.3 \times (80 - 20) = 15 \times 0.3 \times 60 =$$

Result: 270 W is transferred from the heater to the air.

Thermal Radiation

A blackbody with an area of 0.5 m² is at 500 K in a room whose walls are at 300 K. Calculate the net radiative heat transfer. Stefan-Boltzmann constant. $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$.

Solution:

For a blackbody $\varepsilon = 1$:

$$Q = \varepsilon \sigma A(T_1^4 - T_2^4)$$

$$Q = 1 \times 5.67 \times 10^{-8} \times 0.5 \times (500^4 - 300^4)$$

$$500^4 = 6.25 \times 10^{10}$$

$$300^4 = 8.1 \times 10^9$$

$$Q = 2.835 \times 10^{-8} \times (6.25 \times 10^{10} - 8.1 \times 10^9)$$

$$Q = 2.835 \times 10^{-8} \times 5.44 \times 10^{10} \approx 1542 \text{ W}$$

Result: ~1542 W radiated by the body.

Mixing + Specific Heat

Problem:

2 kg of water at 80°C is mixed with 3 kg of water at 20°C in an insulated container. What is the final equilibrium temperature? Specific heat of water $c = 4186 \text{ J/kg} \cdot ^\circ\text{C}$.

Solution:

Heat lost by hot water = Heat gained by cold water

$$m_1 c (T_1 - T_f) = m_2 c (T_f - T_2)$$

Cancel c from both sides:

$$2 \times (80 - T_f) = 3 \times (T_f - 20)$$

$$160 - 2T_f = 3T_f - 60$$

$$160 + 60 = 5T_f$$

$$T_f = \frac{220}{5} = 44^\circ\text{C}$$

Result: 44°C is the equilibrium temperature.

Phase Change + Heating

Problem:

What is the amount of heat required to convert **0.5 kg** of ice at **-10°C** to water at **30°C**.

$C_{\text{Ice}} = 2100 \text{ J/kg} \cdot ^\circ\text{C}$, $C_{\text{Water}} = 4186 \text{ J/kg} \cdot ^\circ\text{C}$, Latent heat of fusion of ice $L_f = 3.34 \times 10^5 \text{ J/kg}$.

Solution:

Three stages

1. Heating ice from -10°C to 0°C:

$$Q_1 = m c_{\text{ice}} \Delta T = 0.5 \times 2100 \times 10 = 10,500 \text{ J}$$

2. Melting ice at 0°C:

$$Q_2 = m L_f = 0.5 \times 3.34 \times 10^5 = 167,000 \text{ J}$$

3. Heating water from 0°C to 30°C:

$$Q_3 = m c_{\text{water}} \Delta T = 0.5 \times 4186 \times 30 = 62,790 \text{ J}$$

$$Q_{\text{total}} = 10,500 + 167,000 + 62,790 = 240,290 \text{ J}$$

Result: 240.3 kJ approximately.

Conduction Through a Glass Window

Problem:

A single-pane glass window has an area of **2.5m²** and a thickness of **5mm**. The temperature on the inner surface is **18°C** and on the outer surface is **2°C**. The thermal conductivity of glass is **k = 0.8 W/mK**. Find the heat loss through the window.

Solution:

$$Q = \frac{kA\Delta T}{L} = \frac{0.8 \times 2.5 \times (18 - 2)}{0.005}$$

$$Q = \frac{0.8 \times 2.5 \times 16}{0.005} = \frac{32}{0.005} = 6400 \text{ W}$$

Result: 6400 W. This is why double glazing is used!

THE SECOND LAW OF THERMODYNAMICS:

INTRODUCTION TO THE SECOND LAW

In Chaps. 4 and 5, we applied the *first law of thermodynamics*, or the *conservation of energy principle*, to processes involving closed and open systems.

As pointed out repeatedly in those chapters, energy is a conserved property, and no process is known to have taken place in violation of the first law of thermodynamics. Therefore, it is reasonable to conclude that a process must satisfy the first law to occur.

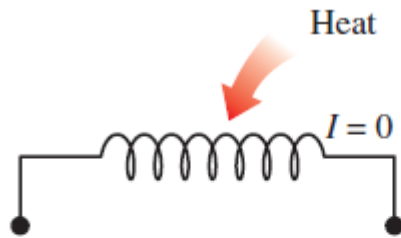
It is common experience that a cup of hot coffee left in a cooler room eventually cools off. This process satisfies the first law of thermodynamics since the amount of energy lost by the coffee is equal to the amount gained by the surrounding air. Now let us consider the reverse process—the hot coffee getting even hotter in a cooler room as a result of heat transfer from the room air. We all know that this process never takes place.



A cup of hot coffee does not get hotter in a cooler room.

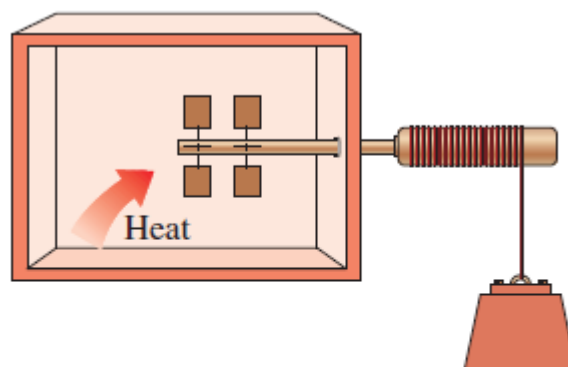
As another familiar example, consider the heating of a room by the passage of electric current through a resistor. Again, the first law dictates that the amount of electric energy supplied to the resistance wires be equal to the amount of energy transferred to the room air as heat. Now let us attempt to reverse this process. It will

come as no surprise that transferring some heat to the wires does not cause an equivalent amount of electric energy to be generated in the wires.



Transferring heat to a wire will not generate electricity.

Finally, consider a paddle-wheel mechanism that is operated by the fall of a mass. The paddle wheel rotates as the mass falls and stirs a fluid within an insulated container. As a result, the potential energy of the mass decreases, and the internal energy of the fluid increases in accordance with the conservation of energy principle. However, the reverse process, raising the mass by transferring heat from the fluid to the paddle wheel, does not occur in nature.



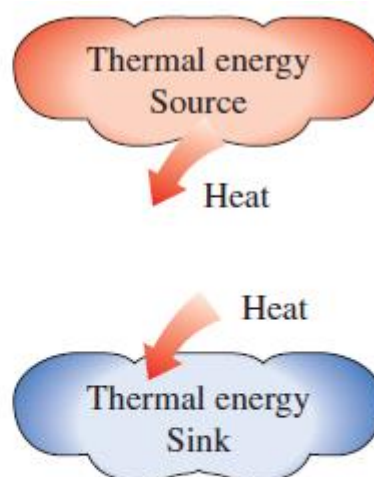
Transferring heat to a paddle wheel will not cause it to rotate.

THERMAL ENERGY RESERVOIRS

A hypothetical body with a relatively large *thermal energy capacity* (mass $\times$ specific heat) that can supply or absorb finite amounts of heat without undergoing any change in temperature. Such a body is called a **thermal energy reservoir**, or just a reservoir. The *atmosphere*, for example, does not warm up as a result of heat losses from residential buildings in winter. Likewise, mega joules of waste energy dumped in large rivers by power plants do not cause any significant change in water temperature.

A body does not actually have to be very large to be considered a reservoir. The air in a room, for example, can be treated as a reservoir in the analysis of the heat dissipation from a TV set in the room, since the amount of heat transfer from the TV set to the room air is not large enough to have a noticeable effect on the room air temperature.

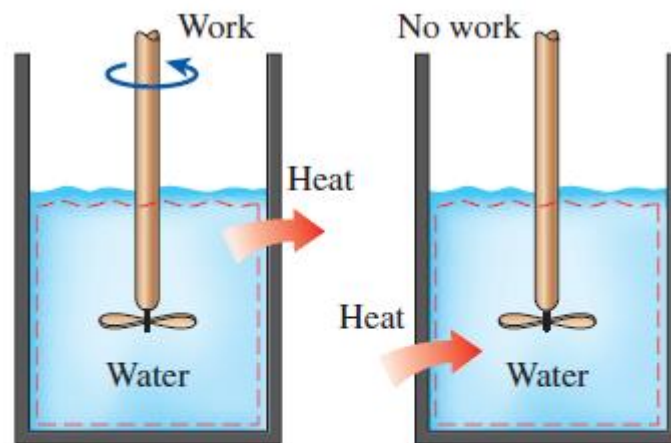
A reservoir that supplies energy in the form of heat is called a **source**, and one that absorbs energy in the form of heat is called a **sink**. Thermal energy reservoirs are often referred to as **heat reservoirs** since they supply or absorb energy in the form of heat.



A source supplies energy in the form of heat, and a sink absorbs it.

HEAT ENGINES

Work can easily be converted to other forms of energy, but converting other forms of energy to work is not that easy. The mechanical work done by the shaft shown in Fig. for example, is first converted to the internal energy of the water. This energy may then leave the water as heat. We know from experience that any attempt to reverse this process will fail. That is, transferring heat to the water does not cause the shaft to rotate.



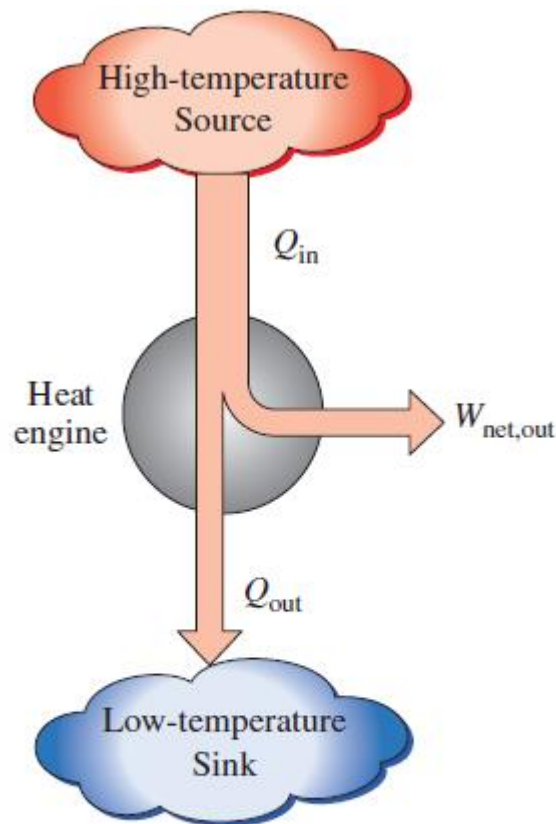
Work can always be converted to heat directly and completely, but the reverse is not true.

From this and other observations, we conclude that work can be converted to heat directly and completely, but converting heat to work requires the use of some special devices. These devices are called **heat engines**.

Heat engines differ considerably from one another, but all can be characterized by the following: They receive heat from a high-temperature source (solar energy, oil furnace, nuclear reactor, etc.).

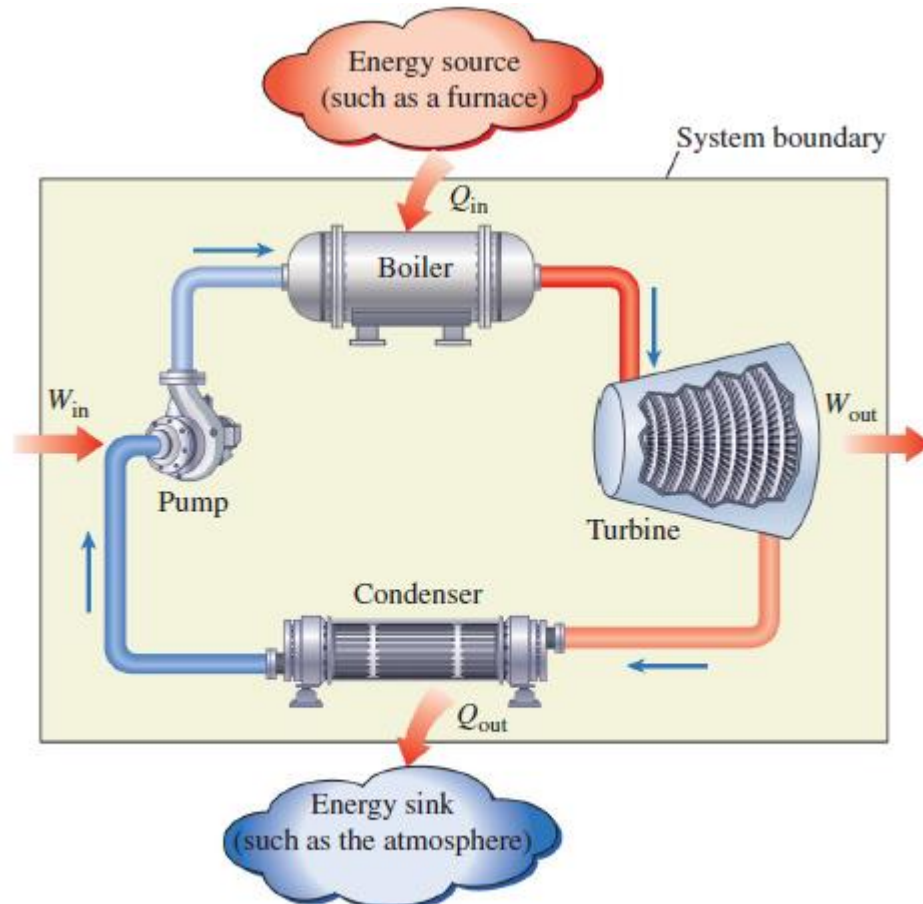
1. They convert part of this heat to work (usually in the form of a rotating shaft).
2. They reject the remaining waste heat to a low-temperature sink (the atmosphere, rivers, etc.).

3. They operate on a cycle.



Part of the heat received by a heat engine is converted to work, while the rest is rejected to a sink.

Heat engines and other cyclic devices usually involve a fluid to and from which heat is transferred while undergoing a cycle. This fluid is called the **working fluid**. The work-producing device that best fits into the definition of a heat engine is the *steam power plant*. The schematic of a basic steam power plant is shown in Fig.



Schematic of a steam power plant.

The various quantities shown on this figure are as follows:

Q_{in} = amount of heat supplied to steam in boiler from a high-temperature source (furnace)

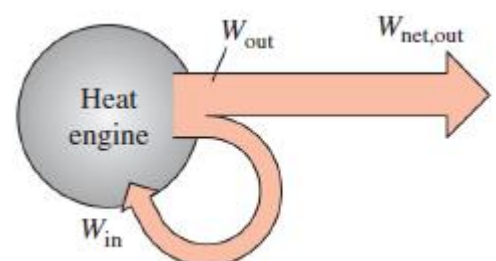
Q_{out} = amount of heat rejected from steam in condenser to a low temperature sink (the atmosphere, a river, etc.)

W_{out} = amount of work delivered by steam as it expands in turbine.

W_{in} = amount of work required to compress water to boiler pressure

The net work output of this power plant is simply the difference between the total work output of the plant and the total work input

$$W_{net,out} = W_{out} - W_{in} \quad (\text{kJ})$$



The net work can also be determined from the heat transfer data alone. The four components of the steam power plant involve mass flow in and out, and therefore should be treated as open systems. These components, together with the connecting pipes, however, always contain the same fluid. No mass enters or leaves this combination system, which is indicated by the shaded area thus, it can be analyzed as a closed system. Recall that for a closed system undergoing a cycle, the change in internal energy ΔU is zero, and therefore the net work output of the system is also equal to the net heat transfer to the system:

$$W_{\text{net,out}} = Q_{\text{in}} - Q_{\text{out}} \quad (\text{kJ})$$

Thermal Efficiency

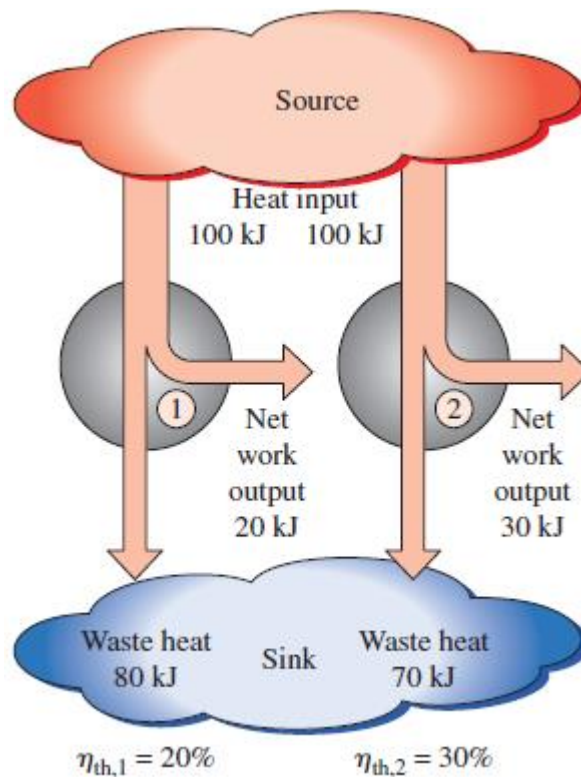
The fraction of the heat input that is converted to net work output is a measure of the performance of a heat engine and is called the **thermal efficiency** ζ_{th}

For heat engines, the desired output is the net work output, and the required input is the amount of heat supplied to the working fluid. Then the thermal efficiency of a heat engine can be expressed as:

$$\text{Thermal efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}$$

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_{\text{in}}}$$

$$\eta_{\text{th}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$$



Cyclic devices of practical interest such as heat engines, refrigerators, and heat pumps operate between a high-temperature medium (or reservoir) at temperature TH and a low-temperature medium (or reservoir) at temperature TL . To bring uniformity to the treatment of heat engines, refrigerators, and heat pumps, we define these two quantities:

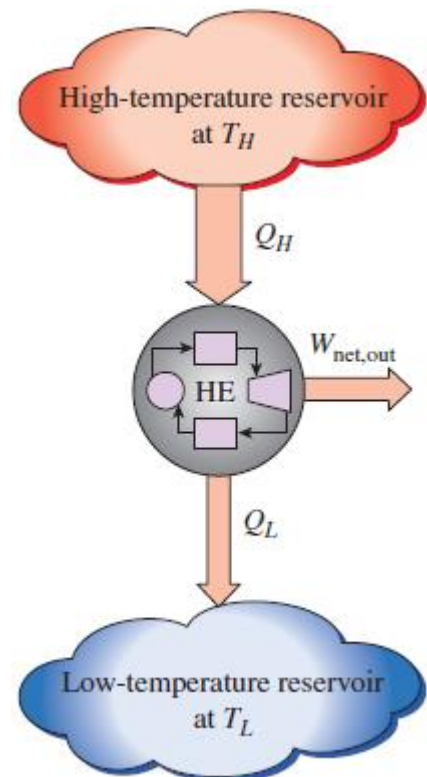
QH = magnitude of heat transfer between the cyclic device and the high temperature medium at temperature TH

QL = magnitude of heat transfer between the cyclic device and the low temperature medium at temperature TL

Notice that both QL and QH are defined as *magnitudes* and therefore are positive quantities. The direction of QH and QL is easily determined by inspection. Then, the net work output and thermal efficiency relations for any heat engine can also be expressed as

$$W_{\text{net,out}} = Q_H - Q_L$$

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_H} \quad \text{or} \quad \eta_{\text{th}} = 1 - \frac{Q_L}{Q_H}$$



The thermal efficiency of a heat engine is always less than unity since both Q_L and Q_H are defined as positive quantities.

Thermal efficiency is a measure of how efficiently a heat engine converts the heat that it receives to work. Heat engines are built for the purpose of converting heat to work, and engineers are constantly trying to improve the efficiencies of these devices since increased efficiency means less fuel consumption and thus lower fuel bills and less pollution.

EXAMPLE

Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

$$\dot{Q}_H = 80 \text{ MW} \quad \text{and} \quad \dot{Q}_L = 50 \text{ MW}$$

The net power output of this heat engine is

$$\dot{W}_{\text{net,out}} = \dot{Q}_H - \dot{Q}_L = (80 - 50) \text{ MW} = \mathbf{30 \text{ MW}}$$

Then the thermal efficiency is easily determined to be

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net,out}}}{\dot{Q}_H} = \frac{30 \text{ MW}}{80 \text{ MW}} = \mathbf{0.375} \text{ (or 37.5\%)}$$

The Second Law of Thermodynamics: Kelvin–Planck Statement

We have demonstrated earlier a heat engine must reject some heat to a low-temperature reservoir in order to complete the cycle. That is, no heat engine can convert all the heat it receives to useful work. The Kelvin–Planck statement of the second law of thermodynamics, which is expressed as follows:

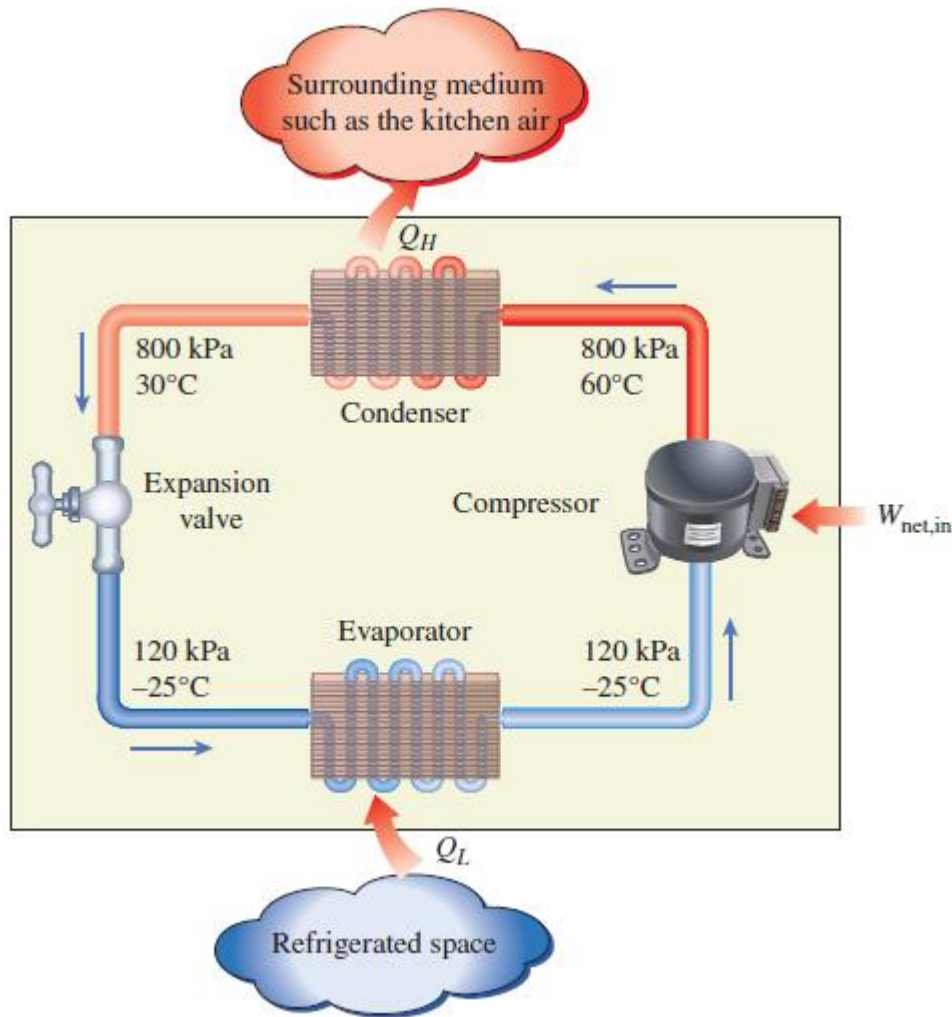
It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.

The Kelvin–Planck statement can also be expressed as *no heat engine can have a thermal efficiency of 100 percent.*

REFRIGERATORS AND HEAT PUMPS

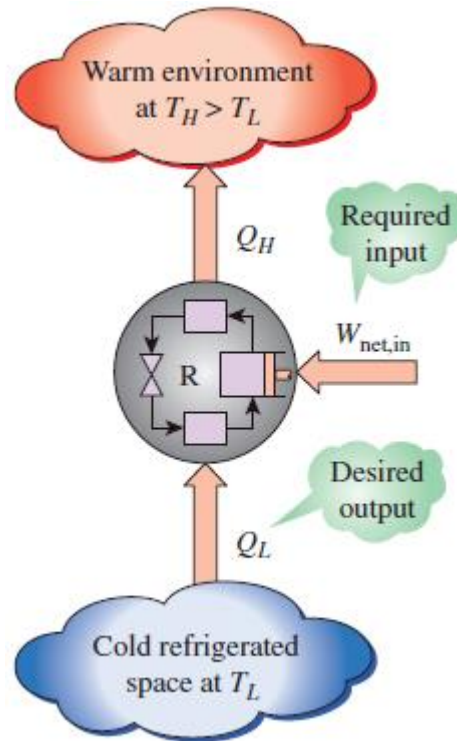
We all know from experience that heat is transferred in the direction of decreasing temperature, that is, from high-temperature mediums to low temperature ones. This heat transfer process occurs in nature without requiring any devices. The reverse process, however, cannot occur by itself.

The transfer of heat from a low-temperature medium to a high-temperature one requires special devices called **refrigerators**. Refrigerators, like heat engines, are cyclic devices. The working fluid used in the refrigeration cycle is called a **refrigerant**. The most frequently used refrigeration cycle is the *vapor-compression refrigeration cycle*, which involves four main components: a compressor, a condenser, an expansion valve, and an evaporator, as shown in Fig.



The refrigerant enters the compressor as a vapor and is compressed to the condenser pressure. It leaves the compressor at a relatively high temperature and cools down and condenses as it flows through the coils of the condenser by rejecting heat to the surrounding medium. It then enters a capillary tube where its pressure and temperature drop drastically due to the throttling effect. The low-temperature refrigerant then enters the evaporator, where it evaporates by absorbing heat from the refrigerated space. The cycle is completed as the refrigerant leaves the evaporator and reenters the compressor.

A refrigerator is shown schematically in Fig. Here Q_L is the magnitude of the heat removed from the refrigerated space at temperature T_L , Q_H is the magnitude of the heat rejected to the warm environment at temperature T_H , and $W_{net,in}$ is the net work input to the refrigerator.



Coefficient of Performance

The *efficiency* of a refrigerator is expressed in terms of the **coefficient of performance** (COP), denoted by COP_R . The objective of a refrigerator is to remove heat (Q_L) from the refrigerated space. To accomplish this objective, it requires a work input of $W_{net,in}$. Then the COP of a refrigerator can be expressed as

$$COP_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{net,in}}$$

This relation can also be expressed in rate form by replacing Q_L by $\dot{Q}_L$ and $W_{net,in}$ by $\dot{W}_{net,in}$.

The conservation of energy principle for a cyclic device requires that

$$W_{net,in} = Q_H - Q_L \quad (\text{kJ})$$

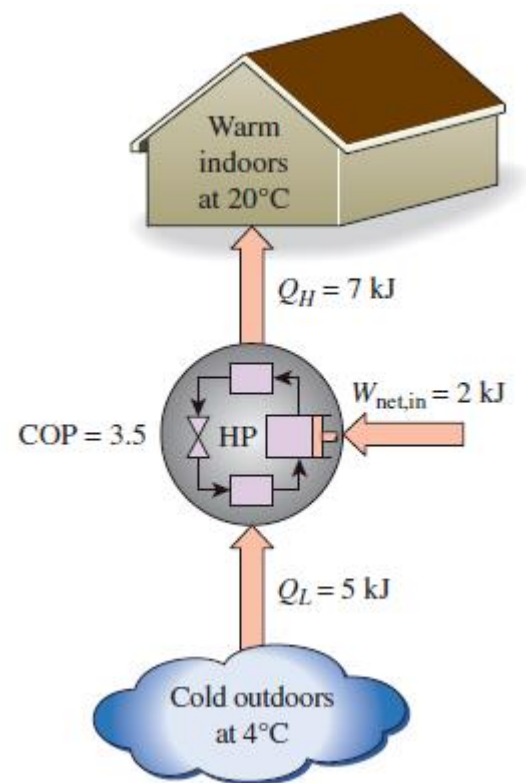
Then the COP relation becomes:

$$\text{COP}_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}$$

Notice that the value of COP_R can be *greater than unity*. That is, the amount of heat removed from the refrigerated space can be greater than the amount of work input.

Heat Pumps

Another device that transfers heat from a low-temperature medium to a high temperature one is the **heat pump**, shown schematically in Fig. Refrigerators and heat pumps operate on the same cycle but differ in their objectives. The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low temperature source, such as well water or cold outside air in winter, and supplying this heat to the high-temperature medium such as a house



The measure of performance of a heat pump is also expressed in terms of the **coefficient of performance** COP_{HP} , defined as

$$\text{COP}_{\text{HP}} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{\text{net,in}}}$$

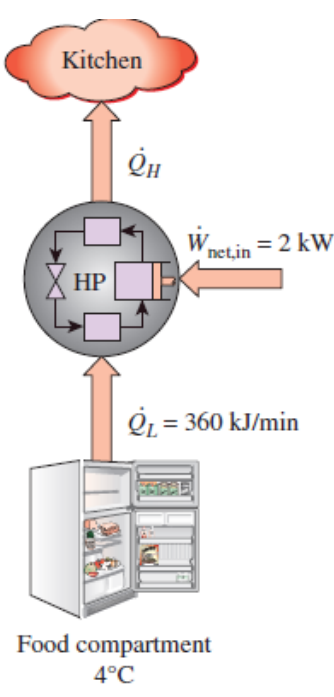
which can also be expressed as:

$$\text{COP}_{\text{HP}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H}$$

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$$

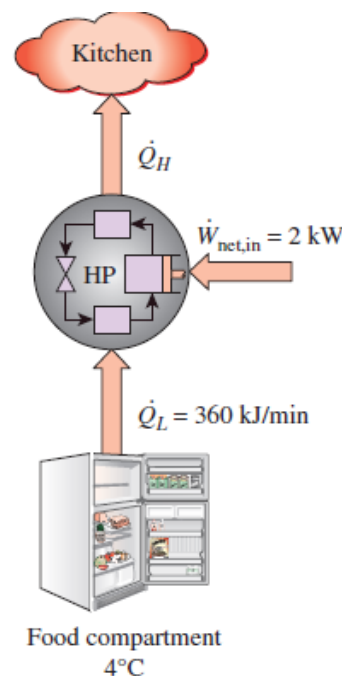
This relation implies that the coefficient of performance of a heat pump is always greater than unity since COP_{R} is a positive quantity.

EXAMPLE

The food compartment of a refrigerator, shown in Fig. , is maintained at 4°C by removing heat from it at a rate of 360 kJ/min. If the required power input to the refrigerator is 2 kW, determine (a) the coefficient of performance of the refrigerator and (b) the rate of heat rejection to the room that houses the refrigerator.

$$\text{COP}_{\text{R}} = \frac{\dot{Q}_L}{\dot{W}_{\text{net,in}}} = \frac{360 \text{ kJ/min}}{2 \text{ kW}} \left(\frac{1 \text{ kW}}{60 \text{ kJ/min}} \right) = 3$$

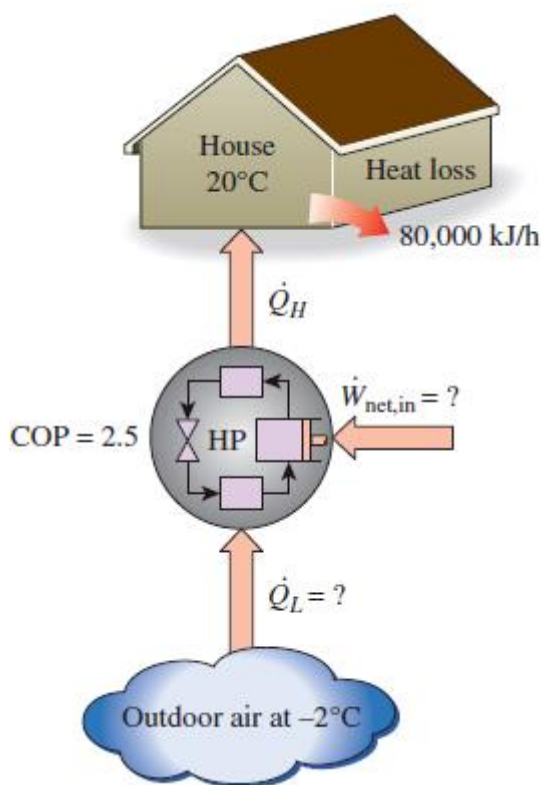
$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{net,in}} = 360 \text{ kJ/min} + (2 \text{ kW}) \left(\frac{60 \text{ kJ/min}}{1 \text{ kW}} \right) = 480 \text{ kJ/min}$$



EXAMPLE

A heat pump is used to meet the heating requirements of a house and maintain it at 20°C. On a day when the outdoor air temperature drops to −2°C, the house is estimated to lose heat at a rate of 80,000 kJ/h. If the heat pump under these conditions has a COP of 2.5, determine (a) the power consumed by the heat pump and (b) the rate at which heat is absorbed from the cold outdoor air.

$$\dot{W}_{\text{net,in}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}} = \frac{80,000 \text{ kJ/h}}{2.5} = \mathbf{32,000 \text{ kJ/h}} \text{ (or 8.9 kW)}$$



$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net,in}} = (80,000 - 32,000) \text{ kJ/h} = \mathbf{48,000 \text{ kJ/h}}$$

The Second Law of Thermodynamics: Clausius Statement

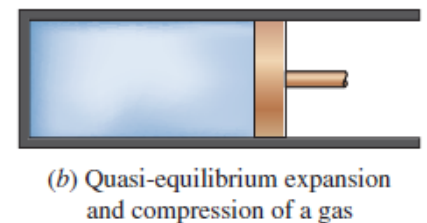
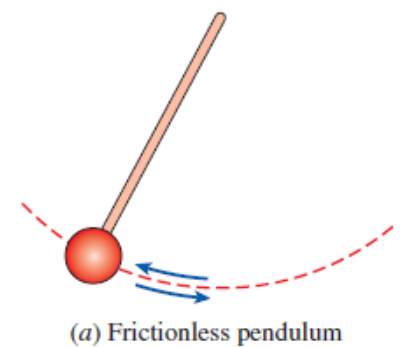
There are two classical statements of the second law—the Kelvin–Planck statement, which is related to heat engines, and the Clausius statement, which is related to refrigerators or heat pumps. The Clausius statement is expressed as follows:

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.

REVERSIBLE AND IRREVERSIBLE PROCESSES

The second law of thermodynamics states that no heat engine can have an efficiency of 100 percent. Then one may ask, what is the highest efficiency that a heat engine can possibly have? Before we can answer this question, we need to define an idealized process first, which is called the *reversible process*.

The processes that were discussed at the beginning of this chapter occurred in a certain direction. Once having taken place, these processes cannot reverse themselves spontaneously and restore the system to its initial state. For this reason, they are classified as *irreversible processes*. Once a cup of hot coffee cools, it will not heat up by retrieving the heat it lost from the surroundings. If it could, the surroundings, as well as the system (coffee), would be restored to their original condition, and this would be a reversible process.



A **reversible process** is defined as a *process that can be reversed without leaving any trace on the surroundings*

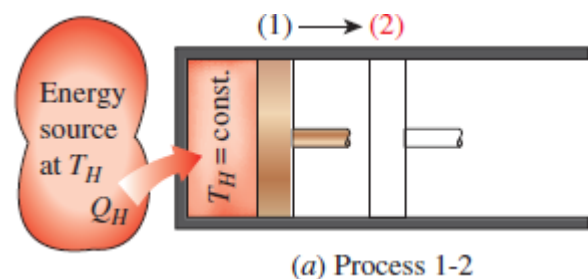
That is, both the system *and* the surroundings are returned to their initial states at the end of the reverse process. This is possible only if the net heat *and* net work exchange between the system and the surroundings is zero for the combined (original and reverse) process. Processes that are not reversible are called **irreversible processes**. Reversible processes actually do not occur in nature. That is, all the processes occurring in nature are irreversible.

THE CARNOT ENGINE

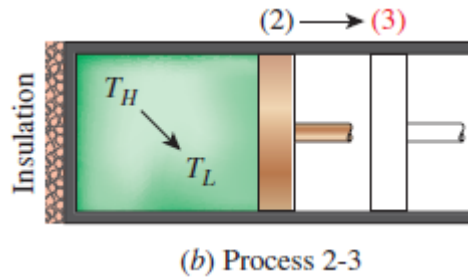
The heat engine that operates the most efficiently between a high-temperature reservoir and a low-temperature reservoir is the *Camot engine*. It is an ideal engine that uses reversible processes to form its cycle of operation; thus it is also called a *reversible engine*. We will determine the efficiency of the Carnot engine and also evaluate its reverse operation. The Carnot engine is very useful, since its efficiency establishes the maximum possible efficiency of any real engine.

The cycle associated with the Carnot engine is shown in Fig, using an ideal gas as the working substance. It is composed of the following four reversible processes:

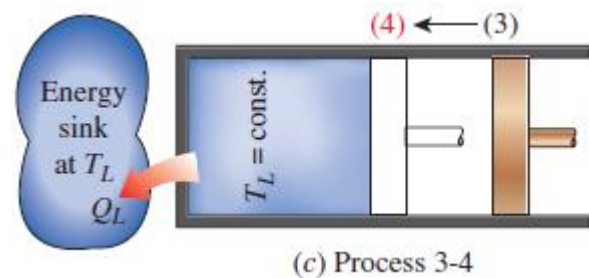
1 - 2: Heat is transferred reversibly from the high-temperature reservoir at the constant temperature T_H . The piston in the cylinder is withdrawn and the volume increases.



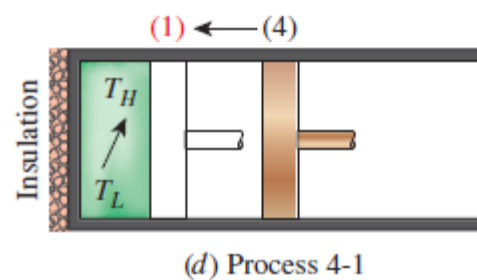
2 - 3: *An adiabatic reversible expansion.* The cylinder is completely insulated so that no heat transfer occurs during this reversible process. The piston continues to be withdrawn, with the volume increasing.



3 - 4: An isothermal compression. Heat is transferred reversibly to the low-temperature reservoir at the constant temperature T_L . The piston compresses the working substance, with the volume decreasing.

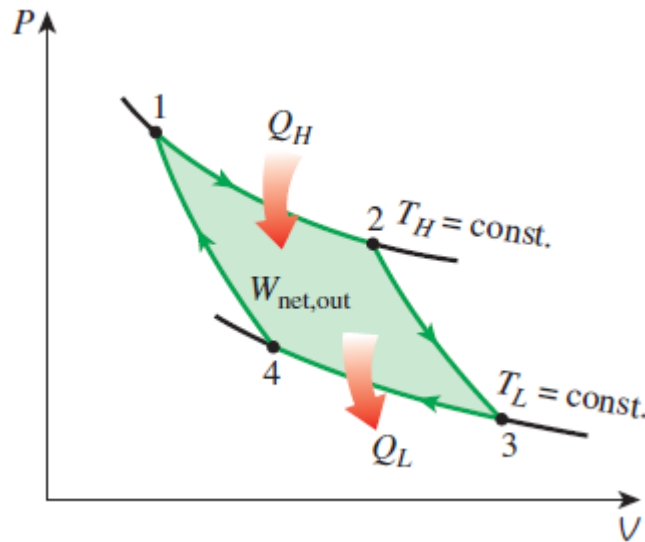


4 - 1: An adiabatic reversible compression. The completely insulated cylinder allows no heat transfer during this reversible process. The piston continues to compress the working substance until the original volume, temperature, and pressure are reached, thereby completing the cycle.



The P - V diagram of this cycle is shown in Fig. Remembering that on a P - V diagram the area under the process curve represents the boundary work, we see that the area under curve 1-2-3 is the work done by the gas during the expansion part of the cycle, and the area under curve 3-4-1 is the work done on the gas during the compression

part of the cycle. The area enclosed by the path of the cycle (area 1-2-3-4-1) is the difference between these two and represents the net work done during the cycle.



Being a reversible cycle, the Carnot cycle is the most efficient cycle operating between two specified temperature limits. Even though the Carnot cycle cannot be achieved in reality.

Applying the first law to the cycle, we note that

$$Q_H - Q_L = W_{\text{net}}$$

where Q_L is assumed to be a positive value for the heat transfer to the low-temperature reservoir.

This allows us to write the thermal efficiency for the Carnot cycle as:

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

where Q_H is heat transferred to the heat engine from a high-temperature reservoir at T_H , and Q_L is heat rejected to a low-temperature reservoir at T_L .

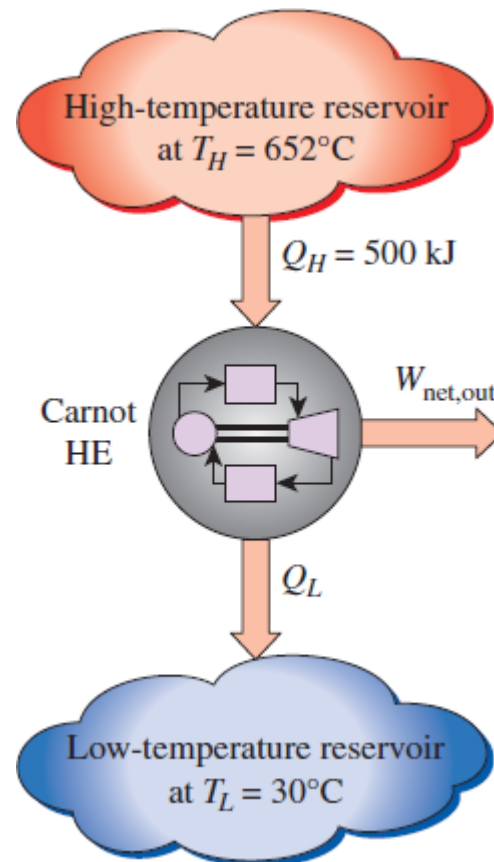
For reversible heat engines, the heat transfer ratio in the above relation can be replaced by the ratio of the absolute temperatures of the two reservoirs. Then the efficiency of a Carnot engine, or any reversible heat engine, becomes

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

This relation is often referred to as the **Carnot efficiency**. *This is the highest efficiency a heat engine operating between the two thermal energy reservoirs at temperatures T_L and T_H can have.*

EXAMPLE

A Carnot heat engine, shown in Fig. , receives 500 kJ of heat per cycle from a high-temperature source at 652°C and rejects heat to a low-temperature sink at 30°C. Determine (a) the thermal efficiency of this Carnot engine and (b) the amount of heat rejected to the sink per cycle.



(a) The Carnot heat engine is a reversible heat engine,

$$\eta_{\text{th}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(30 + 273) \text{ K}}{(652 + 273) \text{ K}} = \mathbf{0.672}$$

(b) The amount of heat rejected Q_L by this reversible heat engine is easily determined

$$Q_{L,} = \frac{T_L}{T_H} Q_{H,\text{rev}} = \frac{(30 + 273) \text{ K}}{(652 + 273) \text{ K}} (500 \text{ kJ}) = \mathbf{164 \text{ kJ}}$$

THE CARNOT REFRIGERATOR AND HEAT PUMP

The Carnot engine, when operated in reverse, becomes a heat pump or a refrigerator, depending on the desired heat transfer. The coefficient of performance for a heat pump becomes:

$$\text{COP}_{\text{HP}} = \frac{Q_H}{W_{\text{net}}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - T_L/T_H}$$

The coefficient of performance for a refrigerator takes the form:

$$\text{COP}_{\text{R}} = \frac{Q_L}{W_{\text{net}}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{T_H/T_L - 1}$$

EXAMPLE A Carnot engine operates between two temperature reservoirs maintained at **200 °C** and **20 °C**, respectively. If the desired output of the engine is **15 kW**, as shown in Fig., determine the heat transfer from the high-temperature reservoir and the heat transfer to the low-temperature reservoir.

The efficiency of a Carnot engine is given by

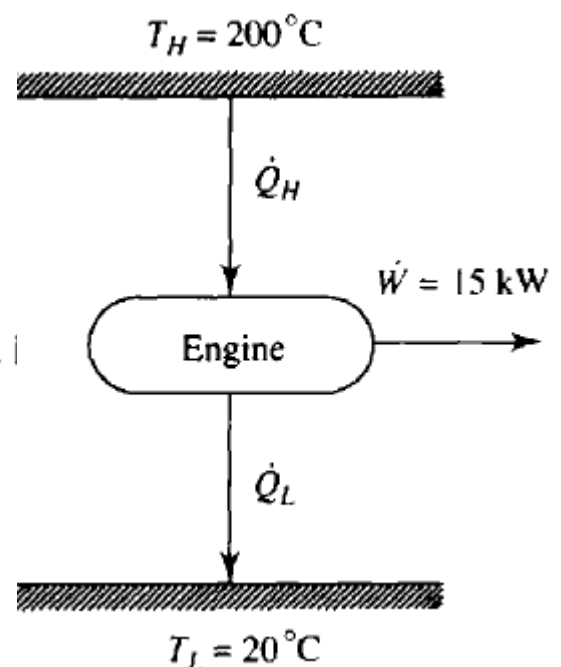
$$\eta = \frac{\dot{W}}{\dot{Q}_H} = 1 - \frac{T_L}{T_H}$$

This gives, converting the temperatures to absolute temperatures,

$$\dot{Q}_H = \frac{\dot{W}}{1 - T_L/T_H} = \frac{15}{1 - 293/473} = 39.42 \text{ kW}$$

Using the first law, we have

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 39.42 - 15 = 24.42 \text{ kW.}$$



EXAMPLE A refrigeration unit is cooling a space to -5°C by rejecting energy to the atmosphere at 20°C . It is desired to reduce the temperature in the refrigerated space to -25°C . Calculate the minimum percentage increase in work required, by assuming a Carnot refrigerator, for the same amount of energy removed.

For a Carnot refrigerator we know that

$$\text{COP} = \frac{Q_L}{W} = \frac{1}{T_H/T_L - 1}$$

For the first situation we have

$$W_1 = Q_L(T_H/T_L - 1) = Q_L(293/268 - 1) = 0.0933Q_L$$

For the second situation there results

$$W_2 = Q_L(293/248 - 1) = 0.181Q_L$$

The percentage increase in work is then

$$\frac{W_2 - W_1}{W_1} = \left(\frac{0.181Q_L - 0.0933Q_L}{0.0933Q_L} \right) (100) = 94.0\%$$

A refrigerator is rated at a COP of 4. The refrigerated space that it cools requires a peak cooling rate of 30 000 kJ/h. What size electrical motor (rated in horsepower) is required for the refrigerator?

The definition of the COP for a refrigerator is $\text{COP} = \dot{Q}_L / \dot{W}_{\text{net}}$. The net power required is then

$$\dot{W}_{\text{net}} = \frac{\dot{Q}_L}{\text{COP}} = \frac{30\,000/3600}{4} = 2.083 \text{ kW} \quad \text{or } 2.793 \text{ hp}$$

A Carnot heat engine produces 10 hp by transferring energy between two reservoirs at 40°F and 212°F . Calculate the rate of heat transfer from the high-temperature reservoir.

The engine efficiency is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{500}{672} = 0.2560$$

A heat engine operates on a Carnot cycle with an efficiency of 75 percent. What COP would a refrigerator operating on the same cycle have? The low temperature is 0°C.

The efficiency of the heat engine is given by $\eta = 1 - T_L/T_H$. Hence,

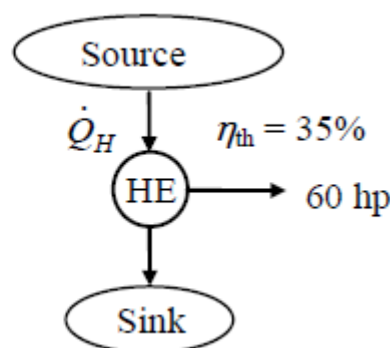
$$T_H = \frac{T_L}{1 - \eta} = \frac{273}{1 - 0.75} = 1092 \text{ K}$$

The COP for the refrigerator is then

$$\text{COP} = \frac{T_L}{T_H - T_L} = \frac{273}{1092 - 273} = 0.3333$$

6–18 The thermal efficiency of a general heat engine is 35 percent, and it produces 60 hp. At what rate is heat transferred to this engine, in kJ/s?

$$\dot{Q}_H = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}} = \frac{60 \text{ hp}}{0.35} \left(\frac{0.7457 \text{ kJ/s}}{1 \text{ hp}} \right) = \mathbf{128 \text{ kJ/s}}$$



6–19 A 600-MW steam power plant, which is cooled by a nearby river, has a thermal efficiency of 40 percent. Determine the rate of heat transfer to the river water. Will the actual heat transfer rate be higher or lower than this value? Why?

$$\dot{Q}_H = \frac{\dot{W}_{\text{net,out}}}{\eta_{\text{th}}} = \frac{600 \text{ MW}}{0.4} = 1500 \text{ MW}$$

The rate of heat transfer to the river water is determined from the first law relation for a heat engine,

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net,out}} = 1500 - 600 = \mathbf{900 \text{ MW}}$$

In reality the amount of heat rejected to the river will be **lower** since part of the heat will be lost to the surrounding air from the working fluid as it passes through the pipes and other components.

6–20 A heat engine that pumps water out of an underground mine accepts 700 kJ of heat and produces 250 kJ of work. How much heat does it reject, in kJ?

Applying the first law to the heat engine gives

$$Q_L = Q_H - W_{\text{net}} = 700 \text{ kJ} - 250 \text{ kJ} = \mathbf{450 \text{ kJ}}$$

6–21 A heat engine with a thermal efficiency of 45 percent rejects 500 kJ/kg of heat. How much heat does it receive?

$$w_{\text{net}} = \eta_{\text{th}} q_H$$

$$q_H - q_L = \eta_{\text{th}} q_H$$

which when rearranged gives

$$q_H = \frac{q_L}{1 - \eta_{\text{th}}} = \frac{500 \text{ kJ/kg}}{1 - 0.45} = \mathbf{909 \text{ kJ/kg}}$$

6–22 A steam power plant with a power output of 150 MW consumes coal at a rate of 60 tons/h. If the heating value of the coal is 30,000 kJ/kg, determine the overall efficiency of this plant.

The rate of heat supply to this power plant is

$$\begin{aligned} \dot{Q}_H &= \dot{m}_{\text{coal}} q_{\text{HV, coal}} \\ &= (60,000 \text{ kg/h})(30,000 \text{ kJ/kg}) = 1.8 \times 10^9 \text{ kJ/h} \\ &= 500 \text{ MW} \end{aligned}$$

Then the thermal efficiency of the plant becomes

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net, out}}}{\dot{Q}_H} = \frac{150 \text{ MW}}{500 \text{ MW}} = 0.300 = \mathbf{30.0\%}$$

6–23 An automobile engine consumes fuel at a rate of 22 L/h and delivers 55 kW of power to the wheels. If the fuel has a heating value of 44,000 kJ/kg and a density of 0.8 g/cm³, determine the efficiency of this engine.

The mass consumption rate of the fuel is

$$\dot{m}_{\text{fuel}} = (\rho \dot{V})_{\text{fuel}} = (0.8 \text{ kg/L})(22 \text{ L/h}) = 17.6 \text{ kg/h}$$

The rate of heat supply to the car is

$$\begin{aligned}\dot{Q}_H &= \dot{m}_{\text{coal}} q_{\text{HV, coal}} \\ &= (17.6 \text{ kg/h})(44,000 \text{ kJ/kg}) \\ &= 774,400 \text{ kJ/h} = 215.1 \text{ kW}\end{aligned}$$

Then the thermal efficiency of the car becomes

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net, out}}}{\dot{Q}_H} = \frac{55 \text{ kW}}{215.1 \text{ kW}} = 0.256 = \mathbf{25.6\%}$$

6–38 Determine the COP of a refrigerator that removes heat from the food compartment at a rate of 5040 kJ/h for each kW of power it consumes. Also, determine the rate of heat rejection to the outside air.

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net, in}}} = \frac{5040 \text{ kJ/h}}{1 \text{ kW}} \left(\frac{1 \text{ kW}}{3600 \text{ kJ/h}} \right) = \mathbf{1.4}$$

The rate of heat rejection to the surrounding air, per kW of power consumed, is determined from the energy balance,

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{net, in}} = (5040 \text{ kJ/h}) + (1 \times 3600 \text{ kJ/h}) = \mathbf{8640 \text{ kJ/h}}$$

6–39 Determine the COP of a heat pump that supplies energy to a house at a rate of 8000 kJ/h for each kW of electric power it draws. Also, determine the rate of energy absorption from the outdoor air.

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net, in}}} = \frac{8000 \text{ kJ/h}}{1 \text{ kW}} \left(\frac{1 \text{ kW}}{3600 \text{ kJ/h}} \right) = \mathbf{2.22}$$

The rate of heat absorption from the surrounding air, per kW of power consumed, is determined from the energy balance,

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net, in}} = (8,000 \text{ kJ/h}) - (1)(3600 \text{ kJ/h}) = \mathbf{4400 \text{ kJ/h}}$$

6–42 An air conditioner removes heat steadily from a house at a rate of 750 kJ/min while drawing electric power at a rate of 6 kW. Determine (a) the COP of this air conditioner and (b) the rate of heat transfer to the outside air.

(a) The coefficient of performance of the air-conditioner (or refrigerator) is determined from its definition,

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net,in}}} = \frac{750 \text{ kJ/min}}{6 \text{ kW}} \left(\frac{1 \text{ kW}}{60 \text{ kJ/min}} \right) = \mathbf{2.08}$$

b) The rate of heat discharge to the outside air is determined from the energy balance,

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{net,in}} = (750 \text{ kJ/min}) + (6 \times 60 \text{ kJ/min}) = \mathbf{1110 \text{ kJ/min}}$$

6-43 A food department is kept at -12°C by a refrigerator in an environment at 30°C . The total heat gain to the food department is estimated to be 3300 kJ/h and the heat rejection in the condenser is 4800 kJ/h. Determine the power input to the compressor, in kW and the COP of the refrigerator.

$$\begin{aligned}\dot{W}_{\text{in}} &= \dot{Q}_H - \dot{Q}_L \\ &= 4800 - 3300 = 1500 \text{ kJ/h} \\ &= (1500 \text{ kJ/h}) \left(\frac{1 \text{ kW}}{3600 \text{ kJ/h}} \right) = \mathbf{0.417 \text{ kW}}\end{aligned}$$

The COP is

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{3300 \text{ kJ/h}}{1500 \text{ kJ/h}} = \mathbf{2.2}$$

6-78 A heat engine operates between a source at 477°C and a sink at 25°C . If heat is supplied to the heat engine at a steady rate of 65,000 kJ/min, determine the maximum power output of this heat engine.

The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{298 \text{ K}}{(477 + 273) \text{ K}} = 0.600 \text{ or } 60.0\%$$

Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{\text{net,out}} = \eta_{\text{th}} \dot{Q}_H = (0.600)(65,000 \text{ kJ/min}) = 39,000 \text{ kJ/min} = \mathbf{653 \text{ kW}}$$

6–81 A heat engine receives heat from a heat source at 1200°C and has a thermal efficiency of 40 percent. The heat engine does maximum work equal to 500 kJ. Determine the heat supplied to the heat engine by the heat source, the heat rejected to the heat sink, and the temperature of the heat sink.

Applying the definition of the thermal efficiency and an energy balance to the heat engine, the unknown values are determined as follows:

$$Q_H = \frac{W_{\text{net}}}{\eta_{\text{th}}} = \frac{500 \text{ kJ}}{0.4} = \mathbf{1250 \text{ kJ}}$$

$$Q_L = Q_H - W_{\text{net}} = 1250 - 500 = \mathbf{750 \text{ kJ}}$$

$$\eta_{\text{th,max}} = 1 - \frac{T_L}{T_H} \longrightarrow 0.40 = 1 - \frac{T_L}{(1200 + 273) \text{ K}} \longrightarrow T_L = 883.8 \text{ K} = \mathbf{611^{\circ}\text{C}}$$

6–91 A Carnot refrigerator operates in a room in which the temperature is 22°C and consumes 2 kW of power when operating. If the food compartment of the refrigerator is to be maintained at 3°C , determine the rate of heat removal from the food compartment.

The coefficient of performance of a Carnot refrigerator depends on the temperature limits in the cycle only, and is determined from

$$\text{COP}_{\text{R,C}} = \frac{1}{(T_H/T_L) - 1} = \frac{1}{(22 + 273\text{K})/(3 + 273\text{K}) - 1} = 14.5$$

The rate of heat removal from the refrigerated space is determined from the definition of the coefficient of performance of a refrigerator,

$$\dot{Q}_L = \text{COP}_{\text{R}} \dot{W}_{\text{net,in}} = (14.5)(2 \text{ kW}) = 29.0 \text{ kW} = \mathbf{1740 \text{ kJ/min}}$$

6–92 An air-conditioning system operating on the reversed Carnot cycle is required to transfer heat from a house at a rate of 750 kJ/min to maintain its temperature at 24°C . If the outdoor air temperature is 35°C , determine the power required to operate this air-conditioning system.

$$\text{COP}_{R,C} = \frac{1}{(T_H/T_L) - 1} = \frac{1}{(35 + 273 \text{ K})/(24 + 273 \text{ K}) - 1} = 27.0$$

The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

$$\dot{W}_{\text{net,in}} = \frac{\dot{Q}_L}{\text{COP}_{R,\text{max}}} = \frac{750 \text{ kJ/min}}{27.0} = 27.8 \text{ kJ/min} = \mathbf{0.463 \text{ kW}}$$

6–93 An inventor claims to have developed a heat pump that produces a 200-kW heating effect for a 293 K heated zone while only using 75 kW of power and a heat source at 273 K. Justify the validity of this claim.

Applying the definition of the heat pump coefficient of performance,

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net,in}}} = \frac{200 \text{ kW}}{75 \text{ kW}} = 2.67$$

The maximum COP of a heat pump operating between the same temperature limits is

$$\text{COP}_{\text{HP,max}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (273 \text{ K})/(293 \text{ K})} = 14.7$$

Since the actual COP is less than the maximum COP, the claim is **valid**.

6–94 A heat pump operates on a Carnot heat pump cycle with a COP of 8.7. It keeps a space at 24°C by consuming 2.15 kW of power. Determine the temperature of the reservoir from which the heat is absorbed and the heating load provided by the heat pump.

The temperature of the low-temperature reservoir is

$$\text{COP}_{\text{HP,max}} = \frac{T_H}{T_H - T_L} \longrightarrow 8.7 = \frac{297 \text{ K}}{(297 - T_L) \text{ K}} \longrightarrow T_L = \mathbf{263 \text{ K}}$$

The heating load is

$$\text{COP}_{\text{HP,max}} = \frac{\dot{Q}_H}{\dot{W}_{\text{in}}} \longrightarrow 8.7 = \frac{\dot{Q}_H}{2.15 \text{ kW}} \longrightarrow \dot{Q}_H = \mathbf{18.7 \text{ kW}}$$

6–100 A refrigerator operating on the reversed Carnot cycle has a measured work input of 200 kW and heat rejection of 2000 kW to a heat reservoir at 27°C. Determine the cooling load supplied to the refrigerator, in kW, and the temperature of the heat source, in °C.

Applying the definition of the refrigerator coefficient of performance,

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net,in}} = 2000 - 200 = \mathbf{1800 \text{ kW}}$$

Applying the definition of the heat pump coefficient of performance,

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net,in}}} = \frac{1800 \text{ kW}}{200 \text{ kW}} = 9$$

The temperature of the heat source is determined from

$$\text{COP}_{R,\text{max}} = \frac{T_L}{T_H - T_L} \longrightarrow 9 = \frac{T_L}{300 - T_L} \longrightarrow T_L = 270 \text{ K} = \mathbf{-3^\circ\text{C}}$$

Entropy

Entropy which is *a measure of* molecular disorder, *or randomness* *molecular* as a system becomes more disordered, the positions of the molecules become less predictable and the entropy increases. Thus, it is not surprising that the entropy of a substance is lowest in the solid phase and highest in the gas phase.

Another important inequality that has major consequences in thermodynamics is the **Clausius inequality** and is expressed

$$\oint \frac{\delta Q}{T} \leq 0$$

Clausius realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property **entropy**. It is designated S and is defined as

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

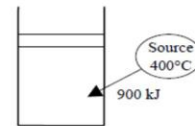
Entropy per unit mass, designated s , is an intensive property and has the unit kJ/kg·K.

The entropy change of a system during a process can be determined by integrating

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)$$

Ex: During the isothermal heat addition process of a Carnot cycle, 900 kJ of heat is added to the working fluid from a source at 400°C. Determine (a) the entropy change of the working fluid, (b) the entropy change of the source, and (c) the total entropy change for the process.

$$\Delta S = \frac{Q}{T}$$



$$\Delta S_{\text{fluid}} = \frac{Q_{\text{fluid}}}{T_{\text{fluid}}} = \frac{Q_{\text{in, fluid}}}{T_{\text{fluid}}} = \frac{900 \text{ kJ}}{673 \text{ K}} = \mathbf{1.337 \text{ kJ/K}}$$

$$(b) \quad \Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = -\frac{Q_{\text{out, source}}}{T_{\text{source}}} = -\frac{900 \text{ kJ}}{673 \text{ K}} = \mathbf{-1.337 \text{ kJ/K}}$$

(c) Thus the total entropy change of the process is

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{fluid}} + \Delta S_{\text{source}} = 1.337 - 1.337 = \mathbf{0}$$

The change in entropy in the reversible process.

1- Iso-choric process (constant volume)

a/for perfect gas

$$\delta Q = \delta W + \delta U \quad \text{wher } \delta W = 0$$

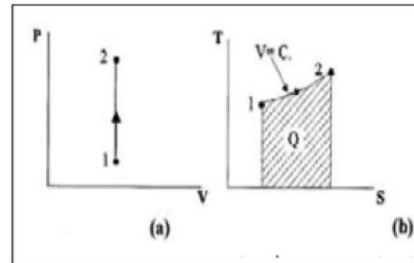
$$\delta Q = C_v \delta T \quad \div T$$

$$\frac{\delta Q}{T} = C_v \frac{\delta T}{T}$$

$$\int_{s_1}^{s_2} ds = C_v \int_{T_1}^{T_2} \frac{\delta T}{T}$$

$$\Delta s = C_v * \ln \frac{T_2}{T_1}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$



$$\frac{T_2}{T_1} = \frac{P_2}{P_1}$$

$$\therefore \Delta s = C_v * \ln \frac{P}{P_1}$$

b/for steam

$$\Delta s = s_2 - s_1 \quad \frac{kJ}{kg \cdot K}$$

2- Iso-baric process (constant pressure)

A/for perfect gas

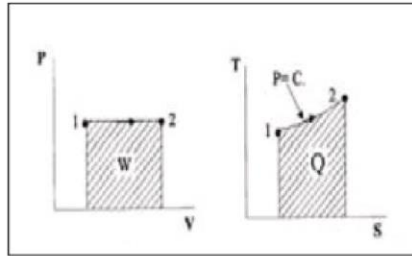
$$\delta Q = \delta W + \delta U$$

$$\delta Q = P\delta v + C_v\delta T \quad \div T$$

$$\frac{\delta Q}{T} = P \frac{\delta v}{T} + C_v \frac{\delta T}{T}$$

$$Pv = RT$$

$$\frac{P}{T} = \frac{R}{V}$$



$$\frac{\delta Q}{T} = R \frac{\delta v}{v} + C_v \frac{\delta T}{T}$$

$$\int_{s_1}^{s_2} \delta s = R \int_{v_1}^{v_2} \frac{\delta v}{v} + C_v \int_{T_1}^{T_2} \frac{\delta T}{T}$$

$$\Delta s = R \ln \frac{v_2}{v_1} + C_v \ln \frac{T_2}{T_1}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Delta s = R \ln \frac{v_2}{v_1} + C_v \ln \frac{v_2}{v_1}$$

$$\Delta s = (R + C_v) \ln \frac{v_2}{v_1}$$

$$\Delta s = C_p \ln \frac{v_2}{v_1} \quad \frac{kJ}{kg.K}$$

b/for steam

$$\Delta s = s_2 - s_1 \quad \frac{kJ}{kg.K}$$

Ex: Two kg of air is heated at constant pressure of 200 kPa to 500°C . Calculate the entropy change if the initial volume is $0.8m^3$.

$$T_1 = \frac{P_1 V_1}{m R} = \frac{200 * 0.8}{2 * 0.287} = 278.7 \text{ K}$$

$$\Delta S = m \left[c \ln \left(\frac{T_2}{T_1} \right) \right]$$

$$\Delta S = 2 * 1.046 * \ln \left(\frac{773}{278.7} \right) = 2.1341 \text{ kJ/K},$$

3) Constant Temperature process (Isothermal Process)

A/for perfect gas

$$\delta Q = \delta W + \delta U$$

$$\delta Q = P \delta v + C_v \delta T \quad \div T$$

$$\frac{\delta Q}{T} = P \frac{\delta v}{T} + C_v \frac{\delta T}{T}$$

$$Pv = RT$$

$$\frac{P}{T} = \frac{R}{V}$$

Science $T_1 = T_2$, $\delta T = 0$

$$\frac{\delta Q}{T} = R \frac{\delta v}{v} -$$

$$\int_{s_1}^{s_2} \delta s = R \int_{v_1}^{v_2} \frac{\delta v}{v} -$$

$$\Delta s = R \ln \frac{v_2}{v_1} -$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\Delta S = R \ln \frac{p_1}{p_2}$$

b/for steam

$$\Delta s = s_2 - s_1 \quad \frac{kJ}{kg.K}$$

Ex: (0.0368) kg of gas has volume (0.03 m³) and pressure (1.05 bar) and temperature (15°C) . This gas is compressed isothermally to (4.2 bar).

Calculate the change in entropy and work don. Take R=0.297 kJ/kg.K

$$\Delta S = -mR \ln \frac{P_2}{P_1}$$

$$\Delta S = -0.0368 * 0.29 \ln \frac{4.2}{1.05}$$

$$\Delta S = -0.01516 \frac{kJ}{kg}$$

$$Q = T \Delta S$$

$$Q = 288(-0.01516)$$

$$Q = -4.37 \text{ kJ}$$

$$W = Q = -4.37 \text{ kJ}$$

$$W = mRT \ln \frac{V_2}{V_1}$$

$$V_2 = \frac{mRT_2}{P_2}$$

$$V_2 = \frac{0.036 * 0.297 * 288}{420}$$

$$V_2 = 0.007 \text{ m}^3$$

$$W = 0.036 * 0.297 * 288 \ln \frac{0.007}{0.03}$$

$$\therefore W = -4.37 \text{ kJ}$$

$$\Delta S = \frac{Q}{T}$$

$$\Delta S = \frac{-4.37}{288} = -0.01516 \text{ kJ/kg}$$