Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

Learning package
In
Mass Transfer
For
second year students

By

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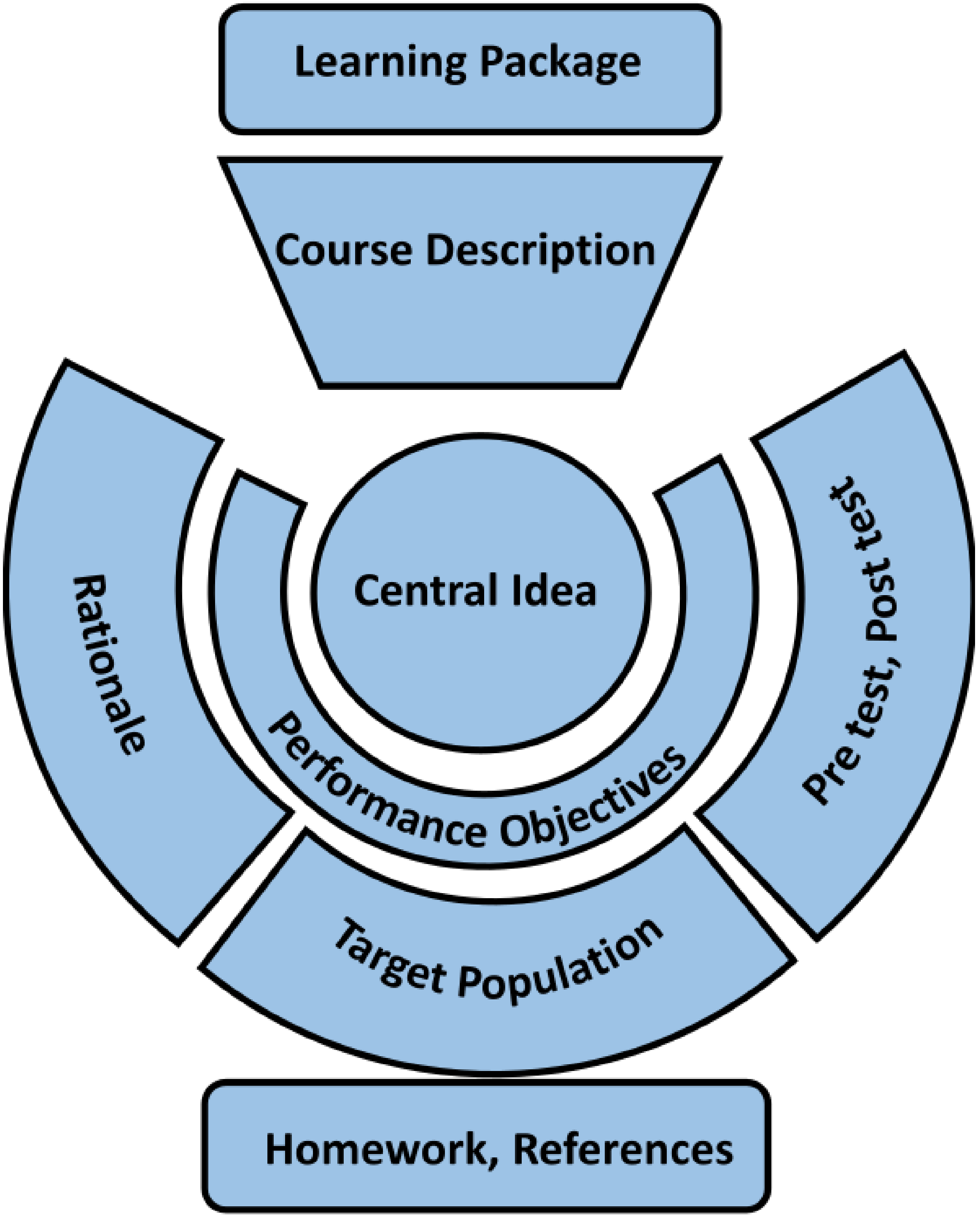
Dep. Of Chemical Industrial Techniques

2025


Learning Package
Course Description
Central Idea
Rationale
Performance Objectives
Pre test, Post test
Target Population
Homework, References

COURSE SPECIFICATION

The term mass transfer is used to denote the transference of a component in a mixture from a region where its concentration is high to a region where the concentration is lower Mass transfer process can take place in a gas or vapour or in a liquid, and it can result from the random velocities of the molecules (molecular diffusion) or from the circulating or eddy currents present in a turbulent fluid (eddy diffusion .

1. Teaching Institution	Technological Institute of Basra
2. University Department/Centre	Department of Chemical Industries Techniques
3. Course title/code	
4. Program(s) to which it contributes	
5. Modes of Attendance offered	In-person in classrooms and online when necessary
6. Semester/Year	2024-2025
7. Number of hours of tuition (total)	60 hours-"4hours weekly, the curriculum is semi-annual."
8. Date of production/revision of this Specification	31/5/2025

9· Learning Outcomes, Teaching, Learning, and Assessment Method

A- Knowledge and Understanding/

1. It is frequently necessary to separate a mixture into its components, and, in a physical process, differences in a particular property are exploited as the basis for the separation process. To explain how heat energy may be transferred.
2. Consideration will be given to the basic principles underlying mass transfer both with and without chemical reaction, and to the models which have been proposed to enable the rates of transfer to be calculated.
3. Mass transfer takes place across a phase boundary where the concentrations on either side of the interface are related by the phase equilibrium relationship.

B. Subject-specific skills

- **Understanding Separation Processes**
  - Analyze and design separation techniques such as distillation, absorption, extraction, leaching, and drying.
- **Application of Mass Transfer Principles**
  - Apply laws like **Fick's Law** to model and solve diffusion problems in gases, liquids, and solids.
- **Design and Analysis of Equipment**
  - Calculate and optimize parameters for mass transfer equipment (e.g., packed columns, tray towers, evaporators, dryers).
- **Equilibrium Analysis**
  - Interpret phase equilibrium data (e.g., vapor-liquid, liquid-liquid, solid-liquid) for process development and simulation.
- **Mass Balance Calculations**
  - Perform mass and component balances for steady-state and unsteady-state systems.
- **Rate-Based Modeling**
  - Develop and use rate-based models to predict mass transfer rates and process efficiency.
- **Problem-Solving and Critical Thinking**

  - Solve complex engineering problems involving multiple phases, non-ideal behavior, and coupled heat and mass transfer.

- **Use of Graphical and Analytical Methods**

  - Employ tools like McCabe-Thiele and Ponchon-Savarit diagrams for stage calculations in distillation and absorption.

- **Equipment Selection and Scale-Up**

  - Choose appropriate separation techniques and scale them up from lab to the industrial scale.

- **Safety and Environmental Considerations**

  - Understand the implications of mass transfer operations on safety, emissions, and sustainability.

Teaching and Learning Methods

In-person lectures, online lectures, quizzes, assignments, and practical work in the laboratory and workshop.

Assessment methods

Oral tests, Written tests, Midterm exams, Final exams, Daily assessment, Electronic (or online) tests.

C. Thinking Skills

1. Continuous evaluation of student behavior during class hours.
2. Direct discussions with students during lectures.
3. Monitoring behavioral changes in some students and following up with them.
4. Implementing mini-projects within the lecture sessions.

Course structure

Week	Hours	Intended Learning Outcomes	Teaching Method	Assessment Method
First	4	Distillation, vapor–liquid equilibrium, relative volatility, methods of obtaining equilibrium curve	Lecture	Quizzes and Assignments
Second	4	Methods of distillation: Differential, Flash, and Rectification	Lecture	Quizzes and Assignments
Third	4	Fractionating columns, calculation of the number of theoretical plates, operating lines	Lecture	Quizzes and Assignments
Fourth	4	Lewis-Sorel and McCabe-Thiele methods for calculating the number of theoretical plates	Lecture	Quizzes and Assignments
Fifth	4	Reflux ratio: importance and minimum value. Batch Distillation: operation at constant product composition	Lecture	Quizzes and Assignments
Sixth	4	Operation at constant reflux ratio, isotropic, extractive, and steam distillation. Mass transfer by diffusion: Fick's law, methods, types of diffusion, and diffusion theories	Lecture	Quizzes and Assignments
Seventh	4	Gas absorption: equilibrium, relation to film theory, solvent selection, operating line equation, column height calculation, absorption in plate towers, theoretical stages	Lecture	Quizzes and Assignments
Eighth	4	Mass transfer equipment: tray towers, tray types, factors affecting tray efficiency, packed columns	Lecture	Quizzes and Assignments
Ninth	4	Liquid–liquid extraction: solvent selection, equilibrium systems, solubility curves, temperature effect on solubility	Lecture	Quizzes and Assignments
Tenth	4	Stage-wise contact: single-stage and multi-stage	Lecture	Quizzes and Assignments
Eleventh	4	Continuous extraction towers, types of extraction equipment	Lecture	Quizzes and Assignments
Twelfth	4	Leaching: influencing factors, theoretical principles, leaching equipment	Lecture	Quizzes and Assignments
Thirteenth–Fourteenth	4	Heat and mass balance for continuous dryers, drying equipment	Lecture	Quizzes and Assignments
Fifteenth	4	Crystallization: theories, crystallization rate, equipment, Drying: equilibrium, drying rate, drying rate curve, drying time,	Lecture	Quizzes and Assignments

		batch and Drying: equilibrium, drying rate, drying rate curve, drying time, batch and continuous drying mechanisms continuous drying mechanisms		

Learning and teaching resources	
Ref.	1-Separation Process Engineering Includes Mass Transfer Analysis Fifth Edition, PHILIP C. Wankat. 2-Heat and Mass Transfer: A Textbook for Students Preparing for B.E. and B.Tech., B.Sc. Engg., and gate examination in SI units Er. R.K. RAJPUT. 3-Mass transfer in chemical engineering Processes edited by Jozef Markos. 4-Volume 4 Petroleum Refining Design and Applications Handbook A.KAYODE COKER. 5-Fundamentals of Heat and Mass Transfer, sixth edition by: Incropera/Dewitt/Bergman/Lavine. 6- Chemical Engineering Coulson and Richardson's volume 1, Fluid Flow, Heat Transfer and Mass Transfer, sixth edition. 7- Principles of Mass transfer and Separation processes Binay k. Dutta.

1/ Overview

1 / A –Target population :-

For students in the second year
Technological Institute of Basra
Dep. Of Chemical Industrial Techniques

1 / B –Rationale:-

Mass transfer is a fundamental process in engineering and science where a chemical species moves from one location to another due to a **concentration gradient**. It plays a critical role in various natural and industrial processes, including **distillation, absorption, drying, extraction, and biological transport**.

1 / C –Central Idea:-

- **Diffusion**: Movement of molecules from high to low concentration (governed by Fick's Laws).
- **Convection**: Mass transfer enhanced by bulk fluid motion.

- **Interphase transfer**: Movement of species between phases (e.g., gas to liquid in absorption).

These processes are essential in operations such as distillation, drying, absorption, extraction, and membrane separation, which are foundational to both chemical and biochemical engineering.

1 / D – Performance Objectives

By the end of this unit, the student will be able to:

1. **Define** key concepts of mass transfer such as diffusion, convection, mass transfer coefficient, and driving force.

2. **Explain** the mechanisms of mass transfer in different phases (gas, liquid, solid) and across phase boundaries.

3. **Apply** Fick's First and Second Laws to solve basic diffusion problems in steady and unsteady states.

4. **Calculate** mass flux and mass transfer rates in systems involving simple geometries and boundary conditions.

5. **Use** dimensionless numbers (e.g., Sherwood, Reynolds, Schmidt) to characterize mass transfer processes.

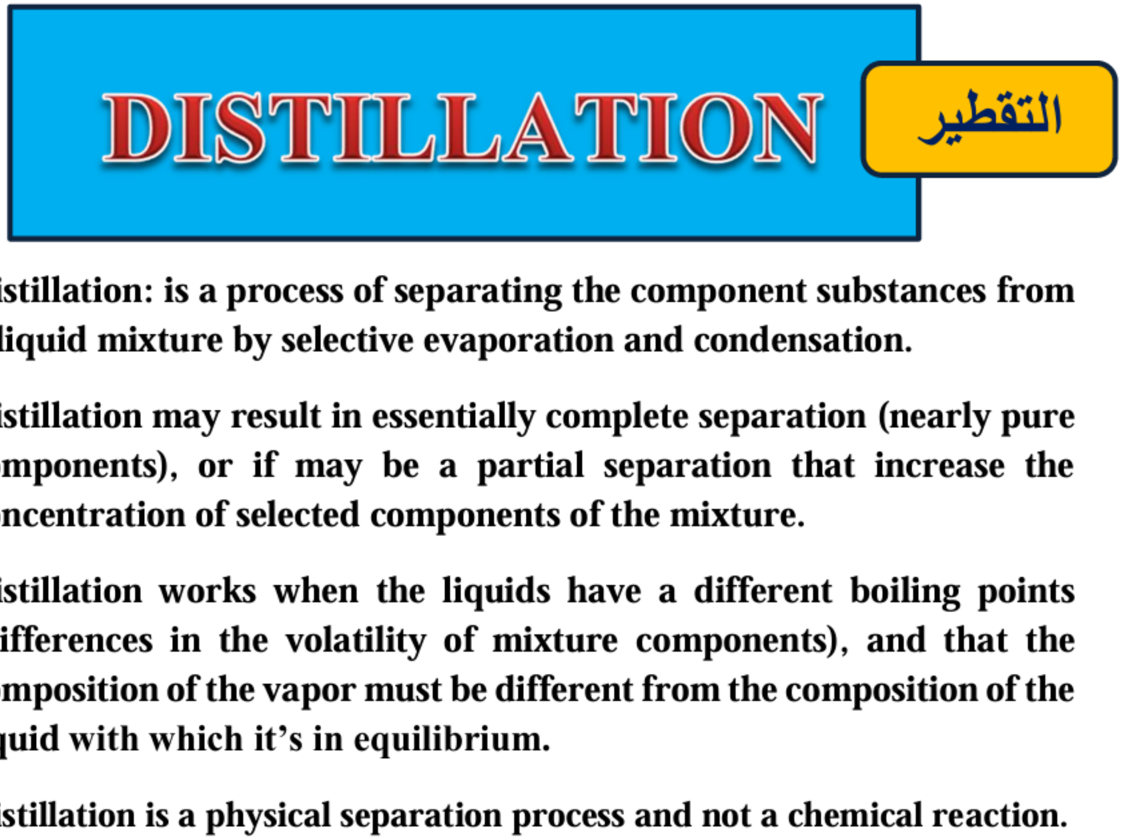
6. **Analyze** the effects of fluid velocity, concentration gradient, and system geometry on mass transfer performance.

7. **Design** or evaluate basic mass transfer equipment (e.g., absorption columns, diffusion cells) under specified conditions.

8. **Compare** mass transfer mechanisms with analogies from heat and momentum transfer.

9. **Interpret** real-life and industrial applications of mass transfer principles, such as in separation processes or environmental engineering.

2/ Pretest :



DISTILLATION التقطير

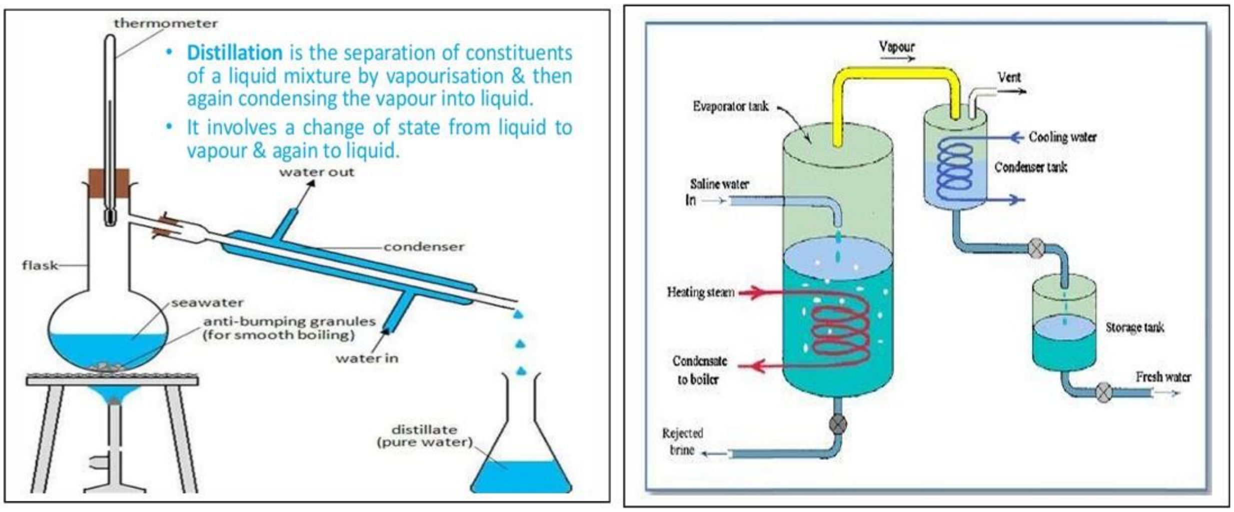
Distillation: is a process of separating the component substances from a liquid mixture by selective evaporation and condensation.

Distillation may result in essentially complete separation (nearly pure components), or if may be a partial separation that increase the concentration of selected components of the mixture.

Distillation works when the liquids have a different boiling points (differences in the volatility of mixture components), and that the composition of the vapor must be different from the composition of the liquid with which it's in equilibrium.

Distillation is a physical separation process and not a chemical reaction.

Figure 1 distillation column



- **Distillation** is the separation of constituents of a liquid mixture by vapourisation & then again condensing the vapour into liquid.
- It involves a change of state from liquid to vapour & again to liquid.

Figure 2 distillation processes

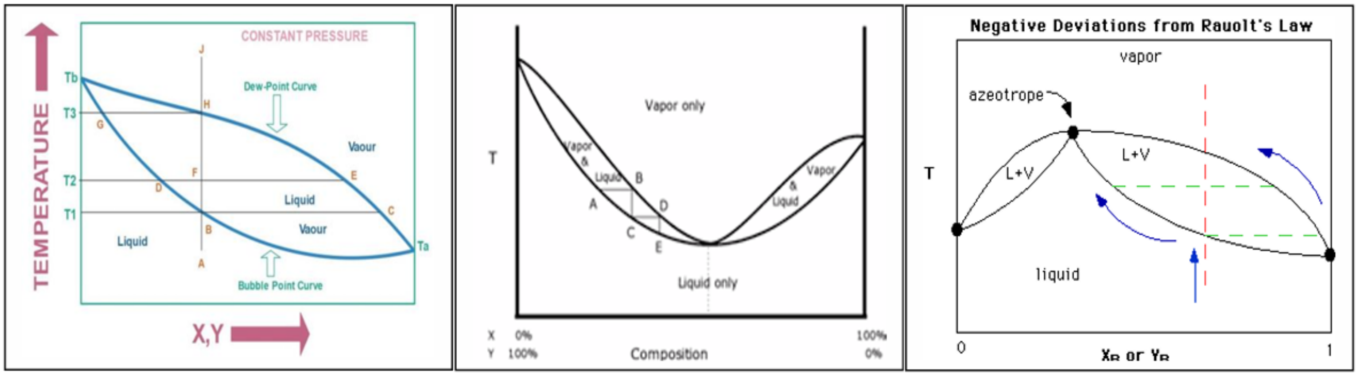


Figure 3 equilibrium curves

هنا في عملية التقطير (distillation) لدينا مزيج من مركبات سائلة والمطلوب فصل احدها عن المزيج بمبدئ التبخير والتقطير والفرق بين هذه العملية والاستخلاص (extraction) هنا بالتقطير نحتاج تسخين اي حرارة وبالاستخلاص لا نحتاج حرارة، فاذا كانت المواد تتأثر بالحرارة فنلجأ للاستخلاص والفرق الثاني بالاستخلاص نحتاج مذيب (يكون مكلف وباهض عادة) وعندما لا يتوفر فنلجأ الى التقطير التي لا نحتاج فيها الى مذيب، وسنعمل هنا على نظام ثنائي فيكون لدينا مركبين الاول الاكثر تطايرية (more volatile component MVC) وهو الاقل درجة غليان والثاني الاقل تطايرية وهو الاعلى درجة غليان وهنا بالتأكيد تكون درجة غليانهما قريبة لانها لو كانت بعيدة لما كنا بحاجة الى برج التقطير وكنا اكتفينا بعملية تبخير بسيطة جدا ثم تكثيف ويتم فصل المواد ومن الجدير بالذكر ان برج التقطير وهو البرج الاطول والأعقد في المصافي.

Vapour – liquid Equilibrium (VLE)

V L E : is a condition in which a liquid and it's vapour are in equilibrium with each other , a condition or state where the rate of evaporation equals the rate of condensation.

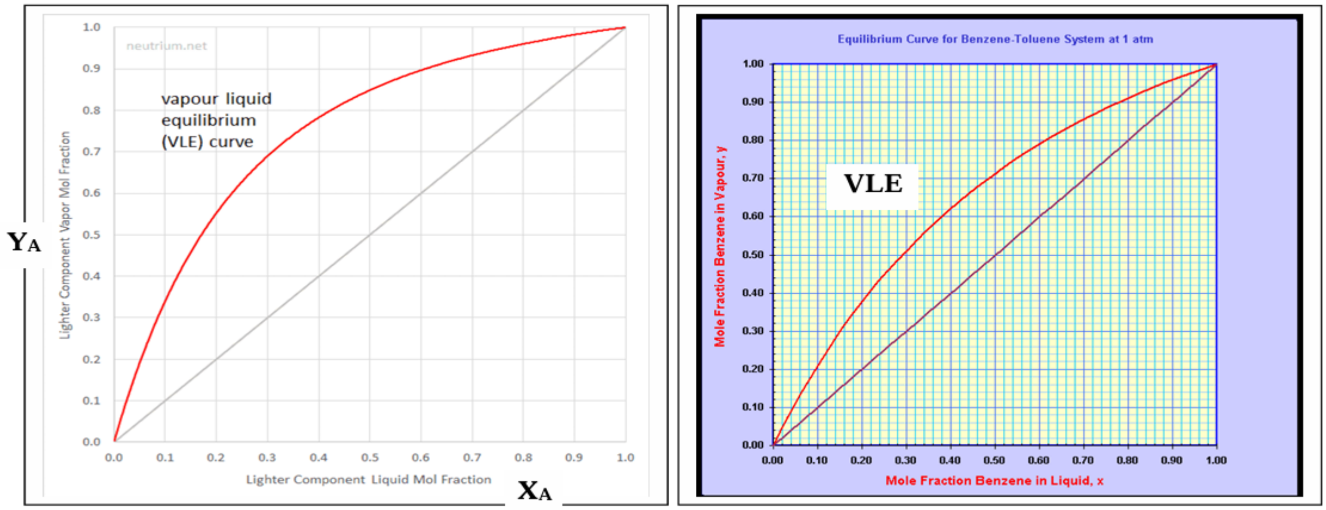


Figure 4 Vapor – Liquid Equilibrium Curves (VLE)

V L E : can be obtained by four ways:

1- Volatility (α): التطايرية

$$\alpha_A = \frac{P_A^o}{P_T} \quad , \quad \alpha_B = \frac{P_B^o}{P_T} \qquad \ldots.. (1)$$

Where:

α_A : volatility of A

α_B : volatility of B

P_A^o : vapor pressure of component A at a certain temp.

P_B^o : vapor pressure of component B at a certain temp.

P_T : total pressure of the system

Relative volatility (α_{AB}) : التطايرية النسبية

$$\propto_{AB} = \frac{\propto_A}{\propto_B} = \frac{P_A^o}{P_B^o} = \frac{Y_A/X_A}{Y_B/X_B} \qquad \text{..... (2)}$$

$$P_A = Y_A\, P_T \Longrightarrow Y_A = \frac{PA}{PT} = \frac{XA\, PA^\circ}{PT} \qquad \text{..... (3)}$$

$$\frac{Y_A}{X_A} = \frac{P_A^o}{PT} \quad \text{and} \quad \frac{Y_B}{X_B} = \frac{P_B{}^\circ}{P_T}$$

$$Y_A = \frac{\propto_{AB}\, X_A}{1+X_A(\propto_{AB}-1)} \qquad \text{..... (4)}$$

$$X_A = \frac{Y_A}{\propto AB-(\propto AB-1)Y_A} \qquad \text{..... (5)}$$

نستخدم احدى المعادلتين حيث
تكون قيم (α_{AB}) معطاة بالسؤال
ويتم فرض قيم X_A ثم نجد قيم
Y_A من المعادلة

$X_A = 0.1, 0.2, 0.3,)$
(….0.9

Example 1: **Draw the equilibrium data for a mixture of A and B , if the $\propto_{AB}$=2. (home work)**

2- For ideal mixture from Dalton's & Raoult's laws for binary mixture

Dalton's Law: $\sum P_i = P_T$ ..... (6)

$P_A + P_B = P_T$

Raolt's law: $P_A = X_A P_A{}^o$ ..... (7)

$P_B = X_B P_B{}^o = (1-X_A)\, P_B{}^o$

$X_A P_A{}^o + (1-X_A)\, P_B{}^o = P_T$

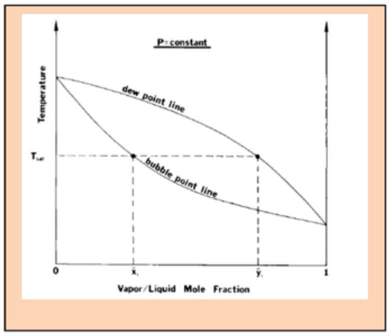
$$X_A = \frac{P_T-P_B{}^\circ}{P_A{}^\circ-P_B{}^\circ} \quad \text{for x} \qquad \text{..... (8)}$$

$$Y_A = \frac{P_A}{P_T} = \frac{X_A P_A{}^\circ}{P_T}$$

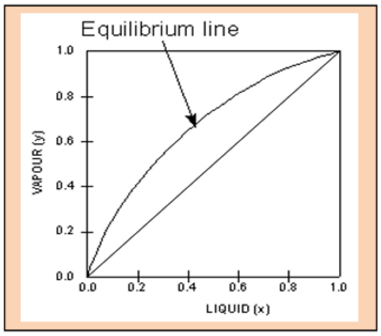
$$Y_A = \frac{P_A{}^\circ}{P_T} X_A \qquad \text{for } Y \qquad \ldots\ldots (8)$$

Example 2: For a mixture of n-heptane (A) and toluene (B) at 373 k and 101.3 KN/m², P_A = 106 KN/m², P_B = 73.7 KN/m², Find Y_A and X_A. (home work).

3- Boiling point diagram



Y_A	X_A
-	-
-	-
-	-
-	-
-	-
-	-
-	-



4- For non-ided mixture

$$\propto AB = \frac{\gamma_A P_A{}^\circ}{\gamma_B P_B{}^\circ} \qquad \ldots\ldots(9)$$

Where:

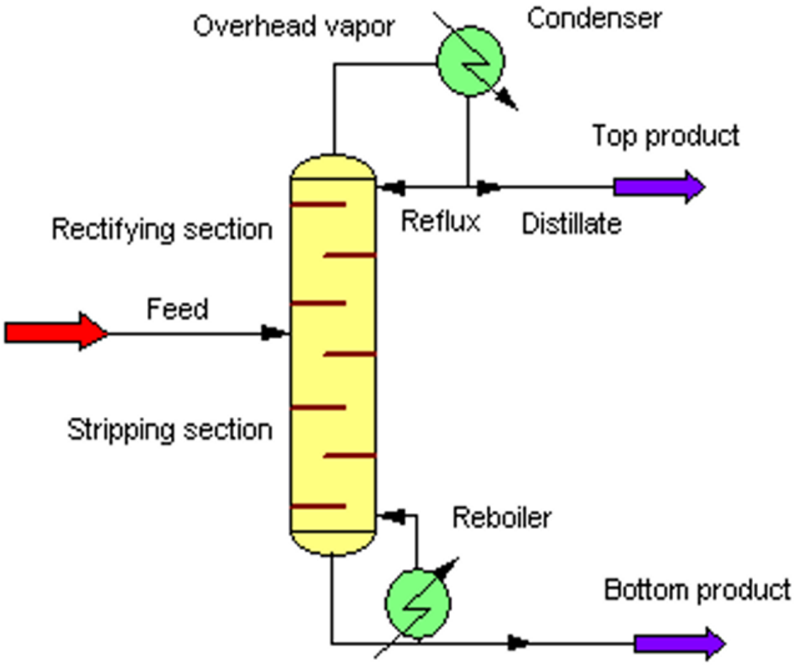
γ_A, γ_B = activity coefficient for A and B

Types of distillation

- Continuous Distillation
- Differential Distillation
- Flash Distillation
- Batch Distillation

1- Continuous Distillation التقطير المستمر

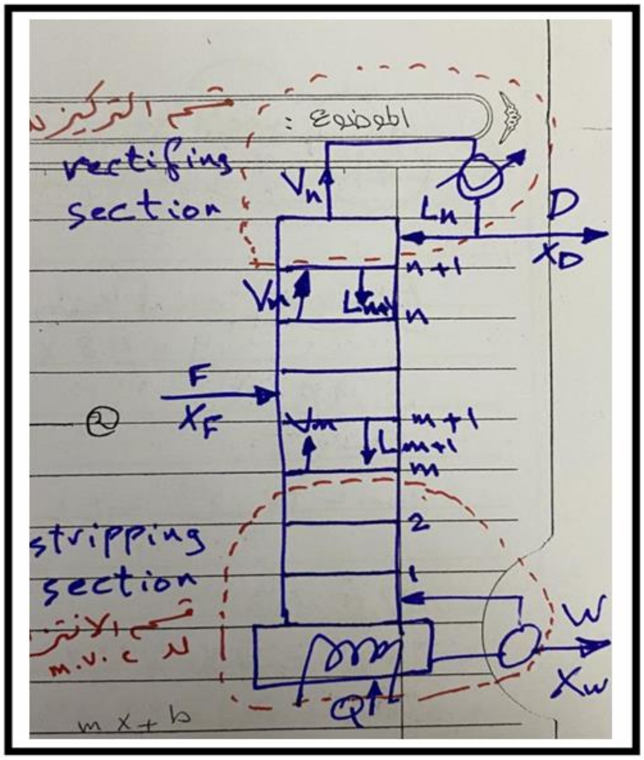
A- Binary System (two components) (Mc-Cabe Thiele Method)



Notes: **(1) here the solution by Mc-Cabe Thiele diagram and the analytical solution by Lowi-Sorel method.**

(2) Number of stages here start from (0) which is the base stage (boiler), it represents the first stage because it included in design.

(a) Rectifying Section



$$R = \frac{Ln}{D} \qquad (10)$$

$$V_n = L_n + D = (R+1)D \qquad (11)$$

M.B. on top section:

$$V_n = L_{n+1} + D$$

$$V_n Y_n = L_{n+1} X_{n+1} + D X_d$$

$$Y_n = \frac{L_{n+1}}{Vn} X_{n+1} + \frac{D}{Vn} X_d$$

Since the mode of diffusion mostly equimolar counter diffusion

$$N_a = -N_b$$

$$\lambda_a = \lambda_b$$

$$L_{n+1} = L_n$$

$$V_{n+1} = V_n$$

$$\boxed{Y_n = \frac{Ln}{Vn} X_{n+1} + \frac{D}{Vn} X_d \qquad (12)}$$

From eqn. (10) → $L_n = R \, . \, D$

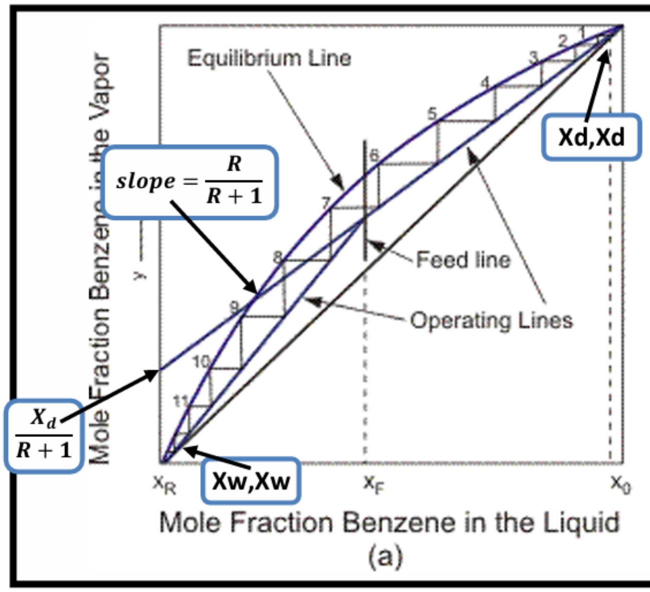
From eqn. (11) → $V_n = L_n + D = R \, . \, D + D$

$$Y_n = \frac{RD}{RD+D} X_{n+1} + \frac{D}{RD+D} Xd$$

$$\boxed{Y_n = \frac{R}{R+1} X_{n+1} + \frac{X_d}{R+1} \qquad (13)}$$

هذه المعادلة تمثل خط التشغيل العلوي

Top operating line



Top operating line

يتم رسم خط التشغيل العلوي من نقطة (Xd,Xd) و ميل مقداره (Ln/Vn)او من نقطتين الاولى (Xd,Xd) والثانية التي تمثل المقطع الصادي intercept [(D/Vn)*Xd] وهنا قد نحتاج لاجراء حسابات لايجاد قيم (Ln وVn) و لتسهيل الحل يتم تحويلهما بدلالة نسبة الراجع (R) فالافضل رسم خط التشغيل العلوي من نقطة (Xd,Xd) و ميل مقداره [R/(R+1)] او من نقطتين الاولى (Xd,Xd) والثانية التي تمثل المقطع الصادي [Xd/(R+1)]

(b) Stripping section

$$L_{m+1} = V_m + W$$

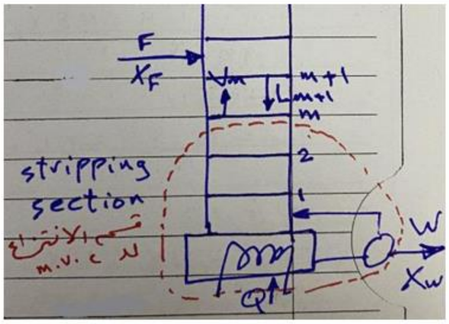
$$L_{m+1} X_{m+1} = V_m Y_m + W.X_w$$

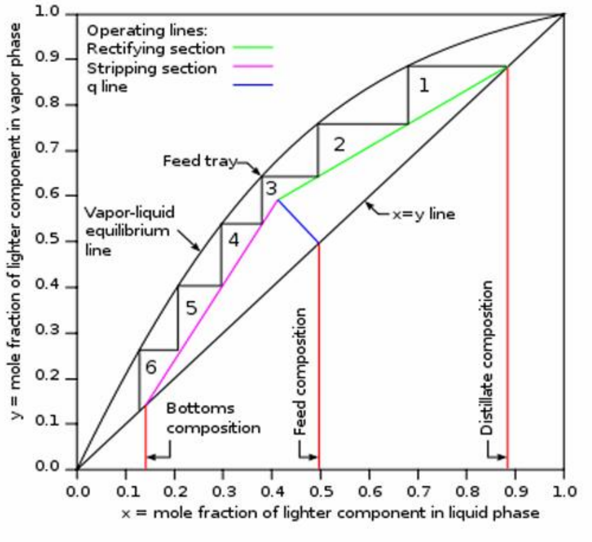
$$Y_m = \frac{L_{m+1}}{V_m} X_{m+1} - \frac{w}{V_m} X_w \qquad (14)$$

هذه المعادلة تمثل خط التشغيل السفلي

Bottom operating line

والذي يتم رسمه من نقطة (Xw,Xw) و نقطة تقاطع خط التشغيل العلوي مع الخط الحراري q-line





(c) q Line الخط الحراري الذي يصف حالة اللقيم

$$q = \frac{heat\ required\ to\ vaporized\ to\ 1\ mole\ of feed}{moler\ latent\ heat} = \frac{c_p(T_{Bp}-T_f)+\lambda}{\lambda}$$

Where :

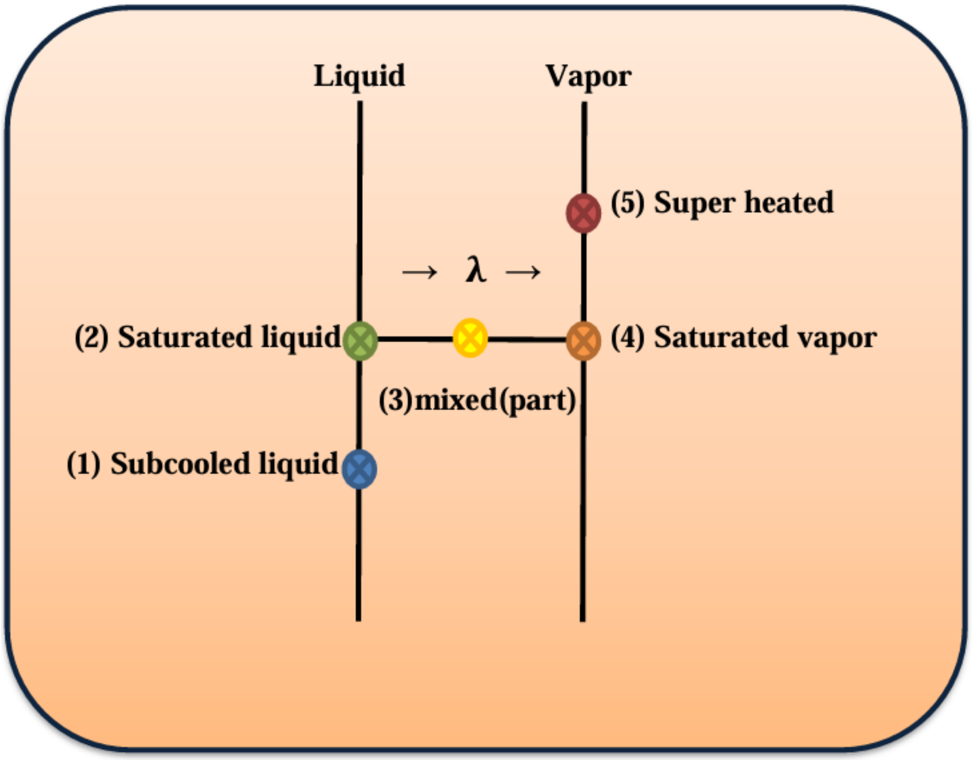
T_{Bp}= boiling point of feed mixture

T_f= feed temp.

λ = latent heat of feed mixture

$$Y_q = \frac{q}{q-1} X_q - \frac{X_f}{q-1} \qquad (15)$$

هذه المعادلة تمثل معادلة الخط الحراري q-pline
الذي يرسم من نقطة (X_f,X_f) وميل مقداره (q/q-1)



1- Subcooled feed

$q = \frac{c_p(T_{Bp}-T_{Ff})+\lambda}{\lambda}$ $\quad\quad q > 1$

slope $= \frac{q}{q-1} = \frac{+}{+}$ (موجب بالربع الاول)

2- Liquid at it’s boiling point (sat liq.)

$q = \frac{0+\lambda}{\lambda} = 1$

Slope $= \frac{q}{q-1} = \frac{1}{0} = \infty$ (vertical line عمودي)

3- Partially vaporized feed

$q = \frac{0+(fraction\ of\ liq.)\lambda}{\lambda} = fraction\ of\ liq.$ $\quad\quad 0 < q < 1$

Slope $= \frac{q}{q-1} = \frac{+}{-} = \frac{-fraction\ of\ liq.}{fraction\ of\ vapour}$ (سالب بالربع الثاني)

4- Saturated vapour

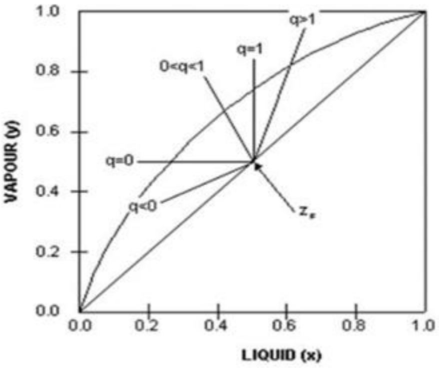
$q = \frac{0}{\lambda} = 0$

Slope $= \frac{q}{q-1} = 0$ (horizontal line افقي)

5- Super heated feed

$q = \frac{c_p(T_{Bp}-T_f)}{\lambda}$ $\quad\quad q < 0$

slope $= \frac{q}{q-1} = \frac{-}{-}$ (موجب بالربع الثالث)



Depending on the state of the feed, the feed lines will have different slopes. For example,

- q = 0 (saturated vapour)
- q = 1 (saturated liquid)
- 0 < q < 1 (mix of liquid and vapour)
- q > 1 (subcooled liquid)
- q < 0 (superheated vapour)

The q-lines for the various feed conditions are shown in the diagram on the left.

(d) No. of plate

$$\text{No. of plate} = \text{No. of stages} - 1 \qquad (16)$$

(e) Calculation of Efficiency , R_{min} , N_{min}

1- Efficieney

I. Column Efficiency

$$\text{Actual no. of plate} = \frac{\text{thearticd no.of plates}}{\eta_c} \qquad (17)$$

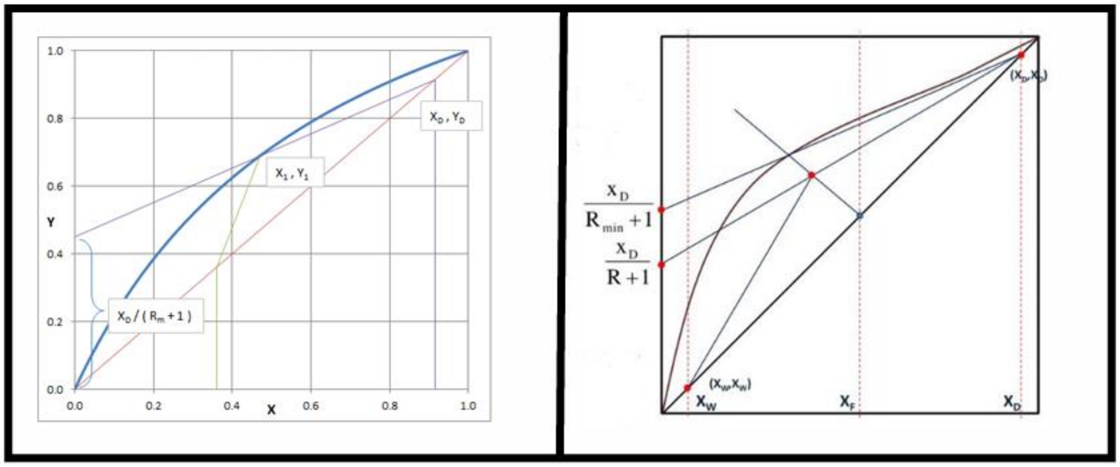
II. Plate efficiency (Murphree efficiency)

$$E_{mv} = \frac{a_b}{a_c} = \frac{y_{n+1} - y_n}{y_e - y_n} \qquad (18)$$

$$E_{mL} = \frac{a_b}{a_c} = \frac{y_{n+1} - X_n}{X_{n+1} - X_e} \qquad (19)$$

2- Minimum reflux ratio R_{min}

i. Calculation of R_{min} by graphical method:



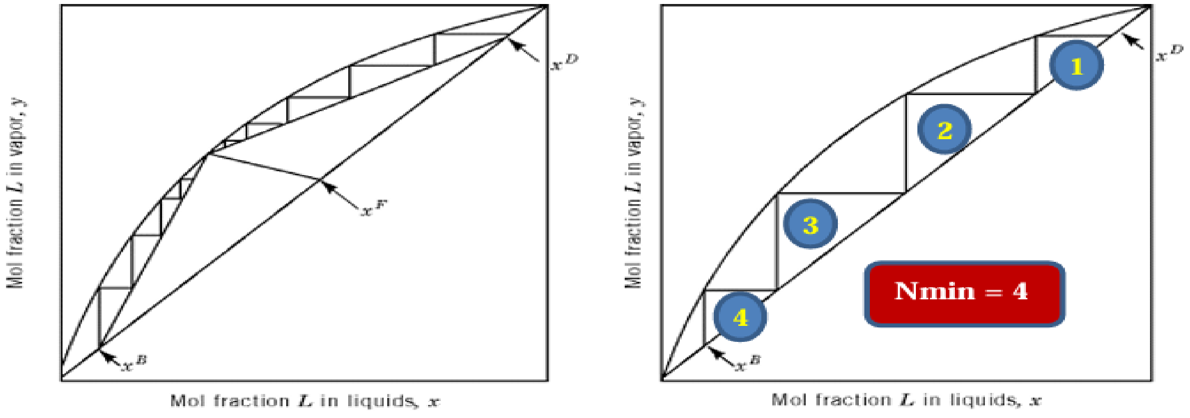
Plot q-line until intersect with ELV curve, plot top operating line from Xd, Xd to the intersection point, get the value of $\frac{XD}{R_{min}+1}$ from graph (on Y axis), then find R_{min}

ii. Calculation of R_{min} by analytical equation:-

$$R_{min} = \frac{1}{\propto - 1} \left[\frac{Xd}{Xf} - \frac{\propto(1-Xd)}{1-Xf}\right] \qquad (20)$$

3- Minimum number of stages N_{min}

i- Calculation of N_{min} by graphical method



(a) (b)

ii- Calculation of N_{min} by analytical equation

$$N_{min} = \frac{\log\left[\left(\frac{xd}{1-xd}\right)\left(\frac{1-Xw}{Xw}\right)\right]}{\log \propto AB} \qquad (21)$$

Example 1:

Binary mixture of n-heptane and n-octane enters partially vaporized (60% liquid). The feed contains 70% n-heptane is to be continuously distilled to give a top product of 95 mol% and 5 mol% bottom, the reflux ratio is 3. The vapor pressure of n-heptane is twice the vapor pressure of n-octane at the same temp.

a. Find the actual no. of plate if the column efficiency is 80%.
b. Determine the min. no. of plate graphically and analytically.
c. Determine the min. reflux ratio R_{min} graphically and analytically.

Solution:

$$\alpha_{AB} = \frac{\propto_A}{\propto_B} = \frac{P_A^o}{P_B^o} = 2$$

$$Y_A = \frac{\alpha_{AB} \,. X_A}{1+X_A(\propto_{AB}-1)} = \frac{2XA}{1+XA}$$

X_A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Y_A	0	0.18	0.33	0.46	0.57	0.66	0.75	0.82	0.88	0.94	1.0

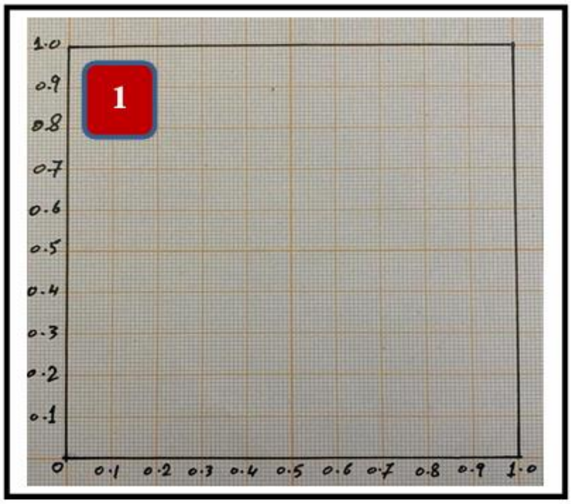
* **Top operating line** $(X_d , X_d) \rightarrow (0.95 , 0.95)$

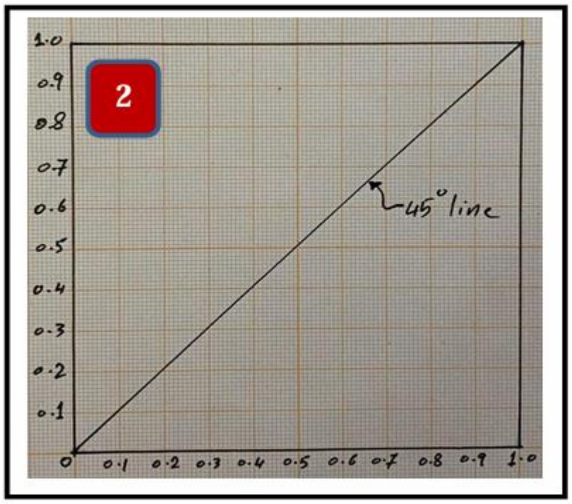
$$\left(0 , \frac{xd}{R+1}\right) \rightarrow \left(0 , \frac{0.95}{3+1}\right) \rightarrow (0 , 0.2375)$$

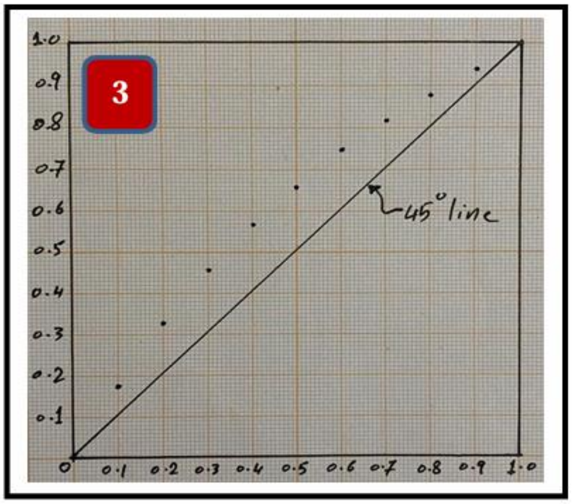
* **q line** $q = \frac{0 + 0.6\,\lambda}{\lambda} = 0.6$

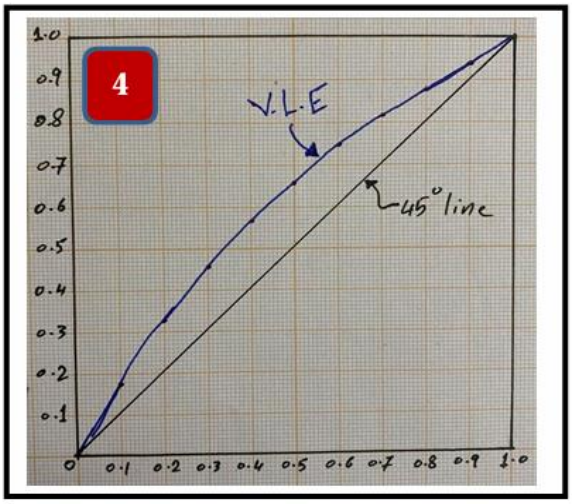
$$\text{Slope} = \frac{q}{q-1} = \frac{-fraction\ of\ liq.}{fraction\ of\ vap.} = -\frac{0.6}{0.4}$$

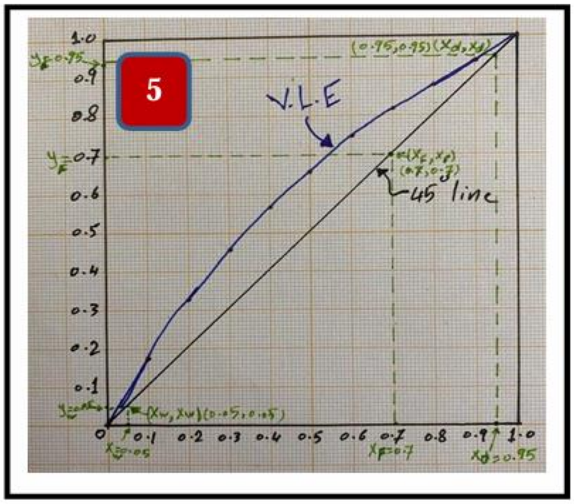
* **Bottom operating line: point $(X_w , X_w) \rightarrow (0.05, 0.05)$ and intersection of q-line and top operating line**

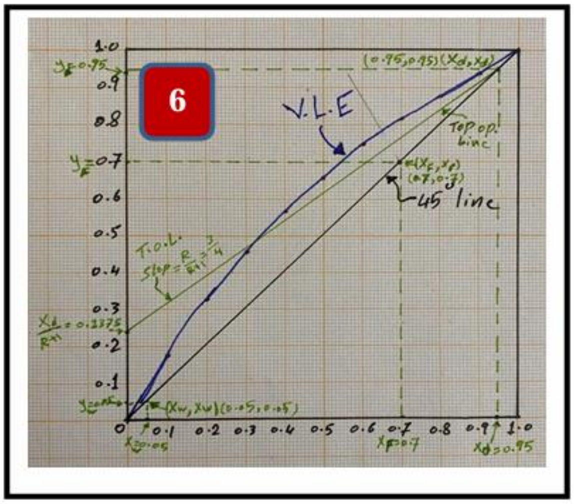

1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

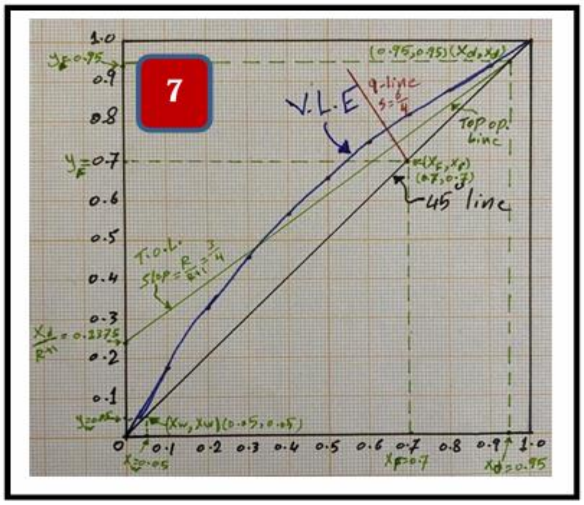

2
45° line
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


3
45° line
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

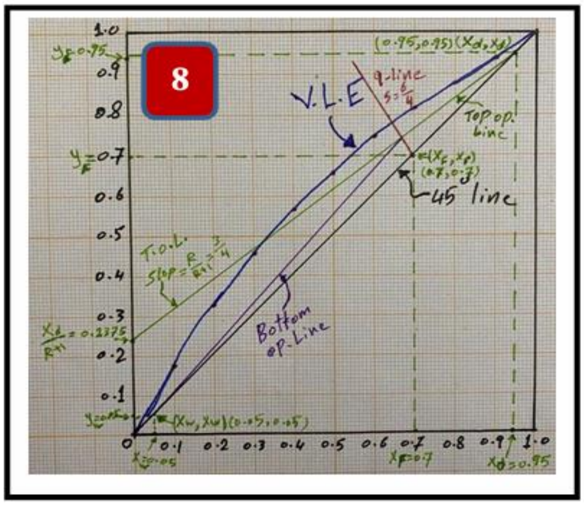

4
V.L.E
45° line
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


5
V.L.E
45 line
yd=0.95
(0.95,0.95)(Xd,Xd)
yF=0.7
(XF,XF)
(0.7,0.7)
yw=0.05
(Xw,Xw)(0.05,0.05)
Xw=0.05
XF=0.7
Xd=0.95
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

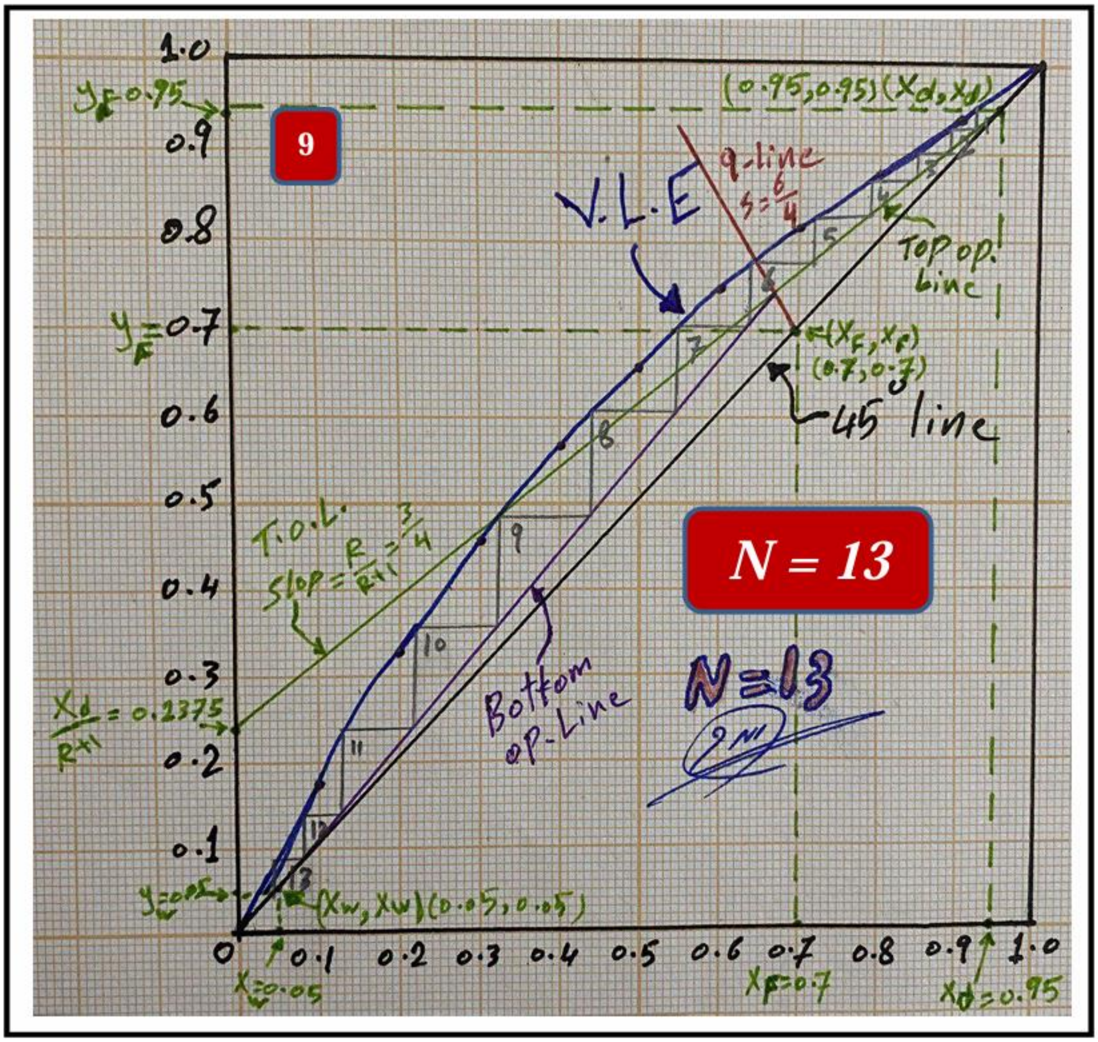

6
V.L.E
45 line
Top op. line
T.O.L.
slop = R/(R+1) = 3/4
Xd/(R+1) = 0.2375
yd=0.95
(0.95,0.95)(Xd,Xd)
yF=0.7
(XF,XF)
(0.7,0.7)
yw=0.05
(Xw,Xw)(0.05,0.05)
Xw=0.05
XF=0.7
Xd=0.95
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


7

1.0
y_F=0.95
(0.95,0.95)(X_d,X_d)
0.9
0.8
V.L.E
q-line S=6/4
Top op. line
y_F=0.7
(X_F,X_F) (0.7,0.7)
0.6
45 line
0.5
T.O.L. Slop = R/(R+1) = 3/4
0.4
0.3
X_d/(R+1) = 0.2375
0.2
0.1
y_w=0.05
(X_w,X_w)(0.05,0.05)
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X_w=0.05
X_F=0.7
X_d=0.95


8

1.0
y_F=0.95
(0.95,0.95)(X_d,X_d)
0.9
0.8
V.L.E
q-line S=6/4
Top op. line
y_F=0.7
(X_F,X_F) (0.7,0.7)
0.6
45 line
0.5
T.O.L. Slop = R/(R+1) = 3/4
0.4
Bottom op-Line
0.3
X_d/(R+1) = 0.2375
0.2
0.1
y_w=0.05
(X_w,X_w)(0.05,0.05)
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X_w=0.05
X_F=0.7
X_d=0.95


9

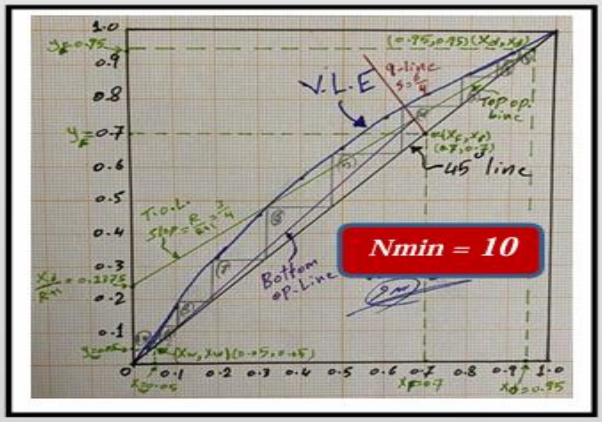
N = 13

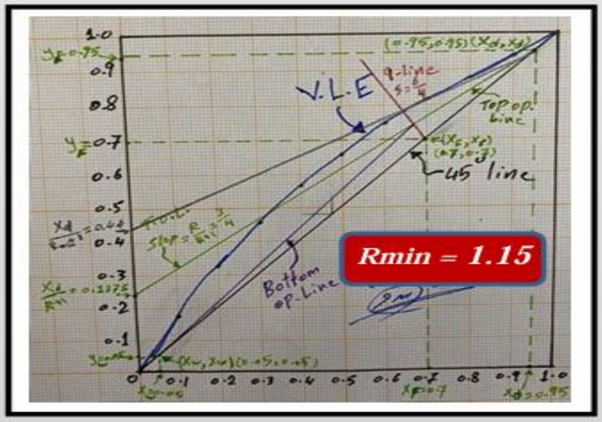
1.0
y_F=0.95
(0.95,0.95)(X_d,X_d)
0.9
q-line S=6/4
V.L.E
0.8
Top op. Line
1
2
3
4
5
6
7
8
9
10
11
12
13
y_F=0.7
(X_F,X_F) (0.7,0.7)
0.6
45 line
0.5
T.O.L. Slop = R/(R+1) = 3/4
0.4
0.3
Bottom op-Line
X_d/(R+1) = 0.2375
0.2
N=13
0.1
y_w=0.05
(X_w,X_w)(0.05,0.05)
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X_w=0.05
X_F=0.7
X_d=0.95

i. **From plot** **theatrical no. of stage** $=13$

theatrical no. of plate $=12$

Actual no. of plate $= \frac{12}{0.8} = 15$





ii. **From plot** $N_{min} = 10$

By equation $N_{min} = \frac{\log\left[\left(\frac{0.95}{1-0.95}\right)\left(\frac{1-0.05}{0.05}\right)\right]}{\log 2} = 8.49 \approx 9$

iii. **From plot** $\frac{xd}{R_{min+1}} = 0.44$

$$\frac{0.95}{R_{min+1}} = 0.44 \rightarrow R_{min} = 1.15$$

By equation:

$$R_{min} = \frac{1}{2-1}\left[\frac{0.95}{0.7} - \frac{2(1-0.95)}{1-0.7}\right] = 1.023$$

$R_{min} = 1.023$

Lewis – Sorel method

Lewis – Sorel method is an analytical way to find number of plates and concentrations at any plate in a symmetrical way as Mic-Cabe Thiele graphical diagrams by using operating lines equations with equilibrium relations as follow:

$X = f(y)$

$Y = f(x)$

طريقة لويس-سوريل هي طريقة تحليلية لايجاد عدد الصواني والتراكيز
على كل صينية بطريقة مناظرة لطريقة مك-كيب ثل التخطيطية و ذلك
باستخدام معادلات خطوط التشغيل مع علاقة التعادل

(1) If it is required to find concentrations (Y & Y) at (for example) plate no.2 from the top, apply equations as follow:

1. $X_d = Y_1$
2. **X_1 found from equilibrium relation**

$$X_1 = \frac{y_1}{\propto - (\propto - 1) y}$$

3. **To find Y2, use top operating line equation**

$$Y_n = \frac{L_n}{V_n} . X_{n+1} + \frac{D}{V_n} X_d \rightarrow Y_2$$

4. **X_2 found from equilibrium relation**

Notes:

a. **If it is required to find no. of plates for rectifying section, we should keep gone until reach X_f or pass it.**
b. **If it is required to find no. of plates for all column, we should use top operating line until reach X_f or pass it, after that we should use bottom operating line until reach X_w or pass it.**

c. **If it is required to find concentrations (Y & Y) at plate no.2 from the bottom, apply equations as follow:**

1. $X_w = X_o$
2. **Y_o found from equilibrium relation**

$$Y_A = \frac{\alpha_{AB} \cdot X_A}{1+X_A(\propto_{AB}-1)}$$

3. **To find X_1, use bottom operating line equation**

$$Y_m = \frac{L_{m+1}}{V_m} X_{m+1} - \frac{w}{V_m} X_w \rightarrow \quad X_1$$

4. **Y_1 found from equilibrium relation**

d. **If the talk (in the problem) about plate number 2 without mention if it is from the top or bottom, we will consider it from bottom.**

e. **In all chapters or subjects of mass transfer involving distillation column, if the amount of feed (F) or products (D or W) not be given, basis should be taken.**

ملاحظات مهمة جداً:

1. اذا قال في السؤال صينية رقم 2 ولم يحدد فانها الصينية الثانية من الاسفل.

2. في جميع مواضيع الهندسة الكيمياوية ومنها جهاز او عمود التقطير، اذا لم تعطى كمية اللقيم (feed F) او احدى النواتج (D , W) فيتم أخذ (basis).

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

Learning package

In

Mass Transfer of liquid-liquid Extraction

For

second year students

By

Ataa Wejood

Dep. Of Chemical Industrial Techniques
2025

1/ Overview

1 / A –Target population :-

For students in the second year
Technological Institute of Basra
Dep. Of Chemical Industrial Techniques

1 / B –Rationale:-

Liquid-liquid extraction (LLE) is a separation process based on the **mass transfer of a solute** from one liquid phase into another immiscible or partially miscible liquid phase. This process relies fundamentally on **concentration gradients** and the **differences in solubility** of the solute in each liquid phase.

Understanding mass transfer in LLE is essential because the **rate and extent of solute transfer** directly affect the efficiency and selectivity of the separation. Engineers must analyze and control factors such as **interfacial area, diffusion coefficients, phase contact time, agitation, and solvent choice** to optimize extraction performance.

Mastery of mass transfer principles in LLE enables:

- Efficient **design of extraction equipment** (e.g., mixers-settlers, packed columns).
- **Minimization of solvent usage**, reducing cost and environmental impact.

- Improvement in **product purity and recovery rates**.
- Proper **scale-up** from laboratory to industrial operations.

2 / C –Central Idea:-

Liquid-liquid extraction is a separation technique that utilizes the **selective mass transfer of a solute** from one liquid phase into another immiscible or partially miscible liquid phase. This process leverages differences in solute solubility and distribution between the two phases to achieve effective separation.

The efficiency of liquid-liquid extraction depends on the **interfacial area, concentration gradients, and mass transfer rates** between the phases. Understanding these factors allows for the design and operation of extraction systems that maximize solute recovery and purity while minimizing solvent usage and operational costs.

In essence, liquid-liquid extraction combines principles of **mass transfer, equilibrium, and fluid dynamics** to separate components based on their preferential solubility, making it an essential method in chemical, pharmaceutical, and environmental processes.

1 / D – Performance Objectives

By the end of this unit, the student will be able to:

After studying liquid-liquid extraction, the student will be able to:

1. **Explain** the fundamental principles of liquid-liquid extraction, including solute distribution between two immiscible liquids.
2. **Describe** the role of mass transfer and equilibrium in the extraction process.
3. **Identify** the key factors affecting extraction efficiency, such as interfacial area, solvent selection, agitation, and phase contact time.
4. **Calculate** the distribution coefficient and understand its significance in predicting solute transfer.
5. **Analyze** batch and continuous liquid-liquid extraction operations.

6. **Design** simple liquid-liquid extraction equipment like mixers and settlers based on mass transfer concepts.
7. **Evaluate** the performance of an extraction process using mass transfer coefficients and equilibrium data.
8. **Apply** safety and environmental considerations in selecting solvents and operating conditions.

2/ Pretest :

Liquid – Liquid Extraction

Liquid-liquid extraction (LLX) is a mass transfer operation in which a solution (called the feed which is a mixture of a solute and a carrier liquid) is brought into intimate contact with a second immiscible or slightly liquid (called the solvent) in order to achieve transfer of the solute (s) from the feed to the solvent. The two liquid phases that have different densities are then separated. The solute – phase (this is the solvent stream, now enriched with the solute) is called the extract; the residual feed stream that may have a little of the solute left in it is called the raffinate. An extraction process generally involves four major steps.

a) Bringing the feed and the solvent into intimate contact by dispersing one phase into the other as droplets.

b) Separation of the extract and the raffinate phases that have different densities.

C) Removal and recovery of the solute from the extract phase in a relatively pure form (by distillation, evaporation, crystalization, etc.).

d) Removal and recovery of the solvent from each phase, usually by distillation.

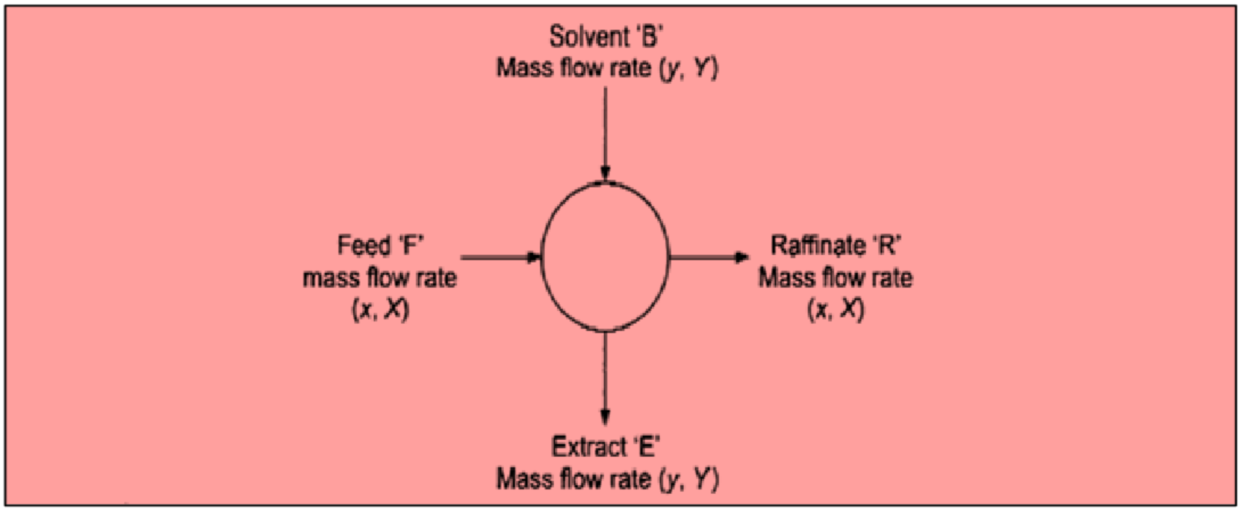


Figure 1.1. Single-stage extraction.

x= mass fraction of Solute in Feed and Raffinate stream

X = the ratio of Solute to solvent in the Raffinate streams.

y= mass fraction of Solute in Solvent and Extract stream.

Y = the ratio of the Solute to Solvent in the Extract streams.

1. Co-current contact with immiscible solvents

In this case, which is illustrated in Figure (1.1), triangular diagrams are not required. If the initial solution contains a mass A of solvent A with a mass ratio X_f of solute, then the selective pure solvent to be added will be a mass S of solvent S. On mixing and separating, a raffinate is obtained with the solvent A containing a mass ratio X_1 of solute, and an extract with the solvent S containing a mass ratio Y_1 of solute. A material balance on the solute gives:

$$A X_f = A X_1 + S Y_1$$

$$-\frac{A}{S} = \frac{Y_1}{X_1 - X_f} \tag{1.1}$$

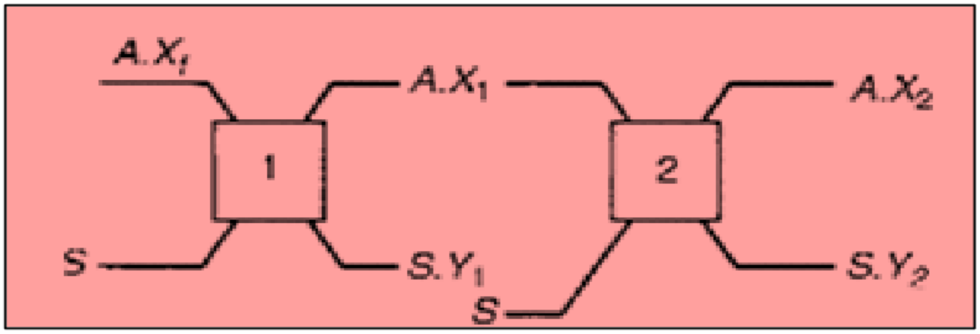


Fig (1.2): Arrangement for multiple-contact extraction in co-current flow.

If the distribution law is followed, then the equilibrium curve becomes a straight line given by $Y = mX$. The material balance on the solute may then be rewritten as:

$$A\,X_f = A\,X_1 + S\,Y_1$$

$$= A\,X_1 + S\,mX_1$$

$$= (A + Sm)\,X_1$$

$$X_1 = \frac{A}{A+Sm}\,X_f$$

If a further mass of **S** is added to raffinate AX_1 to give an extract of composition Y_2 and a raffinate X_2 in a second stage, then:

$$A\,X_1 = A\,X_2 + S\,mX_2 = X_2\,(A + Sm)$$

$$X_2 = \frac{A}{A+Sm}\,X_1 = \left[\frac{A}{A+Sm}\right]^2 X_f$$

For n stages:

$$X_n = \left[\frac{A}{A+Sm}\right]^n X_f$$

And the number of stages is given by:

$$n = \frac{\log \frac{X_n}{X_f}}{\log\left[\frac{A}{A+Sm}\right]} \qquad (1.2)$$

2. Countercurrent contact with immiscible solvents

If a series of mixing and separating vessels is arranged so that the flow is countercurrent, then the conditions of flow may be represented as shown in Figure (1.3), where each circle corresponds to a mixer and a separator. The initial solution F of the solute B in solvent A is fed to the first unit and leaves as raffinate R_1. This stream passes through the units and leaves from the n_{th} unit as stream R_n. The fresh solvent S enters the n_{th} unit and passes in the reverse direction through the units, leaving as extract E_1.

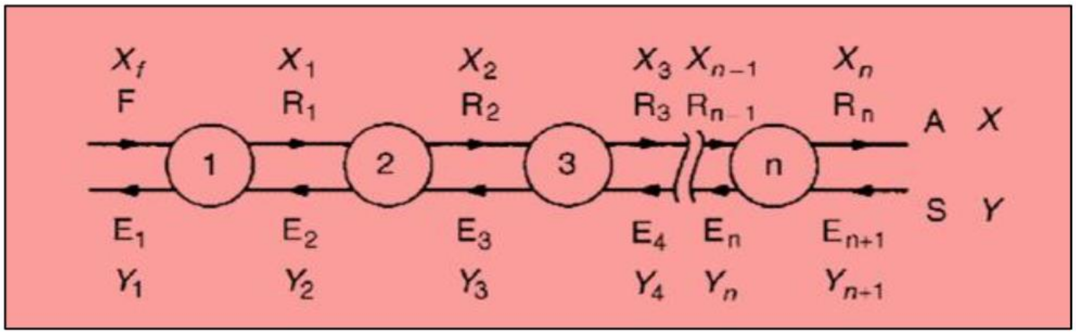


Fig (1.3): Arrangement for multiple-contact extraction in countercurrent flow

The following definitions may be made:

X = the ratio of solute to solvent in the raffinate streams.

Y = the ratio of the solute to solvent in the extract streams.

If the two solvents are immiscible, the solvent in the raffinate streams remains

as A, and the added solvent in the extract streams as S. The material balances

for the solute may then be written as:

(a) For the 1st stage: $AX_f + SY_2 = AX_1 + SY_1$

(b) For the nth stage: $AX_{n-1} + SY_{n+1} = AX_n + SY_n$

(c) For the whole unit: $AX_f + SY_{n+1} = AX_n + SY_1$

or:

$$Y_{n+1} = \frac{A}{S}(X_n - X_f) + Y_1 \qquad \textbf{operating line} \qquad (1.3)$$

This is the equation of a straight line of slope A/S, known as **the operating line**, which passes through the points **(X_f, Y_1)** and **(X_n, Y_{n+1})**. In Figure (1.4), the equilibrium relation, Y_n against X_n, and the operating line are drawn in, and the number of stages required to pass from X_f to X_n is found by drawing in steps between the operating line and the equilibrium curve. In this example, four stages are required, and (X_n , Y_{n+1}) corresponds to (X_4,Y_5). It may be noted that the operating line connects the compositions of the raffinate stream leaving and the fresh solvent stream entering a unit, X_n and Y_{n+1}, respectively.

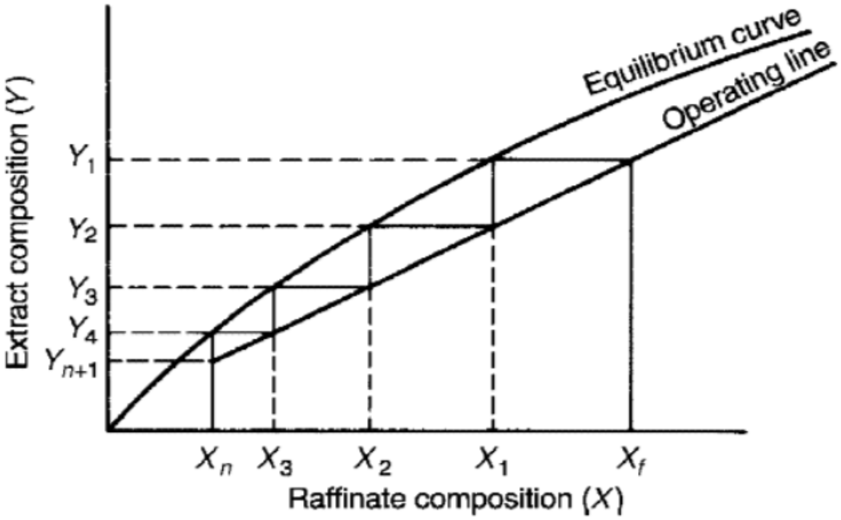


Fig (1.4): Graphical method for determining the number of stage.

EX: 1/A 5% (by weight) solution of acetaldehyde in toluene is extracted with water in co-current unit. If 100 Kg of water is used per stage for 500 kg of feed. The desired outlet in the exiting (raffinate stream) is 0.016 KgC/kgA of acetaldehyde so how many stage will be required. The equilibrium relationship is given by the equation Y= 2.3X.

Solution:

A : toluene, S: water C: acetaldehyde

$F = 500Kg$, $x_f = 0.05$ $X_n = 0.016$ Kgc/KgA

$S = 100Kg$ $Y= 2.3X$

F= 500Kg ⟶ (A= 475 Kg C =25Kg)

$X_f = \frac{0.05}{1-0.05} = 0.052$

$$n = \frac{\log \frac{X_n}{X_f}}{\log[\frac{A}{A+Sm}]}$$

$$n = \frac{\log \frac{0.016}{0.052}}{\log[\frac{475}{475+100*2.3}]} = 2.98$$

Number of stage = 3

EX: 2/ A 100 kg/min of water containing ratio of 1.2 wt% acetic acid is to be extracted with 1-butanol at 1 atm and 26.7 ºC. The ratio of desired outlet concentration in the exiting (raffinate stream) is 0.1 wt% of acetic acid. The solvent of pure 1-butanol is fed conter-currently to the feed with the flow rate of 75 kg/min. Determine the composition of the exiting 1-butanol phase (i.e. the extract phase). Also find the number of equilibrium contacts (stages) needed.

Equilibrium data:

X	0	0.002	0.004	0.006	0.008	0.01	0.012	0.014
Y	0	0.003	0.0065	0.0097	0.012	0.016	0.019	0.023

Solution:

- The inlet ratio of the feed (raffinate phase) is given as 1.2 wt% of acetic acid (or $x_f = 0.012$)

- The outlet ratio of the raffinate phase is given as 0.1 wt% of acetic acid

(or $x_n = 0.001$)

The solvent is pure butanol; thus, $Y_{n+1} = 0$

The concentration of the exiting extract-phase stream can be computed using operating-line equation (Eq. 2.3) as follows

$$Y_{n+1} = \frac{A}{S}(X_n - X_f) + Y_1$$

$$0 = \frac{100}{75}(0.001 - 0.012) + Y_1$$

$$Y_1 = 0.015$$

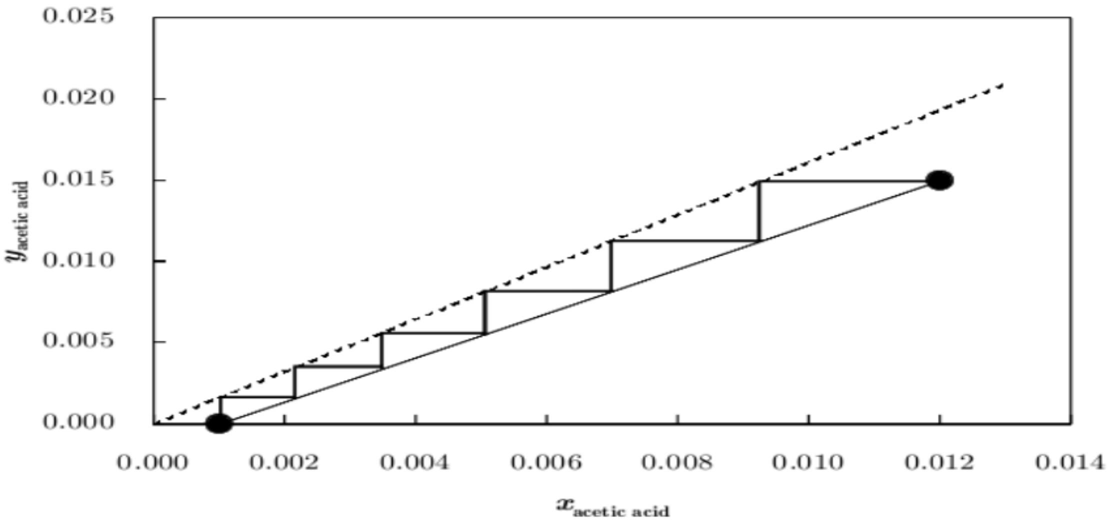
Thus, the co-ordinates for the operating line are

- $(X_f, Y_1) = (0.012, 0.015)$
- $(X_n, Y_{n+1}) = (0.001, 0)$

Plot X vs Y to obtain equilibrium line

Draw an operating line between (X_f, Y_1) and (X_n, Y_{n+1}) and determine the number of stage by stepwise construction.

Number of stage obtained = 6



EX:3

1000Kg per hour of a solution is containing 20% C by weight is to be counter currently extracted with 400 Kg per hour of solvent S. The equilibrium distribution of component C between A and S are as follows;

Wt. of C/ Wt. of A	0.05	0.2	0.3	0.45	0.5	0.54
Wt. of C/ Wt. of S	0.25	0.4	0.5	0.65	0.7	0.74

How many theoretical stage will be required to reduce the concentration of C to 5% in effluent?

Solution.

F = 1000 Kg/h (A= 800 Kg/h , C = 200 Kg/h)

$x_f = 0.2$, $x_n = 0.05$

Assume solvent to be pure, then

Counter current extraction $Y_{n+1} = 0$

Solvent (S) = 400 Kg/h

$X_f = \frac{0.2}{1-0.2}\ 0.25$ $X_n = \frac{0.05}{1-.05} = 0.0526$

$\text{Slop} = \frac{A}{S} = \frac{800}{400} = 2$

$$Y_{n+1} = \frac{A}{S} (X_n - X_f) + Y_1$$

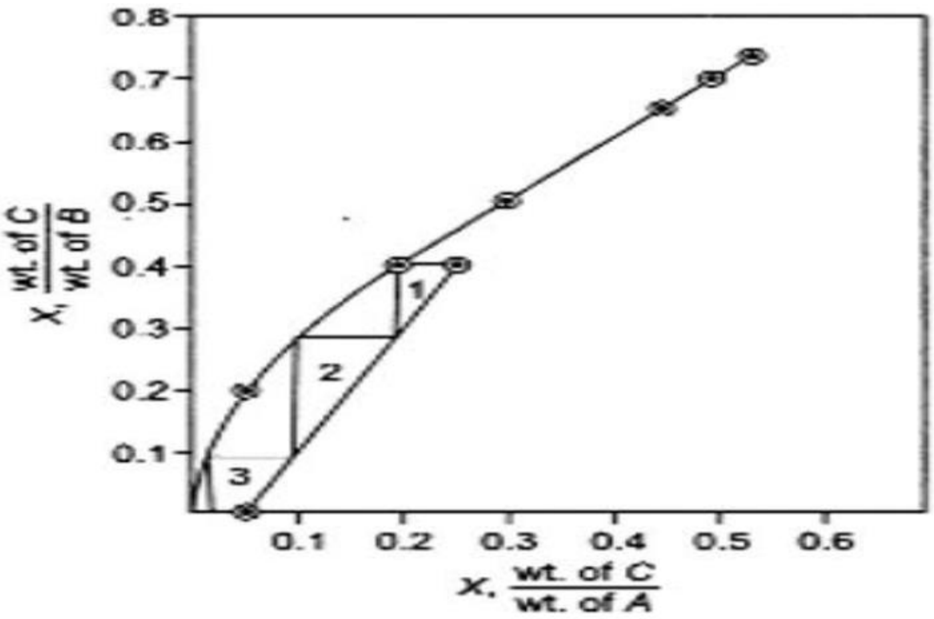
$$0 = 2 (0.0526 - 0.25) + Y_1$$

$$Y_1 = 0.395$$

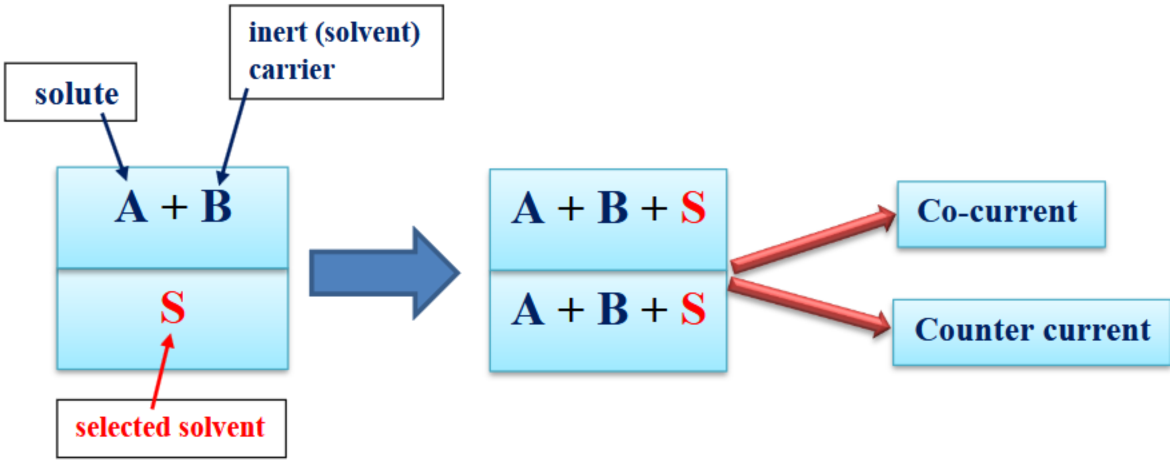
Plot X vs Y to obtain equilibrium line

Draw an operating line between (X_f, Y_1) and (X_n , Y_{n+1}) and determine the number of stage by stepwise construction.

Number of stage obtained = 3


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Y, wt. of C / wt. of B
1
2
3
0.1
0.2
0.3
0.4
0.5
0.6
X, wt. of C / wt. of A

B Partially Miscible Solvent



Here we have mixture of 2 liquids or more, it is desired to separate A from the mixture so it should be select a solvent has special properties (cheap, available, not toxic, not volatile, …), this solvent can dissolve A (desired) and leave B, if not, then it should be select a solvent can dissolve B and leave A, otherwise, we have to select a solvent can dissolve A and little bet of B, this is the meaning of partially miscible.

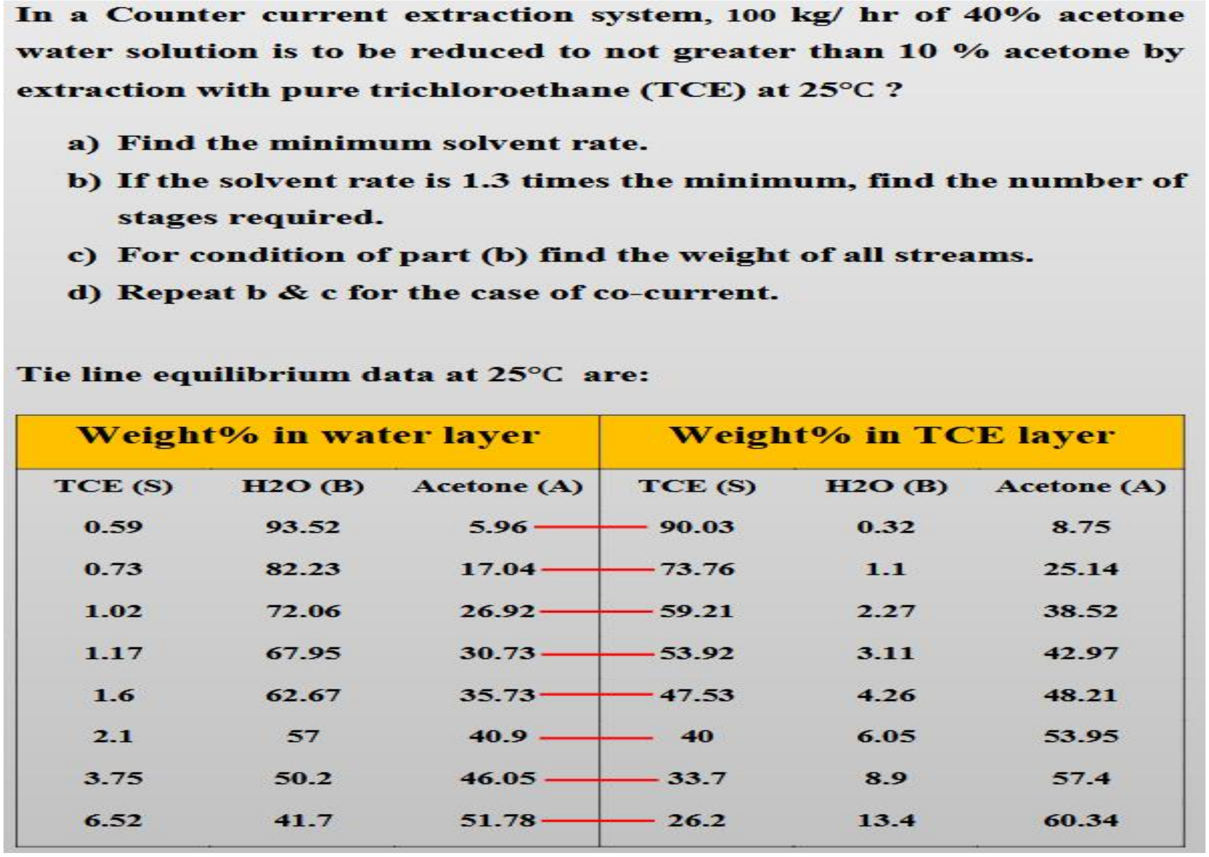
هنا لدينا مزيج من سائلين او اكثر ونريد ان نفصلهما فيجب علينا ان نختار مذيب ذو مواصفات خاصة منها (رخيص-متوفر-غير سام-غير متطاير ...) يقوم بإذابة المادة A المرغوبة ويترك المادة B فاذا لم يتوفر فنختار مذيب يقوم بإذابة المادة B ويترك المادة A المرغوبة وفي حالة عدم توفر الخيارين نضطر لاختيار مذيب يقوم بإذابة المادة A و جزء قليل من المادة B وهذا ما يعرف بالامتزاج الجزئي.

Inert B (solvent carrying solute in the feed) is partially miscible with pure solvent used to extract the solute A, there are two cases depends on the mode of flow:

Case 1: Counter Current Flow (F & S)

To understand this case, we will take the example bellow

Example



In a Counter current extraction system, 100 kg/ hr of 40% acetone water solution is to be reduced to not greater than 10 % acetone by extraction with pure trichloroethane (TCE) at 25°C ?

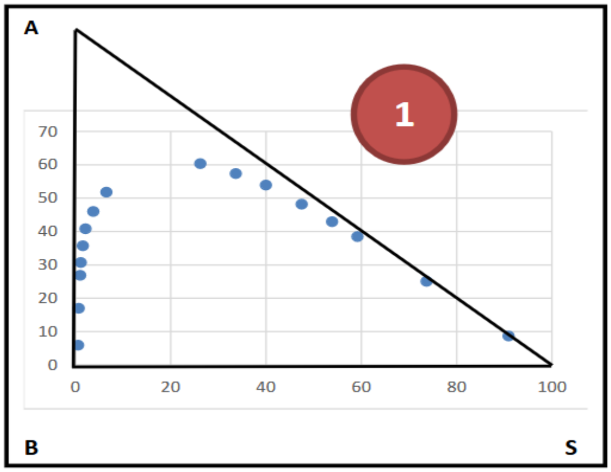
- **a) Find the minimum solvent rate.**
- **b) If the solvent rate is 1.3 times the minimum, find the number of stages required.**
- **c) For condition of part (b) find the weight of all streams.**
- **d) Repeat b & c for the case of co-current.**

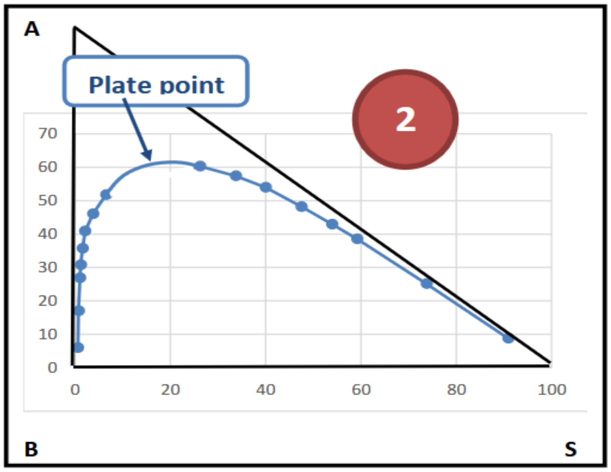
Tie line equilibrium data at 25°C are:

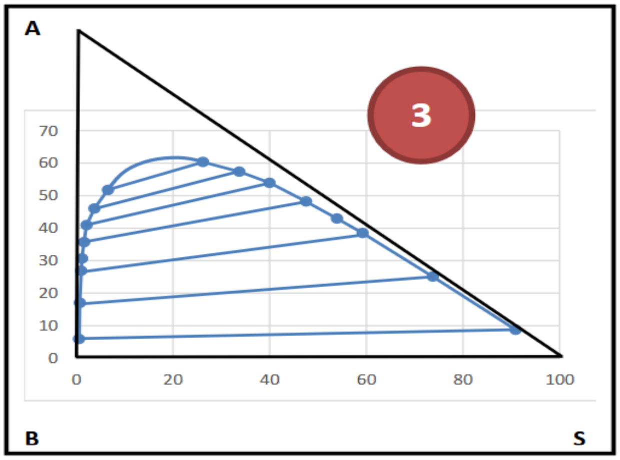
Weight% in water layer			Weight% in TCE layer		
TCE (S)	H2O (B)	Acetone (A)	TCE (S)	H2O (B)	Acetone (A)
0.59	93.52	5.96	90.03	0.32	8.75
0.73	82.23	17.04	73.76	1.1	25.14
1.02	72.06	26.92	59.21	2.27	38.52
1.17	67.95	30.73	53.92	3.11	42.97
1.6	62.67	35.73	47.53	4.26	48.21
2.1	57	40.9	40	6.05	53.95
3.75	50.2	46.05	33.7	8.9	57.4
6.52	41.7	51.78	26.2	13.4	60.34

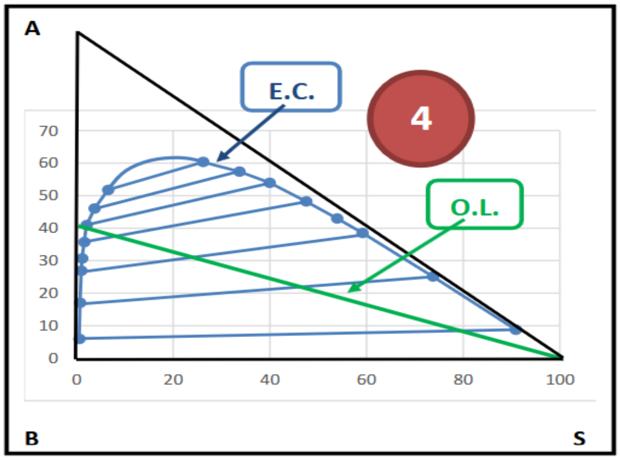
Solution: **The solution is by following these steps:**

1. **Plot equilibrium curve (inside right triangle) from points of solute A and solvent S (from the table of data), each point represents the relation between solute A and solvent S, each two opposite points represent tie line.**
2. **Specify F point on Y axis which is the concentration of A in feed (X_F or X_o) then plot the line F-S representing the operating line.**









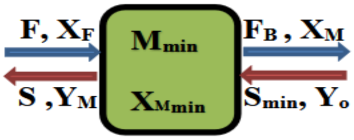
Calculation of minimum solvent rate S_{min}

a. **Plot line equivalent to the nearest tie line from X_F (0.4) to intersect the equilibrium curve in a point, from this point find Y_{min} on Y axis ($Y_{min} = 0.535$).**
b. **Plot the line $Y_{min} - X_n$ which intersect the line $X_F - S$ at point M_{min} then find Xm_{min} from Y axis ($Xm_{min} = 0.32$).**
c. **Finally, S_{min} can be found from material balance**

طريقة ايجاد قيمة S_{min} في حالة الجريان العكسي والموازي

1. نرسم خط موازي لأقرب خط ربط يمر بالنقطة (X_F) ومبتدأً منها ليقطع منحني التعادل من جهة اليمين بالنقطة التي تمثل (Y_{min}) وهي هنا تساوي 0.535 (Y_{min} بحالة توازن مع X_F)
2. نرسم الخط Y_{min}- X_n ليقطع خط التشغيل $X_F - S$ بالنقطة التي تمثل M_{min} ثم نقرأ قيمة Xm_{min} من محور Y وهنا تساوي 0.32
3. أخيرا نجد قيمة S_{min} من موازنة المادة كما موضح أدناه

<u>M.B.</u>



$F + S_{min} = M_{min}$ ….. (1)

$F (X_F) + S_{min} (Y_o) = M_{min} (Xm_{min})$ … . (2)

2 eqns. And 2 unknowns, fin M_{min} and S_{min}

$100 (0.4) + S_{min} (0) = M_{min} (0.32)$

$M_{min} = 40 / 0.32 = 125$ **(sub. in eqn. 1)**

$S_{min} = 125 – 100 = 25$ **kg/hr**

<u>Note:</u> this is general way for calculation of minimum solvent rate S_{min} in cases of co-current and counter current flow.

Calculation of the number of stages and intermediate concentrations

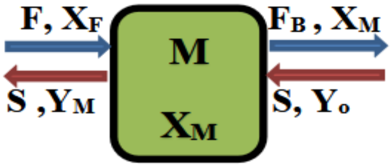
A. Find value of actual amount of solvent (S).

B. specify the point representing (M) on the figure after finding the value of X_M from material balance as follow:

$$S = 1.3\ S_{min} = 1.3 * 25 = 32.5\ kg / hr$$

<u>M.B.</u>

$$F + S = M \qquad ..... (3)$$



$$F (X_F) + S (Y_o) = F(X_M) + S(X_M) \ ...... (4)$$

$$F (X_F) + S (Y_o) = (F + S)\ X_M \quad \text{(divided by S)}$$

$$\frac{F}{S}(X_F) + Y_o = (\frac{F}{S}+1)\ X_M$$

Note: all concentrations should be found (read) from Y axis

$$X_M = \frac{\frac{F}{S}X_F + Y_o}{\frac{F}{S}+1} \qquad ..... (5)$$

$$X_M = \frac{\frac{100}{32.5}(0.4) + 0}{\frac{100}{32.5}+1} = 0.3$$

C. Specify location of X_M on Y axis and plot horizontal line from X_M to intersect the line F-S at point M.

D. Finding location of point P:

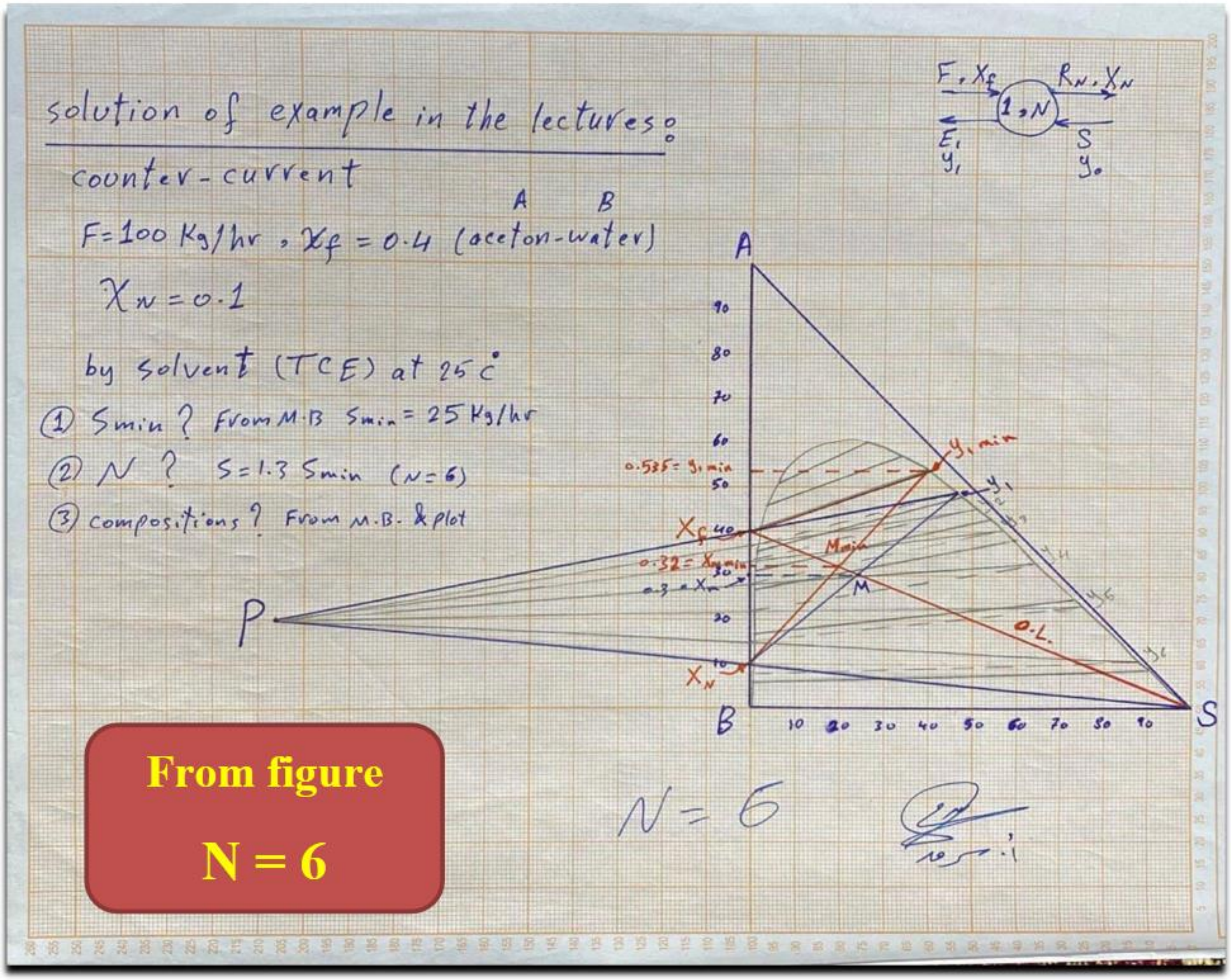
1. **Plot the line M-X_n which intersect the eqm. curve at Y_1 ($Y_1 = 0.48$).**
2. **Plot the lines Y_1-X_F and S-X_n, they intersect at point P outside the triangle.**

E. Find X_1 on the eqm. curve from Y_1 and a line equivalent to the nearest tie line ($X1 = 0.37$).

F. Plot line P-X_1 which intersect eqm, curve at Y_2 ($Y_2 = 0.42$).

G. Find X_2 on the eqm. curve from Y_2 and a line equivalent to the nearest tie line ($X1 = 0.33$).

H. And so on we continue until reach X_n or pass it, number of stages and intermediate concentrations can be found graphically.



solution of example in the lectures:

counter-current

F = 100 Kg/hr , x_f = 0.4 (aceton-water)
A B

X_N = 0.1

by solvent (TCE) at 25°C

① S_{min}? From M.B S_{min} = 25 Kg/hr

② N ? S = 1.3 S_{min} (N = 6)

③ compositions? From M.B. & plot

N = 6

From figure

N = 6

Calculation of inlet and outlet amounts for any stage from these steps:

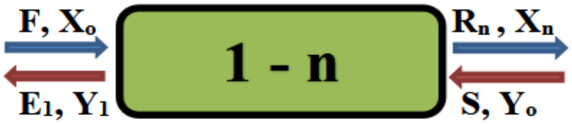
1. **Over all material balance should be done in order to find E_1 & R_n**

O.M.B

$$F + S = R_n + E_1$$

$$100 + 32.5 = R_n + E_1$$

$$R_n + E_1 = 132.5 \qquad ….. (6)$$



Solute M.B

$FX_o + Sy_o = R_n X_n + E_1Y_1$

$(100)\,(0.4) + 0 = 0.1\ R_n + 0.48\ E_1$ ….. (7)

From eqns (6) and (7)

$R_n = 62.1$ kg/s

$E_1 = 70.39$ kg/s

2. To find amounts between stages, material balance on stage 1 should be done in order to find E_2 & R_1

O.M.B on stage 1

$F + E_2 = R_1 + E_1$

$100 + E_2 = R_1 + 70.39$ ….. (8)

Solute M.B

$FX_o + E_2Y_2 = R_1X_1 + E_1Y_1$

$(100)\,(0.4) + E_2\,(0.42) = R_1(0.35) + 70.39\,(0.48)$ ….. (9)

From eqns. (8) and (9)

$R_1 = 98.61$ kg/s

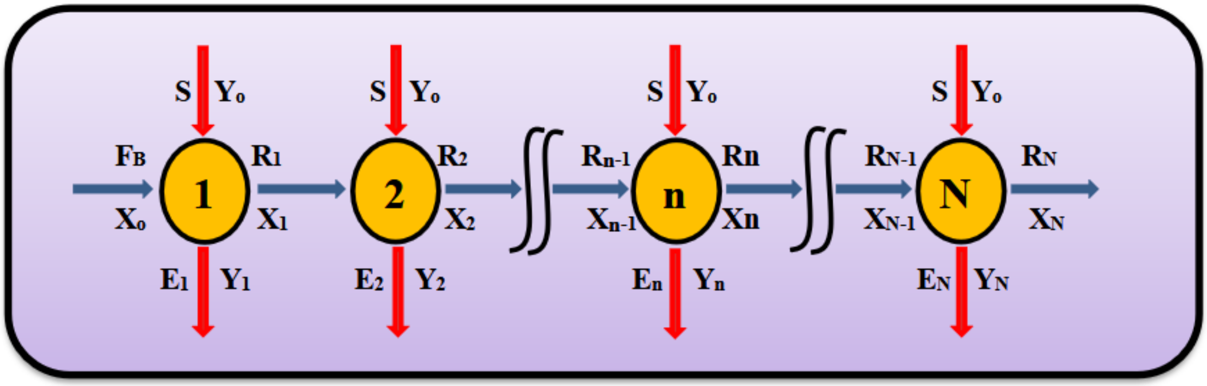
$E_2 = 69$ kg/s

Note: all concentrations should be found from figure & all amounts from material balance as we saw above

And so on, continue making material balance to every stage in order to find the moderate concentrations until reaching the last stage which its concentration get found from previse steps or reaching desired one.

Case 2: Co-Current Flow (F & S)

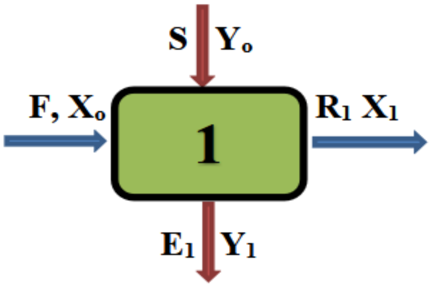
Calculation of minimum amount of solvent (S_{min}) is the same as counter current flow



To find number of stages, these steps should by followed:

a. **Specify F point on Y axis which is the concentration of A in feed ($X_F = 0.4$) then plot the line F-S representing the operating line.**
b. **Find the values of M_1 and X_{M1} from material balance as follow:**

M.B on stage 1 B



$F + S = E_1 + R_1 = M_1$

$100 + 32.5 = M_1 = 132.5$ kg/s

$E_1 + R_1 = 132.5$ kg/s …..(10)

Solute M.B

$FX_o + Sy_o = M_1 X_{M1}$

$E_1Y_1 + R_1 X_1 = M_1 X_{M1}$

$100 (0.4) + 32.5(0) = 132.5 X_{M1}$

$X_{M1} = 0.3$

$E_1Y_1 + R_1 X_1 = M_1 X_{M1}$ …..(11)

c. From X_{M1} (Y axis), plot horizontal line to intersect X_F-S line at M_1.
d. Plot line equivalent to the nearest tie line from M_1 to intersect the equilibrium curve in points X_1 & Y_1 ($Y_1 = 0.37$ & $X_1 = 0.25$).

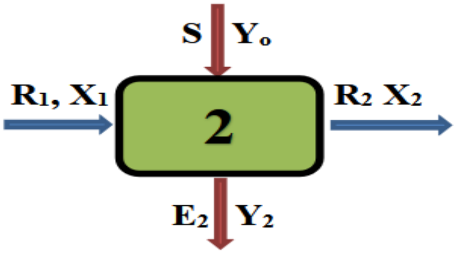
From equations (10) & (11) (2 eqns and 2 unknowns)

$R_1 = 77.29$ kg/s,

$E_1 = 55.2$ kg/s

e. plot the line X_1-S representing the operating line for stage 2.
f. Find the values of M_2 and X_{M2} from material balance as follow:

M.B on stage 2



$R_1 + S = E_2 + R_2 = M_2$

$77.29 + 32.5 = M_2 = 109.79$ kg/s

$E_2 + R_2 = 109.79$ kg/s .....(12)

Solute M.B

$R_1X_1 + Sy_o = E_2Y_2 + R_2 X_2 = M_2 X_{M2}$

$77.29 (0.25) + 32.5(0) = 109.79 X_{M2}$

$X_{M2} = 0.176$

$E_2Y_2 + R_2 X_2 = (109.79) (0.176)$ .....(13)

g. From X_{M2} (Y axis), plot horizontal line to intersect X_1-S line at M_2.
h. Plot line equivalent to the nearest tie line from M_2 to intersect the equilibrium curve in points X_2 & Y_2 ($Y_2 = 0.21$ & $X_2 = 0.13$).
i. And so on until reaching X value to X_n or pass it after that estimate the number of stages.

Calculation of inlet and outlet amounts for any stage by material balance on it:

From equations (12) & (13)

$R_2 = 46.66$ kg / s,

$E_2 = 63.13$ kg / s

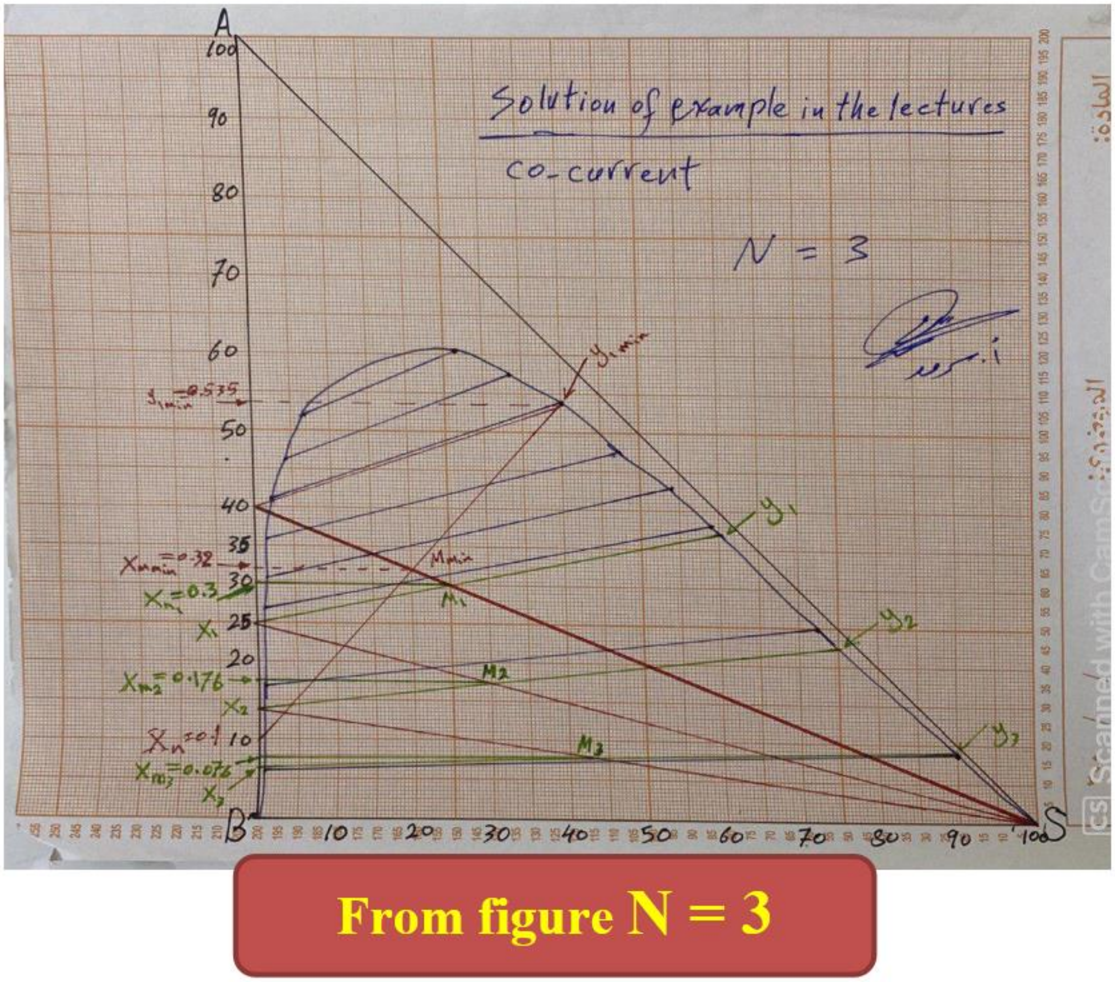
And so on, until reaching the last stage or desired one.

$X_3 = 0.06$, $Y_3 = 0.09$

Note: by reaching to value of X (0.06) less than X_n (0.1), we should stop and account the number of stages.

From graphical figure:

Number of stages = 3



From figure N = 3

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

Learning package
In
Drying operation

For
second year students

By

Ataa Wejood
Dep. Of Chemical Industrial Techniques
2025

1/ Overview

1 / A –Target population :-

For students in the second year
Technological Institute of Basra
Dep. Of Chemical Industrial Techniques

1 / B –Rationale:-

Drying is a crucial unit operation used to remove moisture or liquid from a solid or semi-solid material by vaporization, typically through the application of heat and mass transfer principles. This process is essential in many industries—including food processing, pharmaceuticals, chemicals, and agriculture—to improve product stability, shelf life, and handling.

The rationale for studying drying operations lies in its impact on **product quality, energy efficiency, and process optimization**. Understanding the drying mechanism helps engineers design equipment and select operating conditions that minimize drying time and energy consumption while preserving the material's physical and chemical properties.

Moreover, drying plays a vital role in preventing microbial growth, reducing weight for transportation, and preparing materials for further processing or storage. Mastery of drying principles enables professionals to solve challenges related to uneven drying, product degradation, and scale-up, ensuring sustainable and cost-effective industrial processes.

3 / C –Central Idea:-

Drying is the process of removing moisture from a solid or semi-solid material by transferring liquid as vapor into the surrounding air or gas phase. It involves the simultaneous heat and mass transfer, where heat is supplied to evaporate the moisture, and the vapor is removed to prevent saturation.

The efficiency of drying depends on factors such as temperature, humidity, airflow, and properties of the material. Understanding drying mechanisms enables the design and operation of drying equipment that maximizes moisture removal while preserving product quality and minimizing energy consumption.

In essence, drying is a vital unit operation used to enhance product stability, reduce weight, and prepare materials for storage or further processing in various industries.

The subject of drying operation focuses on the principles and mechanisms of heat and mass transfer involved in moisture evaporation, the factors influencing drying rates, and the design and optimization of drying equipment across various industrial applications.

1 / D – Performance Objectives

After completing the study of drying operations, the student will be able to:

1. **Define** the drying process and explain its industrial importance in various applications.
2. **Describe** the mechanisms of moisture removal, including heat and mass transfer principles involved in drying.
3. **Differentiate** between drying periods (e.g., constant rate period and falling rate period).
4. **Identify** the key factors affecting drying rate, such as temperature, air velocity, humidity, surface area, and material properties.
5. **Classify** different types of dryers (e.g., tray dryer, rotary dryer, fluidized bed dryer) and select suitable equipment for specific materials and processes.
6. **Calculate** drying time and rate using drying curves and moisture content data.

7. **Analyze** energy requirements for different drying operations and propose energy-efficient solutions.
8. **Apply** drying operation knowledge to improve product quality, minimize degradation, and ensure safe moisture levels for storage and transportation.

Drying operation

INTRODUCTION

The drying of materials – whether solids, liquids or slurries – to improve storage life or reduce transportation costs is one of the oldest and most commonly used unit operations. Drying of fruit, meat and various building and craft materials date back before the discovery of fire. The physical laws governing drying remain the same, even though the machinery to accomplish it has improved considerably! Today, dryers are in operation in most manufacturing industries, including chemical, pharmaceutical, process and food. Products that are dried range from organic pigments to proteins, as well as minerals to dairy products. Because of the spectrum of duties required, there is a great variety of dryers available. The correct choice depends on the properties of the feed material and the desired characteristics of the final product. Gases and liquids containing small amounts of water can be dried by adsorption operations.

Purpose of Drying Operations: It is carried out for a number of reasons:

(1) It is used for purifying a crystalline product so that the solvent adhering to the crystals is removed or the solvent can be recovered.

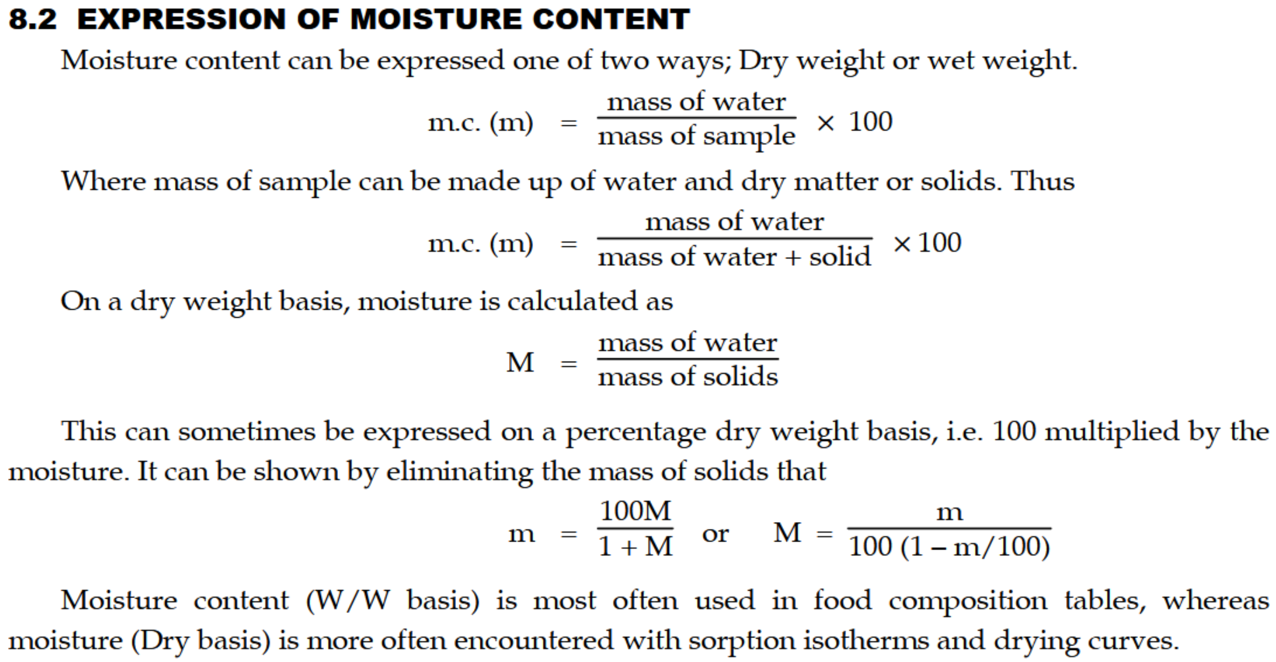
(2) Storage of dry solid as compared to wet solid is easy.

(3) The cost of transportation for dry material would be reduced.

(4) Drying can provided definite desired properties to the material as in the case of pharmaceutical substances such as tablets, syrups etc.

(5) Presence of traces of moisture may lead to problems of corrosion as in the case of chlorine gas. Dry chlorine gas is not corrosive but traces of moisture make it very corrosive.

(6) Sometimes can be an essential part of the process, e.g. drying of paper. However, care must be taken to avoid shrinkage of material or loss of flavor in food product or cracking etc.; which may take place during drying operations.



8.2 EXPRESSION OF MOISTURE CONTENT

Moisture content can be expressed one of two ways; Dry weight or wet weight.

$$\text{m.c. (m)} = \frac{\text{mass of water}}{\text{mass of sample}} \times 100$$

Where mass of sample can be made up of water and dry matter or solids. Thus

$$\text{m.c. (m)} = \frac{\text{mass of water}}{\text{mass of water + solid}} \times 100$$

On a dry weight basis, moisture is calculated as

$$M = \frac{\text{mass of water}}{\text{mass of solids}}$$

This can sometimes be expressed on a percentage dry weight basis, i.e. 100 multiplied by the moisture. It can be shown by eliminating the mass of solids that

$$m = \frac{100M}{1 + M} \quad \text{or} \quad M = \frac{m}{100\,(1 - m/100)}$$

Moisture content (W/W basis) is most often used in food composition tables, whereas moisture (Dry basis) is more often encountered with sorption isotherms and drying curves.

TYPES OF MOISTURE

Equilibrium moisture X*: This is the moisture content of a substance when at equilibrium with a given partial pressure of the vapour.

Bound moisture: This refers to the moisture contained by a substance, which exerts an equilibrium vapour pressure less than that of the pure liquid at the same temperature.

Unbound moisture: This refers to the moisture contained by a substance, which exerts an equilibrium vapour pressure equal to that of the pure liquid at the same temperature.

Free moisture: Free moisture is that moisture contained by a substance in excess of the equilibrium moisture: $X - X^*$. Only free moisture can be evaporated, and the free-moisture content of a solid depends upon the vapour concentration in the gas.

RATE-OF-DRYING CURVE

From the data obtained during such a test, a curve of moisture content as a function of time. This will be useful directly in determining the time required for drying larger batches under the same drying conditions. Much information can be obtained if the data are converted into rates (or fluxes) of drying, expressed as N mass/(area)(time), and plotted against moisture content. This can be done by measuring the slopes of tangents or by determining from the curve small changes in moisture content ΔX for corresponding small changes in time $\Delta\theta$ and calculating the rate as $N = -S_s \Delta X/A \Delta \theta$ Here SS is the mass of dry solid, A is the wet surface over which the gas blows and through which evaporation takes place in the case of cross-air circulation drying. In the case of through-circulation drying, A is the cross section of the bed measured at right angles to the direction of the gas flow. The rate-of-drying curve is sometimes plotted with the ordinate expressed as mass moisture evaporated/ (mass dry solid) (time), which in the present notation is $- d/d\theta$. There are usually two major parts to the rate curve of Fig. 8.3, a period of constant rate and one of falling rate, as marked on the figure. While different solids and different conditions of drying often give rise to curves of very different shape in the falling-rate period, the curve shown occurs frequently. Some of the differences which may arise will be considered later, but for the present let use briefly review the reasons generally advanced for the various parts of the curve shown. If a solid is initially very wet, the surface will be covered with a thin film of liquid, which we shall assume is entirely unbound moisture. When it is exposed to relatively dry air, evaporation will take place from the surface. The rate at which moisture evaporates can be described in terms of a gas mass transfer coefficient KY and the difference in humidity of the gas at the liquid surface YS and in the main stream Y. Thus, for cross-circulation drying.

$$\mathbf{N_C = K_Y (Y_S - Y)}$$

TIME REQUIRED FOR DRYING

If one wishes to determine the time of drying a solid under the same conditions for which a drying curve such as Fig. 8.3 has been completely determined, one need merely read the difference in the times corresponding to the initial and final moisture contents from the curve. Within limits, it is sometimes possible to estimate the appearance of a rate-of-drying curve for conditions different from those used in the experiments. In order to determine the time for drying for such a curve, we proceed as follows.

As evident from above figure, the rate of drying curve consists of two major zones :

(a) a period of constant rate drying.

(b) a period of falling rate drying.

Since the mechanism of drying during the initial adjustment is not very clear, hence this period is not included in the total drying time.

Rate of drying is given by :

$$N = -\frac{L_S}{A} \cdot \frac{dx}{d\theta} \qquad \ldots (8.2)$$

From equation (8.2), the expression for drying time is obtained as :

$$\theta = \frac{L_S}{A} \int_{x_2}^{x_1} \frac{dx}{N} \qquad \ldots (8.3)$$

If drying takes place within the constant rate period only (so that $x_1 > x_o$ and $x_2 > x_C$, $N = N_C$) then equation (8.3) becomes,

$$\theta = \frac{L_S (x_1 - x_2)}{A N_C} \qquad \ldots (8.4)$$

For more general situations involving both constant and falling rate regimes, equation (8.4) can be rewritten to give the constant rate time period as,

$$\boxed{\theta_C = \frac{L_S (x_1 - x_C)}{A N_C}} \qquad \ldots (8.5)$$

Time for the falling rate period is determined with the help of graphical integration of equation (8.5) by determining the area under the curve of $\frac{1}{N}$ Vs x

So,

$$\theta_f = \frac{L_S}{A} \times \text{(area under the curve of Fig. 8.4)}$$

$$\theta_f = \frac{L_S (X_C - X^*)}{AN_C} \cdot \ln \frac{x_1 - x^*}{x_2 - x^*} \qquad (\text{when } x_1 < x_C) \quad \ldots(8.6)$$

If the initial moisture content of the solid is more than the critical moisture, equation (8.6) is written as,

$$\boxed{\theta_f = \frac{L_S (x_C - x^*)}{AN_C} \ln \frac{x_C - x^*}{x_2 - x^*}} \qquad \ldots (8.7)$$

For such a case, the total time for drying with the help of equations (8.5) and (8.7) is given by,

$$\theta_T = \theta_C + \theta_f$$

$$\boxed{\theta_T = \frac{L_S}{AN_C} \left[(x_1 - x_C) + (x_C - x^*) \ln \frac{x_C - x^*}{x_2 - x^*} \right]} \qquad \ldots (8.8)$$

Estimation of drying rate : The heat or mass transfer equation can be used to estimate constant drying rate. We have heat transfer equation :

$$N_C = \frac{\dot{m}}{A} = \frac{h_G \, (T_a - T_i)}{\lambda_i} \qquad \ldots (8.9)$$

where, $\dot{m}$ = Rate of evaporation, kg/hour

A = Drying surface, m^2

T_a = Temperature of air

T_i = Temperature of interface

λ_i = Latent heat at T_i

For air flow parallel to surface the heat transfer coefficient can be estimated by the following dimensional equation :

$$h_G = 0.0176 \, G^{0.8} \qquad \ldots (8.10 \text{ a})$$

where, h_G = heat transfer coefficient, k cal/hr. m^2 °C

G = Mass velocity of air, kg/hr. m^2

When a flow perpendicular to surface is considered, the above equation becomes,

$$h_G = 1.004 \, G^{0.37} \qquad \ldots (8.10 \text{ b})$$

Note that numerical constants of equation (8.10 a) and (8.10 b) are dimensional.

8.10 HEAT TRANSFER IN DRYERS

Drying of solids in a heat transfer process coupled with diffusional transfer of moisture. However, most dryers are designed conveniently on heat transfer conditions :

Heat must be applied for :

(i) heating the feed to vapourisation temperature.
(ii) vapourizing the liquid.
(iii) heating solid to final exit temperature.
(iv) heating vapour to final exit temperature, and

Thus the total rate of heat transfer is,

$$\frac{q_T}{\dot{m}_S} = C_P S (T_{S_b} - T_{S_a}) + X_a C_{P_L} (T_V - T_{S_a}) + (X_a - X_b) \lambda + X_b \cdot C_{P_L} (T_{S_b} - T_V) + (X_a - X_b) C_{P_V} (T_{V_b} - T_V) \quad \ldots (8.11)$$

where, q_T = Amount of heat transfer, k cal/hr.
$\dot{m}_S$ = Amount of bone-dry solid, kg/hr.
X_a = kg moisture/kg dry solid (entry)
X_b = kg moisture/kg dry solid (exit)
C_{P_S} = Specific heat of solid k cal/kg °C
C_{P_L} = Specific heat of liquid, k cal/kg °C
C_{P_V} = Specific heat of vapour k cal/kg °C
T_{S_a} = Inlet solid temperature, °C
T_V = Vapourisation temperature, °C
T_{V_b} = Final vapour (exit) temperature, °C
λ = Latent heat of vapour, k cal/kg

The temperature distribution pattern in a continuous counter-current dryer is given below :

For adiabatic dryers, T_V and T_{V_b} of equation (8.11) become equal to wet bulb temperature and exit temperature of the drying medium respectively. For a continuous adiabatic dryer heat balance is given by,

$$q_T = \dot{m}_g (1 + H_a)\, C_{S_a} (T_{h_a} - T_{h_b}) \quad \dots (8.12)$$

where $\dot{m}_g$ = Mass rate or drying medium, kg of humidity-free/hour

H_a = Humidity of drying medium at entry

C_{S_a} = Humid heat of drying medium at entry

In dryer calculations, the basic heat transfer equation used is,

$$q_T = UA\,\Delta T \quad \dots (8.13)$$

There is uncertainty in area available for heat transfer. So many dryers are designed on the basis of a volumetric heat-transfer coefficient, U_a.

where 'a' is the (unknown) heat transfer 0 area per unit dryer volume.

The governing equation is,

$$q_T = U_a V \cdot \Delta T \quad \dots (8.14)$$

where, U_a = volumetric heat transfer coefficient, k cal/m^2 hr °C

V = dryer volume, m^3

ΔT = average temperature difference, °C

(1) Slabs of paper pulp 1 m × 1 m × 0.015 m is to be dried under constant drying conditions from 66.7% to 30% moisture. The value of equilibrium moisture for the material is 0.5%. If the critical moisture content is 60% and the rate of drying of critical point is 1.5 kg/hr.m²; calculate the drying time. The dry weight of each slab is 2.5 kg. Assume all moisture contents are on wet basis.

Sol. : Consider drying takes place from the two big faces.

So, Area for drying = $(1 \times 1) \times 2 = 2\ m^2$

$$L_S = 2.5\ \text{kg}$$

$$X_1 = \frac{66.7}{100 - 66.7} = 2.0$$

$$X_2 = \frac{30}{100 - 30} = 0.429$$

$$X^* = \frac{0.5}{100 - 0.5} = 0.005$$

$$X_C = \frac{60}{100 - 60} = 1.5$$

$$N_C = 1.5\ \text{kg/hour}\ m^2$$

We have equation for total time required for drying,

$$\theta_T = \theta_C + \theta_f$$

$$= \frac{L_S}{AN_C} \left\{ (X_1 - X_C) + (X_C - X^*)\, ln \frac{X_C - X^*}{X_2 - X^*} \right\}$$

$$= \frac{2.5}{2 \times 1.5} \left\{ (2 - 1.5) + (1.5 - 0.005) \cdot ln \frac{1.5 - 0.005}{0.429 - 0.005} \right\} = 1.987$$

∴ Total drying time = **1.987 hours** **(Ans.)**

(3) Celotax sheets are to be dried by flowing air at 60 °C, 10% relative humidity and a velocity of 10 m/sec. The critical moisture content is 0.35 kg free water/kg dry solid. The rate of drying in the falling rate period can be considered linear. The celotax must be dried from 55% to 12% moisture (Wet basis). Equilibrium moisture content at the conditions of the dryer is 5% (wet basis). The celotax sheets are kept in layer of 6 cm thick in insulated trays. The dry density of solid is 1.38 gm/cc. Calculate the time of drying the sheets.

Data : The rate of drying in gm/cm^2 . hour in the constant rate period is given by :

$$N_C = 0.004\, V^{0.8}\, (p_i - p_g)$$

where V = **Air velocity, m/sec**

p_i and p_g are dew point temperature and vapour pressure of water in mm of Hg in air at the wet bulb temperature respectively.

Sol. : At 60° C and 10% humidity,

$$H = 0.014 \frac{\text{kg water}}{\text{kg dry air}}$$

$$= \frac{\frac{0.014}{18}}{\frac{1}{29}} = 0.023 \frac{\text{kg mole water}}{\text{kg. mole dry air}}$$

If p_g = partial pressure of water vapour,

$$\frac{p_g}{P_T - p_g} = 0.023,$$

where P_T = 760 mm Hg

∴ p_g = 17.09 mm Hg

Corresponding to dew point of the above air,

$$\text{Humidity} = 0.028 \frac{\text{kg water}}{\text{kg dry air}} = 0.046 \frac{\text{kg mole water}}{\text{kg mole dry air}}$$

$$\therefore \quad \frac{p_t}{760 - p_i} = 0.046$$

∴ p_i = 33.42 mm Hg at dew point temperature

Partial pressure of water = vapour pressure

Hence,

$$N_C = 0.004\, V^{0.8}\, (p_i - p_g)$$

$$= 0.004\, (10)^{0.8}\, (33.42 - 17.09)$$

$$= 0.412 \text{ gm/cm}^2\text{ . hr.}$$

where N_C = constant rate drying

$$X_1 = \frac{55}{100 - 55} = 1.222,$$

$$X_C = 0.35,$$

$$X_2 = \frac{12}{100 - 12} = 0.136,$$

$$X^* = \frac{5}{100 - 5} = 0.053$$

Dry density = 1.38 gm/cc

Let us consider a celotax sheet of 1 m × 1 m

$\therefore$ Volume of this sheet = 100 cm × 100 cm × 6 cm = 60,000 cm^3

$$L_S = \text{Dry weight} = 60{,}000 \times 1.38 = 8.28 \times 10^4 \text{ gm}$$

$$\text{Area} = 100 \times 100 = 10^4 \text{ cm}^2$$

(Since the trays are insulated)

Therefore, time required for drying,

$$\theta_T = \theta_C + \theta_f$$

$$= \frac{L_S}{A \cdot N_C} \left\{ (X_1 - X_C) + (X_C - X^*) \, ln \, \frac{X_C - X^*}{X_2 - X^*} \right\}$$

$$= \frac{8.28 \times 10^4}{10^4 \times 0.412} \left\{ (1.222 - 0.35) + (0.35 - 0.053) \, ln \, \frac{0.35 - 0.053}{0.136 - 0.053} \right\}$$

$$\theta_T = \mathbf{25.14 \text{ hours}}$$ **(Ans.)**

(4) It is necessary to dry a batch of 160 kg of a wet material from 30% to 5% moisture content, under constant rate and falling rate period. The falling rate is assumed to be linear. Calculate the total drying time considering an available drying surface of 1 $\mathbf{m^2/40}$ kg of dry solid. A constant drying flux of $\mathbf{3 \times 10^{-4}}$ $\mathbf{kg/m^2}$ is given :

$$\mathbf{X_C = \text{Critical moisture content} = \frac{0.2 \text{ kg moisutre}}{\text{kg solid}}}$$

$$\mathbf{X^* = \text{Equilibrium moisture content} = 0.05}$$

Sol. :

$$\frac{L_S}{A} = \frac{160 \times 0.7}{40} = \frac{112}{40} = 2.8 \text{ m}^2$$

$$X_1 = \frac{30}{100 - 30} = 0.4285 \text{ kg moisture/kg solid}$$

$$X_2 = \frac{5}{100 - 5} = 0.0526 \text{ kg moisture/kg solid}$$

Constant rate period :

$$\theta_C = \frac{L_S (X_1 - X_C)}{AN_C} = \frac{2.8 \, (0.4285 - 0.20)}{3 \times 10^{-4}}$$

$$\theta_C = 2132.66 \text{ sec.}$$

Falling rate period :

$$\theta_f = \frac{L_S (X_C - X^*)}{AN_C} \, ln \left[\frac{X_C - X^*}{X_2 - X^*} \right]$$

$$= \frac{2.8 \, (0.2 - 0.05)}{3 \times 10^{-4}} \, ln \left[\frac{0.2 - 0.05}{0.0526 - 0.05} \right]$$

$$= 5677.17 \text{ sec.}$$

So, the total time required for drying

$$\theta_T = \theta_C + \theta_f = 7809.83 \text{ sec.}$$

$$= \mathbf{2.17 \text{ hours}}$$ **(Ans.)**

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

Learning package
In
Gas-Liquid Absorption

For
second year students

By

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Dep. Of Chemical Industrial Techniques
2025

1/ Overview

1 / A –Target population :-

For students in the second year
Technological Institute of Basra
Dep. Of Chemical Industrial Techniques

1 / B –Rationale:-

Gas-liquid absorption is a key mass transfer operation in which one or more gaseous components are **selectively transferred** from a gas phase into a **liquid solvent**. It plays a vital role in many industrial processes such as **air pollution control, gas purification, chemical synthesis, and environmental protection**.

Understanding the gas-liquid absorption process is essential because it combines principles of **mass transfer, thermodynamics, and fluid dynamics** to achieve efficient solute removal or recovery. The process depends on factors such as **solubility, interfacial area, contact time, and mass transfer coefficients**, which must be optimized for effective design and operation.

Studying this operation equips engineers and scientists with the skills to:

- **Design and analyze absorption columns** (e.g., packed or tray towers),
- **Select appropriate solvents** based on solubility and chemical compatibility,
- **Control emissions of toxic or greenhouse gases**,

- And **improve process sustainability** by integrating absorption with regeneration or chemical reaction.

Therefore, the rationale for studying gas-liquid absorption lies in its **scientific importance**, its **broad industrial applications**, and its **critical role in ensuring safety, environmental compliance, and process efficiency**.

4 / C –Central Idea:-

Gas-liquid absorption is a mass transfer operation in which one or more components of a gas mixture are **transferred into a liquid solvent** due to a difference in concentration. This process relies on the **driving force created by the concentration gradient** between the gas and liquid phases and is often enhanced through equipment that increases **contact surface area and residence time**.

The core of this operation is the **interaction between gas and liquid phases**, where solutes dissolve into the liquid either physically or chemically. It is widely used in industries for **gas purification**, **pollution control**, and **reactive separations**, making it essential for environmental and chemical engineering applications.

The subject of gas-liquid absorption focuses on the **principles of mass transfer between gas and liquid phases**, design and operation of **absorber equipment** (e.g., packed and tray columns), the role of **equilibrium and rate-based models**, and factors that influence the **absorption efficiency**, such as solvent properties, flow rates, pressure, temperature, and interfacial area.

In summary, the central idea of gas-liquid absorption revolves around the **efficient and selective transfer of gaseous components into a liquid phase** to achieve desired separation or purification goals.

1 / D – Performance Objectives

After completing the study of gas-liquid absorption, the student will be able to:

1. **Define** the gas-liquid absorption process and explain its industrial applications (e.g., air pollution control, CO_2 removal, gas purification).
2. **Differentiate** between physical absorption and chemical absorption.
3. **Describe** the mechanisms of mass transfer from the gas phase into the liquid phase, including the role of concentration gradients.
4. **Interpret** equilibrium data such as Henry's law and apply it to gas absorption problems.
5. **Identify** and **evaluate** key factors affecting absorption efficiency, including solvent selection, temperature, pressure, flow rates, and interfacial area.
6. **Classify** different types of absorption equipment (e.g., packed towers, plate columns) and understand their operating principles.
7. **Calculate** mass transfer rates, number of transfer units (NTU), and height of transfer units (HTU) in absorber design.
8. **Design** a basic absorption column using theoretical and empirical correlations.
9. **Analyze** the performance of existing gas absorption systems and suggest improvements for energy efficiency and effectiveness.
10. **Apply** knowledge of gas-liquid absorption to solve real-world problems in environmental and chemical engineering contexts.

Absorption and Stripping (Gas-Liquid Separation)

الامتصاص والانتزاع

Absorption is used to separate gas mixture; remove impurities and pollutants from gas; and recover valuable chemicals.

Thus the species of interest in the gas mixture may be all components, only the component (s) transferred.

The species transferred to the liquid called solute or absorbate.

The liquid called absorbent or solvent.
In stripping (desorption), a liquid mixture is contacted with a gas to selectively remove components by mass transfer from the liquid to gas phase.

Strippers are frequently coupled with absorbers to permit regeneration and recycle of the absorbent. Because stripping is not perfect, absorbent recycled to the absorber contains species percent in the vapor (gas) entering the absorber.

When water is used as the absorbent, it's common to separate the absorbents from the solvent by distillation rather than by stripping.

The more important concepts in absorption

1. definitions and symbols in absorption:

Solute (A) in a mixture of (A + B) will be absorbed in a liquid (C), (B) insoluble in (C), the following symbols will be used :-

G: total gas flow rate (A + B) (kmole/s).

G_S: inert (carrier) gas flow rate of B. (kmole/s).

$\overline{G}$: total gas flow rate (A + B) (Kmole/m^2.s)

$\overline{G_S}$: inert gas flow rate of B.(kmole/ m^2.s)

L: total liquid flow rate (A + B) (kmole/s).

L_S: solvent (absorbate , carrier) flow rate of C only (kmole/s).

$\overline{L}$: total liquid flow rate (A + B) (Kmole/m^2.s)

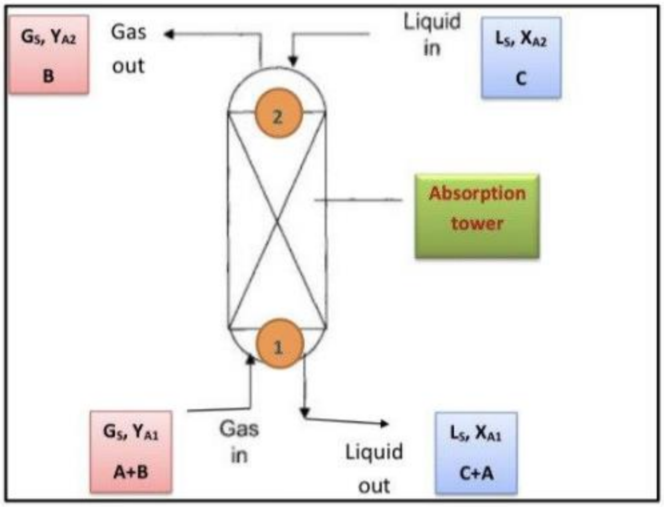
$\overline{L_S}$: solvent flow rate (C) (kmole/m^2.s)

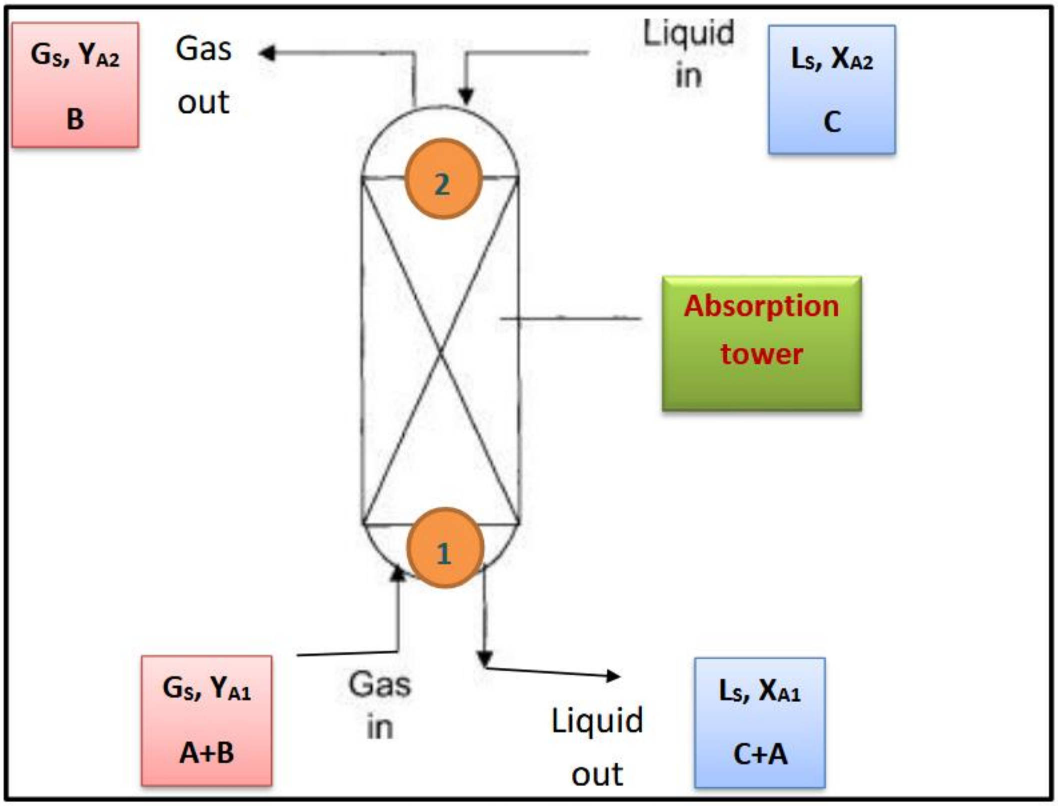
x_A: liquid mole fraction [A / (A+C)]

X_A: liquid mole ratio (A / C).

y_A: : gas mole fraction [A / (A+B)]

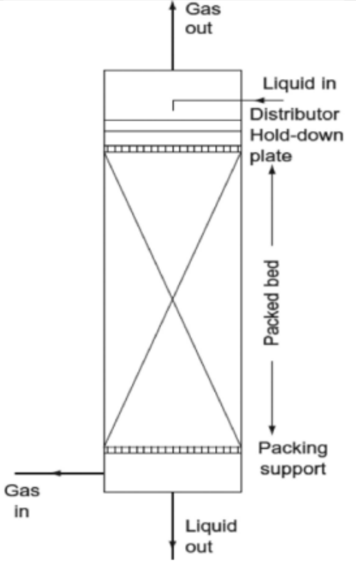
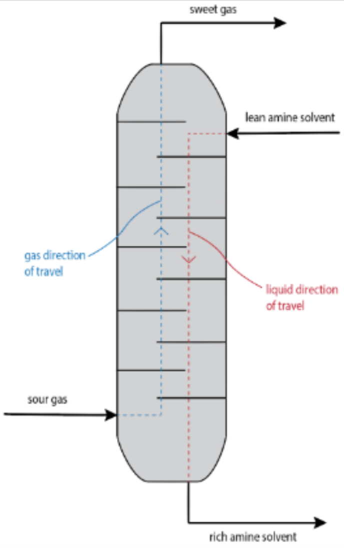
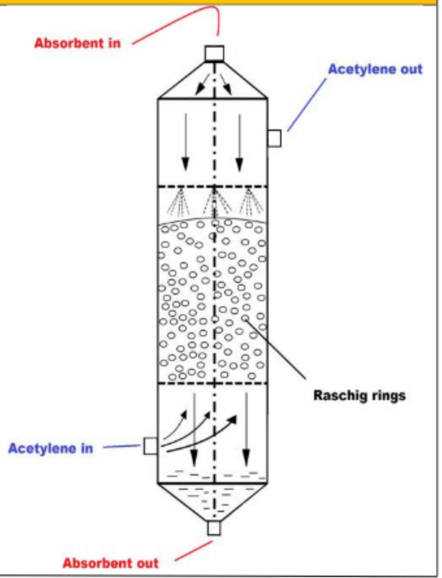
Y_A: gas mole fraction (A / B).





shape 1 Absorption Tower

Types of absorption towers:

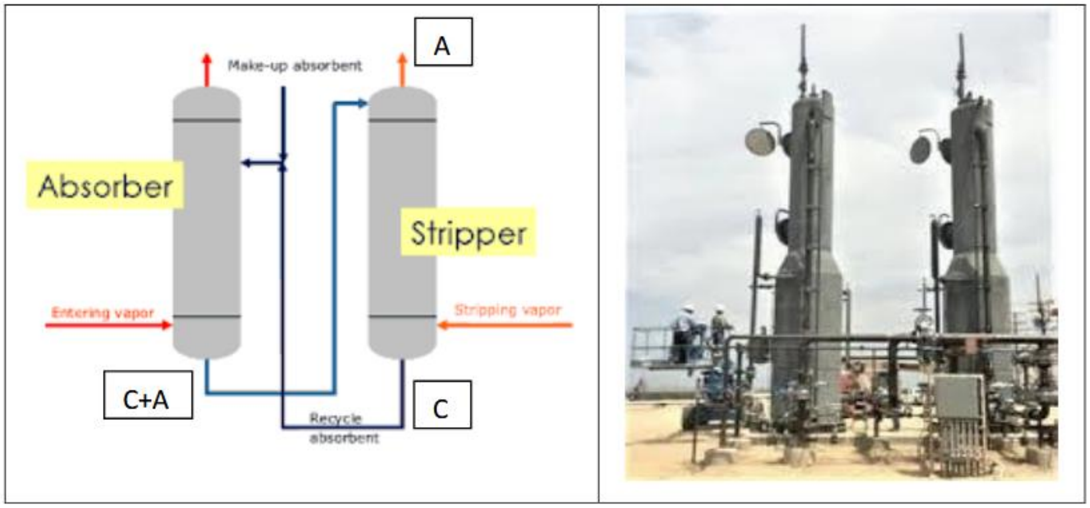
1. Packed tower	2. Tray tower	3. Spray tower
		

Shape 2 Types of absorption towers

Some actual towers:

Shape 3 Actual towers

Absorption towers always followed by stripping towers:



Shape 4 Absorption and stripping towers

Notes:

[1] It is very important to pay attention to the correct units, this represent the key of solution.

[2] Mole fraction (x, y) can be converted to mole ratio (X,Y) (and opposite) as follow:

$$\mathbf{Y} = \frac{y}{1-y} \quad \Longrightarrow \quad y = \frac{\mathbf{Y}}{1+\mathbf{Y}} \qquad (1)$$

$$\mathbf{X} = \frac{x}{1-x} \quad \Longrightarrow \quad x = \frac{\mathbf{X}}{1+\mathbf{X}} \qquad (2)$$

[3] Mole ratio (mole %) can be converted to weight ratio (wt. %) (and opposite) as follow:

$$\text{wt.}\,\% = \frac{mole\,\%\,.(m.wt.)}{\sum mole\,\%\,.(m.wt.)} \qquad (3)$$

$$mole\% = \frac{\frac{wt.\,\%}{m.wt.}}{\sum \frac{wt.\,\%}{m.wt.}} \qquad (4)$$

Where:

m.wt. = molecular weight (kg/kmole)

2. Equilibrium solubility of gases and liquids:

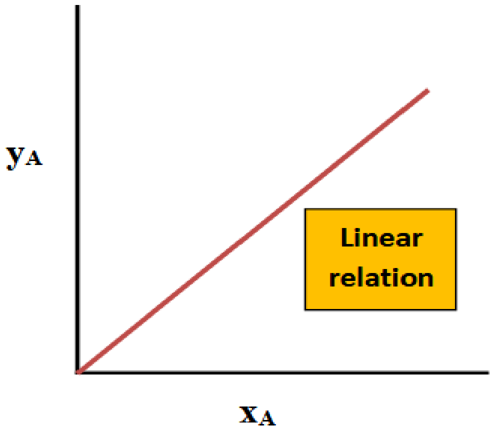
The equilibrium state is the perfect state to describe the column, the more important things in absorption process is knowing the nature of equilibrium relation between solute in gas (Y) and solute in liquid (X), this relation either be linear (ideal solution) where the concentration of solute in gases mixture less than 10%, and can de wrote as:

$$y_A^* = m\, x_A \tag{5}$$

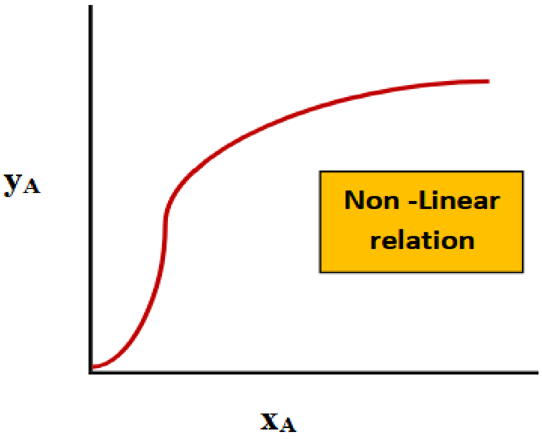
or be non linear (non ideal solution) with high concentration more than 10%, equilibrium information here given by table of data like:

y_A . . . .

x_A . . . .



For ideal solution



For non - ideal solution

Notes about equilibrium relation:

1. **Sometimes, equilibrium relations may be given as a relation between partial pressure and mole ratio so it should be converted as follow:**

$$P_A = H\,X_A \qquad \text{(Henry)}$$

Divided by P_T

$$\frac{P_A}{P_T} = \frac{H}{P_T} X_A$$

$$\boxed{y_A = m\,x_A}$$

$$P_A = P_A^{O}\,X_A \qquad \text{(Raoult)}$$

Divided by P_T

$$\frac{P_A}{P_T} = \frac{P_A^{O}}{P_T} X_A$$

$$\boxed{y_A = m\,x_A}$$

2. **Equilibrium relations may be given as a data, but when plotted it appears that it is linear, in this case we will get the slope and then continue with $y_A = m\,x_A$.**

3. Operating line: الخط التشغيلى

Operating line represent the real relation between yA and xA, this relation may be found from material balance between any point inside the column (tower) and top of the column (tower)

O. M. B. on solute

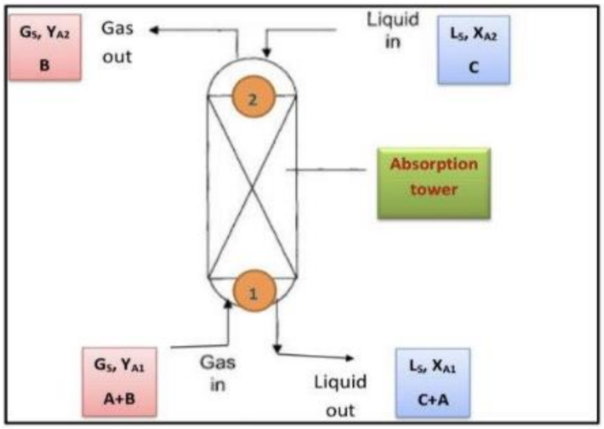
In = Out

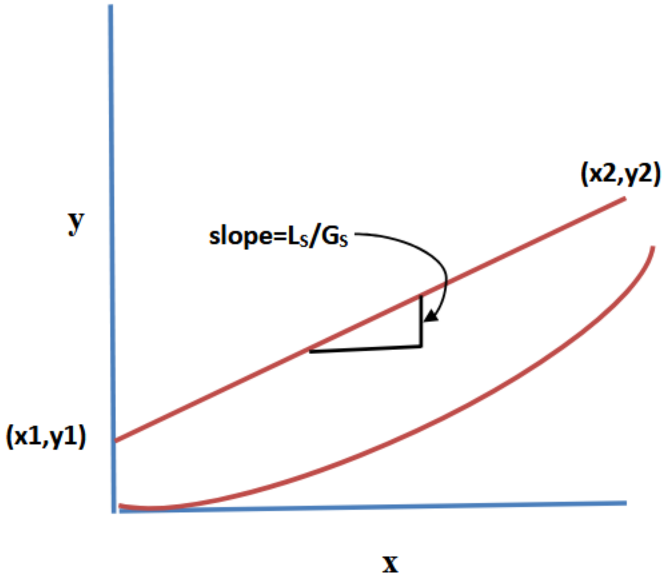
$$G_S \,.\, Y_1 + L_S \,.\, X_2 = G_S \,.\, Y_2 + L_S \,.\, X_1$$

$$G_S \,(Y_1 - Y_2) = L_S \,(X_1 - X_2)$$

$$Y_1 = \frac{L_S}{G_S}(X_1 - X_2) + Y_2 \qquad (6)$$

Operating line equation y = m x





Notes about operating line:

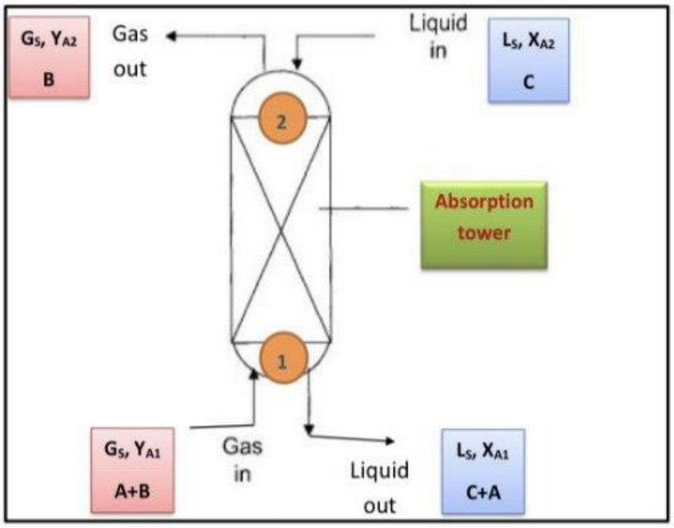
1. **Operating line can be plotted either from two points the bottom of column (x1,y1) and the top of column (x2,y2) or between one point and slope (L_S/G_S).**
2. **Here we will use LS and GS because there values are constant and do not change during the process and they represent the value of inert (carrier) from gases.**
3. **If we need the value of x1, it will be found from mass balance.**
4. **When we make mass balance,**
   - **(a) If the concentration of solute in feed more than 10% we have two choices:**
     Either using G with y
     Or using G_S with Y
   - **(b) If the concentration of solute in feed less than 10% we can consider**
     $Y = y$ and $X = x$
     Where:
     y is mole fraction of solute in gas = part / total (A/ A+B)
     x is mole fraction of solute in liquid = part / total (A/ A+C)
     Y is mole ratio of solute in gas = part A / part B
     X is mole ratio of solute in liquid = part A / part C

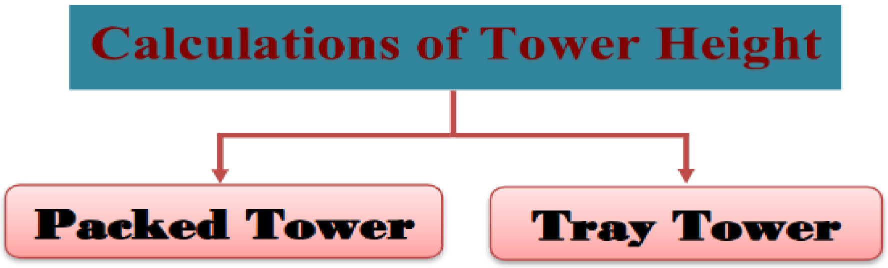
G is the total (A + B)

L is the total (A + C)

G_S is the part (B)

L_S is the part (C)





First: Packed Tower

The tower packing employed in operations involving mass transfer between gases and liquids are designed to carry the liquid as a thin film exposing a large surface area to the gas stream, it preferred for corrosive and foaming liquids and vacuum operation, it normal diameter less than 120 cm. The principle requirement of an efficient packed tower are:

- **Large surface area.**
- **Uniform distribution of liquid and gas over the packing.**
- **Low resistance to the flow of gas through the tower.**

In general

$$Z = HOG * NOG \qquad (7)$$

Where:

HOG: Height of Transfer Unit (H.T.U) (m) or (HETP)

NOG: Number of Transfer Unit (N.T.U)

HETP: Height Equivalent of Theoretical Plates

This is the height of packing that will give the same separation as one theoretical plate. The HETP must be experimentally determined quantity characteristic for each packing, it depends on type and size of packing, also with the flowrate of each fluid and concentration.

Now to derive the relation represent the height of tower (column), material balance will be done on a section inside the tower as follow:

Note: material balance on a section mean the concentration varies along the tower and from one point to another.

M.B on solute:-

$$G_s\, y - G_s\left(y + \frac{dy}{dz}\,\delta z\right) = K_{OG}\,.\,a\,.\,s\,.\,\delta z\,(y - y^*) \qquad (8)$$

Where:

a: interfacial area = active area/volume of column (m^2/m^3).

s: cross-sectional area of column (m^2)

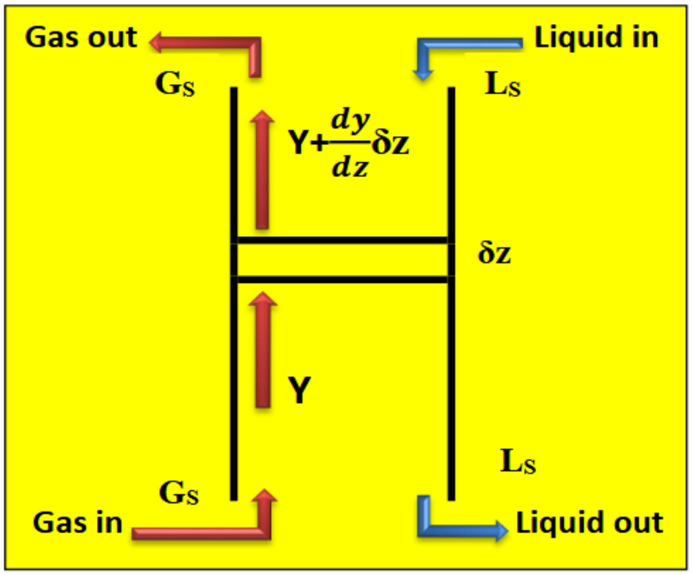
$$-\,G_s\,\frac{dy}{dz}\,\delta z = K_{OG}.a.s.\ \delta z(y - y^*)$$

$$G_s\,\frac{dy}{dz} = -\,K_{OG}.a.s.\ (y - y^*)$$

$$\int_0^z dz = \frac{-G_S}{K_{OG}.a.s}\ .\int_{y_1}^{y_2}\frac{dy}{y - y*}$$

$$Z = \frac{G_S/S}{K_{OG}.a}\ .\int_{y_2}^{y_1}\frac{dy}{y - y*}$$

$$Z = \frac{\overline{G_S}}{K_{OG}.a}\ .\int_{y_2}^{y_1}\frac{dy}{y - y*} \qquad (9)$$



$$Z = HOG\ .\ NOG$$

$$\boxed{HOG = \frac{\overline{G_S}}{K_{OG}.a} \qquad (10)}$$

From equation no. 10, HOG or HTU (Height of Transfer Unit in m) can be calculated in packed tower with any type of equilibrium relation.

But calculation of NOG or NTU (Number of Transfer Unit) which is equal to the integration in equation no. 9 can be found depending on the type of equilibrium relationship (linear or non-linear), so we have two solutions as follow:

A) If the equilibrium relation is linear (y* = m . x)

$$NOG = \int_{y_2}^{y_1} \frac{dy}{y - y^*} \qquad (11)$$

y* should be converted as a function to y

$y^* = m\,x$

to find x

<u>M.B on solute</u>

$Gs\,(y - y_2) = Ls\,(x - x_2)$

$x = \frac{Gs}{Ls}\,(y - y_2) + x_2$

For pure solvent $\longrightarrow$ $x_2 = 0$

$x = \frac{Gs}{Ls}\,(y - y_2)$

$y^* = m \,.\, \frac{Gs}{Ls}\,(y - y_2)$

$$NOG = \int_{y_2}^{y_1} \frac{dy}{y - m\,.\frac{Gs}{Ls}\,(y - y_2\,)}$$

$$= \int_{y_2}^{y_1} \frac{dy}{y - \emptyset\, y + \emptyset\, y_2}$$

$$= \int_{y_2}^{y_1} \frac{dy}{(1 - \emptyset)\, y + \emptyset\, y_2}$$

$$NOG = \frac{1}{1-\emptyset} \ln \frac{(1-\emptyset)y_1+\emptyset y_2}{(1-\emptyset)y_2+\emptyset y_2}$$

$$= \frac{1}{1-\emptyset} \ln \frac{(1-\emptyset)y_1+\emptyset y_2}{y_2}$$

$$NOG = \frac{1}{1-\emptyset} \ln \left[(1-\emptyset)\frac{y_1}{y_2} + \emptyset\right] \qquad (12)$$

Where $\emptyset = m \cdot \frac{Gs}{Ls}$

Conditions

1. Linear → $y^* = m\,x$
2. Pure solvent → $x_2 = 0$

Notes:

1. The value of Ø is always less than 1 **(Ø <1)**
2. If the equilibrium relation is non-linear, the solution should be graphically (by plotting the given data).
3. If the equilibrium relation is linear, the solution should be mathematically (by equations) and it can be graphically.
4. If $x_2 \neq 0$ (i.e. the solvent is not pure or recovered from the stripper) then the derivation of NOG will be repeated with another way:

$$Gs\,(y_1 - y_2) = K_{OG} \cdot a \cdot s \cdot z\,(\Delta y_{LM}) \qquad (13)$$

$$z = \frac{G_S/S}{K_{OG} \cdot a} \cdot \frac{(y_1-y_2)}{\Delta Y_{LM}} \qquad (14)$$

$$Z = HOG \cdot NOG$$

Where:

$$\Delta y_{LM} = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln \frac{(y_1 - y_1^*)}{(y_2 - y_2^*)}}$$

B) If the equilibrium relation is non-linear

In this case, the solution of integration in eq. no. 11 (NOG) will be done either by numerical method (Simpsons Rule) or by graphical mothed as follow:

1. Numerical method (Simpsons Rule)

To find $NOG = \int_{y_2}^{y_1} \frac{dy}{y - y*}$ **, these steps should be followed:**

1. **Plot equilibrium curve (E.C.) from the given values of x and y.**
2. **Plot operating line (O.L.) by slope and point or two points.**
3. **y_1 and y_2 must be given, they tabled in the first column and add some values between them by adding [Δh=(y1-y2)/6] to each one.**
4. **Find y* from the figure for each y by finding real x from (O.L.) and y* from (E.C.), the values of y* tabled in the second column.**
5. **Find [1/(y – y*)] for each y and y*, then tabled in the third column.**
6. **Find NOG from Simpsons rule**

$$NOG = \frac{\Delta h}{3}\left[f_1 + f_n + 2\sum f_{even} + 4\sum f_{odd}\right] \qquad (15)$$

y	y*	1/(y-y*)
y_1	y_1^*	$1/(y_1-y_1^*)= f_1$
y_2	y_2^*	$1/(y_2-y_2^*)= f_2$
y_3	y_3^*	$1/(y_3-y_3^*)= f_3$
y_4	y_4^*	$1/(y_4-y_4^*)= f_4$
y_5	y_5^*	$1/(y_5-y_5^*)= f_5$
y_6	y_6^*	$1/(y_6-y_6^*)= f_6$
y_n	y_n^*	$1/(y_n-y_n^*)= f_n$

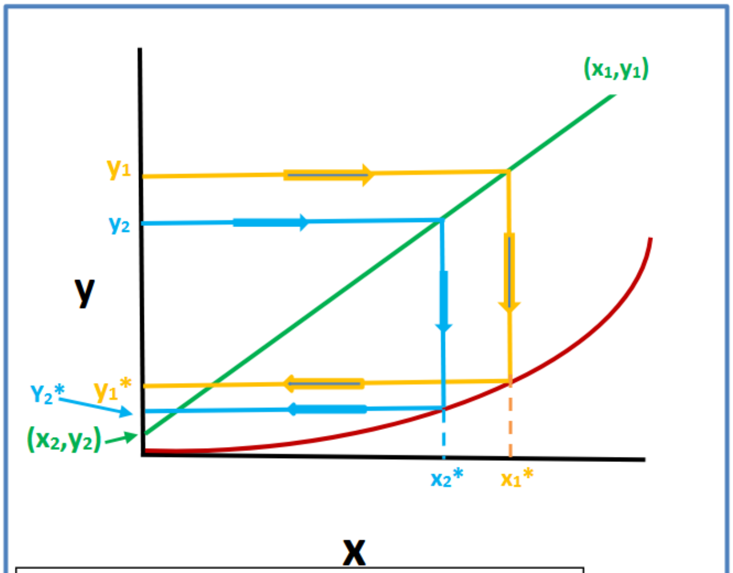


Figure 3 finding y* and x* from y

2. Graphical method

To find $NOG = \int_{y_2}^{y_1} \frac{dy}{y-y*}$

Plot y against $\frac{1}{y-y*}$

NOG = area under the curve

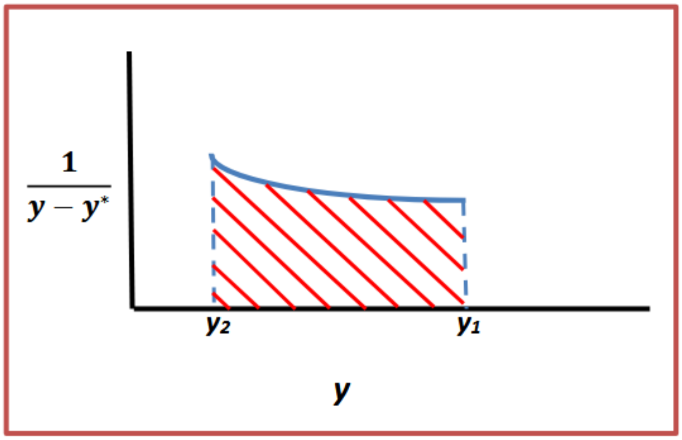


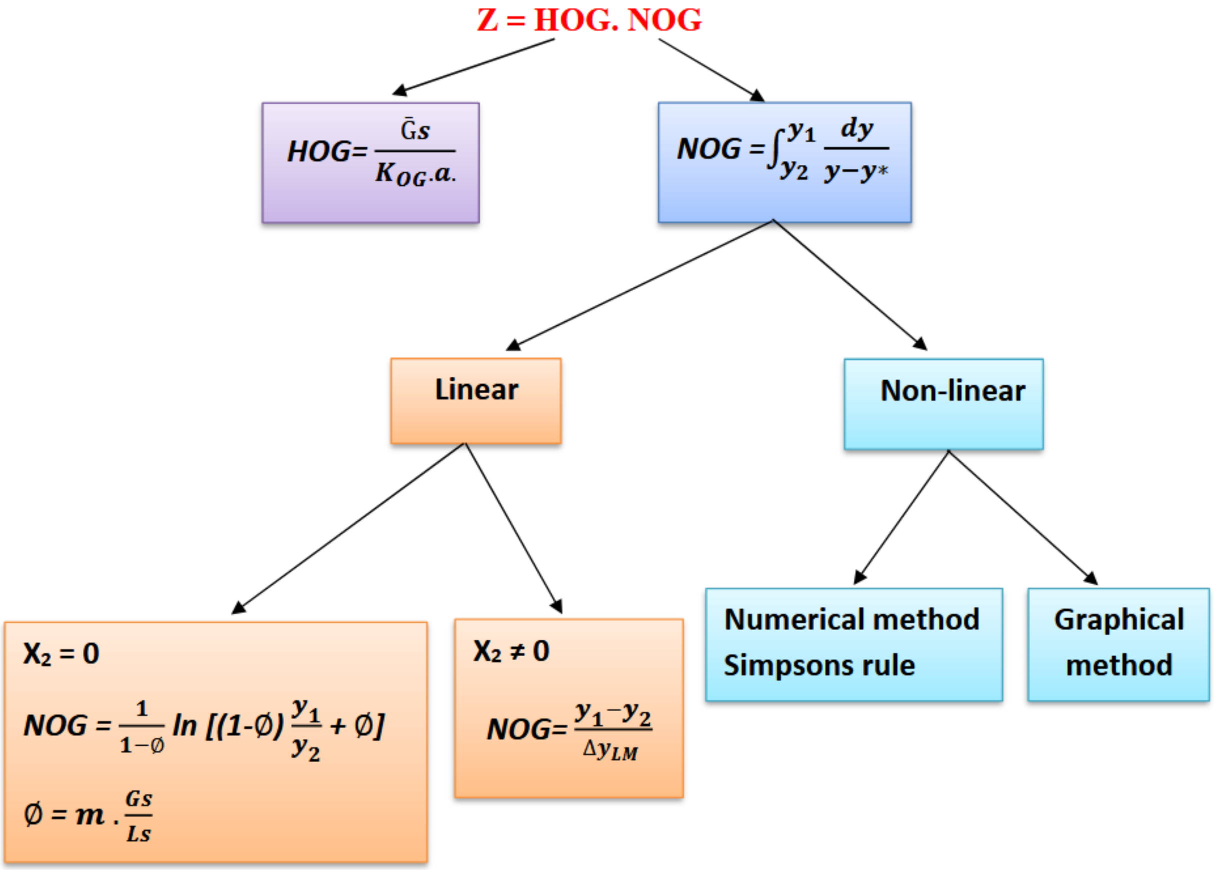
Figure 4 area under curve

summery

Height of packed tower (Absoption)

الخلاصة

Based on gas phase



Z = HOG. NOG

$HOG = \frac{\bar{G}s}{K_{OG}.a.}$

$NOG = \int_{y_2}^{y_1} \frac{dy}{y-y*}$

Linear

Non-linear

$X_2 = 0$

$NOG = \frac{1}{1-\emptyset} \ln [(1-\emptyset)\frac{y_1}{y_2} + \emptyset]$

$\emptyset = m . \frac{Gs}{Ls}$

$X_2 \neq 0$

$NOG = \frac{y_1 - y_2}{\Delta y_{LM}}$

Numerical method
Simpsons rule

Graphical method

Example 1:

Ammonia is to be removed from 10% ammonia–air mixture by scrubbing with water in a packed tower, 99% of ammonia is removed. What is the required height of tower, given the following data:-

- **The eq relation $y^* = 0.8\,x$**
- **Entering gas = 0.95 kg/m^2.s**
- **K_{OG}.a = 0.0008 kmole/m^3.s.kpa**
- **Water rate = 0.65 kg/m^2.s**
- **Mol. Wt. (NH_3) = 17**

Notes: (1) Scrubber is an air pollution control device and it is one of absorption devices

(2) Here the system is gas and liquid, so the process is absorption

(3) From the given equilibrium relation ($y^*=0.8x$), we knew the relation is linear and the value of NOG will be found by eq. 12

Sol:

$$y_2 = (1\text{-recovery})\, y_1 = (1-0.99)\, y_1 = 0.01\, y_1$$

$$\because y_1 = 0.1 \Longrightarrow y_2 = 0.01\,(0.1) = 0.001$$

$$\overline{G_S} = \frac{\overline{G_S}}{m.wt} = \frac{0.95}{0.1(17)+0.9(29)} = 0.0341 \text{ kmol/m}^2\text{.s}$$

$$\overline{L_S} = \frac{\overline{L_S}}{m.wt} = \frac{0.65}{18} = 0.0361 \text{ kmol/m}^2\text{.s}$$

$$HOG = \frac{\overline{G_S}}{K_{OG}.a} = \frac{0.0341}{0.0008 * 101.3} = 0.42m$$

$$NOG = \frac{1}{1-\emptyset} \ln\left[(1-\emptyset)\frac{y_1}{y_2} + \emptyset\right]$$

$$\emptyset = m\,.\frac{Gs}{Ls} = \frac{0.8(0.0341)}{0.0361} = 0.755$$

$$NOG = \frac{1}{1-0.755} \ln\left[(1-0.755)\frac{0.1}{0.001} + 0.755\right] = 13.17$$

$$Z = NOG * HOG = (0.42) * (13.17) = 5.53 \text{ m}$$

Minimum liquid flow rate

To apply perfect process scientifically and economically, minimum liquid flow rate should be taken in the mind because the liquid (solvent) has a special properties that must be kept, the liquid flow should be at the minimum amount that give the best transfer, that happen when x_{A1} (solute concentration in outlet liquid) is in equilibrium with y_{A1} (solute concentration in inlet gas).

The estimation of minimum liquid flow rate depends on the kind of equilibrium relation as follow:

(1) If equilibrium relation is linear ($y^* = m\,x$)

<u>O.M.B on solute</u>

$$G_s\,(y_1-y_2) = L_s\,(x_1-x_2)$$

$$\frac{Ls}{Gs} = \frac{y_1-y_2}{x_1-x_2}$$

1. **For pure solvent → $x_2 = 0$**

$$\frac{Ls}{Gs} = \frac{y_1-y_2}{x_1} \;\ldots\ldots\ldots\ldots\ldots\ldots\;\text{for actual} \qquad (16)$$

For $\left(\frac{Ls}{Gs}\right)_{min}$, above equation become:

$$\left(\frac{Ls}{Gs}\right)_{min} = \frac{y_1-y_2}{x_1^*} = \frac{y_1-y_2}{y_1/m} = m\,\left(\frac{y_1-y_2}{y_1}\right)$$

$$\left(\frac{Ls}{Gs}\right)_{min} = m\,\left(1-\frac{y_2}{y_1}\right) \;\ldots\ldots\;\text{for minimum} \qquad (17)$$

2. **If x_2 has a value → $x_2 \neq 0$**

$$\left(\frac{Ls}{Gs}\right)_{min} = \frac{y_1-y_2}{x_1^*-x_2}$$

$$\left(\frac{Ls}{Gs}\right)_{min} = \frac{y_1-y_2}{\frac{y_1}{m}-x_2} \qquad (18)$$

(2) If equilibrium relation is non-linear

As we know, every non-linear equilibrium relation mean the solution should be graphically by plotting the equilibrium data and get x_1^* from the figure graphically (figure 3)

For pure solvent

$$\left(\frac{Ls}{Gs}\right)_{min} = \frac{y_1 - y_2}{x_1^*} \qquad (19)$$

From figure

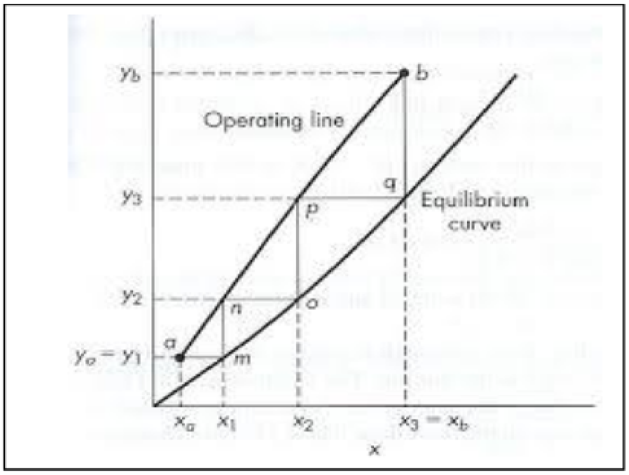


Figure 5 operating line and equilibrium curve

<u>Note:</u> the minimum liquid flow represent ideal state, this state could not be reached unless the contact time between gas and liquid is too big and this required very long column height (infinity), therefor actual minimum liquid flow should be used instead of the ideal minimum liquid flow and related to it, as follow:

$$\left(\frac{Ls}{Gs}\right)_{ac} = constant \left(\frac{Ls}{Gs}\right)_{min} \qquad (20)$$

The above constant in design ranges between (1.1 – 1.5)

Example 2:

A soluble gas is to be absorbed from an air – gas in a packed tower using solute free liquid. The equilibrium relation is y = mx.

(1) Show that the number of overall transfer unit is given by:

$$NOG = \frac{1}{1-\emptyset} \ln \left[(1-\emptyset)\frac{y_1}{y_2} + \emptyset\right] \quad \text{where } \emptyset = m \cdot \frac{Gs}{Ls}$$

(2) If 99% of the solute is to be recovered using a liquid rate of 1.75 times the minimum and the height of the transfer unit is 1.0m. What packed height is required?

Sol:

(1) The derivation is above in lucture

(2)

$y_2 = (1\text{-recovery})\, y_1 = (1-0.99)\, y_1 = 0.01\, y_1$

$$\left(\frac{Ls}{Gs}\right)_{min} = m\left(1-\frac{y_2}{y_1}\right) = m\left(1-\frac{0.01y_1}{y_1}\right) = 0.99\, m$$

$$\left(\frac{Ls}{Gs}\right)_{ac} = 1.75 \left(\frac{Ls}{Gs}\right)_{min} = 1.75 * 0.99\, m = 1.7325\, m$$

$$\emptyset = m * \frac{Gs}{Ls} = \frac{m}{1.7325} = 0.577$$

Here the value of Ø is less than 1, that's mean the solution is right

$$NOG = \frac{1}{1-\emptyset} \ln \left[(1-\emptyset)\frac{y_1}{y_2} + \emptyset\right]$$

$$NOG = \frac{1}{1-0.577} \ln \left[(1-0.577)\frac{y_1}{0.01y_1} + 0.577\right] = 8.88$$

$$Z = HOG \,.\, NOG$$

$$= 1 * 8.88 = 8.88 \text{ m}$$

Note: In this problem the values of y_1,y_2 and m did not given, it is supposed that these parameters will be eliminate through the solution.

Second: Tray Tower

The height of tray tower equal to number of trays multiplied by the distance between each two consecutive trays, this distance is often approximate about 0.5 m.

To find the height, number of trays should be estimated which depends on the type of equilibrium relation as follow:

1. If the equilibrium relation is linear y* = m x

$$N = \frac{\ln\left[\frac{Y_N - A}{B}\right]}{\ln \varnothing} \qquad (18\ a)$$

Where

$$A = Y_o - B \qquad (19)$$

$$B = \frac{Y_O - m\,X_1}{1 - \varnothing} \qquad (20)$$

If it is required to find the concentration between trays, Equation (18) can be written as:

$$Y_N = A + B\,\varnothing^N \qquad (18\ b)$$

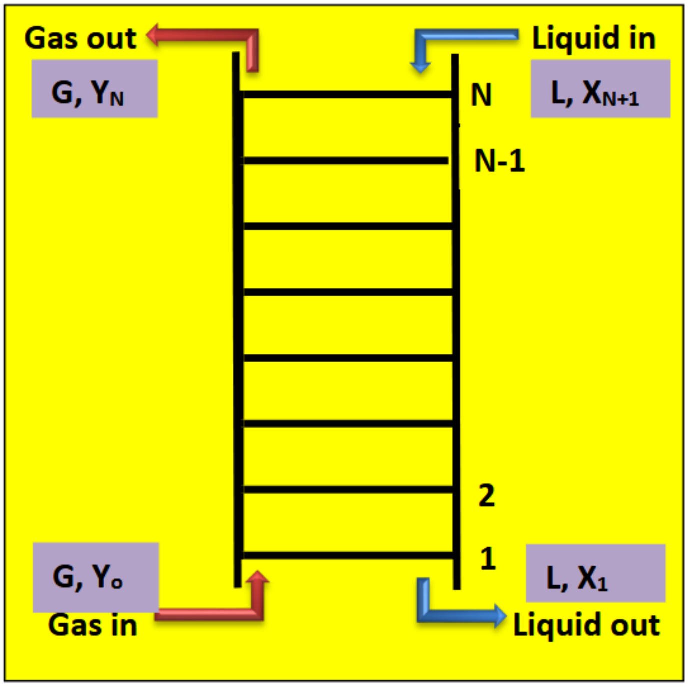


Figure 8 Tray Tower

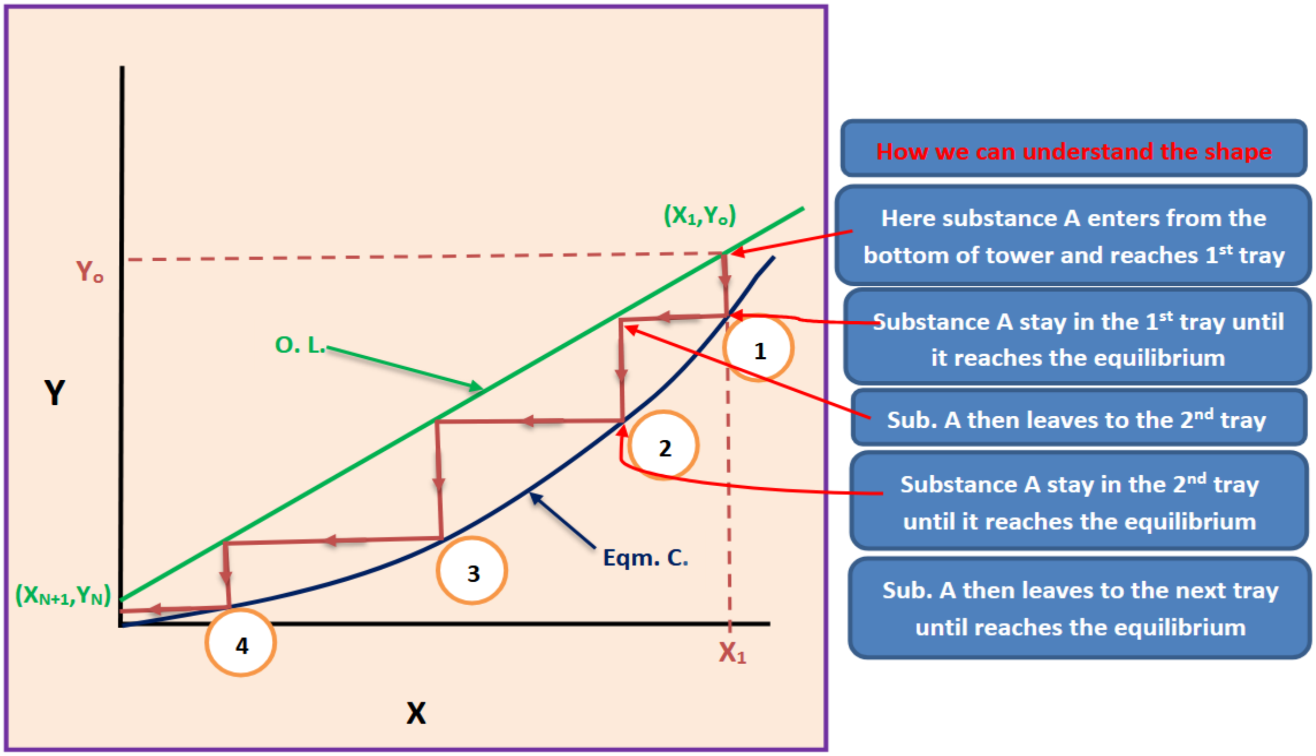
2. If equilibrium relation is non linear

x . . . .

y . . . .

The solution is by applying the following steps:

1. **Plot the equilibrium data**
2. **Plot the operating line**
3. **By stepping off, find number of trays.**



How we can understand the shape

Here substance A enters from the bottom of tower and reaches 1st tray

Substance A stay in the 1st tray until it reaches the equilibrium

Sub. A then leaves to the 2nd tray

Substance A stay in the 2nd tray until it reaches the equilibrium

Sub. A then leaves to the next tray until reaches the equilibrium

Figure 9 number of trays in Tray Tower

Efficiency

Efficiency can be divided into two types:

1. Column Efficiency (Ec) كفاءة العمود او البرج

Refers to the efficiency of whole column or tower, this can be found from the following relation:

$$Column\ Efficiency = \frac{theoretical\ no.of\ trays}{actual\ no.of\ trays}$$

$$Actual\ no.\ of\ trays = \frac{theoretical\ no.of\ trays}{column\ efficiency} \qquad (19)$$

Theoretical no. of trays can be found from mathematical relations and figures depends on the equilibrium relation.

2. Plate (Tray) Efficiency (E_m) كفاءة الصينية او الصفيحة

Plate efficiency can be divided into two types:

1. Plate (tray) efficiency based on gas phase (E_{mv})

This can be applied graphically by following these steps (fig.11):

1) **Plot both O. L. and E. C. from given data.**

2) **Choose point c at random location on E. C.**

3) **Plot vertical line from point c to intersect O. L. at point a.**

4) **Measure the distance ac, then multiply it by E_{mv} to find the distance ab which it plot from point a toward point c.**

5) **Repeat steps 2-4 for many points.**

6) **Plot new E. C. from the new points b.**

7) **Actual no. of trays can be found by stepping of between O. L. and new E. C.**

Where: (E_{mv}) E = efficiency, m = Merphy (scientist name), v = vapour (gas phase)

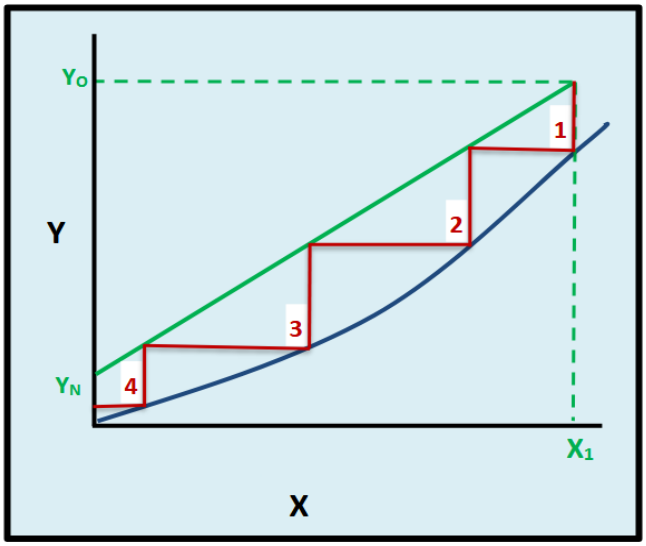


Figure 10 Theoretical number of plate

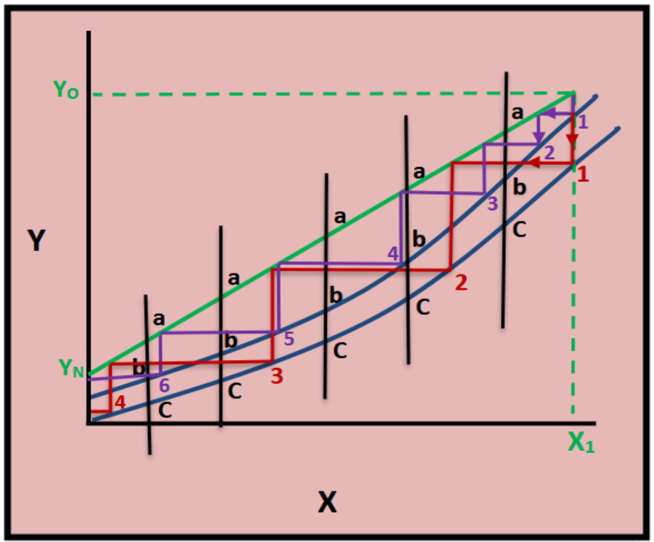


Figure 11 Plate efficiency based on gas phase (E_{mv})

Plate (tray) efficiency based on liquid phase E_{mL}

This can be applied graphically by following these steps (fig.12):

1) **Plot both O. L. and E. C. from given data.**
2) **Choose point c at random location on E. C.**
3) **Plot horizontal line from point c to intersect O. L. at point a.**
4) **Measure the distance ac, then multiply it by E_{mL} to find the distance ab which it plot from point a toward point c.**
5) **Repeat steps 2-4 for many points.**
6) **Plot new E. C. from the new points b.**
7) **Actual no. of trays can be found by stepping of between O. L. and new E. C.**

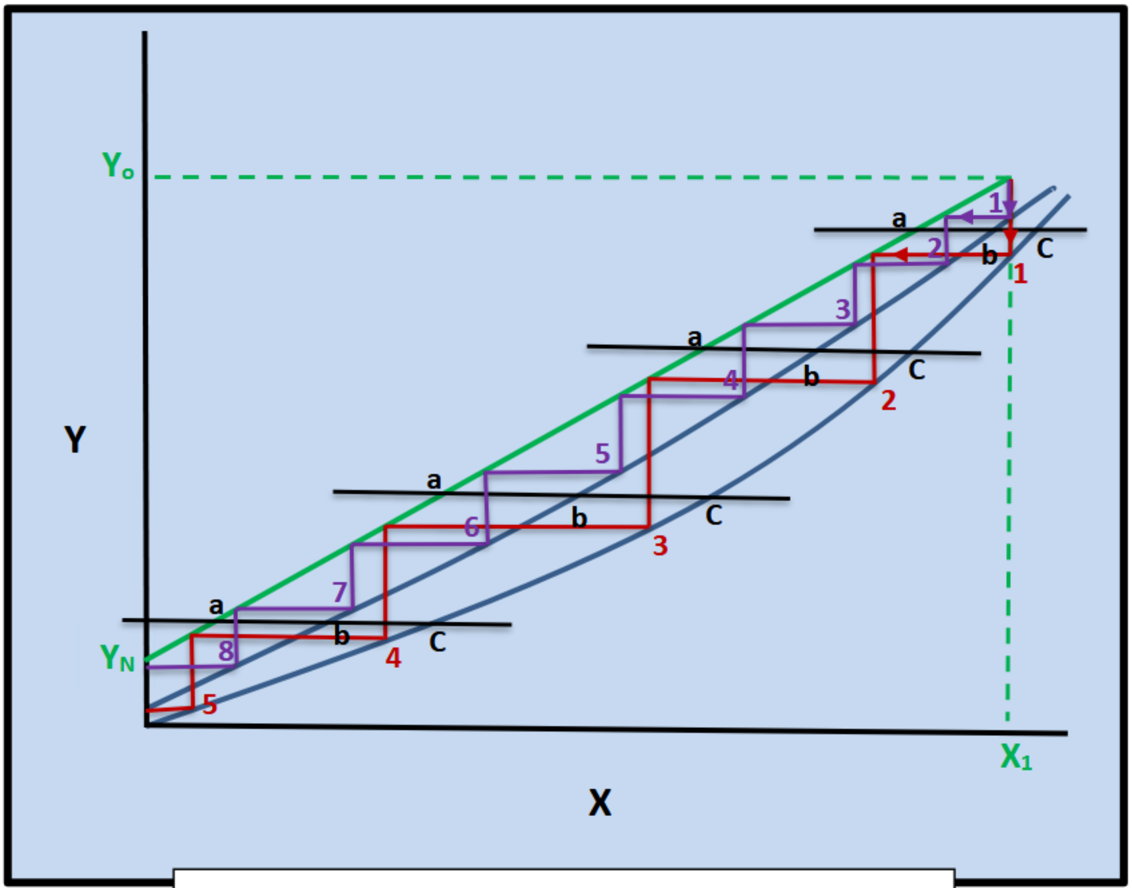


Figure 12 Plate efficiency based on liquid phase (E_{mL})

Example 3:

A mixture of ammonia and air is scrubbed in a tray column with fresh water. If ammonia concentration is reduced from 5% to 0.01%. The water and air rates are 0.65 and 0.4 kg/m^2.s respectively. The equilibrium relation can be written as $Y_e = X$ where Y_e and X are mole ratio in gas phase and liquid phase respectively. Calculate the theoretical number of plates by:

a) Analytical method.
b) Graphical method.

Sol:

Notes: (1) From the first statement, we understand that the system is contain of gas and liquid, that's mean absorption process.

(2) Scrubbed (scrubbing) mean we deal with absorption process because this device is often used to control on air pollution by liquid.

$Y_o = 0.05$, $Y_N = 0.0001$

Since the concentration is $\leq$ 10%

$G_S = G$, $L_S = L$, $Y = y$, $X = x$

$G_S = 0.4/29 = 0.01379$ kmol/m^2.s

$L_S = 0.65/18 = 0.0361$ kmol/ m^2.s

a- Analytical method

The eqm is linear $\longrightarrow$ $N = \dfrac{\ln\left[\frac{Y_{N-A}}{B}\right]}{\ln \emptyset}$

From M.B.

$G_S\,(Y_o - Y_N) = L_S\,(X_1 - X_{N+1})$ **($X_{N+1} = 0$ fresh water)**

$0.0137\,(0.05 - 0.0001) = 0.0361\,(X_1)$

$X_1 = 0.01893$

Now, $\emptyset = m\,.\,\dfrac{Gs}{Ls}$ **$= 1{*}0.01379/0.01361 = 0.382$**

$$B = \frac{Y_o - m\,X_1}{1 - \emptyset} = (0.05 - 0.01893) / (1 - 0.882) = 0.0502$$

$$A = Y_o - B = 0.05 - 0.0502 = 0.0002$$

$$N = \frac{\ln\left[\frac{0.0001 - 0.0002}{0.0502}\right]}{\ln 0.382} = 5.08 \cong 5$$

b- Graphical method

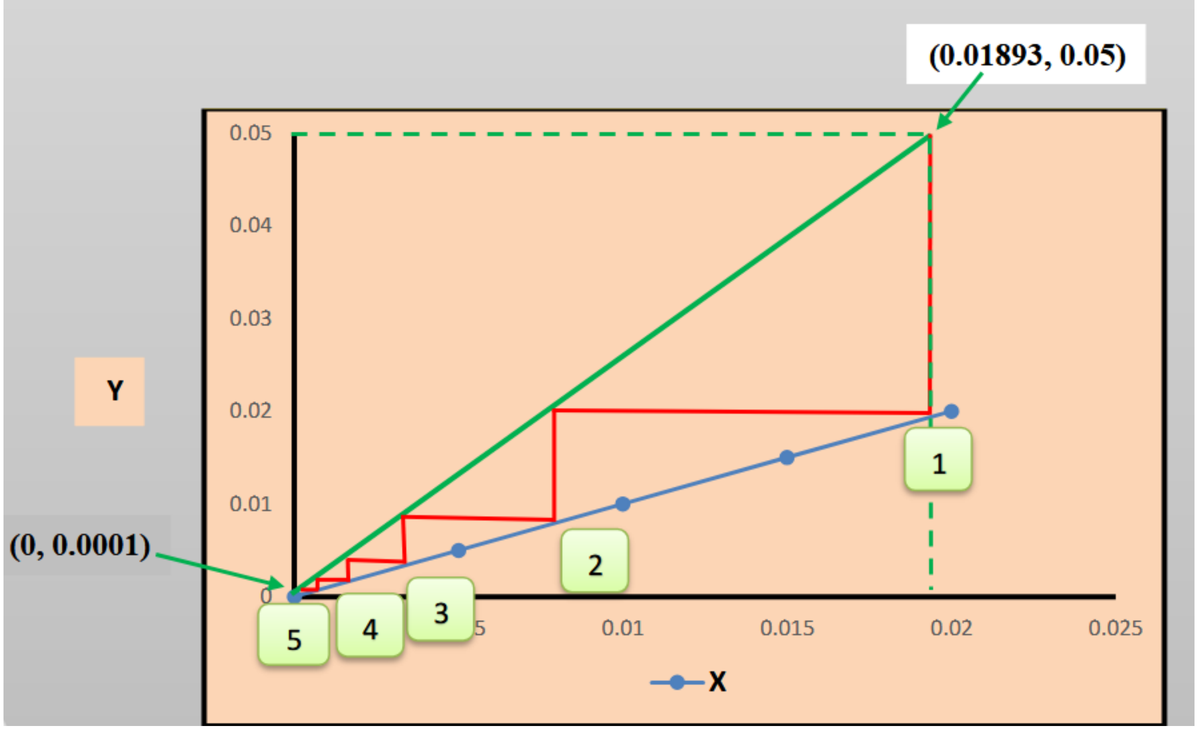
Plot operating line from (X_1 , Y_o) $\longrightarrow$ **(0.01893, 0.05)**

and (X_{N+1} , Y_N) $\longrightarrow$ **(0, 0.0001)**

plot eqm data from $Y_e = X$

X	**0.005**	**0.01**	**0.015**	**0.02**
Y_e	**0.005**	**0.01**	**0.015**	**0.02**

By stepping off $N = 5$



Example 4:

It is desired to absorb 90% of acetone in a gas containing 1 mole % acetone in air in a counter current stage tower. The total inlet gas flow rate to the tower is 30 kmol/hr, and the inlet of pure water flow rate to the tower is 90 kmol/hr, the process is to operate at 300 °k and 1 atm. , the equilibrium relation for the acetone in the gas liquid is (($Y^* = 2.53\ X$)).

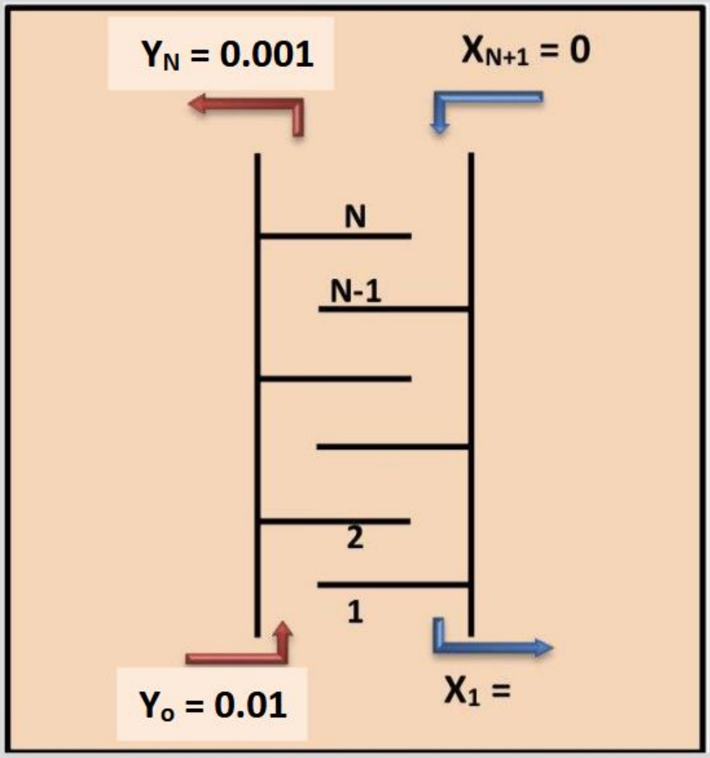
(a) Determine the number of theoretical stage required for this separation.

(b) If the overall tower efficiency is 65%, find the actual stage required.

Sol:

$Y_N = (1- \text{recovery})\ Y_o = (1-0.9)\ 0.01 = 0.001$

a) $\emptyset = \frac{m.G}{L} = \frac{2.53*30}{90} = 0.84$



Eqm. relation is linear

and $X_2 = 0$ (tray tower)

$$N = \frac{\ln\left[\frac{Y_N - A}{B}\right]}{\ln \emptyset} \text{…..(*)}$$

$$Y_N = A + B\,\emptyset^N \text{ ….(**)}$$

At $N = 0$, $Y_N = Y_o$

sub in eq. (**)

$Y_o = A+B\,(\emptyset)^o$

$Y_o = A+B \rightarrow B = Y_o - A$

$B = 0.01 - A$ ……..(1)

At $N = 1$, $Y_N = Y_1$ *sub in eq. (**)*

$Y_1 = A+B\,(\emptyset)$

$m\,X_1 = A + B\,(0.84)$

X_1 *should be found*

<u>O.M.B.</u>

$G_S (Y_o - Y_N) = L_S (X_1 - X_{N+1})$ ($X_{N+1} = 0$ pure water)

$$X_1 = \frac{G}{L} (Y_o - Y_N) + X_{N+1}$$

$$X_1 = \frac{30}{90} (0.01 - 0.001) + 0$$

$$X_1 = 0.003$$

$2.53 * 0.003 = A + B (0.84)$

$7.59*10^{-3} = A + B (0.84)$ …..(2) sub. in eqn 1

$7.59*10^{-3} = A + (0.01 - A) 0.84 = 0.16A$

$A = -0.005$ $B = 0.017$ sub in eqn (*)

$$N_{theo} = \frac{\ln[\frac{0.001 - 0.005}{0.015}]}{Ln 0.84}$$

$N_{theo} = 5.3$ units

b) Overall tower efficiency 65%

$$\text{Actual N} = \frac{theortical\ N}{colum\ efficiency}$$

$$N_{act} = \frac{5.3}{0.65}$$

$N_{act} = 8.2 \cong 9$ units