Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

Learning package

Heat Transfer

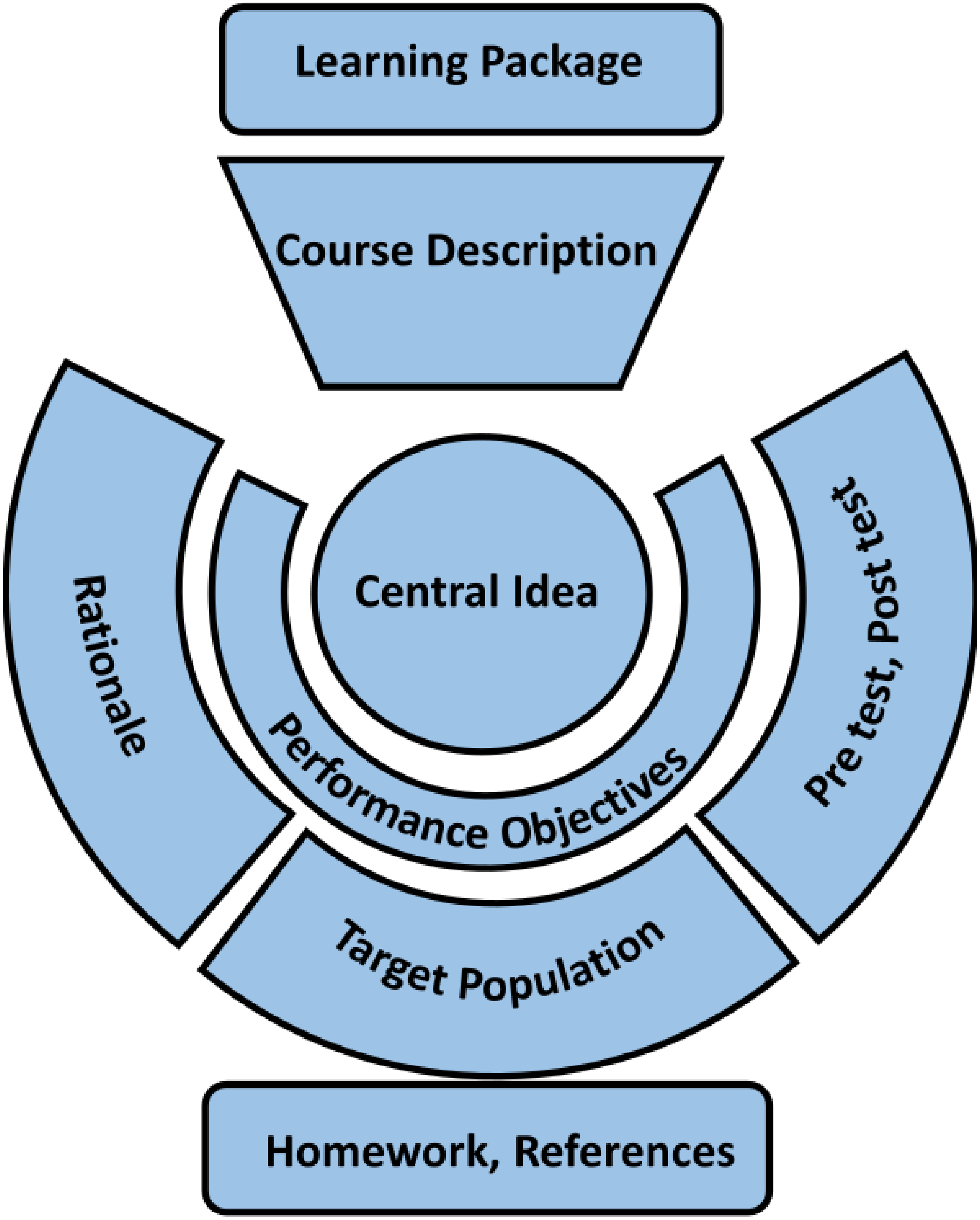
For

Second year students

By

Ataa Wejood
Dep. Of Chemical Industrial Techniques

2025


Learning Package
Course Description
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Rationale
Performance Objectives
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Target Population
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Course Description

Course Name:
Heat Transfer
Course Code:
Semester / Year:
Semester
Description Preparation Date:
30/ 05/ 2025
Available Attendance Forms:
Attendance only
Number of Credit Hours (Total) / Number of Units (Total)
60 hours/4 hour weekly/4 unit
Course administrator's name (mention all, if more than one name)
Name: Ataa Wejood Email: awejood@lecturers.stu.edu.iq
Course Objectives

1. **Understanding Fundamental Principles** Grasp the core laws governing heat transfer—conduction, convection, and radiation—and their roles in natural and engineered systems.
2. **Analyzing Conduction Phenomena** Study steady-state and transient conduction in various geometries, employing analytical and numerical methods to solve related problems.
3. **Exploring Convective Heat Transfer** Examine both natural and forced convection scenarios, including internal flows (e.g., ducts) and external flows (e.g., over surfaces), and learn to calculate associated heat transfer coefficients.
4. **Investigating Radiative Heat Exchange** Understand the principles of thermal radiation, including

blackbody and graybody radiation, and apply these concepts to calculate radiative heat transfer between surfaces.

5. **Evaluating Heat Exchanger Performance** Analyze the operation of heat exchangers using methods such as the log mean temperature difference and effectiveness-NTU approaches.
6. **Developing Problem-Solving Skills** Apply theoretical knowledge to practical engineering problems, enhancing analytical and computational abilities pertinent to heat transfer applications.
7. **Conducting Experimental Measurements** Gain experience in designing and performing experiments to measure heat transfer parameters, interpreting data to validate theoretical models.

Teaching and Learning Strategies

1. Cooperative Concept Planning Strategy.
2. Brainstorming Teaching Strategy.
3. Note–taking Sequence Strategy.

Course Structure

Week	Hours	Intended Learning Outcomes	Unit / Course Topic	Teaching Method	Assessment Method
1	4	Units: Unit systems, conversion, thermal units, methods of heat transfer	Introduction to Heat Transfer	Presentation & Explanation	In-class and Homework Assignments
2	4	Basics of the heat conduction equation, thermal conductivity, conduction through a single plane wall	Heat Conduction	Presentation & Explanation	In-class and Homework Assignments
3	4	Conduction through a composite plane wall, thermal (heat) resistance, conduction through a single cylindrical wall	Heat Conduction	Presentation, Explanation, and Drawing	In-class and Homework Assignments
4–5	4	Conduction through a composite cylindrical wall, conduction through a spherical wall, heat	Heat Convection	Presentation, Explanation, and Drawing	In-class and Homework Assignments

		transfer by convection, free and forced convection, heat transfer coefficient, forced convection inside and outside tubes			
6–7	4	Combined heat transfer by conduction and convection, heat transfer between two fluids through plane and cylindrical walls	Heat Convection	Presentation, Explanation, and Drawing	In-class and Homework Assignments
8	4	Overall heat transfer coefficient, types of heat exchangers, energy balance for double-pipe heat exchangers, LMTD	Heat Convection	Presentation, Explanation, and Drawing	In-class and Homework Assignments
9–10	4	Shell and tube heat exchangers, types of tube arrangements, correction factors (Uc, Ud, Rf)	Heat Convection	Presentation, Explanation, and Drawing	In-class and Homework Assignments
11	4	Heat transfer with phase change, condensation of vapors, film and dropwise condensation, heat transfer to boiling liquids	Heat Conduction and Convection	Presentation, Explanation, and Drawing	In-class and Homework Assignments
12	4	Heat transfer by radiation, absorptivity, reflectivity, transmissivity, Kirchhoff's Law, Stefan-Boltzmann Law, radiation between real surfaces	Heat Radiation	Presentation, Explanation, and Drawing	In-class and Homework Assignments
13	4	Evaporation, types of evaporators, performance of evaporators, economy, Duhring's Rule	Heat Transfer Applications	Presentation, Explanation, and Drawing	In-class and Homework Assignments
14	4	Mass and energy balance for a single evaporator	Heat Transfer Applications	Presentation, Explanation, and Drawing	In-class and Homework Assignments
15	4	Multi-effect evaporators, methods of feeding, capacity and economy,	Heat Transfer Applications	Presentation, Explanation, and Drawing	In-class and Homework Assignments

		energy balance, vapor recompression			

Course Evaluation
Distribution as follows: 20 points for Midterm Theoretical Exams for the first semester, 20 points for Midterm Practical Exams for the first semester, 10 points for Daily Exams and Continuous Assessment and 50 points for the Final Exam.

Learning and Teaching Resources	
Ref.	1-Heat Transfer, Tenth Edition J. P. Holman Department of Mechanical Engineering Southern Methodist University. 2-Heat and Mass Transfer: A Textbook for Students Preparing for B.E. and B.Tech., B.Sc. Engg., and gate examination in SI units Er. R.K. RAJPUT. 3-Process Heat Transfer, principles and applications, ROBERT W. SERTH Department of Chemical and Natural Gas Engineering 4-Volume 4 Petroleum Refining Design and Applications Handbook A.KAYODE COKER. 5-Fundamentals of Heat and Mass Transfer, sixth edition by: Incropera/Dewitt/Bergman/Lavine. 6- Chemical Engineering Coulson and Richardson's volume 1, Fluid Flow, Heat Transfer, and Mass Transfer, sixth edition.

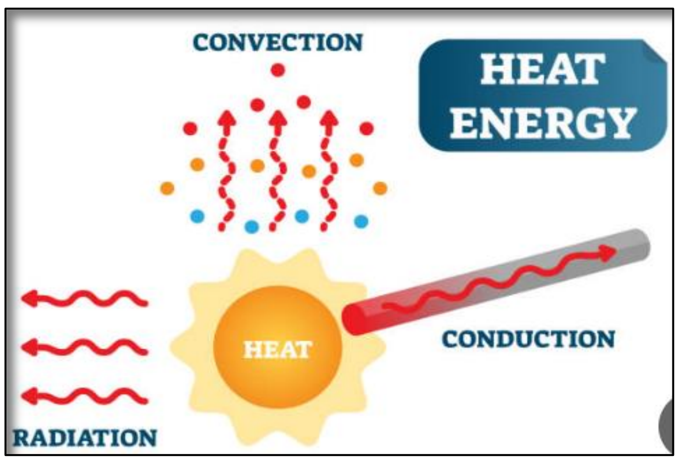
Learning package

In

Heat Transfer

For

Students of the Second Year



By

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Assistant

Dep. Of Chemical Industrial Techniques

2025

1/ Overview

1 / A –Target population :-

For students of second year
Technological Institute of Basra
Dep. of Chemical Industrial Techniques

1 / B –Rationale:-

Heat transfer by conduction is a fundamental process where thermal energy moves through a material without any bulk movement of the material itself. This mechanism is most effective in solids, particularly metals, due to their tightly packed atomic structures. Heat transfer is the flow of thermal energy from hotter to cooler regions, accomplished through conduction (direct contact), convection (fluid movement), and radiation (the emission of electromagnetic waves). **Understanding these allows us to engineer efficient systems, optimize comfort, conserve energy, and explain natural phenomena.**

1 / C –Central Idea:-

Heat flows through a material by direct contact, from the warmer region to the cooler region, via microscopic collisions or energy-carrying particles, without any bulk movement of the material itself.

- **Mechanism**

- Thermal energy moves as fast-moving (hotter) atoms or electrons collide with neighboring slower (cooler) ones, transferring kinetic energy step by step
- In metals, **free electrons** also carry heat efficiently; in insulators, heat moves via **lattice vibrations (phonons)** .

- **No macroscopic flow**

  - Unlike convection, the material doesn't move—only energy is transmitted through stationary matter.

- **Fourier's law**

  - The heat flow rate Q˙\dot{Q}Q˙ depends on:
    - **Thermal conductivity** (k): how well the material conducts heat
    - **Area** (A) of the cross-section
    - **Temperature difference** (ΔT\Delta TΔT)
    - **Thickness** (L) of the material
  - Mathematically:

- **Material matters**

  - Metals (e.g., copper, silver) have high k → excellent conductors
  - Non-metals (e.g., wood, air) have low k → act as insulators

- **Ubiquitous applications**

  - Cooking utensils: stovetop pans, hot handles
  - Building insulation: resisting heat flow through walls
  - Electronics cooling: using heat sinks to conduct heat away from processors

Heat is moved through a fluid (liquid or gas) by the bulk movement of the fluid itself, driven by temperature-induced density differences or external forces. This transfer combines two components:

- **Diffusion (conduction)** within the fluid
- **Advection**: the movement of warmer (or cooler) fluid carrying heat from one place to another
- • **Natural (free) convection**: Hot fluid becomes less dense and rises, while cooler fluid sinks, creating circulation, like rising air by a heater or boiling water currents.
- • **Forced convection**: Fluid movement is driven by external means (e.g., fans, pumps)—as seen in car radiators, hair dryers, and convection ovens.
- • **Efficiency**: Convection is often the dominant and more efficient form of heat transfer in fluids compared to conduction alone.

Mathematical Model

- Governed by Newton's law of cooling:

  Q˙=h A (Tsurface−Tfluid) \dot{Q} = h \, A \, (T_{\text{surface}} - T_{\text{fluid}})Q˙=hA(Tsurface−Tfluid)

- Here, **h** is the heat transfer coefficient, depending on fluid properties, flow velocity, surface geometry, etc.

1. **What it is**
   Radiation involves the emission of electromagnetic waves (primarily infrared, but also visible and ultraviolet) by any object with a temperature above absolute zero. These waves travel through space and may be absorbed, reflected, or transmitted by other objects.
2. **How it depends on temperature**
   The energy radiated per unit area is given by the **Stefan–Boltzmann law**:
   E=εσT4 E = \varepsilon \sigma T^4E=εσT4
   where:
   - ε\varepsilonε = emissivity (0 to 1)
   - σ\sigma = Stefan–Boltzmann constant
   - TTT = absolute temperature in kelvins
3. **No medium needed**
   Unlike conduction and convection, radiation occurs in a vacuum—this is how solar energy crosses the 93 million miles to warm Earth.
4. **Surface properties matter**
   An object's **emissivity** determines how efficiently it emits radiation: black bodies (ε=1\varepsilon=1ε=1) radiate most, shiny metals much less
5. **Influencing factors**
   The net radiative heat exchange depends on:
   - Temperature differences raised to the fourth power
   - Emissivity values of participating surfaces
   - Geometry and orientation (view factors) between objects

🌐 Everyday Examples

- Sunlight warming your skin through space's vacuum
- Feeling heat from a hot stove or fire without direct contact
- Thermal imaging cameras detecting infrared radiation.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

a. **Explain** the physical mechanism of heat conduction at the molecular level.
b. **Define** key terms such as thermal conductivity, temperature gradient, and steady-state heat flow.
c. **Apply** Fourier's Law of Heat Conduction to calculate heat transfer rates in simple geometries.
d. **Identify** the factors that influence the rate of heat conduction in materials.
e. **Differentiate** between good and poor thermal conductors and explain practical applications.
f. **Solve** basic engineering problems involving 1D steady-state heat conduction through plane walls.
g. **Identify and define** each heat transfer mode (conduction, convection, radiation) and distinguish between them.
h. **Explain fundamental mechanisms**, such as molecular collisions (conduction), fluid motion (convection), and electromagnetic waves (radiation) .
i. **Recognize real-world examples**, like heat moving through a pan (conduction), boiling water stirring (convection), and sun warming the Earth (radiation) .
j. **Apply mathematical models**:
    a. Fourier's law for conduction
    b. Newton's law of cooling for convection
    c. Stefan–Boltzmann law for radiation
k. **Predict and contrast efficiency** among materials and modes based on conductivity, emissivity, fluid velocity, etc.

Introduction

1. Dimensions and Units

Useful conversion factors

Physical quantity	Symbol	SI to English conversion	English to SI conversion
Length	L	$1\ m = 3.2808\ ft$	$1\ ft = 0.3048\ m$
Area	A	$1\ m^2 = 10.7639\ ft^2$	$1\ ft^2 = 0.092903\ m^2$
Volume	V	$1\ m^3 = 35.3134\ ft^3$	$1\ ft^3 = 0.028317\ m^3$
Velocity	v	$1\ m/s = 3.2808\ ft/s$	$1\ ft/s = 0.3048\ m/s$
Density	ρ	$1\ kg/m^3 = 0.06243\ lb_m/ft^3$	$1\ lb_m/ft^3 = 16.018\ kg/m^3$
Force	F	$1\ N = 0.2248\ lb_f$	$1\ lb_f = 4.4482\ N$
Mass	m	$1\ kg = 2.20462\ lb_m$	$1\ lb_m = 0.45359237\ kg$
Pressure	p	$1\ N/m^2 = 1.45038 \times 10^{-4}\ lb_f/in^2$	$1\ lb_f/in^2 = 6894.76\ N/m^2$
Energy, heat	q	$1\ kJ = 0.94783\ Btu$	$1\ Btu = 1.05504\ kJ$
Heat flow	q	$1\ W = 3.4121\ Btu/h$	$1\ Btu/h = 0.29307\ W$
Heat flux per unit area	q/A	$1\ W/m^2 = 0.317\ Btu/h \cdot ft^2$	$1\ Btu/h \cdot ft^2 = 3.154\ W/m^2$
Heat flux per unit length	q/L	$1\ W/m = 1.0403\ Btu/h \cdot ft$	$1\ Btu/h \cdot ft = 0.9613\ W/m$
Heat generation per unit volume	$\dot{q}$	$1\ W/m^3 = 0.096623\ Btu/h \cdot ft^3$	$1\ Btu/h \cdot ft^3 = 10.35\ W/m^3$
Energy per unit mass	q/m	$1\ kJ/kg = 0.4299\ Btu/lb_m$	$1\ Btu/lb_m = 2.326\ kJ/kg$
Specific heat	c	$1\ kJ/kg \cdot {}^\circ C = 0.23884\ Btu/lb_m \cdot {}^\circ F$	$1\ Btu/lb_m \cdot {}^\circ F = 4.1869\ kJ/kg \cdot {}^\circ C$
Thermal conductivity	k	$1\ W/m \cdot {}^\circ C = 0.5778\ Btu/h \cdot ft \cdot {}^\circ F$	$1\ Btu/h \cdot ft \cdot {}^\circ F = 1.7307\ W/m \cdot {}^\circ C$
Convection heat-transfer coefficient	h	$1\ W/m^2 \cdot {}^\circ C = 0.1761\ Btu/h \cdot ft^2 \cdot {}^\circ F$	$1\ Btu/h \cdot ft^2 \cdot {}^\circ F = 5.6782\ W/m^2 \cdot {}^\circ C$
Dynamic		$1\ kg/m \cdot s = 0.672\ lb_m/ft \cdot s$	
Viscosity	μ	$= 2419.2\ lb_m/ft \cdot h$	$1\ lb_m/ft \cdot s = 1.4881\ kg/m \cdot s$
Kinematic viscosity and thermal diffusivity	ν, α	$1\ m^2/s = 10.7639\ ft^2/s$	$1\ ft^2/s = 0.092903\ m^2/s$

$$1\ Btu = 778.16\ lb_f \cdot ft$$
$$1\ Btu = 1055\ J$$
$$1\ kcal = 4182\ J$$
$$1\ lb_f \cdot ft = 1.356\ J$$
$$1\ Btu = 252\ cal$$
$${}^\circ F = \tfrac{9}{5}{}^\circ C + 32$$
$${}^\circ R = {}^\circ F + 459.69$$
$$K = {}^\circ C + 273.16$$
$${}^\circ R = \tfrac{9}{5} K$$

Multiplier factors for SI units.

Multiplier	Prefix	Abbreviation
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-18}	atto	a

SI quantities used in heat transfer.

Quantity	Unit abbreviation
Force	N (newton)
Mass	kg (kilogram mass)
Time	s (second)
Length	m (meter)
Temperature	°C or K
Energy	J (joule)
Power	W (watt)
Thermal conductivity	$W/m \cdot °C$
Heat-transfer coefficient	$W/m^2 \cdot °C$
Specific heat	$J/kg \cdot °C$
Heat flux	W/m^2

2. Modes of Heat Transfer

Heat transfer which is defined as the transmission of energy from one region to another as a result of temperature gradient takes place by the following modes:

1- Conduction
2- Convection
3- Radiation

Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. Example: The water in a boiler shell receives its heat from the fire-bed by conducted, convected and radiated heat from fire to the shell, conducted heat through the shell and conducted and convected heat from the inner shell wall, to the water. Heat always flows in the direction of lower temperature.

The above three modes are similar in that a temperature differential must exist and the heat exchange is in the direction of decreasing temperature; each method, however, has different controlling laws.

2.1 Heat Transfer by Conduction

2.1.1 Fourier's Laws of Heat Conduction

Fourier's law of heat conduction is an empirical law based on observation and states as follows:

The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of heat flow.

Mathematically, it can be represented by the equation :

$$Q \propto A \cdot \frac{dt}{dx}$$

where, Q = Heat flow through a body per unit time (in watts), W,

A = Surface area of heat flow (*perpendicular to the direction of flow*), m^2,

dt = Temperature difference of the faces of block (homogeneous solid) of thickness 'dx' through which heat flows, °C or K, and

dx = Thickness of body in the direction of flow, m.

Thus,

$$Q = -k \cdot A \frac{dt}{dx}$$

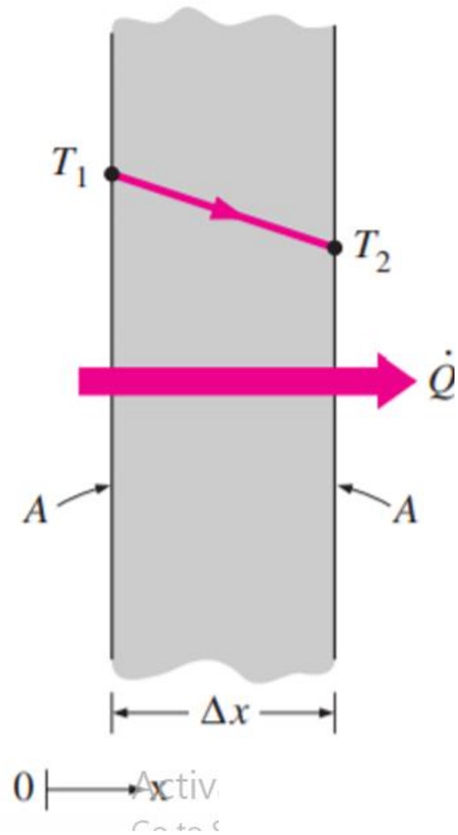
where, k = Constant of proportionality and is known as *thermal conductivity of the body*.

The – ve sign of k [eqn. (1.1)] is to take care of the decreasing temperature alongwith the direction of increasing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is *always negative along positive x direction and, therefore, the value as Q becomes* + ve.

Assumptions :

The following are the assumptions on which Fourier's law is based :

1. Conduction of heat takes place under *steady state conditions*.
2. The heat flow is unidirectional.
3. The temperatures gradient is *constant* and the temperature profile is *linear*.
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is *constant in all directions*).



Example 1.1. *Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick, whose one face is maintained at 350°C and the other face at 50°C. Take thermal conductivity of copper as 370 W/m°C.*

Solution. Temperature difference, $dt\ (= t_2 - t_1) = (50 - 350)$

Thickness of copper plate, L = 45 mm = 0.045 m

Thermal conductivity of copper, k = 370 W/m°C

Rate of heat-transfer per unit area, q :

From Fourier's law

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L} \qquad \text{...(Eqn. 1.1)}$$

or,

$$q = \frac{Q}{A} = -k\frac{dt}{dx}$$

$$= -370 \times \frac{(50 - 350)}{0.045}$$

$$= 2.466 \times 10^6 \text{ W/m}^2 \text{ or}$$

2.466 MW/m² (Ans.)

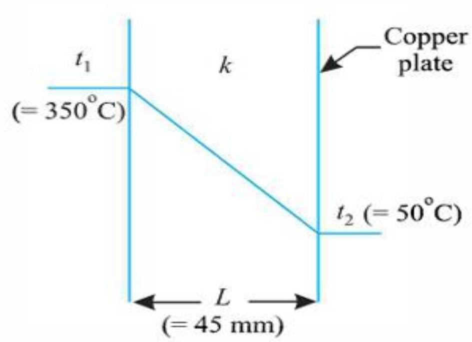


Fig. 1.8.

Example 1.2. *A plane wall is 150 mm thick and its wall area is 4.5 m^2. If its conductivity is 9.35 W/m°C and surface temperatures are steady at 150°C and 45°C, determine :*

(*i*) *Heat flow across the plane wall;*

(*ii*) *Temperature gradient in the flow direction.*

Solution. Thickness of the plane wall,

$$L = 150 \text{ mm} = 0.15 \text{ m}$$

Area of the wall, $A = 4.5 \text{ m}^2$

Temperature difference, $dt = t_2 - t_1 = 45 - 150 = -105°\text{C}$

Thermal conductivity of wall material,

$$k = 9.35 \text{ W/m°C}$$

(*i*) **Heat flow across the plane wall, Q :**

As per Fourier's law,

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

$$= -9.35 \times 4.5 \times \frac{(-105)}{0.15} = \mathbf{29452.5\ W}$$

(*ii*) **Temperature gradient, $\frac{dt}{dx}$:**

From Fourier's law, we have

$$\frac{dt}{dx} = -\frac{Q}{kA} = \frac{29452.5}{9.35 \times 4.5} = \mathbf{-\ 700°C/m}$$

Example 1.3. *The following data relate to an oven :*

Thickness of side wall of the oven = 82.5 mm
Thermal conductivity of wall insulation = 0.044 W/m°C
Temperature on inside of the wall = 175°C
Energy dissipated by the electrical coil within the oven = 40.5 W

Determine the area of wall surface, perpendicular to heat flow, so that temperature on the other side of the wall does not exceed 75°C.

Solution. *Given* : $x = 82.5$ mm $= 0.0825$ *m*; $k = 0.044$ W/m°C; $t_1 = 175$°C; $t_2 = 75$°C; $Q = 40.5$W

Area of the wall surface, A :

Assuming one-dimentional steady state heat conduction,

Rate of electrical energy dissipation in the oven.

= Rate of heat transfer (conduction) across the wall

i.e.
$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{x} = \frac{kA\,(t_1 - t_2)}{x}$$

or,
$$40.5 = \frac{0.044\ A\ (175 - 75)}{0.0825}$$

or,
$$A = \frac{40.5 \times 0.0825}{0.044\ (175 - 75)} = \mathbf{0.759\ m^2}$$

2.2 Thermal Conductivity of Materials

Thermal Conductivity of material can be defined as follows:

The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference.

$$k = \frac{Q}{A} \cdot \frac{dx}{dt}$$

The value of $k = 1$ when $Q = 1$, $A = 1$ and $\frac{dt}{dx} = 1$

Now $k = \frac{Q}{1} \cdot \frac{dx}{dt}$ (unit of k : $\mathrm{W} \times \frac{1}{\mathrm{m}^2} \times \frac{\mathrm{m}}{\mathrm{K\,(or\ °C)}} = \mathrm{W/mK.}$ or $\mathrm{W/m°C}$)

Thermal conductivity (a property of material) depends essentially upon the following *factors* :

(*i*) Material structure (*ii*) Moisture content

(*ii*) Density of the material (*iv*) Pressure and temperature (operating conditions).

Thermal conductivities (average values at normal pressure and temperature) of some common materials are as under :

Material	Thermal conductivity (k) (W/mK)	Material	Thermal conductivity (k) (W/mK)
1. *Silver*	410	8. *Asbestos sheet*	0.17
2. *Copper*	385	9. *Ash*	0.12
3. *Aluminium*	225	10. *Cork, felt*	0.05 – 0.10
4. *Cast iron*	55–65	11. *Saw dust*	0.07
5. *Steel*	20–45	12. *Glass wool*	0.03
6. *Concrete*	1.20	13. *Water*	0.55 – 0.7
7. *Glass (window)*	0.75	14. *Freon*	0.0083

Example _ The wall of a furnace is constructed from 15 cm thick fire brick having constant thermal conductivity of 1.6 W/m.K. The two sides of the wall are maintained at 1400 K and 1100 K, respectively. What is the rate of heat loss through the wall which is50 cm × 3 m on a side?

Solution

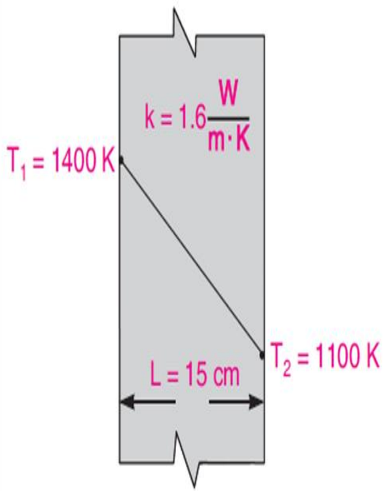
Given : A furnace wall with

T1 = 1400 K, T2 = 1100 K

A = 50 cm × 3 m = 0.5 × 3 = 1.5 m2

k = 1.6 W/m.K

L = 15 cm = 0.15 m



To find : Heat loss through the wall.

Assumptions :

1. Steady state conditions.
2. One dimensional heat conduction through the wall.
3. Constant properties.

Analysis : According to Fourier law of heat conduction, equation (1.9)

$$Q = kA\,\frac{(T_1 - T_2)}{L}$$

Using numerical values

$$Q = \frac{(1.6\ \text{W/m.K}) \times (1.5\ \text{m}^2) \times (1400\ \text{K} - 1100\ \text{K})}{(0.15\ \text{m})}$$

= **4800 W. Ans.**

2-Heat Transfer by Convection

"Convection" is the transfer of heat within a fluid by mixing of one portion of the fluid with another**.**

- Convection is possible only in a fluid medium and is directly linked with the transport of medium itself.
- Convection constitutes the macroform of the heat transfer since macroscopic particles of a fluid moving in space cause the heat exchange.

- The effectiveness of heat transfer by convection depends largely upon the mixing motion of fluid.

Free or natural convection: occurs when the fluid circulates by virtue of the natural differences in densities of hot and cold fluids; the denser portions of the fluid move downward because of greater force of gravity, as compared with the force on the less dense.

Forced convection: when the work is done to blow or pump the fluid, it is said to be forced convection.

The rate equation for the convective heat transfer (regardless of particular nature) between a surface and an adjacent fluid is prescribed by *Newton's law of cooling* (*Refer Fig. 1.9*)

$$Q = hA\,(t_s - t_f) \qquad \text{...(1.6)}$$

where,
Q = Rate of conductive heat transfer,
A = Area exposed to heat transfer,
t_s = Surface temperature,
t_f = Fluid temperature, and
h = Co-efficient of convective heat transfer.

The units of h are,

$$h = \frac{Q}{A\,(t_s - t_f)} = \frac{\text{W}}{\text{m}^2\,°\text{C}} \quad \text{or} \quad \text{W/m}^2\,°\text{C}$$

or, $\text{W/m}^2\text{K}$

The coefficient of convective heat transfer 'h' (also known as *film heat transfer coefficient*) may be defined as "*the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time.*"

The value of 'h' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.* viscosity, density, specific heat etc.).

(*ii*) Nature of fluid flow.

(*iii*) Geometry of the surface.

(*iv*) Prevailing thermal conditions.

Since 'h' depends upon several factors, it is difficult to frame a single equation to satisfy all the variations, however, by dimensional analysis an equation for the purpose can be obtained.

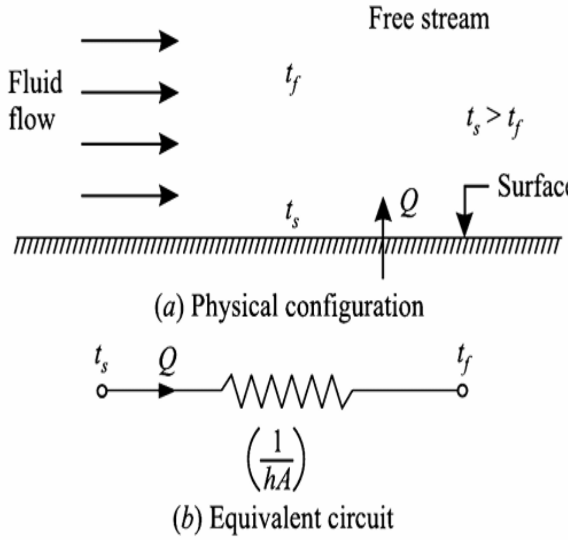


Fig. 1.9. Convective heat-transfer

The *mechanisms of convection* in which phase changes are involved lead to the important *fields of boiling and condensation*. Refer Fig. 1.9 (*b*). The quantity $\frac{1}{hA}\left[Q = \frac{t_s - t_f}{(1/hA)} \text{ ...Eqn (1.6)}\right]$ is called **convection thermal resistance** [$(R_{th})_{conv.}$] to heat flow.

Example 1.4. *A hot plate 1m × 1.5 m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer coefficient is 20W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5\ \text{m}^2$

Plate surface temperature, $t_s = 300°\text{C}$

Temperature of air (fluid), $t_f = 20°\text{C}$

Connvective heat-transfer coefficient, $h = 20\ \text{W/m}^2\ °\text{C}$

Rate of heat transfer, Q :

From Newton's law of cooling,

$$Q = hA\,(t_s - t_f)$$

$$= 20 \times 1.5\,(300 - 20) = 8400\ \text{W or } \mathbf{8.4\ kW}$$

Example 1.5. *A wire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100°C. Under the condition if convective heat transfer coefficient is 4500 W/m²°C find how much electric power must be supplied to the wire to maintain the wire surface at 120°C ?*

Solution. Diameter of the wire, $d = 1.5$ mm $= 0.0015$ m

Length of the wire, $L = 150$ mm $= 0.15$ m

$\therefore$ Surface area of the wire (exposed to heat transfer),

$$A = \pi\, d\, L = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4}\ \text{m}^2$$

Wire surface temperature, $t_s = 120°C$

Water temperature, $t_f = 100°C$

Convective heat transfer coefficient, $h = 4500\ \text{W/m}^2\ °C$

Electric power to be supplied :

Electric power which must be supplied = Total convection loss (Q)

$$\therefore \quad Q = hA\,(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}\,(120 - 100) = \mathbf{63.6\ W}$$

3-Heat Transfer by Radiation

''Radiation'' is the transfer of heat through space or matter by means other than conduction or convection.

Radiation heat is thought of as electromagnetic waves or quanta (as convenient) an emanation of the same nature as light and radio wave. All bodies radiate heat; so, a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits. Radiant energy (being electromagnetic radiation) requires no medium for propagation and will pass through vacuum.

Laws of Radiation :

1. **Wien's law.** It states that *the wavelength λ_m corresponding to the maximum energy is inversely proportional to the absolute temperature T of the hot body.*

$$\text{i.e.,} \quad \lambda_m \propto \frac{1}{T} \quad \text{or,} \quad \lambda_m T = \text{constant} \qquad \text{...(1.7)}$$

2. **Kirchhoff's law.** It states that the *emissivity of the body at a particular temperature is numerically equal to its absorptivity for radiant energy from body at the same temperature.*

3. **The Stefan-Boltzmann law.** The law states that *the emissive power of a black body is directly proportional to fourth power of its absolute temperature.*

$$\text{i.e.,} \quad Q \propto T^4 \qquad \text{...(1.8)}$$

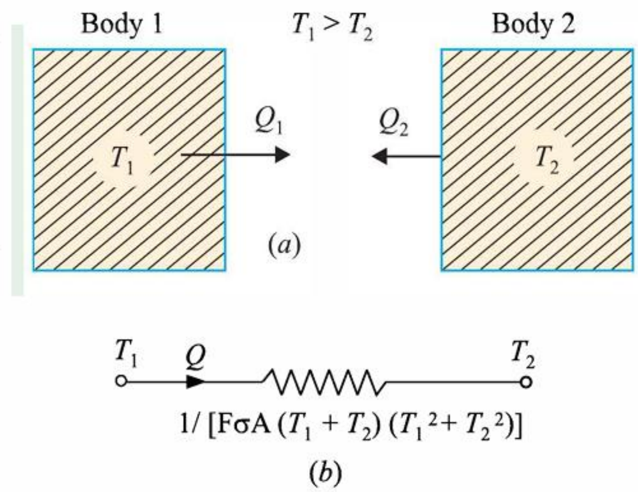


Fig. 1.10. Heat transfer by radiation.

Refer Fig. 1.10 (*a*)

$$Q = F \sigma A (T_1^4 - T_2^4) \qquad \text{...(1.9)}$$

where, F = A factor depending on geometry and surface properties,

σ = Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ W/m²K⁴,

A = Area, m², and

T_1, T_2 = Temperatures, degrees kelvin (K).

This equation can also be rewritten as :

$$Q = \frac{T_1 - T_2}{1/[F \sigma A (T_1 + T_2)(T_1^2 + T_2^2)]} \qquad \text{...(1.10)}$$

where denomenator is **radiation thermal resistance**, $(R_{th})_{rad}$. [Fig. 1.10 (*b*)]

$$\text{i.e.,} \quad (R_{th})_{rad} = 1/[F \sigma A (T_1 + T_2)(T_1^2 + T_2^2)]$$

The values of F are available for simple configurations in the form of charts and tables.

$F = 1$ ... for simple cases of black surface enclosed by other surface

F = emissivity (ε) ... for non-black surface enclosed by other surface.

[*Emissivity* (ε) is defined as the ratio of heat radiated by a surface to that of an ideal surface.]

Example 1.6. *A surface having an area of 1.5 m² and maintained at 300°C exchanges heat by radiation with another surface at 40°C. The value of factor due to the geometric location and emissivity is 0.52. Determine :*

(*i*) *Heat lost by radiation,*

(*ii*) *The value of thermal resistance, and*

(*iii*) *The value of equivalent convection coefficient.*

Solution. *Given* : $A = 1.5$ m²; $T_1 = t_1 + 273 = 300 + 273 = 573$K; $T_2 = t_2 + 273 = 40 + 273 = 313$K; $F = 0.52$.

(*i*) Heat lost by radiation, Q :

$$Q = F\sigma A(T_1^4 - T_2^4) \quad \text{...[Eqn. (1.9)]}$$

(where $\sigma = 5.67 \times 10^{-8}$ W/m² K⁴)

or, $$Q = 0.52 \times 5.67 \times 10^{-8} \times 1.5\,[(573)^4 - (313)^4]$$

$$= 0.52 \times 5.67 \times 1.5 \left[\left(\frac{573}{100}\right)^4 - \left(\frac{313}{100}\right)^4\right]$$

(Please note this step)

or, $$Q = \mathbf{4343\ W}$$

(*ii*) The value of thermal resistance, (Rth)rad :

We know that, $$Q = \frac{(T_1 - T_2)}{(R_{th})_{rad}} \quad \text{...[Eqn. (1.10)]}$$

$$\therefore \quad (R_{th})_{rad.} = \frac{(T_1 - T_2)}{Q} = \frac{(573 - 313)}{4343} = \mathbf{0.0598\ °C/W}$$

(*iii*) The value of equivalent convection coefficient, hr :

$$Q = h_r A(t_1 - t_2)$$

or, $$h_r = \frac{Q}{A(t_1 - t_2)} = \frac{4343}{1.5\,(300 - 40)} = \mathbf{11.13\ W/m^2\ °C}$$

[Alternatively, $$h_r = F\sigma(T_1 + T_2)(T_1^2 + T_2^2) \quad \text{...From eqn. (1.10)}$$

$$= 0.52 \times 5.67 \times 10^{-8}(573 + 313)(573^2 + 313^2)$$

$$= \mathbf{11.13\ W/m^2\ °C}$$]

Example 1.7. *A carbon steel plate (thermal conductivity = 45 W/m°C) 600 mm × 900 mm × 25 mm is maintained at 310°C. Air at 15°C blows over the hot plate. If convection heat transfer coefficient is 22 W/m² °C and 250 W is lost from the plate surface by radiation, calculate the inside plate temperature.*

Solution. Area of the plate exposed to heat transfer,

$$A = 600 \text{ mm} \times 900 \text{ mm} = 0.6 \times 0.9 = 0.54 \text{ m}^2$$

Thickness of the plate, $L = 25 \text{ mm} = 0.025 \text{ m}$

Surface temperature of the plate, $t_s = 310°C$

Temperature of air (fluid), $t_f = 15°C$

Convective heat transfer coefficient,

$$h = 22 \text{ W/m}^2 °C$$

Heat lost from the plate surface by radiation,

$$Q_{rad.} = 250 \text{W}$$

Thermal conductivity, $k = 45 \text{ W/m °C}$

Inside plate temperature, t_i :

In this case the heat conducted through the plate is removed from the plate surface by a *combination of convection and radiation*.

Heat conducted through the plate = Convection heat losses + radiation heat losses.

or,
$$Q_{cond.} = Q_{conv.} + Q_{rad.}$$

$$-kA \frac{dt}{dx} = hA(t_s - t_f) + F\sigma A (T_s^4 - T_f^4)$$

or,
$$-45 \times 0.54 \times \frac{(t_s - t_i)}{L} = 22 \times 0.54 (310 - 15) + 250 \text{ (given)}$$

or,
$$-45 \times 0.54 \times \frac{(310 - t_i)}{0.025} = 22 \times 0.54 \times 295 + 250$$

or,
$$972 (t_i - 310) = 3754.6$$

or,
$$t_i = \frac{3754.6}{972} + 310 = \mathbf{313.86°C}$$

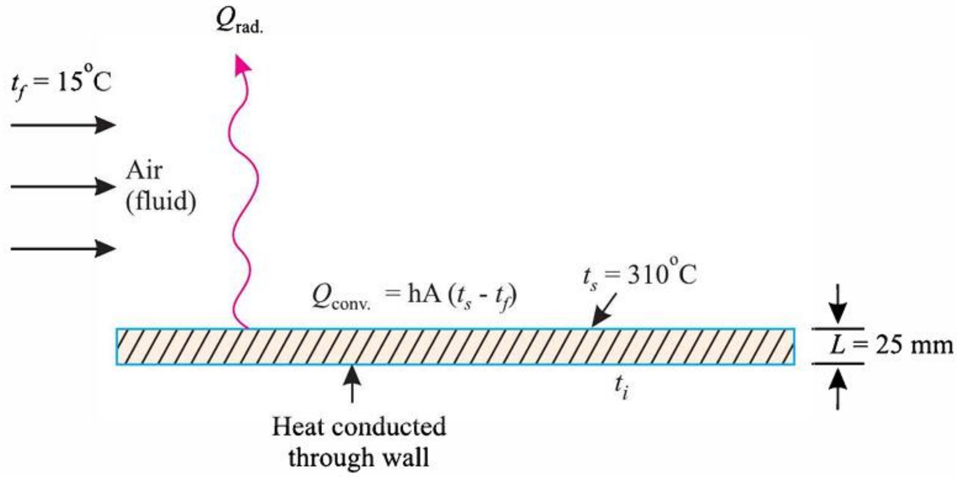


Fig. 1.11. Combination of conduction, convection and radiation heat transfer.

Example 1.8. *A surface at 250°C exposed to the surroundings at 110°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 75W/m²°C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 10W/m°C, what is the temperature gradient at the surface in the solid ?*

Solution. Temperature of the surface, $t_s = 250°C$

Temperature of the surroundings, $t_{sur} = 110°C$

The convection co-efficient, $h = 5W/m^2°C$

Radiation factor, $F = 1$

Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

Conductivity of the solid, $k = 10W/m°C$

Temperature gradient, $\boldsymbol{\dfrac{dt}{dx}}$ **:**

Heat conducted through the plate = Convection heat losses + radiation heat losses

i.e., $\quad Q_{cond.} = Q_{conv.} + Q_{rad.} \quad - kA\dfrac{dt}{dx} = hA\,(t_s - t_{sur}) + F\sigma A\,(T_s^4 - T_{sur}^{\ 4})$

Substituting the values, we have

$$-10 \times \frac{dt}{dx} = 75\,(250 - 110) + 1 \times 5.67 \times 10^{-8}\,[(250 + 273)^4 - (110 + 273)^4]$$

$$-10 \times \frac{dt}{dx} = 10500 + 5.67\left[\left(\frac{523}{100}\right)^4 - \left(\frac{383}{100}\right)^4\right]$$

$$= 10500 + 3022.1 = 13522.1$$

$$\therefore \qquad \frac{dt}{dx} = -\frac{13522.1}{10} = \mathbf{-\,1352.21\ °C/m}$$

Determine the heat transfer by convection over a surface of 0.75 m^2 if the surface is at 200°C and the fluid is at 80°C. The value of convective heat transfer is 25 W/m^2 °C. [**Ans.** 2.25 kW]

A surface of area 3m^2 and at 200°C exchanges heat with another surface at 30°C by radiation. If the value of factor due to the geometric location and emissivity is 0.69, determine :

(*i*) The rate of heat transfer,

(*ii*) The value of thermal resistance, and

iii) The equivalent convection coefficient. [**Ans.** (*i*) 4885.6W; (*ii*) 0.0348 °C/W; (*iii*) 9.58 W/m^2 °C]

A surface at 200°C exposed to the surroundings at 60°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 80W/m^2 °C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 15W/m °C what is the temperature gradient at the surface ? (**Ans.** – 889.4 °C/m)

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

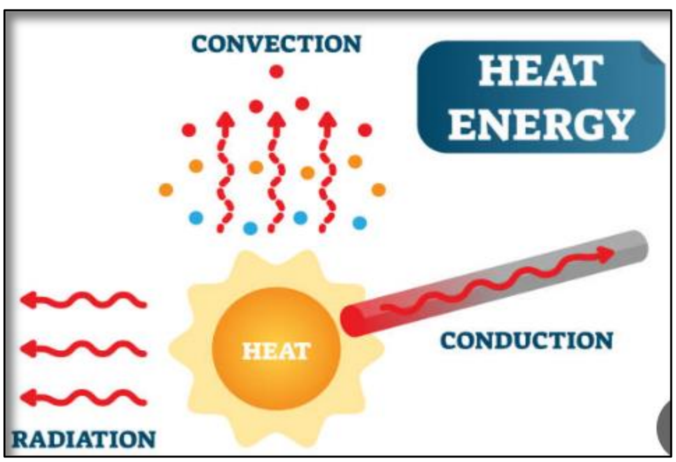
Learning package

In

General heat conduction equation in cylindrical coordinates

For

Students of the Second Year



By

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Assistant

1/ Overview

1 / A –Target population :-

For students of second year
Technological Institute of Basra
Dep. of Chemical Industrial Techniques

1 / B –Rationale:-

The **central idea** behind the general heat conduction equation in cylindrical coordinates is to **apply energy conservation to a small cylindrical volume**, using a coordinate system that perfectly matches the geometry of cylindrical objects (pipes, rods, wires). This leads to an equation that naturally incorporates how geometry affects heat flow and makes solving for temperature distributions much more straightforward in these shapes.

- **Coordinate system** fits the object's shape, making math simpler.
- **Energy conservation** applied at a microscopic control volume yields the full PDE.
- **General terms** allow modeling real-world effects (anisotropy, generation, unsteady behavior).

- **Special-case reductions** allow for simpler analytical or numerical solutions in practice.

1 / C –Central Idea:-

The **central idea** behind the general heat conduction equation in cylindrical coordinates is to **apply energy conservation to a small cylindrical volume**, using a coordinate system that perfectly matches the geometry of cylindrical objects (pipes, rods, wires). This leads to an equation that naturally incorporates how geometry affects heat flow and makes solving for temperature distributions much more straightforward in these shapes.

1 / D – Performance Objectives

Here are clear **performance objectives** for mastering the general heat conduction equation in **cylindrical coordinates**, suitable for both educational and engineering contexts:

Learning Objectives

1. **Understand when to use cylindrical coordinates**
   - Identify geometries (cylindrical rods, pipes, wires) where radial dependence and curvature matter
2. **Formulate the PDE from physical principles**
   - Derive the general heat conduction PDE using an energy balance on a differential cylindrical element (accounting for radial, angular, and axial heat fluxes, heat generation, and transient energy storage)
3. **Recognize all terms and their physical meaning**
   - Explain terms: radial/axial/angular conduction, time dependence ($\rho cp \partial T/\partial t$)(\rho c_p\partial T/\partial t)($\rho cp \partial T/\partial t$), and internal heat source ($\dot{q}$)(\dot q)($\dot{q}$)
4. **Apply special-case simplifications**
   - Reduce the general equation to common forms (e.g., steady-state, 1D radial only, no generation) for analyzing practical problems

General heat conduction equation in cylindrical coordinates

In case there are no heat sources present and the flow is steady and one- dimensional, the equation reduced to:

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} = 0$$

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{dt}{dr} = 0$$

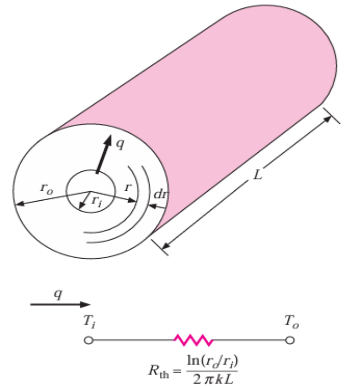
$$\frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{dt}{dr}\right) = 0$$

Since $\frac{1}{r} \neq 0$, therefore,

$$\frac{d}{dr}\left(r \cdot \frac{dt}{dr}\right) \quad \text{or} \quad r \cdot \frac{dt}{dr} = \text{constant}$$

Consider a long cylinder of inside radius $r_{i,}$ outside radius r_o, and length L, such as the shown in figure. We expose this cylinder to a temperature differential T_i-T_o and ask what the heat flow will be. For a cylinder with length very large to a diameter, it may be assumed that the heat flows only in the radial direction so that only the space coordinate needed to specify the system is r. Again Fourier' s law is used by interesting the proper area relation. The area for heat flow in the cylindrical system is

Figure 2-3 | One-dimensional heat flow through a hollow cylinder and electrical analog.



$$A_r = 2\pi rL$$

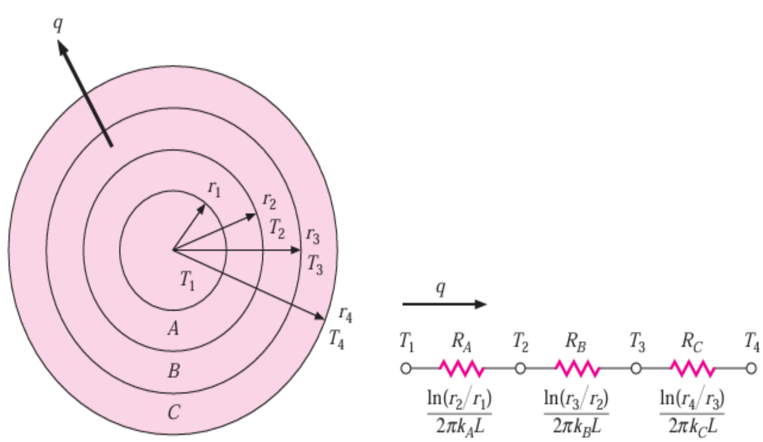
so that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -2\pi krL \frac{dT}{dr}$$

Figure 2-4 | One-dimensional heat flow through multiple cylindrical sections and electrical analog.



with the boundary conditions

$$T = T_i \qquad \text{at } r = r_i$$
$$T = T_o \qquad \text{at } r = r_o$$

The solution to Equation (2-7) is

$$q = \frac{2\pi k L\,(T_i - T_o)}{\ln(r_o/r_i)} \qquad \textbf{[2-8]}$$

and the thermal resistance in this case is

$$R_{\text{th}} = \frac{\ln(r_o/r_i)}{2\pi k L}$$

The thermal-resistance concept may be used for multiple-layer cylindrical walls just as it was used for plane walls. For the three-layer system shown in Figure 2-4 the solution is

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln(r_2/r_1)/k_A + \ln(r_3/r_2)/k_B + \ln(r_4/r_3)/k_C} \qquad \textbf{[2-9]}$$

General heat conduction equation in spherical coordinates

In case there are no heat sources present and the flow is steady and one- dimensional, the equation reduced to:

$$\frac{1}{r^2}\cdot\frac{d}{dr}\left(r^2\cdot\frac{dt}{dr}\right) = 0$$

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o} \qquad \textbf{[2-10]}$$

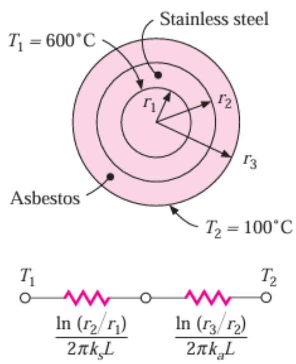
Heat conduction in cylinders and spheres

- Consider steady heat conduction through a hot water pipe. Heat is continuously lost to the outdoors through the wall of the pipe.
- The wall of the pipe, whose thickness rather small, separates two fluids at different temperatures, and thus the temperature gradient in the radial direction will be relatively large.
- Further, if the fluid temperatures inside and outside the pipe remains constant, then heat transfer through the pipe is steady.

EXAMPLE 2-2 Multilayer Cylindrical System

A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19$ W/m · °C] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2$ W/m · °C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.

Figure Example 2-2



■ Solution

Figure Example 2-2 shows the thermal network for this problem. The heat flow is given by

$$\frac{q}{L} = \frac{2\pi\,(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi\,(600 - 100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

where T_a is the interface temperature, which may be obtained as

$$T_a = 595.8^\circ\text{C}$$

The largest thermal resistance clearly results from the insulation, and thus the major portion of the temperature drop is through that material.

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Department of Chemical Industrial Techniques

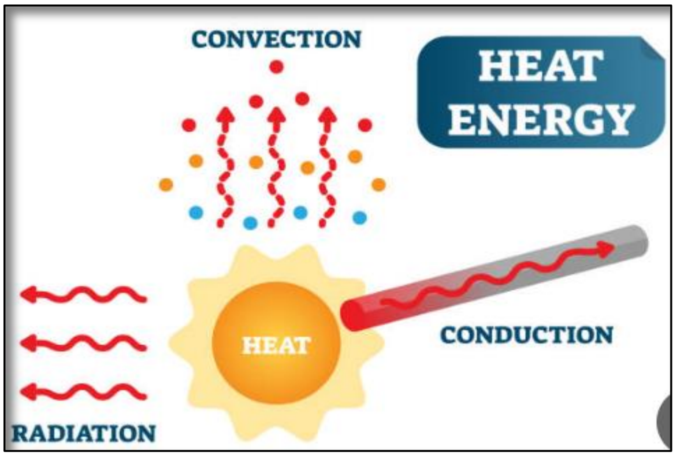
Learning package

In

The Overall Heat Transfer Coefficient

For

Students of the Second Year



By

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Dep. Of Chemical Industrial Techniques

2025

<u>1/ Overview</u>

1 / A –Target population :-

For students of second year
Technological Institute of Basra
Dep. of Chemical Industrial Techniques

1 / B –Rationale:-

The **overall heat transfer coefficient (U)** serves as a **single, equivalent measure of heat transfer performance** across a composite boundary—such as fluid–solid–fluid systems in heat exchangers or layered walls. Here's the rationale behind its use:

- The overall coefficient **quantifies the total heat transfer effectiveness** of a system incorporating multiple modes and layers of heat transfer.

- It supports **streamlined design**, **performance comparison**, and **maintenance diagnostics**.

- U elegantly blends **conduction**, **convection**, and **resistance concepts** into a single, practical parameter.

1 / C –Central Idea:-

The **central idea** of the **overall heat transfer coefficient (U)** is straightforward:

> U consolidates all thermal resistances—through solid materials, fluid films, fouling layers, etc.—into a single value that represents how effectively heat transfers across a composite boundary.

The **overall heat transfer coefficient** is a powerful, concise metric that captures all resistances in a heat transfer path. It simplifies complex, multi-layer systems into:

Q=UAΔTQ = U A \Delta TQ=UAΔT

—making it indispensable in engineering applications like thermal system design, performance evaluation, and maintenance planning.

1 / D – Performance Objectives

There are well-defined **performance objectives** for mastering the **overall heat transfer coefficient (U)**, framed for both educational and practical engineering contexts:

Learning Objectives

1. **Define and interpret U**
    - Clearly articulate what the overall heat transfer coefficient represents—the reciprocal of the total thermal resistance per unit area across a composite system.
2. **Understand its mathematical form**
    - Recognize the formula
3. **Explain physical interpretation and applications**
    - Explain why U aggregates multiple resistances (conduction, convection, fouling) into one value, and how it simplifies understanding heat transfer across boundaries.
4. **Relate U to heat transfer rate and design equations**
5. **Guide design and performance evaluation**
    - Understand how U's value influences design choices in apparatus like heat exchangers and building insulation.
    - **Estimate and compare U values**

    - Use empirical correlations or thermophysical data to calculate U and benchmark against expected ranges for different exchanger types.
    - **Perform detailed U-based design**
    - Given a desired heat duty (Q) and temperature difference, determine the minimum surface area (A) or necessary U covering counter- or co-current flows.
    - **Monitor and diagnose performance via U trending**
    - Calculate field U using measured data and compare with design U to detect fouling or degradation.
    - **Optimize thermal performance**
    - Use U as the objective in multi-criteria optimization—e.g., maximizing U while limiting pressure drop and cost, using surface enhancements or flow modifications.
    - **Evaluate fouling and maintenance impact**
    - Quantify fouling resistance and its effect on U; use this to plan maintenance or estimate cleaning intervals based on U decline.

The Overall Heat Transfer Coefficient

Consider the plane wall shown in Figure 2-5 exposed to a hot fluid A on one side and a cooler fluid B on the other side. The heat transfer is expressed by

$$q = h_1 A\,(T_A - T_1) = \frac{kA}{\Delta x}\,(T_1 - T_2) = h_2 A\,(T_2 - T_B)$$

The heat-transfer process may be represented by the resistance network in Figure 2-5, and the overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A} \qquad \textbf{[2-12]}$$

Figure 2-5 | Overall heat transfer through a plane wall.

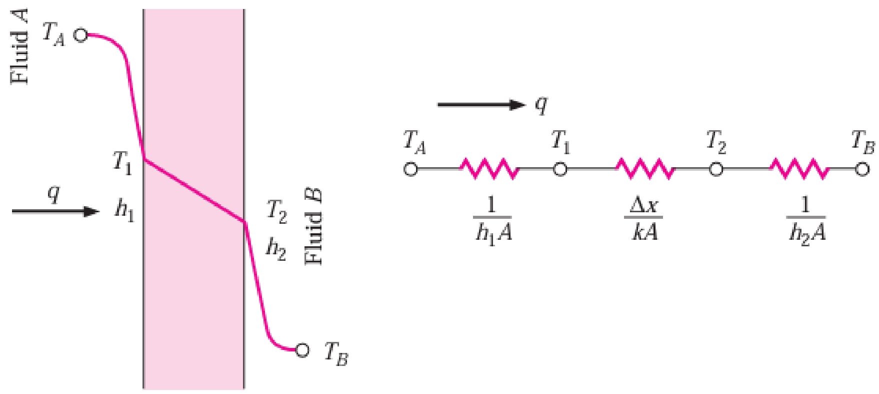
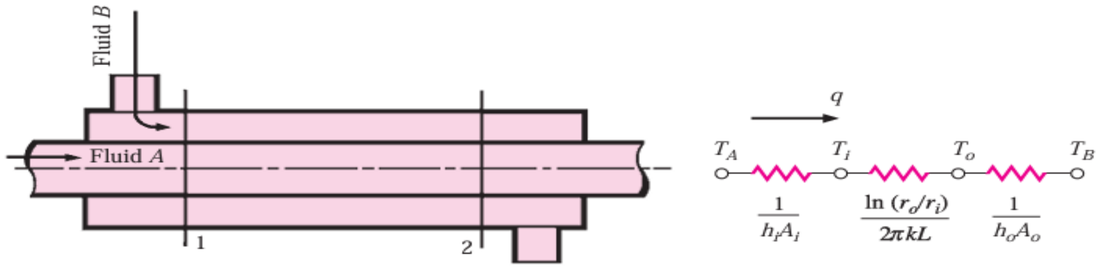


Figure 2-6 | Resistance analogy for hollow cylinder with convection boundaries.



Observe that the value $1/hA$ is used to represent the convection resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient U, defined by the relation

$$q = UA\,\Delta T_{\text{overall}} \qquad \textbf{[2-13]}$$

where A is some suitable area for the heat flow. In accordance with Equation (2-12), the overall heat-transfer coefficient would be

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

The overall heat-transfer coefficient is also related to the R value of Equation (2-6) through

$$U = \frac{1}{R \text{ value}}$$

For a hollow cylinder exposed to a convection environment on its inner and outer surfaces, the electric-resistance analogy would appear as in Figure 2-6 where, again, T_A and T_B are the two fluid temperatures. Note that the area for convection is not the same for both fluids in this case, these areas depending on the inside tube diameter and wall thickness. The overall heat transfer would be expressed by

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}} \qquad \textbf{[2-14]}$$

in accordance with the thermal network shown in Figure 2-6. The terms A_i and A_o represent the inside and outside surface areas of the inner tube. The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly,

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi k L} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}} \qquad \textbf{[2-15]}$$

$$U_o = \frac{1}{\dfrac{A_o}{A_i}\dfrac{1}{h_i} + \dfrac{A_o \ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o}} \qquad \textbf{[2-16]}$$

The general notion, for either the plane wall or cylindrical coordinate system, is that

$$UA = 1/\Sigma\, \mathrm{R_{th}} = 1/\mathrm{R_{th,overall}}$$

Overall Heat-Transfer Coefficient for a Tube

EXAMPLE 2-5

Water flows at 50°C inside a 2.5-cm-inside-diameter tube such that $h_i = 3500\ \mathrm{W/m^2 \cdot {}^\circ C}$. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m · °C. The outside of the tube loses heat by free convection with $h_o = 7.6\ \mathrm{W/m^2 \cdot {}^\circ C}$. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20°C.

■ Solution

There are three resistances in series for this problem, as illustrated in Equation (2-14). With $L = 1.0$ m, $d_i = 0.025$ m, and $d_o = 0.025 + (2)(0.0008) = 0.0266$ m, the resistances may be calculated as

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ {}^\circ\mathrm{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L} = \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\ {}^\circ\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575\ {}^\circ\mathrm{C/W}$$

Clearly, the outside convection resistance is the largest, and *overwhelmingly so*. This means that it is the controlling resistance for the total heat transfer because the other resistances (in series) are negligible in comparison. We shall base the overall heat-transfer coefficient on the outside tube area and write

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T \qquad \textbf{[a]}$$

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)} = 7.577\ \mathrm{W/m^2 \cdot {}^\circ C}$$

or a value very close to the value of $h_o = 7.6$ for the outside convection coefficient. The heat transfer is obtained from Equation (*a*), with

$$q = U A_o\, \Delta T = (7.577)\pi(0.0266)(1.0)(50 - 20) = 19\ \mathrm{W}\ \text{(for 1.0 m length)}$$

■ Comment

This example illustrates the important point that many practical heat-transfer problems involve multiple modes of heat transfer acting in combination; in this case, as a series of thermal resistances. It is not unusual for one mode of heat transfer to dominate the overall problem. In this example, the total heat transfer could have been computed very nearly by just calculating the free convection heat loss from the outside of the tube maintained at a temperature of 50°C. Because the inside convection and tube wall resistances are so small, there are correspondingly small temperature drops, and the outside temperature of the tube will be very nearly that of the liquid inside, or 50°C.

Example 2.35. *A thick walled tube of stainless steel with 20 mm inner diameter and 40 mm outer diameter is convered with a 30 mm layer of asbestos insulation (k = 0.2 W/m°C). If the inside wall temperature of the pipe is maintained at 600°C and the outside insulation at 1000°C, calculate the heat loss per metre of length.* **(AMIE Summer, 1997)**

Solution. Refer to Fig. 2.48.

Given:
$$r_1 = \frac{20}{2} = 10\,\text{mm} = 0.01\text{ m}$$
$$r_2 = \frac{40}{2} = 20\,\text{mm} = 0.02\text{ m}$$
$$r_3 = 20 + 30 = 50\,\text{mm} = 0.05\text{m}$$
$$t_1 = 600°\text{C}$$
$$t_3 = 1000°\text{C}$$
$$k_B = 0.2\text{ W/m°C}$$

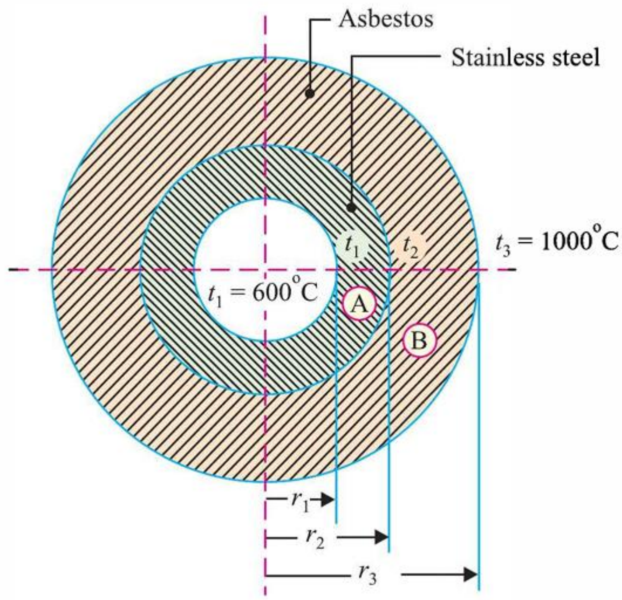


Fig. 2.48.

Heat transfer per metre of a length, Q/L:

$$Q = \frac{2\pi L(t_1 - t_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$

Since the thermal conductivity of stainless steel is not given, therefore, neglecting the resistance offered by stainless steel to heat transfer across the tube, we have

$$\frac{Q}{L} = \frac{2\pi(t_1 - t_3)}{\frac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi(600 - 1000)}{\frac{\ln(0.05/0.02)}{0.2}}$$

$= -$**548.57 W/m (Ans.)**

Negative sign indicates that the heat transfer takes place *radially inward.*

Example 2.48. *Hot air at a temperature of 65°C is flowing through a steel pipe of 120 mm diameter. The pipe is covered with two layers of different insulating materials of thickness 60 mm and 40 mm, and their corresponding thermal conductivities are 0.24 and 0.4 W/m°C. The inside and outside heat transfer coefficients are 60 W/m°C and 12 W/m°C respectively. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe.*

Solution. Refer to Fig. 2.61.

Given :

$$r_1 = \frac{120}{2} = 60\,\text{mm} = 0.06\,\text{m}$$

$$r_2 = 60 + 60 = 120\,\text{mm} = 0.12\,\text{m}$$

$$r_3 = 60 + 60 + 40 = 160\,\text{mm} = 0.16\,\text{m}$$

$$k_A = 0.24\,\text{W/m°C}; \qquad k_B = 0.4\,\text{W/m°C}$$

$$h_{hf} = 60\,\text{W/m}^2\text{°C}; \qquad h_{cf} = 12\,\text{W/m}^2\text{°C}$$

$$t_{hf} = 65°\text{C}; t_{cf} = 20°\text{C}$$

Length of pipe, $L = 60$ m

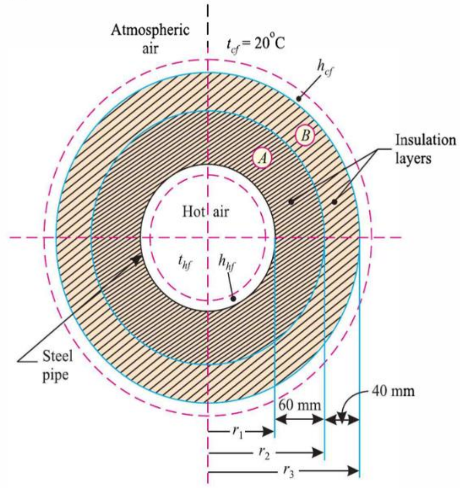


Fig. 2.61.

Rate of heat loss, Q :

Rate of heat loss is given by

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf} \cdot r_3}\right]} \qquad \text{[Eqn. (2.69)]}$$

$$= \frac{2\pi \times 60 (65 - 20)}{\left[\frac{1}{60 \times 0.06} + \frac{\ln(0.12/0.06)}{0.24} + \frac{\ln(0.16/0.12)}{0.4} + \frac{1}{12 \times 0.16}\right]}$$

$$= \frac{16964.6}{0.2777 + 2.8881 + 0.7192 + 0.5208} = 3850.5\,\text{W}$$

i.e., Rate of heat loss = **3850.5 W** **(Ans.)**

Example 2.53. *A steam pipe (k = 45 W/m°C) having 70 mm inside diameter and 85 mm outside diameter is lagged with two insulation layers; the layer in contact with the pipe is 35 mm asbestos (k = 0.15 W/m°C) and it is covered with 25 mm thick magnesia insulation (k = 0.075 W/m°C). The heat transfer coefficients for the inside and outside surfaces are 220 W/m²°C and 6.5 W/m²°C respectively. If the temperature of steam is 350°C and the ambient temperature is 30°C, calculate* :

(*i*) *The steady loss of heat for* 50 *m length of the pipe*;

(*ii*) *The overall heat transfer coefficients based on inside and outside surfaces of the lagged steam main.*

Solution. Refer to Fig. 2.67.

$$r_1 = \frac{70}{2} = 35\,\text{mm or } 0.035\,\text{m}$$

$$r_2 = \frac{85}{2} = 42.5\,\text{mm or } 0.0425\,\text{m}$$

$$r_3 = 42.5 + 35$$
$$= 77.5\,\text{mm or } 0.0775\,\text{m}$$

$$r_4 = 77.5 + 25$$
$$= 102.5\,\text{mm or } 0.1025\,\text{m}$$

$$L = 50\,\text{m}$$

$$k_A = 45\,\text{W/m°C}$$

$$k_B = 0.15\,\text{W/m°C}$$

$$k_C = 0.075\,\text{W/m°C}$$

Temperature of steam,

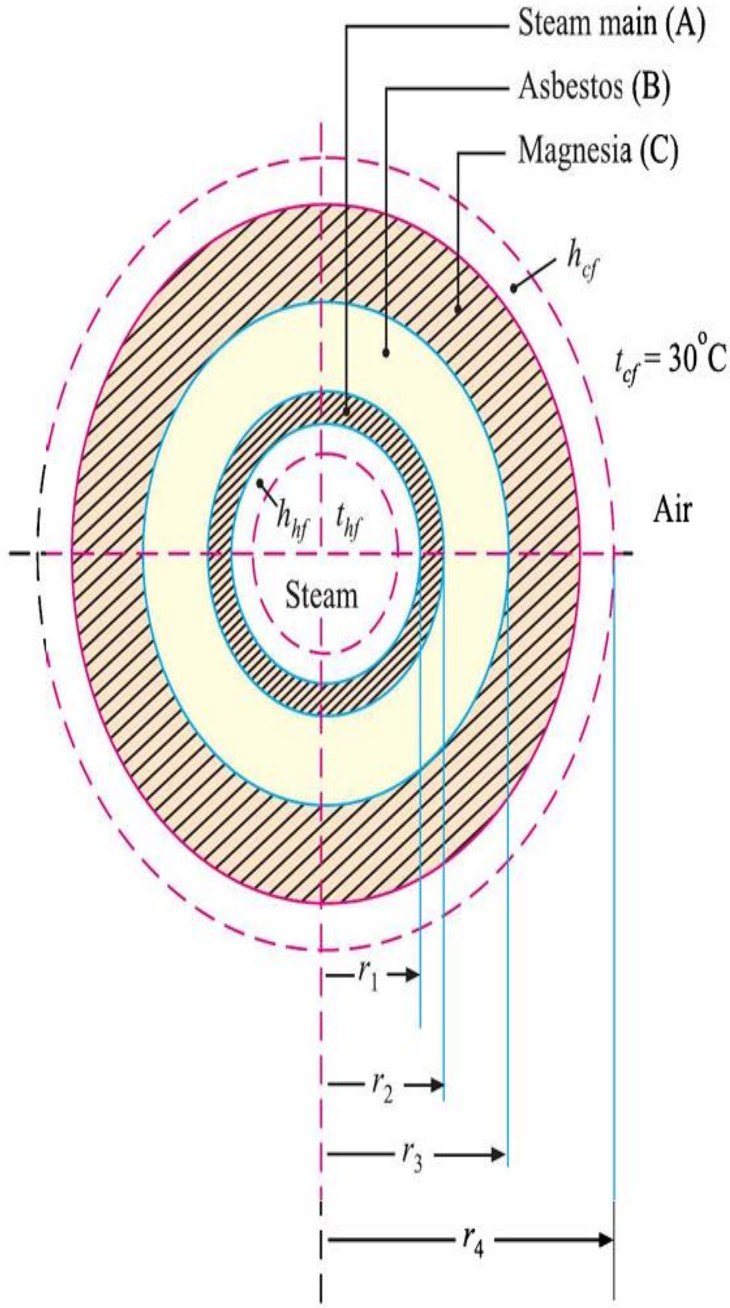
$$t_{hf} = 350°\text{C}$$

Ambient temperature,

$$t_{cf} = 30°\text{C}$$

$$h_{hf} = 220\,\text{W/m}^2\text{°C},$$

$$h_{cf} = 6.5\,\text{W/m}^2\text{°C}.$$



(*i*) **Loss of heat, Q :** **Fig. 2.67.**

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\frac{1}{h_{hf} \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{\ln(r_4/r_3)}{k_C} + \frac{1}{h_{cf} \cdot r_4}}$$

$$= \frac{2\pi \times 50(350 - 30)}{\frac{1}{220 \times 0.035} + \frac{\ln(0.0425/0.035)}{45} + \frac{\ln(0.0775/0.0425)}{0.15} + \frac{\ln(0.1025/0.0775)}{0.075} + \frac{1}{6.5 \times 0.1025}}$$

$$= \frac{100530.96}{0.129870 + 0.00431 + 4.00516 + 3.72779 + 1.50094} = 10731.23 \text{ W}$$

i.e., Loss of heat for 50 m of length = **10731.23 W** **(Ans.)**

(*ii*) **The overall heat transfer coefficients, U_o, U_i :**

The loss of heat can also be expressed as follows :

$$Q = U_o A_o \Delta t = U_i A_i \Delta t$$

Where U_o and U_i are the overall heat transfer co-efficients based on the outside area A_o and inside area A_i respectively.

$\therefore$

$$U_0 = \frac{Q}{A_o \cdot \Delta t} = \frac{10731.23}{2\pi r_4 L \times \Delta t}$$

$$= \frac{10731.23}{2\pi \times 0.1025 \times 50(350 - 30)} = \mathbf{1.0414 \ W/m^2 \, ^\circ C} \quad \textbf{(Ans.)}$$

Similarly,

$$U_i = \frac{Q}{A_i \cdot \Delta t} = \frac{10731.23}{2\pi r_1 L \times \Delta t} = \frac{10731.23}{2\pi \times 0.035 \times 50(350 - 30)}$$

$$= \mathbf{3.05 \ W/m^2 \, ^\circ C} \quad \textbf{(Ans.)}$$

Example 2.61. *A spherical shaped vessel of 1.4 m diameter is 90 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 220°C. Thermal conductivity of the material of the sphere is 0.083 W/m°C.*

Solution. Refer to Fig. 2.78.

$$r_2 = \frac{1.4}{2} = 0.7\,\text{m}.$$

$$r_1 = 0.7 - \frac{90}{1000} = 0.61\,\text{m}$$

$$t_1 - t_2 = 220°\text{C};$$

$$k = 0.083\ \text{W/m°C}$$

The rate of heat transfer/leakage is given by

$$Q = \frac{(t_1 - t_2)}{\left[\dfrac{(r_2 - r_1)}{4\pi k\, r_1 r_2}\right]} \qquad \text{...Fig. (2.76)}$$

$$= \frac{220}{\left[\dfrac{(0.7 - 0.61)}{4\pi \times 0.083 \times 0.61 \times 0.7}\right]}$$

$$= 1088.67\ \text{W}$$

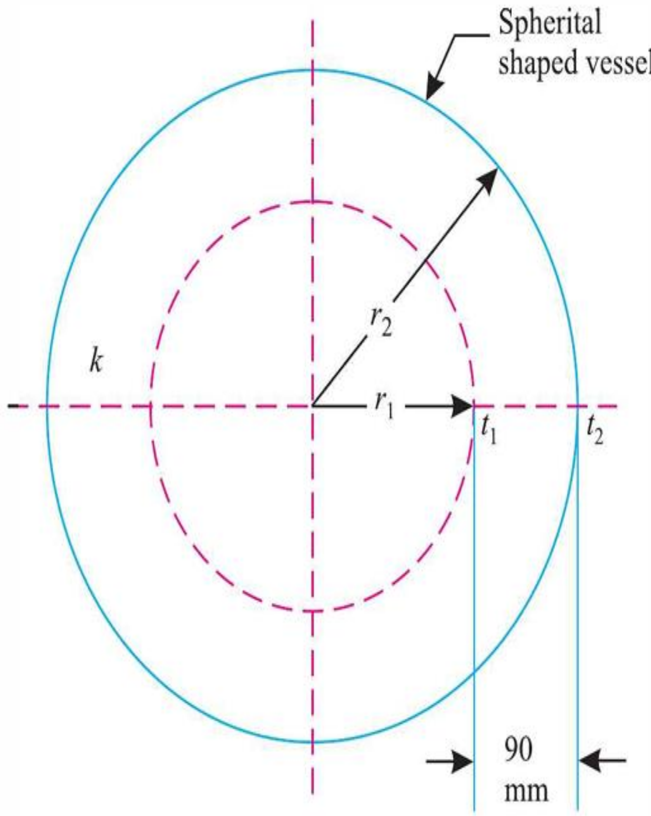


Fig. 2.78.

i.e., Rate of heat leakage = **1088.67 W** **(Ans.)**

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

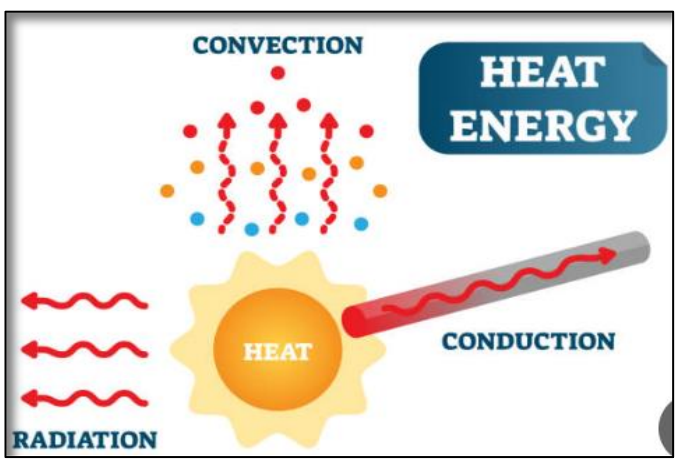
Learning package

In

HEAT EXCHANGER

For

Students of the Second Year



By
Ataa Wejood
Assistant
Dep. Of Chemical Industrial Techniques

2025

1/ Overview

1 / A –Target population :-

For students of second year
Technological Institute of Basra
Dep. of Chemical Industrial Techniques

1 / B –Rationale:-

A **heat exchanger** is an engineered device designed to **efficiently transfer heat between two isolated media**, minimizing energy loss, optimizing thermal control, and enhancing system performance. Through clever design (surface area, flow paths, materials) and engineering tools (effectiveness, pinch methods), they ensure optimal thermal behavior across diverse applications—from industrial processes to everyday appliances.

1 / C –Central Idea:-

A **heat exchanger** is a device engineered to **transfer thermal energy from one fluid to another without mixing them**, using conduction through a separating wall and convection on each fluid interface. It's designed to **maximize heat transfer efficiency** while maintaining fluid isolation and minimizing energy penalty.

- **Maximized surface area**: Shell-and-tube, plate-and-frame, or fin-tube arrangements increase contact area between fluids and barrier surfaces.
- **Optimal flow configuration**:

Counterflow arrangements maintain higher temperature gradients along the exchanger and thus higher overall heat transfer effectiveness.

- **Material selection**: High-conductivity metals and corrosion-resistant alloys ensure efficient conduction and long-term reliability.

1 / D – Performance Objectives

Learning Objectives (Foundational Understanding)

1. **Define the purpose and core function**
   - Clearly explain that heat exchangers transfer thermal energy between two fluid streams without mixing them, using conduction through a separating wall and convection at each fluid–wall interface.
2. **Differentiate between designs and flow configurations**
   - Identify common types (e.g., shell-and-tube, plate, regenerative) and distinguish between parallel-flow, counterflow, and crossflow setups.
3. **Explain heat transfer mechanisms and parameters**
   - Describe the roles of conduction, convection, overall heat transfer coefficient (U), and LMTD in the thermal performance of exchangers.
4. **Recognize factors affecting performance**
   - Understand influences such as fouling, pressure drop, flow rates, fluid properties, and surface enhancements.
5. **Discuss maintenance, safety, and inspection protocols**
   - Identify common failure modes and the importance of cleaning, inspection, and compliance with standards for safe operation

HEAT EXCHANGER

A Heat Exchanger may be defined as an equipment which transfers the energy from the hot fluid to a cold fluid, with maximum rate and minimum investment and running costs.

In heat exchangers the temperature of each fluid changes as it passes through the exchangers, and hence the temperature of the driving wall between the fluids also changes along the length of the exchangers.

Examples of Heat Exchangers

1. Intercoolers and preheaters;
2. Condensers and boilers in steam plant;
3. Condensers and evaporators in refrigeration units;
4. Regenerators

5. Automobile radiators;
6. Oil coolers of heat engine;
7. Milk chiller of a pasteurizing plant;
8. Several other industrial processes.

Types of Heat Exchangers

In order to meet widely varying application, several types of heat exchangers have been developed which are classified on the basis of nature of heat exchange process, relative direction of fluid motion, design and constructional features, and physical state of fluids.

1. **Nature of heat exchange process**

   Heat Exchangers, on the basis of nature of heat exchange process, are classified as follows:

   - Direct contact (or open) heat exchangers.
   - Indirect contact heat exchangers.
     - Regenerators
     - Recuperators

- Direct contact (or open) heat exchangers.

  Examples: cooling towers, jet condensers, direct contact feed heaters.

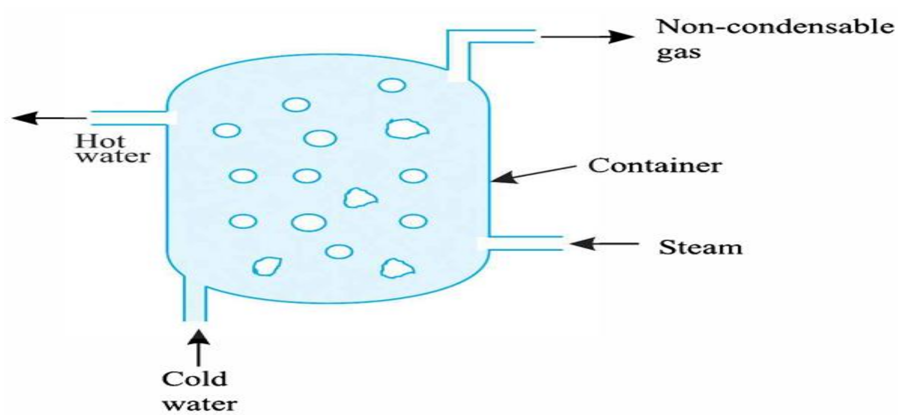


Fig. 10.1. Direct contact or open heat exchanger.

- **Indirect contact heat exchangers**.

exchangers :

In this type of heat exchanger, the heat transfer between two fluids could be carried out by transmission through wall which separates the two fluids. This type includes the following :

(*a*) Regenerators.

(*b*) Recuperators or surface exchangers.

Oil cooler.

(*a*) *Regenerators*: In a *regenerator* type of heat exchanger the *hot and cold fluids pass alternately through a space containing solid particles* (*matrix*), *these particles providing alternately a sink and a source for heat flow*.

Examples : (*i*) I.C. engines and gas turbines; (*ii*) Open hearth and glass melting furnaces; (*iii*) Air heaters of blast furnaces.

A regenerator generally operates periodically (the solid matrix alternately stores heat extracted from the hot fluid and then delivers it to the cold fluid). However, in some regenerators the matrix is made to rotate through the fluid passages arranged side by side which makes the heat exchange process *continuous*.

The performance of these regenerators is affected by the following *parameters* :

(*i*) Heat capacity of regenerating material,

(*ii*) The rate of absorption, and

(*iii*) The release of heat.

Advantages **:**

1. Higher heat transfer coefficient;
2. Less weight per kW of the plant;
3. Minimum pressure loss;
4. Quick response to load variation;
5. Small bulk weight;
6. Efficiency quite high.

***Disadvantages* :**

1. Costlier compared to recuperative heat exchangers.
2. Leakage is the main trouble, therefore, perfect sealing is required.

(*b*) *Recuperators*: '*Recuperator*' is the most important type of heat exchanger in which the flowing fluids exchanging heat are on either side of *dividing wall* (in the form of pipes or tubes generally). These heat exchangers are used when two fluids cannot be allowed to mix *i.e.*, when the mixing is undesirable.

Examples : (*i*) Automobile radiators, (*ii*) Oil coolers, intercoolers, *air preheaters*, economisers, superheaters, condensers and surface feed heaters of a steam power plant, (*iii*) Milk chiller of pasteurising plant, (*iv*) Evaporator of an ice plant.

***Advantages* :**

1. Easy construction;
2. More economical;
3. More surface area for heat transfer;
4. Much suitable for stationary plants.

***Disadvantages* :**

1. Less heat transfer coefficient;
2. Less generating capacity;
3. Sooting problems.

The flow through *direct heat exchangers and recuperators* may be treated as *steady state* while through regenerators the *flow is essentially transient.*

2. Relative direction of fluid motion :

According to the relative directions of two fluid streams the heat exchangers are classified into the following *three* categories :

(*i*) Parallel flow or unidirection flow (*ii*) Counter-flow

(*iii*) Cross-flow.

(*i*) *Parallel flow heat exchangers* :

In a *parallel flow heat exchanger*, as the name suggests, the two fluid streams (hot and cold) travel in the *same direction*. The two streams enter at one end and leave at the other end. The flow arrangement and variation of temperatures of the fluid streams in case of parallel flow heat exchangers, are shown in Fig. 10.2. It is evident from the Fig.10.2 (*b*) that the *temperature difference* between the hot and cold fluids goes on *decreasing* from inlet to outlet. Since this type of heat exchanger needs a large area of heat transfer, therefore, it is *rarely used in practice*.

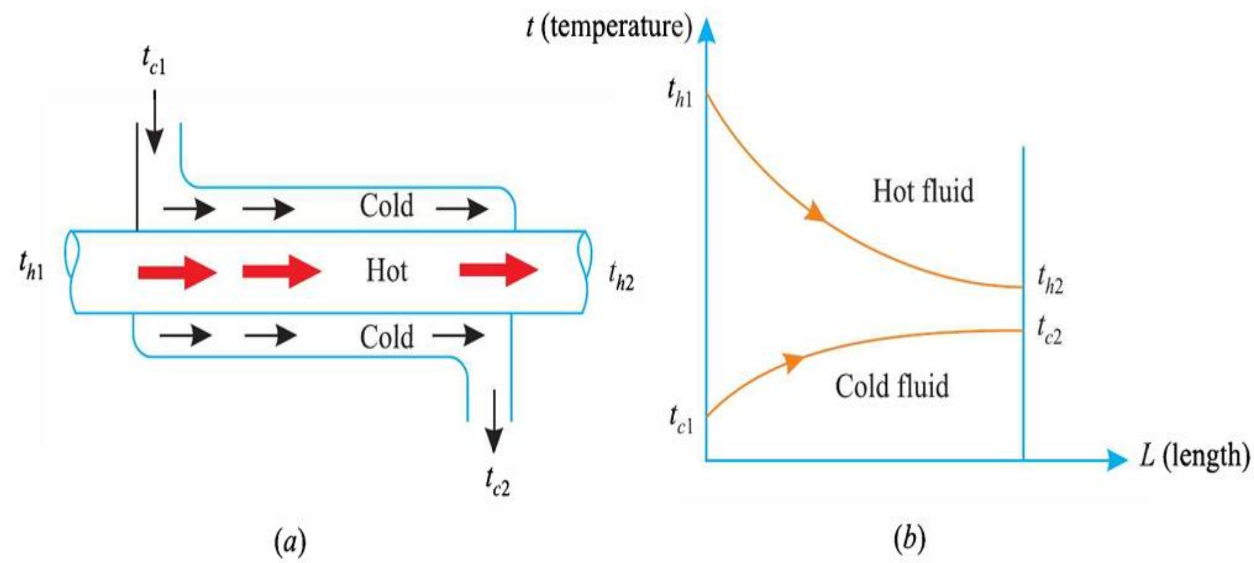


Fig. 10.2. Parallel flow heat exchanger.

Examples : Oil coolers, oil heaters, water heaters etc.

As the two fluids are separated by a wall, this type of heat exchanger may be called parallel flow recuperator or surface heat exchanger.

(*ii*) *Counter-flow heat exchangers* :

In a counter-flow heat exchanger, the two fluids flow in opposite directions. The hot and cold fluids enter at the opposite ends. The flow arrangement and temperature distribution for such a heat exchanger are shown schematically in Fig. 10.3. The *temperature difference* between the two fluids remains more or less *nearly constant*. This type of heat exchanger, due to counter flow, gives *maximum rate of heat transfer for a given surface area*. Hence such heat exchangers are *most favoured* for heating and cooling of fluids.

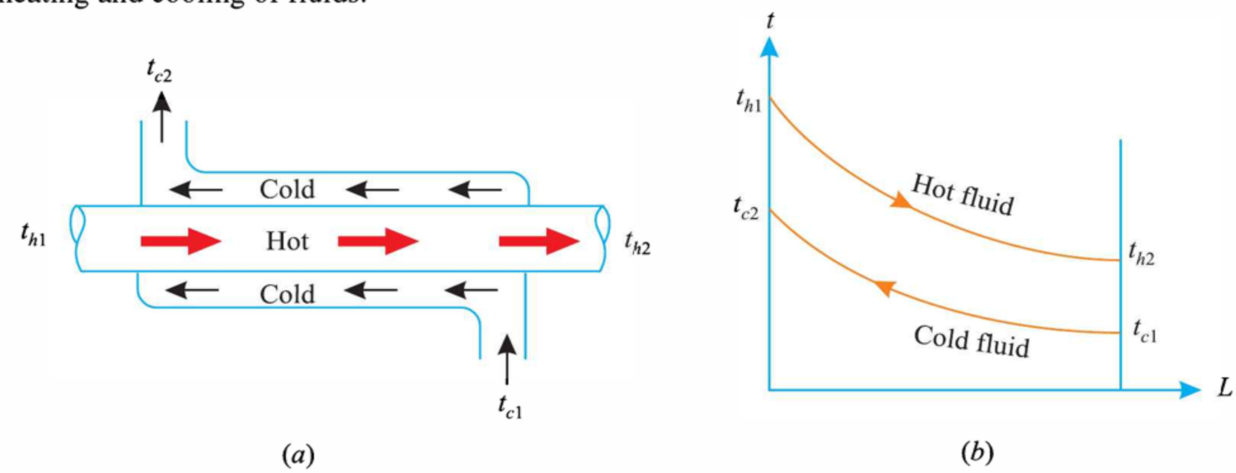


Fig. 10.3. Counter-flow heat exchanger

(*iii*) *Cross-flow heat exchanger :*

In cross-flow heat exchangers, the *two fluids* (*hot and cold*) *cross one another in space*, *usually at right angles*. Fig. 10.4 shows a schematic diagram of common arrangements of cross-flow heat exchangers.

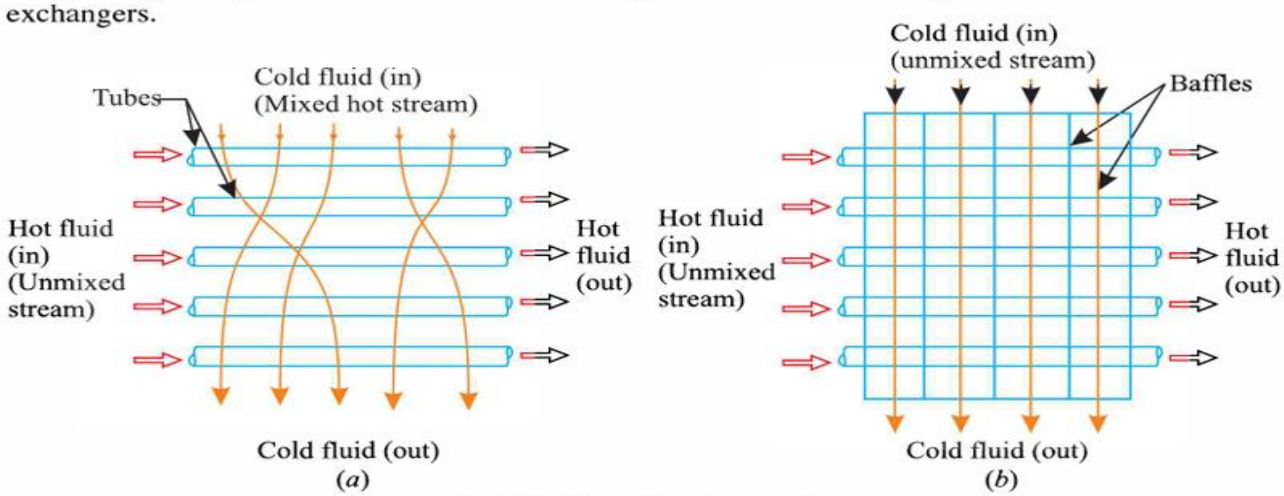


Fig. 10.4. Cross-flow heat exchangers

- Refer Fig. 10.4 (*a*) : *Hot fluid* flows in the the separate tubes and there is no mixing of the fluid streams. The *cold fluid* is perfectly mixed as it flows through the exchanger. The temperature of this mixed fluid will be uniform across any section and will vary only in the direction of flow.

Examples : The cooling unit of refrigeration system etc.

- Refer Fig. 10.4 (*b*) : In this case each of the fluids follows a prescribed path and is unmixed as it flows through heat exchanger. Hence the temperature of the fluid leaving the heater section is not uniform.

Examples : Automobile radiator etc.

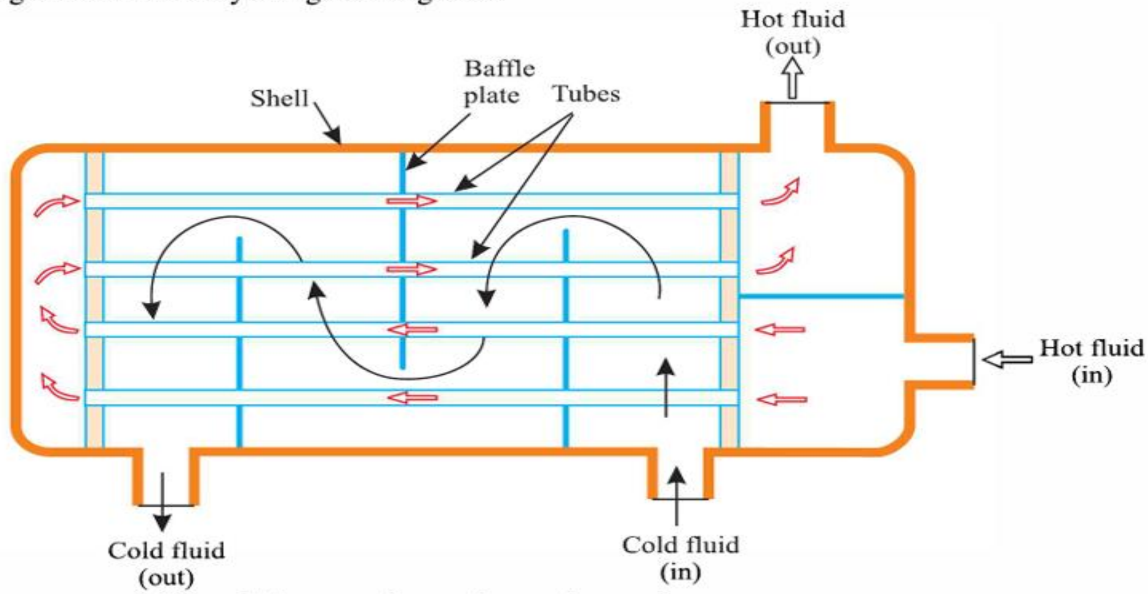
- In yet another arrangement, *both the fluids are mixed* while they travel through the exchanger; consequently the temperature of both the fluids is uniform across the section and varies only in the direction in which flow takes place.

3. Design and constructional features :

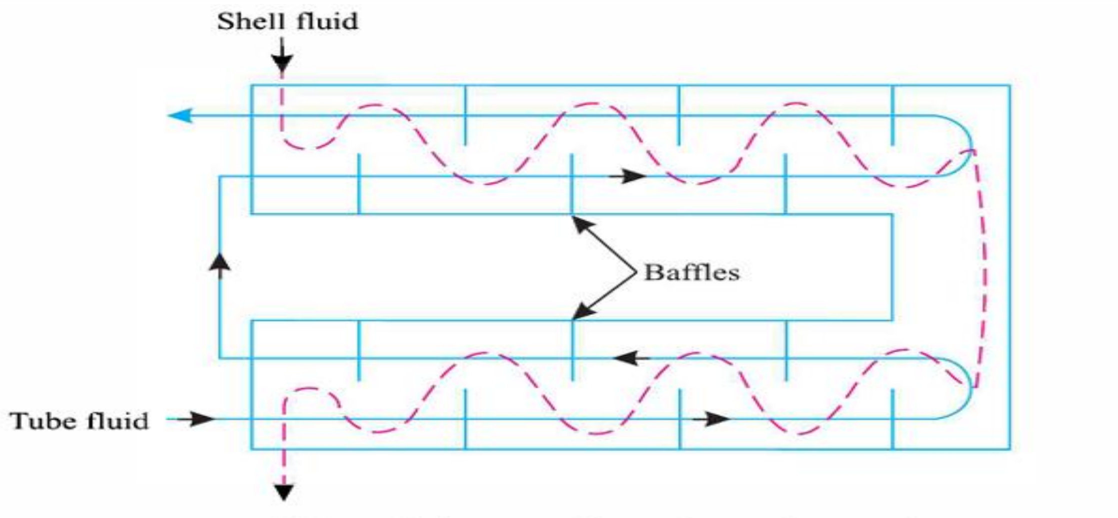
On the basis of design and constructional features, the heat exchangers are classified as under :

(*i*) *Concentric tubes* :

In this type, two concentric tubes are used, each carrying one of the fluids. The direction of flow may be parallel or counter as depicted in Fig. 10.2 (*a*) and Fig. 10.3 (*a*). The effectiveness of the heat exchanger is increased by using swirling flow.



(*a*) One-shell pass and two-tube pass heat exchanger



(*b*) Two-shell pass and four-tube pass heat exchanger

Fig. 10.5. Shell and tube heat exchangers.

(*ii*) *Shell and tube* :

In this type of heat exchanger one of the fluids flows through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it flows over the outside surface of the tubes. Such an arrangement is employed where *reliability* and *heat transfer effectiveness are important*. With the use of multiple tubes heat transfer rate is amply improved due to increased surface area.

(*iii*) *Multiple shell and tube passes* :

Multiple shell and tube passes are used for *enhancing the overall heat transfer*. *Multiple shell pass* is possible where the fluid flowing through the shell is re-routed. The shell side fluid is forced to flow back and forth acros the tubes by baffles. *Multiple tube pass* exchangers are those which re-route the fluid through tubes in the opposite direction.

(*iv*) *Compact heat exchangers* :

There are special purpose heat exchangers and have a very large transfer surface area per unit volume of the exchanger. They are generally employed when convective heat transfer coefficient associated with one of the fluids is much smaller than that associated with the other fluid.

Example : Plate-fin, flattened fin tube exchangers etc.

(4) Physical state of fluids

Depending upon the physical state of fluids the heat exchangers are classified as follows :

(*i*) Condensers (*ii*) Evaporators

(*i*) *Condensers.* In a condenser, the condensing fluid remains at constant temperature throughout the exchanger while the temperature of the colder fluid gradually increases from inlet to outlet. The hot fluid loses latent part of heat which is accepted by the cold fluid (Refer Fig. 10.6).

This photo shows the heat exchanger side of the open system with wood boiler.

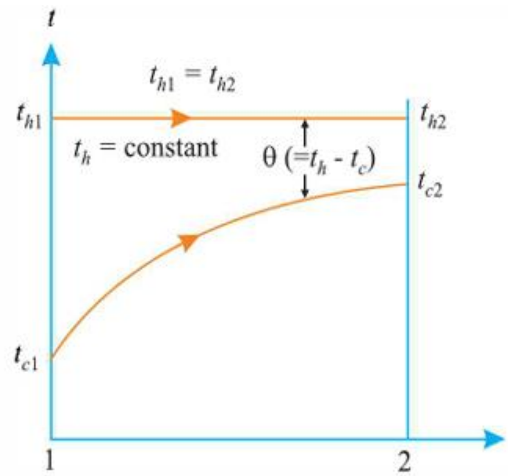


Fig. 10.6. Temperature distribution in a condenser.

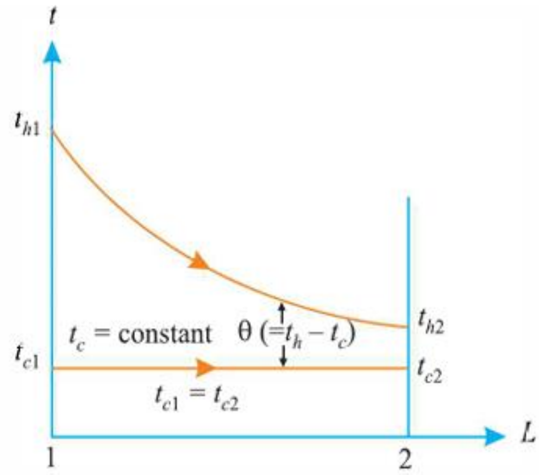


Fig. 10.7. Temperature distribution in an evaporator.

(*ii*) *Evporators.* In this case, the boiling fluid (cold fluid) remains at constant temperature while the temperature of hot fluid gradually decreases from inlet to outlet. (Refer Fig. 10.7).

Heat Exchangers Analysis

For designing or predicting the performance of a heat exchanger it is necessary that the total heat transfer may be related with its governing parameters : (*i*) U (overall heat transfer coefficient) due to various modes of heat transfer, (*ii*) A total surface area of the heat transfer, and (*iii*) t_1, t_2 (the inlet and outlet fluid temperatures). Fig. 10.8 shows the overall energy balance in a heat exchanger.

Let, $\dot{m}$ = Mass flow rate, kg/s,

c_p = Specific heat of fluid at constant pressure, J/kg°C,

t = Temperature of fluid, °C, and

Δt = Temperature drop or rise of a fluid across the heat exchanger.

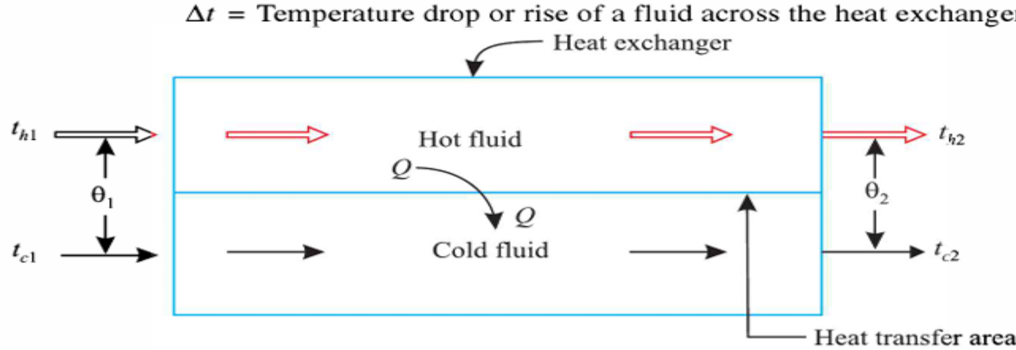


Fig. 10.8. Overall energy balance in a heat exchanger.

Subscripts *h* and *c* refer to the *hot* and *cold* fluids respectively; subscripts 1 and 2 correspond to the *inlet* and *outlet* conditions respectively.

Assuming that there is no heat loss to the surroundings and potential and kinetic energy changes are negligible, from the energy balance in a heat exchanger, we have :

Heat given up by the hot fluid, $Q = m_h\, c_{ph}\, (t_{h1} - t_{h2})$ ...(10.1)

Heat picked up by the cold fluid, $Q = m_c\, c_{pc}\, (t_{c2} - t_{c1})$ ...(10.2)

Total heat transfer rate in the heat exchanger, $Q = U A\, \theta_m$ ...(10.3)

where, U = Overall heat transfer coefficient between the two fluids,

A = Effective heat transfer area, and

θ_m = *Appropriate mean value of temperature difference or logarithmic mean temperature difference* (*LMTD*).

Logarithmic Mean Temperature Difference (LMTD)

Logarithmic mean temperature difference (LMTD) *is defined as* that *temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference.*

In order to derive expression for *LMTD* for various types of heat exchangers, the following **assumptions** are made :

1. The overall heat transfer coefficient U is constant.
2. The flow conditions are steady.
3. The specific heats and mass flow rates of both fluids are constant.
4. There is no loss of heat to the surroundings, due to the heat exchanger being perfectly insulated.
5. There is no change of phase either of the fluid during the heat transfer.
6. The changes in potential and kinetic energies are negligible.
7. Axial conduction along the tubes of the heat exchanger is negligible.

Logarithmic Mean Temperature Difference (LMTD) (Parallel Flow)

Refer Fig. 10.9, which shows the flow arrangement and distribution of temperature in a single-pass parallel flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U\, dA\, (t_h - t_c) = U \,.\, dA \,.\, \Delta t$$

As a result of heat transfer dQ through the area dA, the hot fluid is cooled by dt_h whereas the cold fluid is heated up by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\dot{m}_h \,.\, c_{ph} \,.\, dt_h = \dot{m}_c \,.\, c_{pc} \,.\, dt_c = U \,.\, dA \,.\, (t_h - t_c) \quad ...(10.4)$$

(Here d_{th} is – ve and d_{tc} is + ve)

or, $$dt_h = -\frac{dQ}{\dot{m}_h\, c_{ph}} = -\frac{dQ}{C_h}$$

and, $$dt_c = \frac{dQ}{\dot{m}_c\, c_{pc}} = \frac{dQ}{C_c}$$

where, $C_h = \dot{m}_h\, c_{ph}$ = Heat capacity or water equivalent of hot fluid, and

$C_c = \dot{m}_c\, c_{pc}$ = Heat capacity or water equivalent of cold fluid.

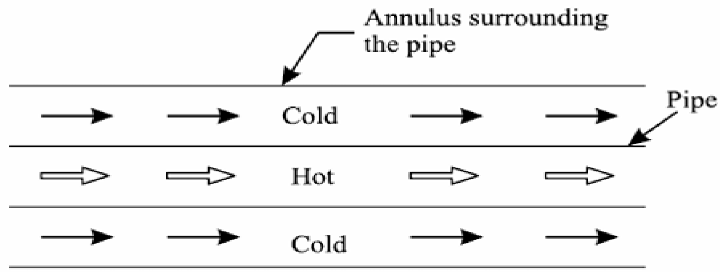
$\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of fluids and c_{ph} and c_{pc} are the respective specific heats.

$$\therefore \quad dt_h - dt_c = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

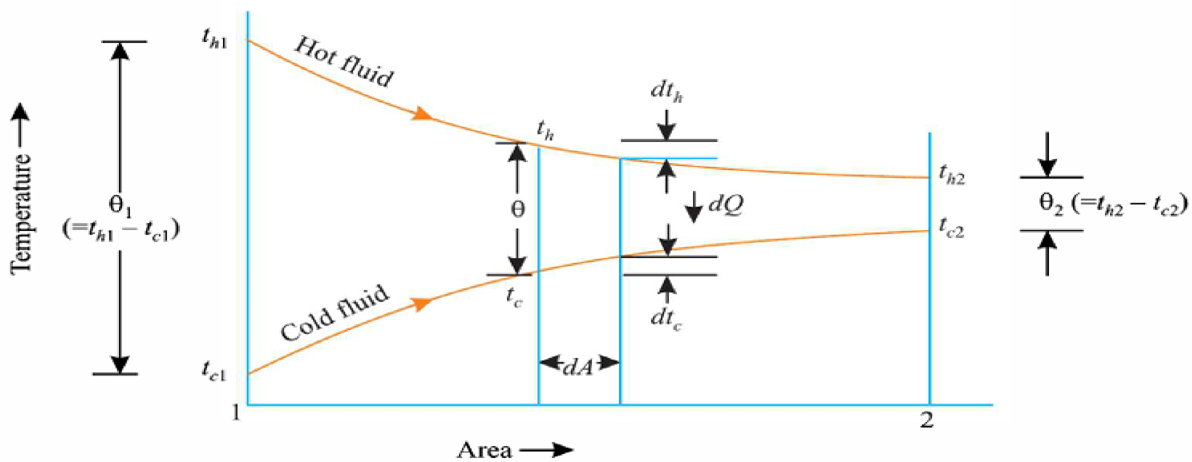
$$d\theta = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad ...(10.5)$$

Substituting the value of dQ from eqn. (10.4) the above equation becomes

$$d\theta = -U \,.\, dA\, (t_h - t_c)\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$



(*a*) Flow arrangement



(*b*) Temperature distribution

Subscripts h, c refer to : *hot* and *cold* fluids
Subscript 1, 2 refer to : *inlet* and *oulet* conditions.

Fig. 10.9. Calculation of LMTD for a parallel flow heat exchanger.

or, $$d\theta = -U \,.\, dA \,.\, \theta \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or, $$\frac{d\theta}{\theta} = -U \,.\, dA \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Integrating between inlet and outlet conditions (*i.e.* from $A = 0$ to $A = A$), we get

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \int_{A=0}^{A=A} U \,.\, dA$$

or, $$ln\,(\theta_2/\theta_1) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad ...(10.6)$$

Now, the total heat transfer rate between the two fluids is given by

$$Q = C_h\,(t_{h1} - t_{h2}) = C_c\,(t_{c2} - t_{c1}) \quad ...(10.7)$$

or, $$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q} \quad ...[10.7\ (a)]$$

$$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q} \quad ...[10.7\ (b)]$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. (10.6), we get

$$\ln(\theta_2/\theta_1) = -UA\left[\frac{t_{h1}-t_{h2}}{Q} + \frac{t_{c2}-t_{c1}}{Q}\right]$$

$$= \frac{UA}{Q}\left[(t_{h2}-t_{c2}) - (t_{h1}-t_{c1})\right] = \frac{UA}{Q}(\theta_2-\theta_1)$$

$$Q = \frac{UA\,(\theta_2-\theta_1)}{\ln(\theta_2/\theta_1)}$$

The above equation may be written as

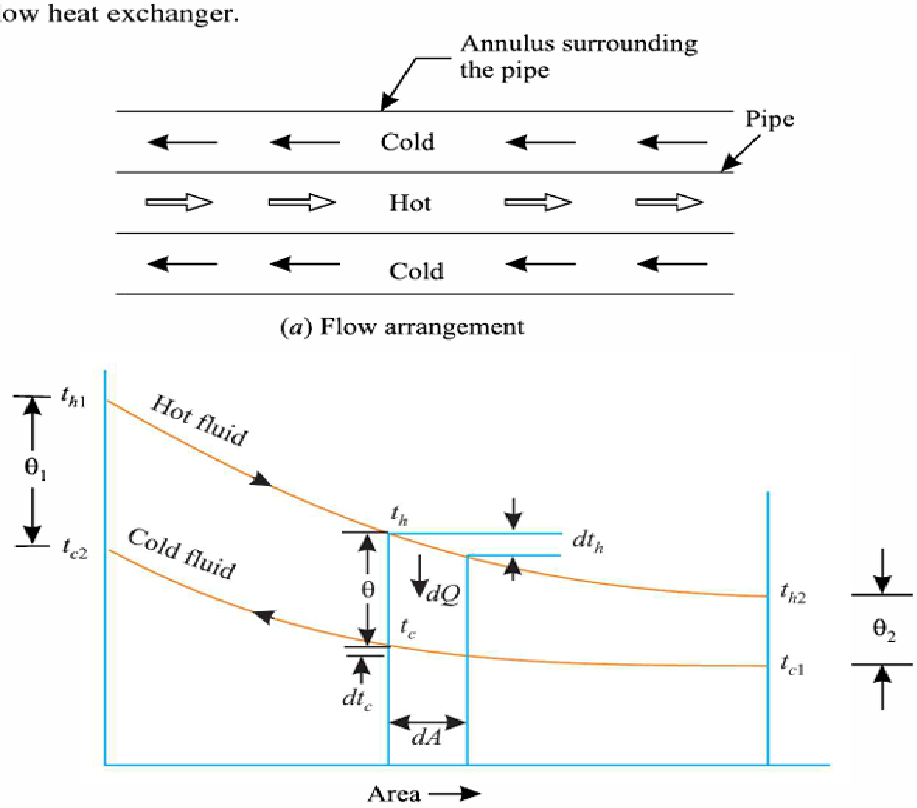
$$Q = U\,A\,\theta_m \qquad \text{...(10.8)}$$

where, $$\theta_m = \frac{\theta_2-\theta_1}{\ln(\theta_2/\theta_1)} = \frac{\theta_1-\theta_2}{\ln(\theta_1/\theta_2)} \qquad \text{...(10.9)}$$

θ_m is called the *logarithmic mean temperature difference* (*LMTD*).

Logarithmic Mean Temperature Difference (LMTD) for (counter-flow)

Refer Fig. 10.10, which shows the flow arrangement and temperature distribution in a single-pass counter-flow heat exchanger.



(*b*) Temperature distribution

Fig.10.10. Calculation of LMTD for a counter-flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U\,.\,dA\,(t_h - t_c) = U.\,dA\,.\,\Delta t \qquad \text{...(10.11)}$$

In a counter-flow system, the temperatures of both the fluids *decrease* in the direction of heat exchanger length, hence the – ve signs.

$$\therefore \qquad dt_h = -\frac{dQ}{\dot{m}_h\, c_{ph}} = -\frac{dQ}{C_h}$$

and,

$$dt_c = -\frac{dQ}{\dot{m}_c\, c_{pc}} = -\frac{dQ}{C_c}$$

$$\therefore \qquad dt_h - dt_c = -\,dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or,

$$d\theta = -\,dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \qquad ...(10.13)$$

Inserting the value of dQ from eqn. (10.11), we get

$$d\theta = -\,U\;dA\;(t_h - t_c)\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= -\,U\,dA\,.\,\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or,

$$\frac{d\theta}{\theta} = -\,U\,dA\,.\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Integrating the above equation from $A = 0$ to $A = A$, we get

$$ln\;(\theta_2/\theta_1) = -\,U\,.\,A\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \qquad ...(10.14)$$

Now, the total heat transfer rate between the two fluids is given by

$$Q \;=\; C_h\,(t_{h1} - t_{h2}) = C_c\,(t_{c2} - t_{c1}) \qquad ...(10.15)$$

or,

$$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q} \qquad ...[10.15\;(a)]$$

or,

$$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q} \qquad ...[10.15\;(b)]$$

substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. (10.14), we get

---

$$ln\;(\theta_2/\theta_1) = -\,U\,A\left[\frac{t_{h1} - t_{h2}}{Q} - \frac{t_{c2} - t_{c1}}{Q}\right]$$

$$= -\frac{UA}{Q}\,[(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})] = -\frac{UA}{Q}\,(\theta_1 - \theta_2) = \frac{UA}{Q}\,(\theta_2 - \theta_1)$$

or,

$$Q = \frac{UA\;(\theta_2 - \theta_1)}{ln\;(\theta_2/\theta_1)}$$

Since,

$$Q \;=\; U\,A\,\theta_m$$

$$\therefore \qquad \theta_m = \frac{\theta_2 - \theta_1}{\ln\;(\theta_2/\theta_1)} = \frac{\theta_1 - \theta_2}{\ln\;(\theta_1/\theta_2)} \qquad ...(10.16)$$

A special case arises when $\theta_1 = \theta_2 = \theta$ in case of a *counter-flow* heat exchanger. In such a case, we have

$$\theta_m = \frac{\theta - \theta}{\ln\;(\theta/\theta)} = \frac{0}{0}$$

This value is *indeterminate*. The value of θ_m for such a case can be found by applying *L'* Hospital's rule :

$$\lim_{\theta_2 \to \theta_1} \frac{\theta_2 - \theta_1}{ln\,(\theta_2/\theta_1)} = \lim_{(\theta_2/\theta_1) \to 1} \frac{\theta_1 \left[\dfrac{\theta_2}{\theta_1} - 1\right]}{ln\,(\theta_2/\theta_1)}$$

Let $(\theta_2/\theta_1) = R$. Therefore, the above expresion can be written as

$$\lim_{R \to 1} \frac{\theta\,(R - 1)}{ln\,(R)}$$

Differentiating the numerator and denomenator with respect to R and taking limits, we get

$$\lim_{(R \to 1)} \frac{\theta}{(1/R)} = \theta$$

Hence, when $\theta_1 = \theta_2$ eqn. (10.3) becomes

$$Q = U\,A\,\theta$$

θ_m *(LMTD) for a counter-flow unit is always greater than that for a parallel flow unit*; hence counter-flow heat exchanger can transfer *more* heat than parallel-flow one; in other words a counter-flow heat exchanger needs a *smaller heating surface for the same rate of heat transfer*. For this reason, the *counter-flow arrangement is usually used.*

When the temperature variations of the fluids are relatively small, then temperature variation curves are approximately straight lines and adequately accurate results are obtained by taking the *arithmatic mean temperature difference (AMTD)*.

$$AMTD = \frac{t_{h1} + t_{h2}}{2} - \frac{t_{c1} + t_{c2}}{2} = \frac{(t_{h1} - tc_1) + (t_{h2} - tc_2)}{2} = \frac{\theta_1 + \theta_2}{2} \qquad ...(10.17)$$

However, practical considerations suggest that the logarithmic mean temperature difference (θ_m) should be invariably used when $\dfrac{\theta_1}{\theta_2} > 1.7$.

OVERALL HEAT TRANSFER COEFFICIENT

In a heat exchanger in which two fluids are separated by a **plane wall** as shown in the Fig. 10.11, the overall heat transfer coefficient is given by

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{L}{k} + \dfrac{1}{h_o}} \qquad ...(10.18)$$

If the fluids are separated by a **tubewall** as shown in Fig. 10.12 the overall heat transfer coefficient is given by,

Inner surface :

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{r_i}{k}\ \ln\,(r_o/r_1) + (r_i/r_o) \times \dfrac{1}{h_o}} \qquad ...(10.19)$$

Outer surface :

$$U_0 = \frac{1}{(r_o/r_i)\dfrac{1}{h_c} + \dfrac{r_o}{k}\ \ln\,(r_o/r_i) + \dfrac{1}{h_o}} \qquad ...(10.20)$$

where,

$$U_i A_i = U_o A_o \qquad ...(10.21)$$

$$A_i = 2\,\pi r_i\,L; \qquad A_o = 2\pi r_o\,L$$

It may be noted that eqns. (10.20) and (10.21) are valid only for *clean and uncorroded surface*.

Consideration of fouling or scaling. In a heat exchanger, during normal operation the tube surface gets covered by deposits of ash, soot, dirt and scale etc. This *phenomenon of rust formation and deposition of fluid impurities is called* **fouling.** Due to these surface deposits the thermal resistance is increased and eventually the performance of the heat exchanger lowers. Since it is difficult to ascertain the thickness and thermal conductivity of the scale deposits, the effect of scale on heat flow is considered by specifying an equivalent *scale heat transfer coefficient* h_s. If h_{si} and h_{so} be the heat transfer coefficients for the scale deposited on the inside and outside surfaces respectively, then the thermal resistances to scale formation on the inside surface (R_{si}) and outside surface (R_{so}) are given by

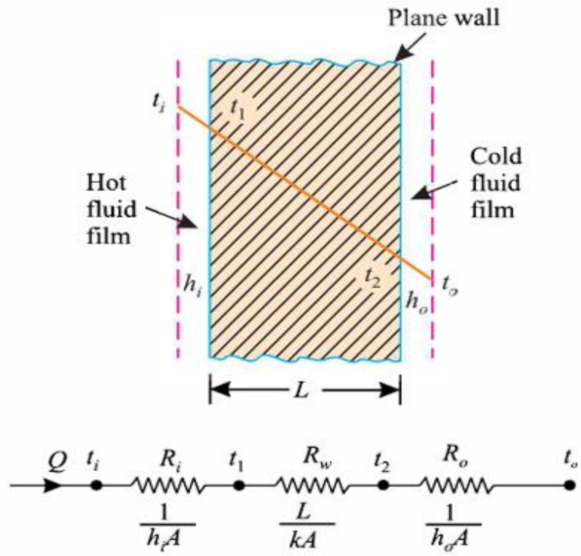


Fig. 10.11. Overall heat transfer coefficient of two fluids separated by a plane wall.

$$R_{si} = \frac{1}{A_i\, h_{si}} \qquad ...(10.22)$$

$$R_{si} = \frac{1}{A_i\, h_{si}} \qquad ...(10.22)$$

$$R_{so} = \frac{1}{A_o\, h_{so}} \qquad ...(10.23)$$

The *reciprocal of scale heat transfer coefficient*, h_s is called the *fouling factor*, R_f. Thus

$$R_f = \frac{1}{h_s}\ \mathrm{m^{2}\,{}^\circ C/W} \qquad ...(10.24)$$

Fouling factors are determined experimentally by testing the heat exchanger in both the clean and dirty conditions. The fouling factor, R_f is thus defined as :

$$R_f\left(=\frac{1}{h_s}\right) = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \qquad ...(10.25)$$

Some typical values (approximate) of R_f are given in table 10.1. Representative values of overall heat transfer coefficient, U is given in table 10.2.

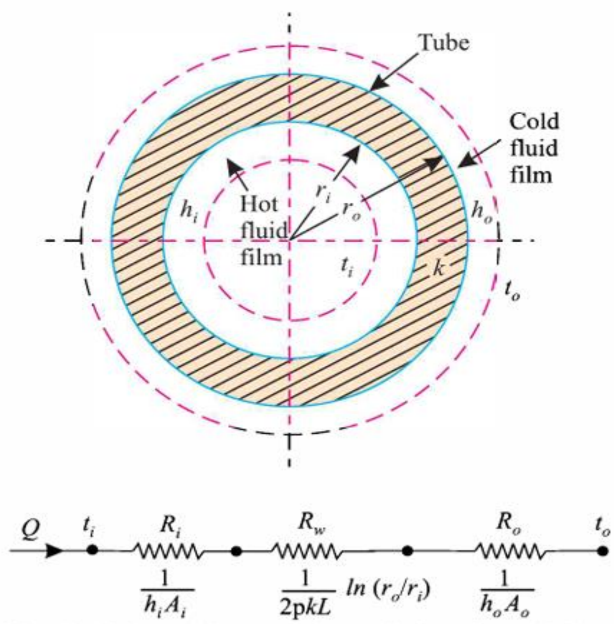


Fig. 10.12. Overall heat transfer coefficient of two fluids flowing inside and outside a tube.

The heat transfer, considering the thermal resistance due to scale formation, is given by :

$$Q = \frac{(t_i - t_o)}{\dfrac{1}{A_i\, h_i} + \dfrac{1}{A_i\, h_{si}} + \dfrac{1}{2\pi Lk}\ ln\ (r_o/r_i) + \dfrac{1}{A_o\, h_{so}} + \dfrac{1}{A_o\, h_o}} \qquad ...(10.26)$$

The overall heat transfer coefficients, U based on the inner and outer surfaces of the inner tube are given by,

$$U_i = \frac{1}{\frac{1}{h_i} + R_{f_i} + \frac{r_i}{k} \, ln \, (r_o/r_i) + (r_i/r_o) \, R_{f_o} + (r_i/r_o) \, \frac{1}{h_o}} \quad ...(10.27)$$

$$U_o = \frac{1}{(r_o/r_i) \, \frac{1}{h_i} + (r_o/r_i) \, R_{fi} + \frac{r_o}{k} \, ln \, (r_o/r_1) + R_{fo} + \frac{1}{h_o}} \quad ...(10.28)$$

In case the tube is thin walled and the thermal resistances due to tube wall thickness and scale formed are neglected, then the overall heat transfer coefficient based on outer surface is given by :

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} \quad ...(10.29)$$

When only fouling factors are neglected, we have

$$U_o = \frac{1}{(r_o/r_i) \, \frac{1}{h_i} + \frac{r_o}{k} \, ln \, (r_o/r_i) + \frac{1}{h_o}} \quad ...(10.30)$$

Points worth noting :

1. The overall heat transfer coefficient depends upon the following *factors* :
   (*i*) The flow rate,
   (*ii*) The properties of the fluid,
   (*iii*) The thickness of material,
   (*iv*) The surface condition of the tubes, and
   (*v*) The geometrical configuration of the heat exchanger.
2. The overall heat transfer coefficient U will generally decrease when any of the fluids (e.g. tars, oils or any of the gases) having low values of heat transfer coefficient, h flows on one side of the exchanger.
3. The highly conducting liquids such as water and liquid metals give much higher values of heat transfer coefficient, h and overall heat transfer coefficient, U. In case of boiling and condensation processes also, the values of U are high.
4. All the thermal resistances in the heat exchanger must be low for its efficient and effective design.

Table 10.1. Fouling factors

S.No.	Fluid	Fouling factor, $R_f = \frac{1}{h_s}$ (m^2°C/W)
1.	Sea water	0.0001 (below 50°C)
		0.0002 (above 50°C)
2.	Clean river and lake water	0.0002 – 0.0006
3.	Well water	0.0004
4.	Distilled water	0.0001
5.	Treated boiler feed water	0.0001 – 0.0002
6.	Worst water used in heat exchangers	< 0.0002
7.	Fuel oil and crude oil	0.0009
8.	Industrial liquids	0.0002

Fouling processes :

1. Precipitation or crystallization fouling.
2. Sedimentation or particulate fouling.
3. Chemical reaction fouling or polymerisation.
4. Corrosion fouling.
5. Biological fouling.
6. Freeze fouling.

Parameters affecting fouling :

- Velocity
- Temperature
- Water chemistry
- Tube material.

Prevention of fouling :

The following methods may be used to keep fouling *minimum* :

1. Design of heat exchanger.
2. Treatment of process system.
3. By using cleaning system.

Properties to be considered for selection of materials for heat exchangers :

- Physical properties
- Mechanical properties
- Climatic properties
- Chemical environment
- Quality of surface finish
- Service life
- Freedom from noise
- Reliability.

Common failures in heat exchangers :

- Chocking of tubes either expected or extraordinary.
- Excessive transfer rates in heat exchanger.
- Increasing the pump pressure to maintain throughout.
- Failure to clean tubes at regularly scheduled intervals.
- Excessive temperatures in heat exchangers.
- Lack of control of heat exchangers atmosphere to retard scaling.
- Increased product temperature over a safe design limit.
- Unexpected radiation from refractory surfaces.
- Unequal heating around the circumference or along the length of tubes.

Example 10.1. *For what value of end temperature differences ratio* $\frac{\theta_1}{\theta_2}$, *is the arithmatic mean temperature difference 5 per cent higher than the log-mean temperature difference?*

Solution. The arithmetic mean temperature difference $(\bar{\theta})$ and log-mean temperature difference (θ_m) ratio may be written as

$$\frac{\bar{\theta}}{\theta_m} = \frac{\left(\frac{\theta_1 + \theta_2}{2}\right)}{\left[\frac{\theta_1 - \theta_2}{ln\ (\theta_1/\theta_2)}\right]} = \frac{(\theta_1 + \theta_2)}{2\ (\theta_1 - \theta_2)} \times ln\ (\theta_1/\theta_2)$$

It is given that $\bar{\theta}$ is to be 5 percent higher than θ_m

$$\therefore \qquad \frac{\bar{\theta}}{\theta_m} = 1.05 = \frac{(\theta_1/\theta_2) + 1}{2\ [(\theta_1/\theta_2) - 1]}\ ln\ (\theta_1 / \theta_2)$$

or, $$\frac{(\theta_1/\theta_2) + 1}{(\theta_1/\theta_2) - 1}\ ln\ (\theta_1/\theta_2) = 2 \times 1.05 = 2.1$$

By hit and trial method, we get

$$\frac{\theta_1}{\theta_2} = \mathbf{2.2} \quad \textbf{(Ans.)}$$

Thus the simple arithmetic mean temperature difference gives results to within 5 percent when end temperature differences vary by *no more than a factor of 2.2.*

Example 10.2. (*a*) *Derive an expression for the effectiveness of a parallel flow heat exchanger in terms of the number of transfer units, NTU, and the capacity ratio* C_{min}/C_{max}.

(*b*) *In a parallel flow double-pipe heat exchanger water flows through the inner pipe and is heated from 20°C to 70°C. Oil flowing through the annulus is cooled from 200°C to 100°C. It is desired to cool the oil to a lower exit temperature by increasing the length of the heat exchanger. Determine the minimum temperature to which the oil may be cooled.* **(U.P.S.C., 1995)**

Solution. (*a*) Refer Article 10.7.

(*b*) Using subscripts *h* and *c* for oil and water respectively, we have

$$t_{h1} = 200°\text{C};\ t_{h2} = 100°\text{C};$$

$$t_{c1} = 20°\text{C};\ t_{c2} = 70°\text{C} \qquad \text{...(Given)}$$

Now, $Q = \dot{m}_h c_{ph}\ (t_{h1} - t_{h2}) = \dot{m}_c c_{pc}\ (t_{c2} - t_{c1})$

or, $\dot{m}_h c_{ph}\ (200 - 100) = \dot{m}_c c_{pc}\ (70 - 20)$

or, $$\frac{\dot{m}_c c_{pc}}{\dot{m}_h c_{ph}} = \frac{100}{50} = 2$$

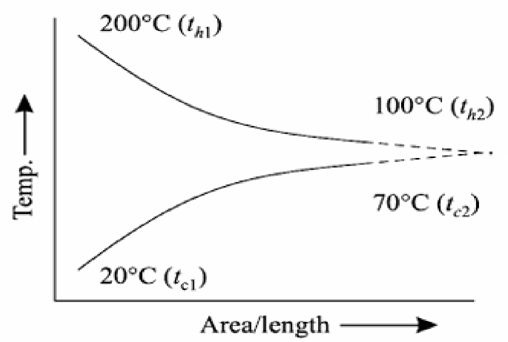


Fig. 10.13.

Let '*t*' be the lowest temperature to which oil may be cooled and this will be the highest temperature of water too (Refer Fig. 10.13).

Hence, $$\dot{m}_h c_{ph}\ (200 - t) = \dot{m}_c c_{pc}\ (t - 20)$$

or, $$(200 - t) = \frac{\dot{m}_c c_{pc}}{\dot{m}_h c_{ph}}\ (t - 20) = 2\ (t - 20)$$

or, $$200 - t = 2t - 40$$

or, $$t = \mathbf{80°C} \quad \textbf{(Ans.)}$$

Eample 10.3. *The flow rates of hot and cold water streams running through a parallel flow heat exchanger are 0.2 kg/s and 0.5 kg/s respectively. The inlet temperatures on the hot and cold sides are 75°C and 20°C respectively. The exit temperature of hot water is 45°C. If the individual heat transfer coefficients on both sides are 650 W/m²°C, calculate the area of the heat exchanger.*

Solution. *Given* : $\dot{m}_h = 0.2$ kg/s; $\dot{m}_c = 0.5$ kg/s; $t_{h1} = 75°$ C; $t_{h2} = 45°$C; $t_{c1} = 20°$C; $h_i = h_o =$ 650 W/m²°C.

The area of heat exchanger, *A* :

The heat exchanger is shown diagrammatically in Fig. 10.14.

The heat transfer rate,

$$Q = \dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2})$$

$$= 0.2 \times 4.187 \times (75 - 45) = 25.122 \text{ kJ/s}$$

Heat lost by hot water = Heat gained by cold water

$$\dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2}) = \dot{m}_c \times c_{pc} \times (t_{c2} - t_{c1})$$

$$0.2 \times 4.187 \times (75 - 45) = 0.5 \times 4.187 \times (t_{c2} - 20)$$

$$\therefore \quad t_{c2} = 32°\text{C}$$

Logarithmic mean temperature difference (*LMTD*) is given by

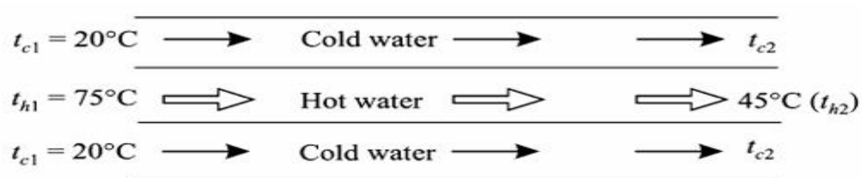
$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \quad \text{...[Eqn. (10.9)]}$$

or,

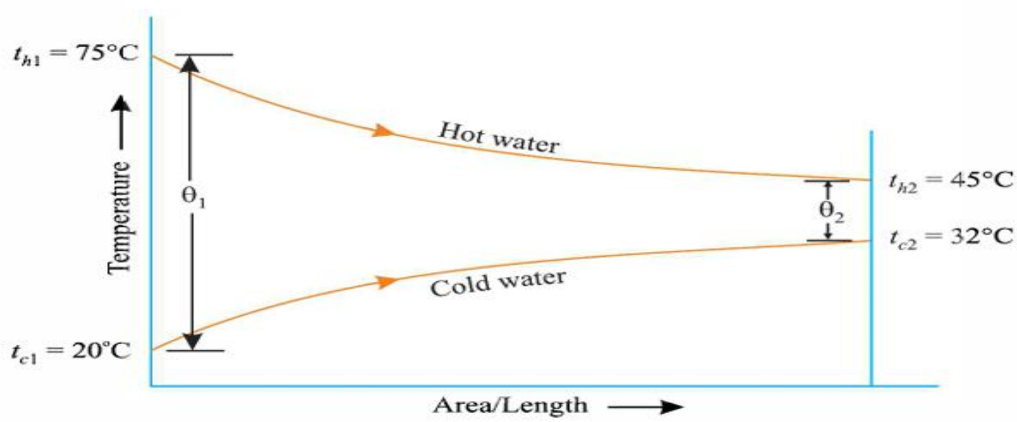
$$\theta_m = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln[(t_{h1} - t_{c1})/(t_{h2} - t_{c2})]}$$

$$= \frac{(75 - 20) - (45 - 32)}{\ln[(75 - 20)/(45 - 32)]}$$

$$= \frac{55 - 13}{\ln(55/13)} = 29.12°\text{C}$$



(*a*) Flow arrangement



(*b*) Temperature distribution

Fig. 10.14. Parallel flow heat exchanger.

Overall heat transfer coefficient U is calculated from the relation,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$= \frac{1}{650} + \frac{1}{650} = \frac{1}{325}$$

$\therefore$ $U = 325 \text{ W/m}^2\text{°C}$

Also, $Q = U A \theta_m$

or, $A = \dfrac{Q}{U\theta_m} = \dfrac{25.122 \times 1000}{325 \times 29.12} = \mathbf{2.66\ m^2}$ **(Ans.)**

Example 10.4. *The following data relate to a parallel flow heat exchanger in which air is heated by hot exhaust gases.*

Heat transferred per hour	*...155450 kJ*
Inside heat transfer coefficient	*...120 W/m²°C*
Outside heat transfer coefficient	*...195 W/m²°C*
Inlet and outlet temperatures of the hot fluid	*...450°C and 250°C, respectively*
Inlet and outlet temperatures of the cold fluid	*...60°C and 120°C, respectively*
Inside and outside diameters of the tube	*...50 mm and 60 mm, respectively.*

Calculate the length of the tube required for the necessary heat transfer to occur. Neglect the tube resistance.

Solution. *Given* : $Q = 155450$ kJ/h; $h_i = 120$ W/m²°C; $h_o = 195$ W/m²°C; $t_{h1} = 450$°C; $t_{h2} = 250$°C; $t_{c1} = 60$°C; $t_{c2} = 120$°C; $d_i = 50$ mm = 0.05m; $d_o = 60$ mm = 0.06 m.

Length of each tube, L :

Logarithmic mean temperature difference (*LMTD*) is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln[(t_{h1} - t_{c1})/(t_{h2} - t_{c2})]}$$

$$= \frac{(450 - 60) - (250 - 120)}{\ln[(450 - 60)/(250 - 120)]} = \frac{390 - 130}{ln[(390/130)]} = 236.66°\text{C}$$

The overall heat transfer coefficient, U is given by

$$\frac{1}{U} = \frac{r_o}{r_i}\frac{1}{h_i} + \frac{1}{h_o}$$

$$= \frac{0.03}{0.025} \times \frac{1}{120} + \frac{1}{195} = 0.01513$$

$\therefore$ $U = 66.09 \text{ W/m}^2\text{°C}$

Total heat transfer rate is given by

$$Q = U A \theta_m = U \times (\pi d_o L) \times \theta_m$$

or, $L = \dfrac{Q}{U \times \pi d_o \times \theta_m} = \dfrac{155450 \times (1000/3600)}{60.09 \times \pi \times 0.06 \times 236.66} = \mathbf{14.65\ m}$ **(Ans.)**

Example 10.6. *In a certain double pipe heat exchanger hot water flows at a rate of 5000 kg/h and gets cooled from 95°C to 65°C. At the same time 50000 kg/h of cooling water at 30°C enters the heat exchanger. The flow conditions are such that overall heat transfer coefficient remains constant at 2270 W/m² K. Determine the heat transfer area required and the effectiveness, assuming two streams are in parallel flow. Assume for the both the streams* c_p = *4.2 kJ/kg K.* **(GATE, 1997)**

Solution. *Given* : $\dot{m}_h = \frac{50000}{3600} = 13.89$ kg/s; $t_{h1} = 95°C$; $t_{h2} = 65°C$;

$$\dot{m}_c = \frac{50000}{3600} = 13.89 \text{ kg/s}; \ t_{c1} = 30°C; \ U = 2270 \text{ W/m}^2\text{ K};$$

$$c_{ph} = c_{pc} = 4.2 \text{ kJ/kg or } 4200 \text{ J/kg K.}$$

Q = Heat lost by hot water = Heat gained by cold water.

$$\dot{m}_h\, c_{ph} \times (t_{h1} - t_{h2}) = \dot{m}_c\, c_{pc} \times (t_{c2} - t_{c1})$$

or, $\quad 13.89 \times 4200 \times (95 - 65) = 13.89 \times 4200 \times (t_{c2} - 30)$

$\therefore \quad t_{c2} = 60°C$

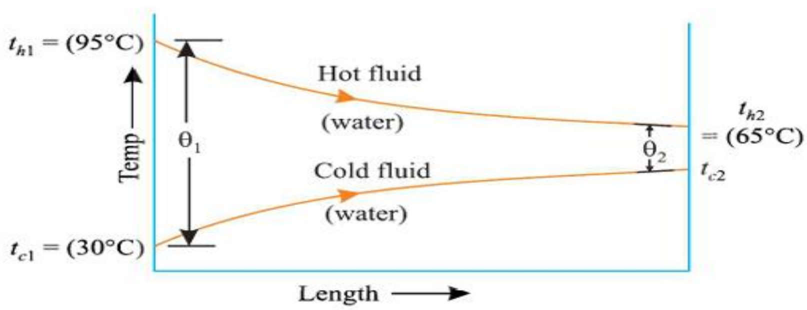


Fig. 10.16. Parallel-flow heat-exchanger.

Log mean temperature difference,

$$LMTD,\ \theta_m = \frac{(\theta_1 - \theta_2)}{\ln(\theta_1/\theta_2)}$$

$$= \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln\left(\dfrac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}\right)}$$

$$= \frac{(95 - 30) - (65 - 60)}{\ln\left(\dfrac{95 - 30}{65 - 60}\right)} = \frac{60}{0.583} = 23.4°C$$

Also, $\quad Q = UA\theta_m$

or, $\quad 13.89 \times 4200 \times (95 - 65) = 2270 \times A \times 23.4$

Heat transfer area, $\quad$ **A = 32.95 m² (Ans.)**

Also, $\quad Q_{\text{actual}} = \dot{m}_h\, c_{ph}\ (t_{h1} - t_{h2})$ and $Q_{\max} = \dot{m}_c\, c_{pc}\ (t_{h1} - t_{c1})$

$\therefore$ Effectiveness of the heat exchanger,

$$\varepsilon = \frac{Q_{\text{actual}}}{Q_{\max}} = \frac{\dot{m}_h c_{ph}\ (t_{h1} - t_{h2})}{\dot{m}_h c_{ph}\ (t_{h1} - t_{c1})} = \frac{95 - 65}{95 - 30} = \mathbf{0.461} \qquad \textbf{(Ans.)}$$

Example 10.8. *In a counter-flow double pipe heat exchanger, water is heated from 25°C to 65°C by an oil with a specific heat of 1.45 kJ/kg K and mass flow rate of 0.9 kg/s. The oil is cooled from 230°C to 160°C. If the overall heat transfer coefficient is 420 W/m²°C, calculate the following :*

(*i*) *The rate of heat transfer,*

(*ii*) *The mass flow rate of water, and*

(*iii*) *The surface area of the hat exchanger.*

Solution. *Given* : $t_{c1} = 25°C; t_{c2} = 65°C, c_{ph} = 1.45$ kJ/kg K; $\dot{m}_h = 0.9$ kg/s;

$t_{h1} = 230°C; t_{h2} = 160°C, U = 420$ W/m²°C.

(*i*) The rate of heat transfer, Q :

$$Q = \dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2})$$

or, $$Q = 0.9 \times (1.45) \times (230 - 160) = \mathbf{91.35\ kJ/s} \quad \textbf{(Ans.)}$$

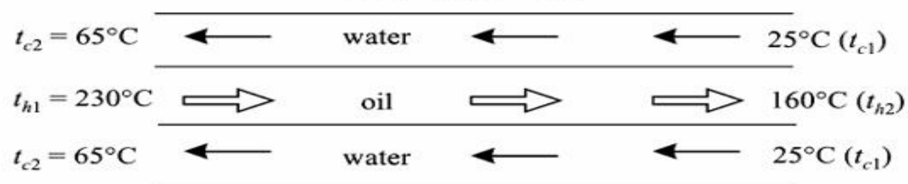
(*ii*) The mass flow rate of water, $\dot{m}_c$:

Heat lost by oil (hot fluid) = Heat gained by water (cold fluid)

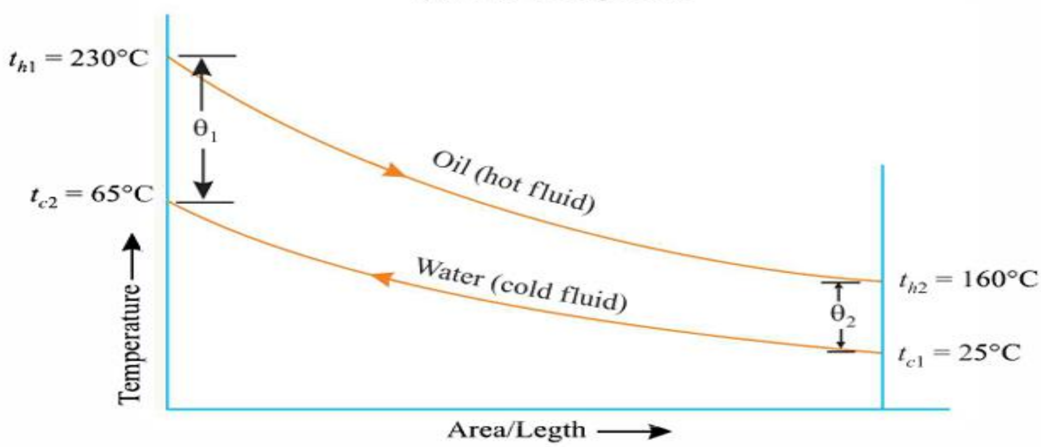
$$\dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2}) = \dot{m}_c \times c_{pc} \times (t_{c2} - t_{c1})$$

$$91.35 = \dot{m}_c \times 4.187\,(65 - 25)$$

$$\therefore \quad \dot{m}_c = \frac{91.35}{4.187 \times (65 - 25)} = \mathbf{0.545\ kg/s\ (Ans.)}$$



(*a*) Flow arrangement



(*b*) Temperature distribution

(*iii*) The surface area of heat exchanger, *A* :

Logarithmic mean temperature difference (LMTD) is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln (\theta_1/\theta_2)}$$

$$= \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\ln [(t_{h1} - t_{c2})/(t_{h2} - t_{c1})]} = \frac{(230 - 65) - (160 - 25)}{\ln [(230 - 65)/(160 - 25)]}$$

or, $$\theta_m = \frac{165 - 135}{\ln [(165/135)]} = 149.5°\text{C}$$

Also, $$Q = U A \theta_m$$

or, $$A = \frac{Q}{U \theta_m} = \frac{91.35 \times 10^3}{420 \times 149.5} = \mathbf{1.45\ m^2} \quad \textbf{(Ans.)}$$

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Chemical Industrial Techniques

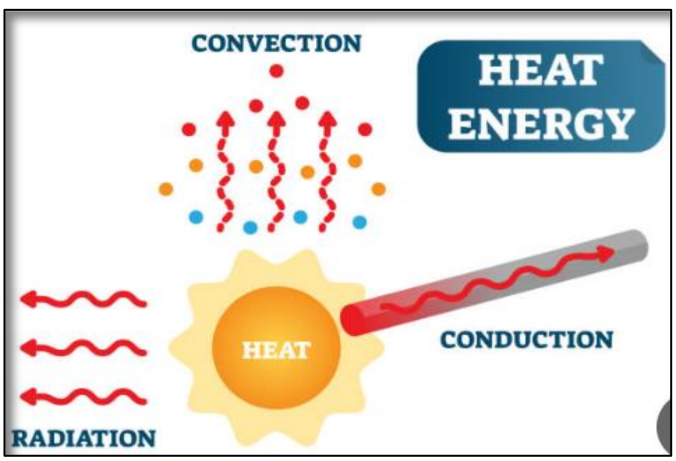
Learning package

In

Correction Factor of Multi-Pass Arrangement

For

Students of the Second Year



By

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Assistant

Dep. Of Chemical Industrial Techniques

2025

1/ Overview

1 / A –Target population :-

For students of second year
Technological Institute of Basra
Dep. of Chemical Industrial Techniques

1 / B –Rationale:-

The **correction factor** F (sometimes F_t) in a **multi-pass shell-and-tube heat exchanger** serves to **adjust the ideal log mean temperature difference (LMTD) for deviations from perfect counter-flow behavior**. It ensures more accurate predictions of heat transfer rate when the actual flow configuration is complex.

1 / C –Central Idea:-

A **heat exchanger** is a device engineered to **transfer thermal energy from one fluid to another without mixing them**, using conduction through a separating wall and convection on each fluid interface. It's designed to **maximize heat transfer efficiency** while maintaining fluid isolation and minimizing energy penalty.

- The LMTD formula assumes a **pure counter-current flow**, which provides the maximum possible driving temperature difference throughout the exchanger.
- Real-world designs often use **multi-pass arrangements**—multiple shell-side or tube-side passes, cross-flow sections, or recirculation loops—that deviate from counter-flow. This reduces the effective driving temperature difference.
- The correction factor FFF **adjusts the ideal LMTD downward** to account for these non-idealities, making the heat transfer prediction realistic.
  $Q=U\,A\,F\,\Delta T_{LM}$ counterflow

1 / D – Performance Objectives

After studying this topic, the student will be able to:

1. **Define** the concept of the correction factor (F) in multi-pass heat exchanger arrangements.
2. **Differentiate** between ideal counterflow and actual multi-pass configurations in terms of thermal efficiency.
3. **Identify** various heat exchanger configurations (e.g., 1-2, 2-4, 1-4 passes) and understand how they affect the correction factor.
4. **Utilize** the correction factor charts or equations to determine F for given shell-and-tube arrangements.
5. **Calculate** the effective Log Mean Temperature Difference (LMTD) using the correction factor.
6. **Evaluate** the impact of the correction factor on the overall performance and design of a heat exchanger.
7. **Apply** correction factors appropriately when designing or analyzing shell-and-tube heat exchangers with complex flow arrangements.

Correction Factor of Multi-Pass Arrangement

The expression $\theta_m = \dfrac{(\theta_1 - \theta_2)}{ln\ (\theta_1/\theta_2)}$ for *LMTD* is essentially valid for single-pass heat exchangers. The analytical treatment of multiple pass shell and tube heat exchangers and cross-flow heat exchangers is much more difficult than single pass cases; such cases may be analysed by using the following equation :

$$Q = UAF\,\theta_m \qquad ...(10.31)$$

where *F* is the *correction factor;* the correction factors have been published in the form of charts by Bonman, Mueller and Nagle and by *TEMA*.

Correction factors for several common arrangements have been given in Figs. 10.40 to 10.43. The data is presented as a function of two non-dimensional variables namely the temperature ratio *P* and the capacity ratio *R*.

Temperature ratio, P : It is defined as the *ratio of the rise in temperature of the cold fluid to the difference in the inlet temperatures of the two fluids*. Thus:

$$P = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} \qquad ...(10.32)$$

where subscripts h and c denote the hot and cold fluids respectively, and the subscripts 1 and 2 refer to the inlet and outlet conditions respectively.

The temperature ratio P *indicates cooling or heating effectiveness* and it can vary from *zero* for a constant temperature of one of the fluids to *unity* for the case when inlet temperature of the hot fluid equals the outlet temperature of the cold fluid.

Capacity ratio R : The ratio of the products of the mass flow rate times the heat capacity of the fluids is termed as capacity ratio R. Thus :

$$R = \frac{\dot{m}_c \cdot c_{pc}}{\dot{m}_h \cdot c_{ph}} \qquad \text{...(10.33)}$$

Since, $\dot{m}_c \cdot c_{pc} \cdot (t_{c2} - t_{c1}) = \dot{m}_h \cdot c_{ph} \cdot (t_{h1} - t_{h2})$

or,

$$R = \frac{\dot{m}_c \cdot c_{pc}}{\dot{m}_h \cdot c_{ph}} = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}}$$

$$= \left[\frac{\text{Temperature drop of the hot fluid}}{\text{Temperature rise in the cold fluid}}\right] \qquad \text{...(10.34)}$$

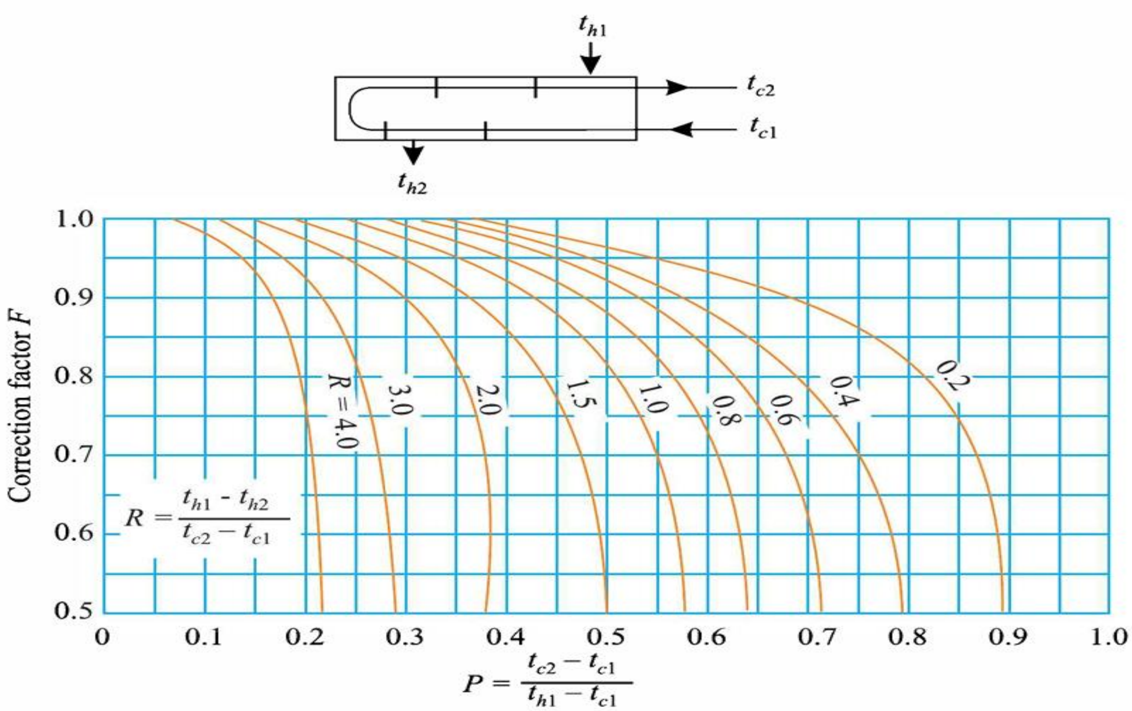


Fig. 10.40. Correction factor plot for heat exchanger with one shell pass and two, four or any multiple of tube passes.

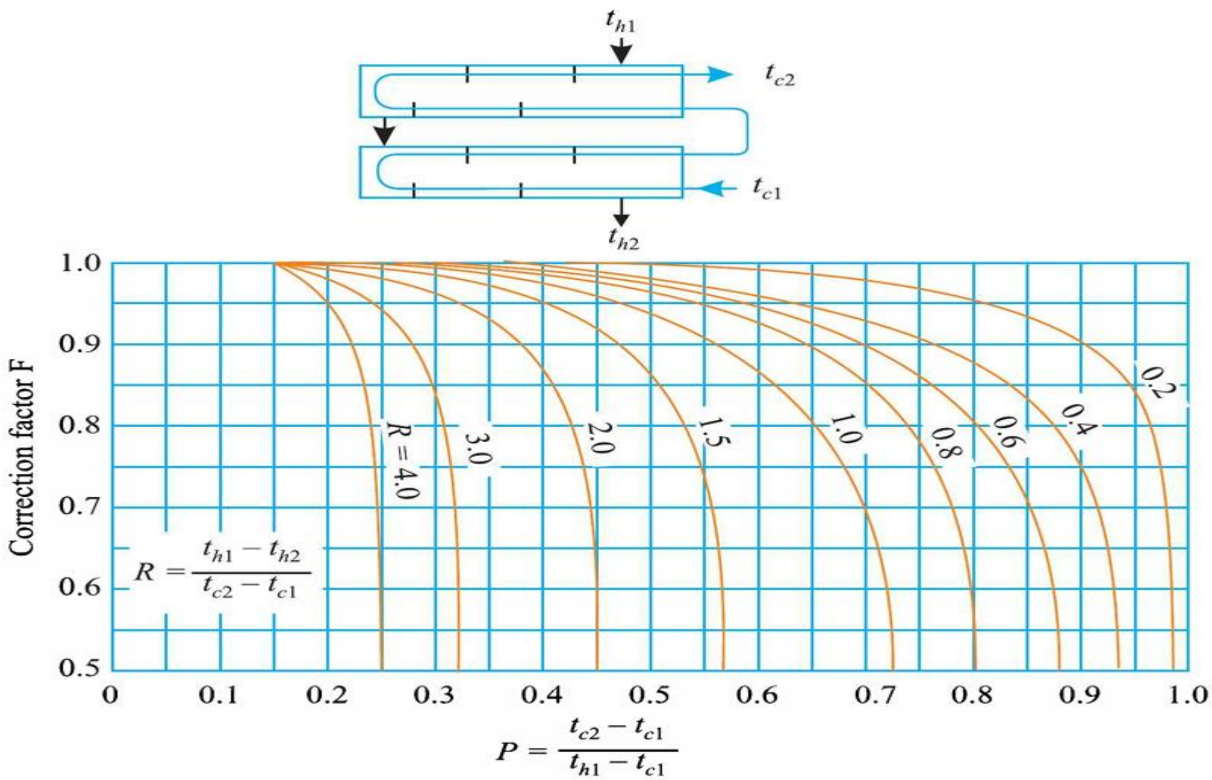


Fig. 10.41. Correction factor plot for heat exchanger with two shell passes and two, four, eight or any multiple of tube passes.

For these correction factor plots it is immaterial whether the hot fluid flows in the shell or the tubes. The value of the correction factor indicates the *performance level of a given arrangement for the given terminal fluid temperatures*. The *correction factor F is always less than unity as no arrangement can be more effective than the conventional counter flow.*

Example 10.31. *Calculate for the following cases, the surface area required for a heat exchanger which is required to cool 3200 kg/h of benzene (c_p = 1.74 kJ/kg°C) from 72°C to 42°C. The cooling water (c_p = 4.18 kJ/kg°C) at 15°C has a flow rate of 2200 kg/h.*

(i) Single pass counter-flow,

(ii) 1-4 exchanger (one-shell pass and four-tube passes), and

(iii) Cross flow single pass with water mixed and benzene unmixed.

For each configuration, the overall heat transfer coefficient may be taken as 0.28 kW/m²°C.

Solution. *Given* : $\dot{m}_h = \frac{3200}{3600} = 0.889$ kg/s; c_{ph} = 1.74 kJ/kg°C; t_{h1} = 72°C, t_{h2} = 42°C;

$\dot{m}_w = \dot{m}_c = \frac{2000}{3600} = 0.611$ kg/s, c_{pc} = 4.18 kJ/kg°C, t_{c1} = 15°C, U = 280 W/m²°C

Surface area required, *A* :

Using energy balance on both the fluids, we have

$$\dot{m}_h \, c_{ph} \, (t_{h1} - t_{h2}) = \dot{m}_c \, c_{pc} \, (t_{c2} - t_{c1})$$

$$0.889 \times 1.74 \, (72 - 42) = 0.611 \times 4.18 \, (t_{c2} - 15)$$

$$\therefore \quad t_{c2} = 33.2°\text{C}$$

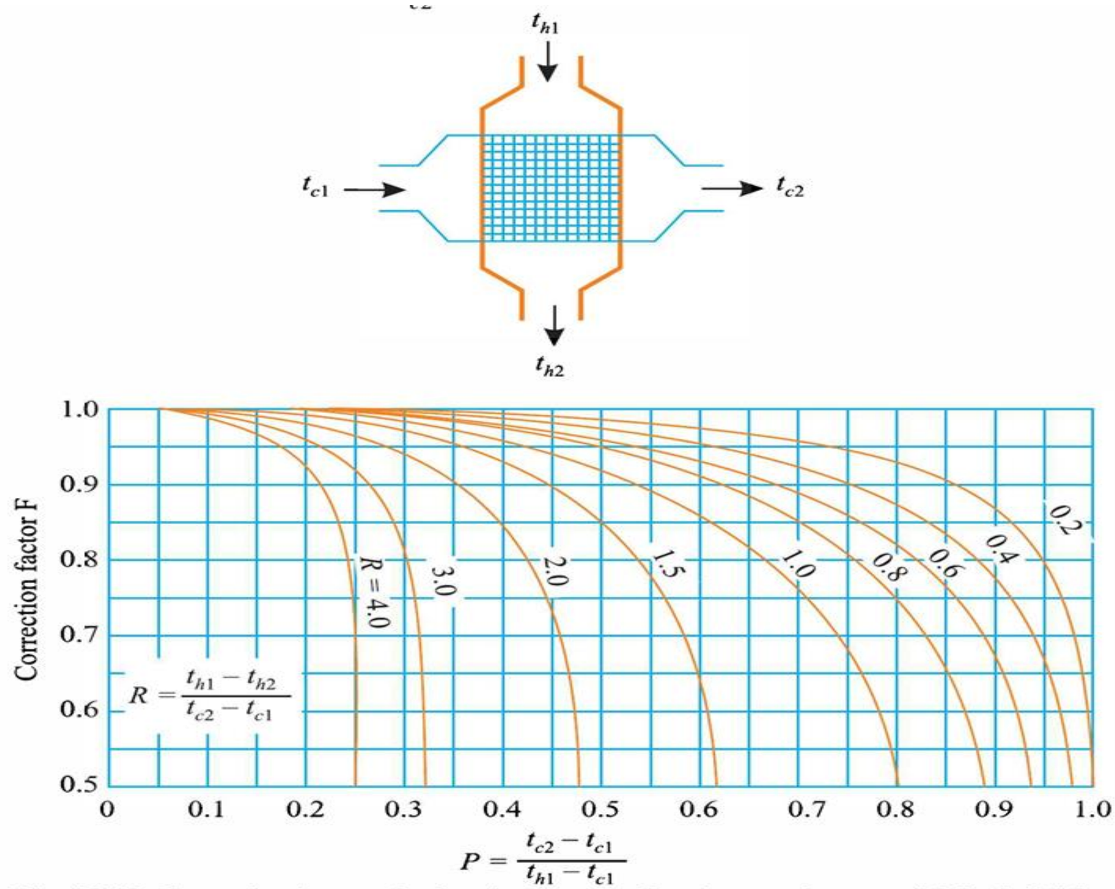


Fig. 10.42. Correction factor plot for single cross-flow heat exchanger with both fluids unmixed.

An energy balance on the hot fluid yields the total heat transfer,

$$Q = \dot{m}_h \; c_{ph} \, (t_{h1} - t_{h2}) = 0.889 \times 1.74 \, (72 - 42) = 46.4 \text{ kW}$$

(*i*) *Single-pass counter-flow :*

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln (\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{ln \, [(t_{h1} - t_{c2})/(t_{h2} - t_{c1})]}$$

$$= \frac{(72 - 33.2) - (42 - 15)}{\ln [(72 - 33.2)/(42 - 15)]} = \frac{38.8 - 27}{\ln [(38.8/27)]} = 32.5°\text{C}$$

$\therefore$ Area of the exchanger, $A = \dfrac{Q}{U \, \theta_m} = \dfrac{46.4}{0.28 \times 32.5} = \mathbf{5.1 \ m^2}$ **(Ans.)**

(*ii*) *1-4 exchanger* :

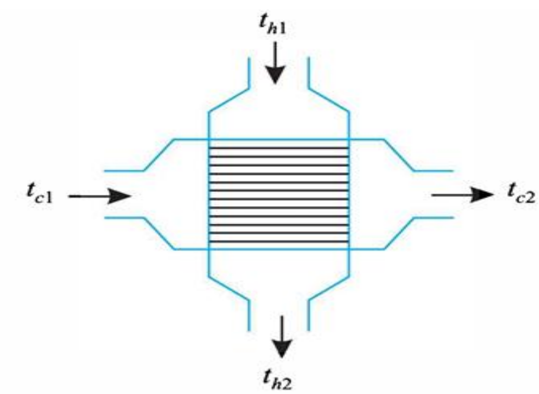
Since the number of passes is more than one hence θ_m (*LMTD*) needs correction factor, *F*. To know the correction factor we have to know first *P* (temperature ratio) and (capacity ratio) *R*.

$$P = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{(33.2 - 15)}{(72 - 15)} = 0.32$$

$$R = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = \frac{(72 - 42)}{(33.2 - 15)} = 1.65$$

Using $P = 0.32$ and $R = 1.65$ the correction factor F from Fig. 10.40 is read as

$$F \simeq 0.9$$



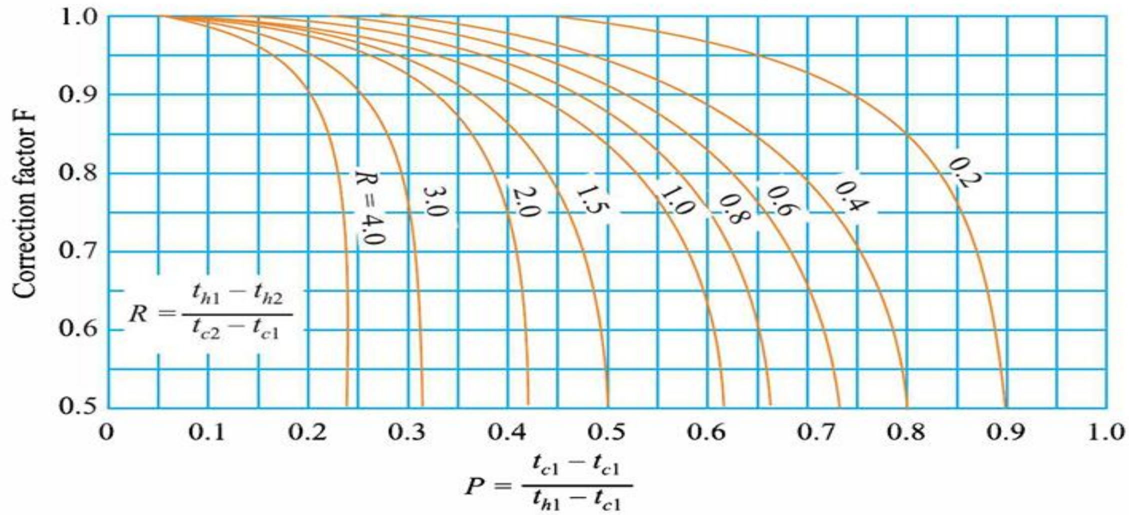


Fig. 10.43. Correction factors plot for single-pass cross-flow heat exchanger, one fluid mixed and the other unmixed.

$\therefore$ Area of the exchanger, $$A = \frac{Q}{F U \theta_m} = \frac{46.4}{0.9 \times 0.28 \times 32.5} = \mathbf{5.66\ m^2}\ \textbf{(Ans.)}$$

(*iii*) *Cross-flow single-pass with water mixed and benzene unmixed :*

Using $P = 0.32$ and $R = 1.65$ the correction factor F from Fig. 10.43 is read as

$$F \simeq 0.92$$

$\therefore$ Area of the exchanger, $$A = \frac{Q}{F U \theta_m} = \frac{46.4}{0.92 \times 0.28 \times 32.5} = \mathbf{5.54\ m^2}\quad \textbf{(Ans.)}$$

Example 10.32. *It is required to design a shell-and-tube heat exchanger for heating 2.4 kg/s of water from 20°C to 90°C by hot engine oil (c_p = 2.4 kJ/kg°C) flowing through the shell of the heat exchanger. The oil makes a single pass entering at 145°C and leaving at 90°C with an average heat transfer coefficient of 380 W/m²°C. The water flows through 12 thin-walled tubes of 25 mm diameter with each-tube making 8-passes through the shell. The heat transfer coefficient on the water side is 2900 W/m²°C. Calculate the length of the tube required for the heat exchanger to accomplish the required water heating.*

Solution. *Given* : $\dot{m}_w = \dot{m}_c$ = 2.4 kg/s, t_{c1} = 20°C, t_{c2} = 90°C; t_{h1} = 145°C, t_{h2} = 90°C,

c_{ph} = 2.4 kJ/kg°C, d = 25mm = 0.025 m, N = 12; h_i = 2900 W/m°C, h_0 = 380 W/m²°C

Length of the tube L :

The overall heat transfer coefficient, neglecting thermal resistance of the tube, is given by

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

or, $$U = \frac{h_i \times h_o}{h_i + h_o} = \frac{2900 \times 380}{2900 + 380} = 335.97\ \text{W/m}^2\,°\text{C}$$

or, $$U = \frac{h_i \times h_o}{h_i + h_o} = \frac{2900 \times 380}{2900 + 380} = 335.97 \text{ W/m}^2\text{°C}$$

The parameters required to get the correction factors are :

$$P = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{(90 - 20)}{(145 - 20)} = 0.56$$

$$R = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = \frac{(145 - 90)}{(90 - 20)} = 0.786$$

Form Fig. 10.40, $F \simeq 0.82$

For the conventional counter-flow arrangement

$$\theta_1 = t_{h1} - t_{c2} = 145 - 90 = 55\text{°C}$$

$$\theta_2 = t_{h2} - t_{c1} = 90 - 20 = 70\text{°C}$$

$$\therefore \quad \theta_m = \frac{(\theta_1 - \theta_2)}{\ln (\theta_1/\theta_2)} = \frac{55 - 70}{\ln (55/70)} = 103.6\text{°C}$$

The heat transfer rate is given by

$$Q = \dot{m}_c \ c_{pc} (t_{c2} - t_{c1})$$
$$= 2.4 \times 4.18 \times 10^3 (90 - 20) = 702240 \text{ W}$$

Also, $$Q = F\,U\,A\,\theta_m, \text{ where } A = \text{heating surface}$$

$$\therefore \quad A = \frac{Q}{F\,U\,\theta_m} = \frac{702240}{0.82 \times 335.97 \times 103.6} = 24.6 \text{ m}^2$$

But, $$A = \pi\,d\,L \times N$$

$$\therefore \quad L = \frac{A}{\pi\,d\,N} = \frac{24.6}{\pi \times 0.025 \times 12} \ 26.1 \text{ m}$$

$$\text{The shell length} = \frac{26.1}{8} = \mathbf{3.26\ m\ (Ans.)}$$

Example 10.14. *In a shell and tube counter-flow heat exchanger water flows through a copper tube 20 mm I.D. (internal diameter) and 23 mm O.D. (outer diameter), while oil flows through the shell. Water enters at 20°C and comes out at 30°C, while oil enters at 75°C and comes out at 60°C. The water and oil side film coefficients are 4500 and 1250 W/m²°C respectively. The thermal conductivity of the tube wall is 355 W/m°C. The fouling factors on the water and oil sides may be taken to be 0.0004 and 0.001 respectively. If the length of the tube is 2.4 m, calculate the following :*

(*i*) *The overall heat transfer coefficient;*

(*ii*) *The heat transfer rate.*

Solution. *Given* : d_i = 20 mm = 0.02 m; d_o = 23 mm = 0.023 m; t_{c1} = 20°C; t_{c2} = 30°C;
t_{h1} = 75°C; t_{h2} = 60°C, h_i = 4500 W/m²°C; h_o = 1250 W/m²°C;
k = 355 W/m°C; R_{fi} = 0.0004; R_{fo} = 0.001; L = 2.4 m

(*i*) The overall heat transfer coefficient, U_o :

The overall heat transfer coefficient *based on outer surface of inner pipe is* given by,

$$\frac{1}{U_o} = \frac{r_o}{r_i} \times \frac{1}{h_i} + \frac{r_o}{r_i} R_{fi} + \frac{r_o}{k} \ln (r_o/r_i) + R_{fo} + \frac{1}{h_o} \qquad \text{...[Eqn. (10.28)]}$$

(R_f stands for fouling factor)

$$= \left[\frac{(0.023/2)}{(0.02/2)}\right] \times \frac{1}{4500} + \left[\frac{(0.023/2)}{(0.02/2)}\right] \times 0.0004 + \frac{(0.023/2)}{355} \ln \left[\frac{(0.023/2)}{(0.02/2)}\right] + 0.001 + \frac{1}{1250}$$

$= 10^{-4}$ [2.555 + 4.6 + 0.04527 + 10 + 8] = 0.00252

$= 10^{-4}$ [2.555 + 4.6 + 0.04527 + 10 + 8] = 0.00252

$$U_o = \frac{1}{\quad}$$

= 396.8 W/m²°C (Ans.)

(*ii*) The heat transfer rate, *Q* :

Area, $A_o = \pi \, d_o L$

$$= \pi \times 0.023 \times 2.4$$

$$= 0.1734 \text{ m}^2$$

Logarithmic mean temperature difference (*LMTD*) is given by,

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)}$$

$$= \frac{(75-30)-(60-20)}{\ln[(75-3)/(60-20)]}$$

$$= \frac{45-40}{\ln(45/40)} = 42.45°\text{C}$$

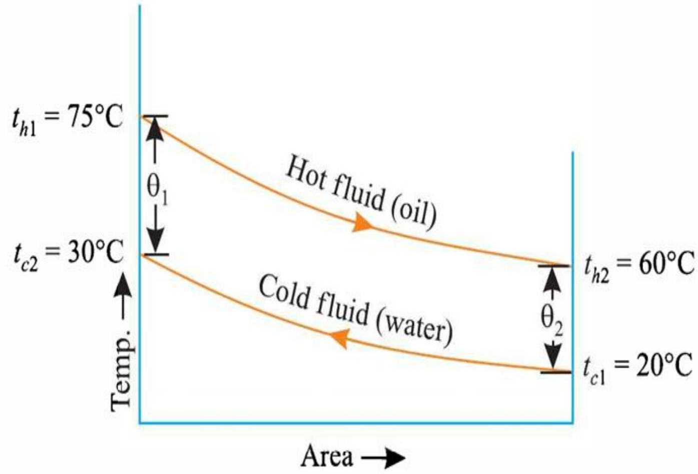


Fig. 10.22.

The heat transfer rate is given by

$$Q = U_o A_o \theta_m$$

or $= 396.8 \times 0.1734 \times 42.15 =$ **2900 W** **(Ans.)**

Example 10.30. *A multipass heat exchanger has two passes on shell side and four passes on the tube side. The oil is passed through the tubes and cooled from 135°C to 52°C. The cooling water passing through shells enters at 13°C and leaves at 31°C. Calculate the heat transfer rate using the following data :*

h_i (oil) = 270 W/m²°C, h_o (water) = 965 W/m²°C

h (scale on water side) = 2840 W/m²°C

Number of tubes per pass = 120

Length and outer diameter of each tube are 2 m and 2.54 cm respectively.

Thickness of the tube = 1.65 mm

LMTD correction factor = 0.98

Neglect the tube wall resistance. **(A.U. Winter, 2000)**

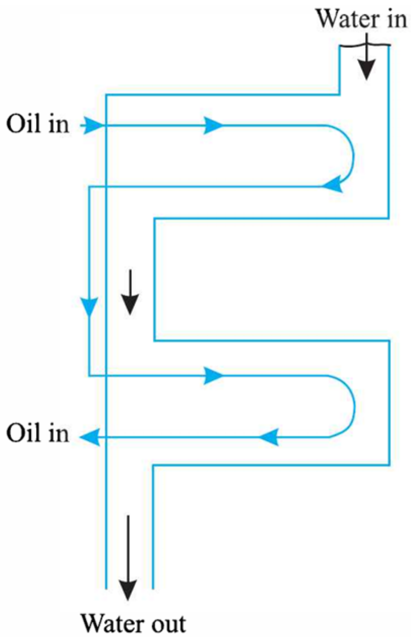
Solution. *Given* : No. of passes two on shell side and four on tube side.

$t_{c1} = 13°\text{C}$; $t_{c2} = 31°\text{C}$; $t_{h1} = 135°\text{C}$; $t_{h2} = 52°\text{C}$; h_i (oil) = 270 W/m²°C;

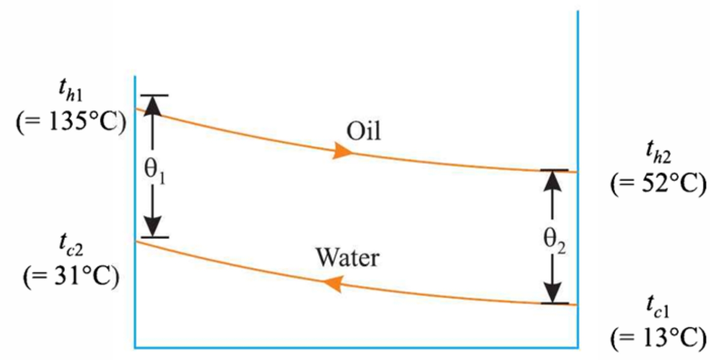
h_o (water) = 965 W/m²°C; h (scale on water side) = 2840 W/m²°C, $N_p = 120$;

$d_o = 2.54$ cm = 0.0254 m; $L = 2$ m; $d_i = 0.0254 - 2 \times 0.00165 = 0.0221$ m;

LMTD correction factor, $F = 0.98$



(*a*) Arrangement



(*b*) Temperature distribution

Fig. 10.39.

Rate of heat transfer, Q :

$$LMTD,\ \theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln\left(\dfrac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}\right)}$$

$$= \frac{(135 - 31) - (52 - 13)}{\ln\left(\dfrac{135 - 31}{52 - 13}\right)} = \frac{65}{0.9808} = 66.3°\text{C}$$

$$(\theta_m)_{\text{actual}} = 66.3 \times F = 66.3 \times 0.98 = 65°\text{C}$$

The overall heat transfer coefficient,

$$U_o = \frac{1}{\dfrac{r_o}{r_i} \times \dfrac{1}{h_i} + \left(\dfrac{r_o}{r_i}\right) \times R_{fi} + \dfrac{1}{h_o}}$$

where, R_{fi} (inside fouling factor) $= \dfrac{1}{2840} = 0.000352$

Substituting the values, we have

$$U_o = \frac{1}{\left(\dfrac{0.0254}{0.0221}\right) \times \dfrac{1}{270} + \dfrac{0.0254}{0.0221} \times 0.000352 + \dfrac{1}{965}} \qquad \left(\because \frac{r_o}{r_i} = \frac{d_o}{d_i}\right)$$

$$= \frac{1}{0.004257 + 0.0004046 + 0.00104} = 175.4 \text{ W/m}^2 \text{°C}$$

$\therefore$ Rate of heat transfer, $Q = U_o A_o (\theta_m)_{\text{actual}}$

$$= 175.4 \times [\pi D_o L \times (N_p \times 4)] \times 65$$

$$= 175.4 \times [\pi \times 0.0254 \times 2 \times (120 \times 4)] \times 65$$

$$= 873369 \text{ W} \quad \text{or} \quad \textbf{873.369 kW (Ans.)}$$