

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

Fluid Flow

For

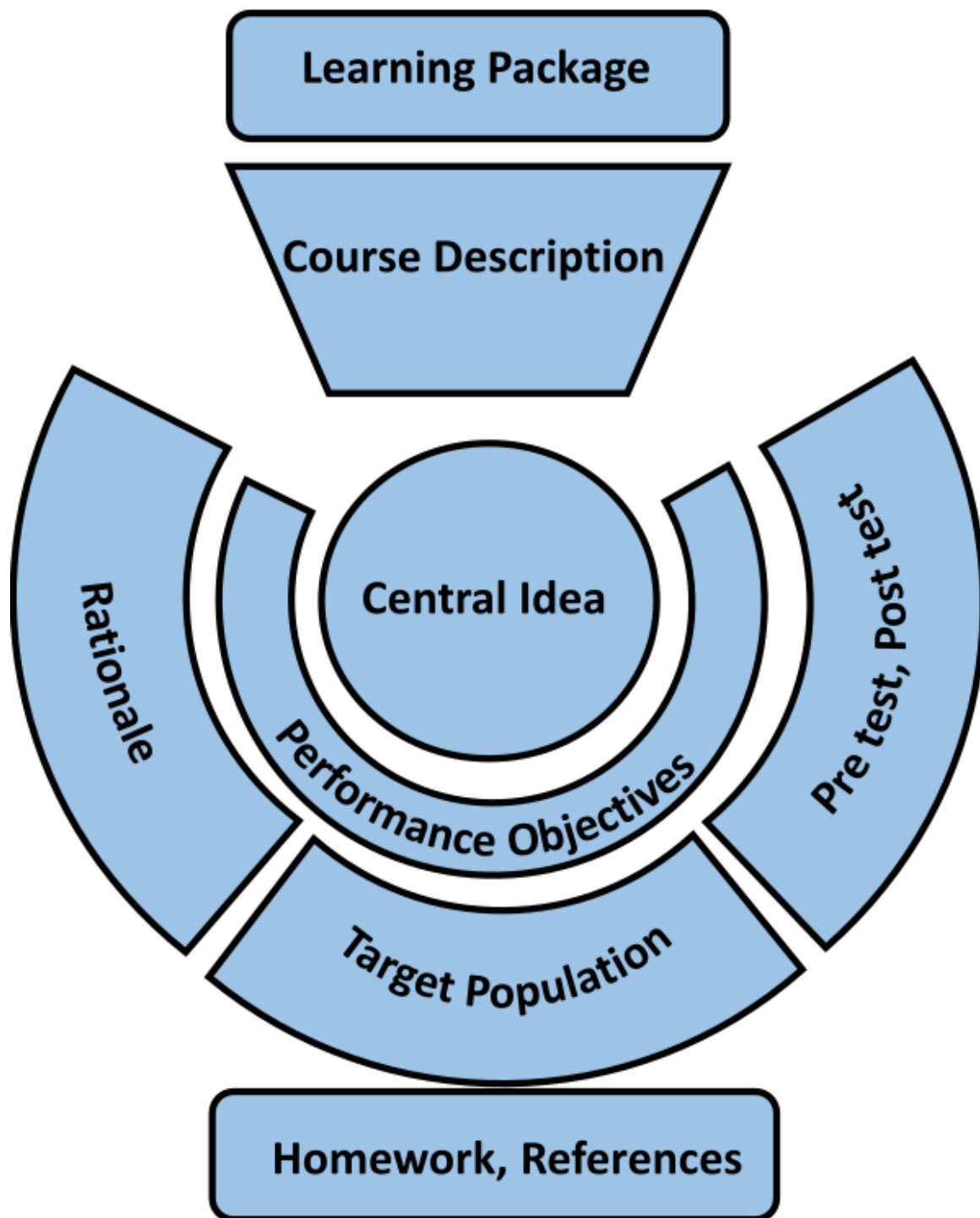
First year students

By

Halah Kadhim Mohsin

Dep. Of Chemical industries Techniques

2025



Course Description

Course Name:	
Fluid flow	
Course Code:	
Semester / Year:	
Semester	
Description Preparation Date:	
24/ 06/ 2025	
Available Attendance Forms:	
Attendance only	
Number of Credit Hours (Total) / Number of Units (Total)	
60 hours/4 hour weekly/4 unit	
Course administrator's name (mention all, if more than one name)	
Name: Halah Kadhim Mohsin	
Email: hala.alkhafaji@stu.edu.iq	
Course Objectives	
1. Developing Basic Understanding of Fluid Mechanics: Enabling students to comprehend fundamental fluid properties, fluid statics (pressure distribution, buoyancy), and the basic principles of fluid flow (types of flow, continuity). 2. Applying Theoretical Concepts in Fluid Dynamics: Enhancing the ability to apply conservation laws (mass and energy, including the extended Bernoulli equation) to analyze fluid flow in various engineering systems. 3. Developing Problem-Solving and Analytical Skills: Equipping students with the ability to solve practical fluid mechanics problems, including calculating flow rates, pressure drops (major and minor losses), and power requirements for pumps	

and turbines in complex pipe networks and porous media (e.g., packed beds). 4. Fostering Practical Application: Providing a foundation understanding necessary for practical applications and potential laboratory experiments related to fluid measurements and system performance. 5. Enhancing Critical Thinking and Analytical Skills: Encouraging students to engage in critical evaluation and analytical problem-solving for various fluid flow scenarios and system designs Developing Practical Skills and Tools Usage: Equipping students with the ability to apply theoretical knowledge to practical engineering problems, including the use of charts, tables, and computational tools (where applicable) for analysis of fluid systems.	
Teaching and Learning Strategies	
1. Interactive Lectures and Problem-Solving Sessions: Engaging students through direct instruction, derivation of equations, and collaborative problem-solving exercises. 2. Conceptual Questioning and Discussion: Fostering deeper understanding by encouraging students to explain fluid phenomena in their own words and discuss various approaches to problems. 3. Illustrative Examples and Case Studies: Applying theoretical concepts to real-world engineering applications to demonstrate relevance and practical implications. 4. Hands-on Laboratory Experiments (Optional/If available): Providing practical experience in measuring fluid properties, flow rates, and pressure drops to validate theoretical understanding.	

Course Structure					
Weeks	Hours	Required Learning Outcomes	Unit subject name or	Learning method	Evaluation method

1	4hours	Understand basic concepts of fluid mechanics, distinguish between different fluid types, and apply fundamental units and dimensions.	1. Introduction to Fluid Mechanics
 2. Units and Dimensions	1. Lectures and discussions on fundamental concepts.
 2. Solving basic problems individually and in groups.	Weekly Quizzes
2	4hours	Define and calculate key fluid properties (density, specific weight, viscosity, surface tension, compressibility) and understand their significance.	3. Fluid Properties	1. Interactive lectures with real-world examples of fluid properties.
 2. Problem-solving exercises focused on property calculations.	Homework Assignments
3	4hours	Apply pressure concepts, calculate pressure variation in static fluids, and use manometers for pressure measurement.	4. Fluid Statics: Pressure and its Measurement	1. Detailed derivations and conceptual explanations.
 2. Practical examples of pressure measurement devices.	Quizzes on pressure concepts
4	4hours	Understand buoyant force, calculate forces on submerged/floating bodies, and analyze stability.	5. Fluid Statics: Buoyancy and Stability	1. Application of principles to real-life scenarios (e.g., ships, balloons).
 2. Group problem-solving on buoyancy-related challenges.	Monthly Exam 1
5	4hours	Classify fluid flow types	6. Fluid Flow Concepts &	1. Visual aids (animations,	Problem Set Submission

		(steady/unsteady, uniform/non-uniform, laminar/turbulent), describe streamlines, and apply the continuity equation.	Types
 7. The Continuity Equation	diagrams) for flow types and streamlines.
 2. Derivation and application of the continuity equation in various contexts.	
6	4hours	State and apply the ideal Bernoulli's equation, understanding its assumptions and limitations.	8. Energy of Flowing Fluid: Ideal Bernoulli's Theorem	1. Step-by-step derivation and assumption analysis.
 2. Solving basic Bernoulli problems without losses.	Weekly Quizzes
7	4hours	Apply ideal Bernoulli's equation to practical devices like Venturi meters, Pitot tubes, siphons, and tank efflux.	9. Practical Applications of Bernoulli's Equation	1. Analysis of flow measurement devices.
 2. Solving application-oriented problems.	Homework Assignment
8	2 hours	Understand the limitations of ideal Bernoulli's equation for real fluids and apply the extended Bernoulli (Energy) equation.	10. Bernoulli's Equation Correction: The Energy Equation	1. Introduction of head loss, pump, and turbine terms.
 2. Setting up energy equations for simple real systems.	Monthly Exam 2
9	4hours	Calculate major head losses due to friction in pipes using the Darcy-Weisbach equation, determining friction factor.	11. Energy Loss in Pipes: Major Losses (Friction)	1. Detailed explanation of Darcy-Weisbach and Moody chart/correlations.
 2. Practice problems	Quizzes on friction factor determination

				on calculating major losses.	
10	4hours	Calculate minor head losses due to pipe fittings, valves, entrances, and exits using loss coefficients (K _L).	12. Energy Loss in Pipes: Minor Losses (Fittings and Valves)	1. Discussion of various fittings and their associated K _L values. 2. Calculation of minor losses in typical configurations.	Problem Set Submission
11	4hours	Integrate major and minor loss calculations to determine total head loss in a system and apply the full Energy Equation.	13. Total Head Loss & Combined Energy Equation Applications	1. Comprehensive examples combining all loss types. 2. Team-based problem-solving on integrated systems.	Homework Assignment / Project Part 1
12	4hours	Analyze fluid flow through packed beds, understand bed characteristics (voidage, particle size), and apply the Ergun equation.	14. Fluid Flow Through Packed Beds	1. Introduction to porous media concepts. 2. Application of the Ergun equation to practical scenarios.	Weekly Quizzes
13	4hours	Apply the Energy Equation to analyze complex piping systems for flow between tanks, including iterative solutions.	15. Fluid Flow Between Tanks	1. Detailed problem-solving sessions for multi-tank systems. 2. Iterative calculation practice.	Final Project Part 2 / Presentation (if applicable)
14	4hours	Review key concepts, prepare for final assessment, and explore advanced	16. Course Review & Advanced Topics/Case Studies /	1. Comprehensive review of all major topics. 2. Q&A	Final Exam (Covers all course material)

		topics or case studies in fluid mechanics.	Final Exam Prep	session for exam preparation. 3. Overview of specialized topics (e.g., compressible flow, open channel flow - brief).	
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Distribution as follows: 20 points for Midterm Theoretical Exams for the first semester, 20 points for Midterm Practical Exams for the first semester, 10 points for Daily Exams and Continuous Assessment, and 50 points for the Final Exam.

Learning and Teaching Resources

Ref.

1. **Fundamentals of Fluid Mechanics** – Munson, Young, Okiishi.
2. **Introduction to Fluid Mechanics** – Fox & McDonald.
3. كتاب "ميكانيك الموائع" – تأليف د. محمد تقي الكامل .
4. كتاب جريان الموائع – المعاهد التقنية – العراق .

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package
In
Units and Unit Systems

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students, or anyone seeking a fundamental understanding of measurement units in engineering mechanics and fluid mechanics.

1 / B – Rationale: A solid grasp of measurement units and the ability to convert between them is crucial for accurate problem-solving in engineering and science fields, especially in fluid mechanics where various physical quantities interact.

1 / C – Central Idea: To ensure accuracy and consistency in engineering calculations, it is essential to master various unit systems, particularly the International System (SI), and to be able to efficiently convert between them.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Define the concept of a unit and its importance in physical measurements.
- Identify the base and derived units in the International System (SI).
- Recognize other common unit systems (e.g., Imperial/US Customary System).
- Perform accurate conversions between units within the same system and between different systems.
- Apply conversions to practical examples and problems.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is the base SI unit for length?
2. How many centimeters are in one meter?

3. If a force is 10 pounds (lbs), what is its value in Newtons (N)? (Given: $1 \text{ lb} \approx 4.448 \text{ N}$)
4. What is the main difference between mass and weight in terms of units?
5. Why is it important to use correct units in engineering problems?

3 / Lecture (Content):

Introduction: Units are the language of science and engineering. Without standardized and universally understood units of measurement, it would be impossible to communicate accurately, replicate experiments, or build complex systems. This lecture will cover the fundamentals of unit systems, focusing on the International System (SI) and the ability to perform conversions.

I. Definition of a Unit and Its Importance

- **Unit:** A standard measure or a fixed quantity used as a standard to measure quantities of the same kind.
- **Importance:**
 - **Standardization of Measurements:** Allows for precise and objective comparison of quantities.
 - **Consistency:** Ensures that results are reproducible and understandable by others.
 - **Accuracy:** Reduces errors and increases the reliability of calculations and designs.

II. International System of Units (SI - *Système International d'Unités*)

This is the most widely used system of measurement globally, based on seven base units:

Base Quantity	Base Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

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- **Derived Units:** These units are formed by combining base units.
 - Force: Newton (N) = $\text{kg} \cdot \text{m}/\text{s}^2$
 - Pressure: Pascal (Pa) = N/m^2
 - Energy/Work: Joule (J) = $\text{N} \cdot \text{m}$
 - Power: Watt (W) = J/s
 - Density: Kilogram per cubic meter (kg/m^3)
- **SI Prefixes:** Used to simplify the expression of very large or very small quantities.

Prefix	Symbol	Value
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Hecto	h	10^2
Deca	da	10^1
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}

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III. Other Unit Systems

1. Imperial/US Customary System:

- Mainly used in the United States and a few other countries.
- Based on units like: foot (ft) for length, pound-mass (lbm) for mass, second (s) for time, pound-force (lbf) for force.
- **Important Note:** Distinguish between pound as a unit of mass (lbm) and pound as a unit of force (lbf). ($1 \text{ lbf} = 32.174 \text{ lbm} \cdot \text{ft}/\text{s}^2$)

2. Centimeter-Gram-Second (CGS) System:

- An older system, still used in some scientific contexts.
- Based on centimeter (cm) for length, gram (g) for mass, and second (s) for time.

IV. Conversion Between Systems and Examples

Basic Principle of Conversion: We use conversion factors that are equal to "1" to express a quantity with the same value but in different units. Example: $1\text{ m} = 100\text{ cm}$, so the conversion factor $(100\text{ cm} / 1\text{ m}) = 1$ or $(1\text{ m} / 100\text{ cm}) = 1$.

Solved Examples:

Example 1: Length Conversion

- **Question:** How many meters are in 5 kilometers?
- **Solution:** We know that $1\text{ km} = 1000\text{ m}$ $5\text{ km} \times 1\text{ km} / 1000\text{ m} = 5000\text{ m}$

Example 2: Speed Conversion

- **Question:** Convert a speed of 60 miles per hour (mph) to meters per second (m/s).
- **Given:**
 - $1\text{ mile} = 1609.34\text{ m}$
 - $1\text{ hour} = 3600\text{ seconds}$
- **Solution:** $60\text{ mph} = 60\text{ hourmiles} = 60\text{ hourmiles} \times 1\text{ mile} / 1609.34\text{ m} \times 3600\text{ s} / 1\text{ hour} = 3600 / 60 \times 1609.34\text{ m} = 26.822\text{ m/s}$

Example 3: Pressure Conversion

- **Question:** Convert a pressure of 14.7 pounds per square inch (psi) to kilopascals (kPa).
- **Given:**
 - $1\text{ psi} = 6894.76\text{ Pa}$
 - $1\text{ kPa} = 1000\text{ Pa}$
- **Solution:** $14.7\text{ psi} = 14.7 \times 6894.76\text{ Pa} = 101352.972\text{ Pa}$ To convert to kilopascals: $101352.972\text{ Pa} \times 1000\text{ Pa} / 1\text{ kPa} = 101.353\text{ kPa}$

Example 4: Density Conversion

- **Question:** Convert a density of 62.4 pounds per cubic foot (lbm/ft³) to kilograms per cubic meter (kg/m³).
- **Given:**
 - $1\text{ lbm} = 0.453592\text{ kg}$
 - $1\text{ ft} = 0.3048\text{ m}$

- **Solution:** $62.4 \text{ ft}^3 \text{ lbm} = 62.4 \text{ ft}^3 \text{ lbm} \times 1 \text{ lbm} 0.453592 \text{ kg} \times (0.3048 \text{ m} 1 \text{ ft})^3$
 $= 62.4 \times 0.453592 \times (0.3048)^3 \text{ m}^3 \text{ kg} = 62.4 \times 0.453592 \times 35.3147 \text{ m}^3 \text{ kg}$
 $= 999.6 \text{ m}^3 \text{ kg}$ (very close to the density of water)

V. Important Considerations in Conversion

- **Accuracy and Significant Figures:** Pay attention to the number of significant figures in the original values and conversion factors.
- **Dimensional Analysis:** Always ensure that the final units match the required units.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. List three base units in the SI system and the quantities they measure.
2. How many grams are in 2.5 kilograms?
3. If an area is 1000 square inches (in^2), what is its value in square meters (m^2)? (Given: 1 in = 2.54 cm)
4. Differentiate between Newton and pound-force in terms of the system they belong to.
5. Explain why unit conversion is a critical step in solving physical and engineering problems.

5 / Homework:

1. Convert the following:
 - a. 1500 liters (L) to cubic meters (m^3) (Given: 1 m^3 = 1000 L)
 - b. 75 degrees Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$) and Kelvin (K) (Formulas: $^{\circ}\text{C} = (^{\circ}\text{F} - 32) \times 5/9$, $\text{K} = ^{\circ}\text{C} + 273.15$)
 - c. 200 kilogram-force (kgf) to Newtons (N) (Given: 1 kgf = 9.80665 N)
 - d. 30 meters per second (m/s) to kilometers per hour (km/h)
2. If a tank has a volume of 500 cubic feet (ft^3), what is its volume in liters?
3. A force of 50 Newtons (N) is applied over an area of 20 square centimeters (cm^2). Calculate the resulting pressure in Pascals (Pa) and pounds per square inch (psi).
4. A car weighs 3000 pounds-mass (lbm). What is its mass in kilograms (kg) and its weight in Newtons (N) on Earth?

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Typically Chapter 1)
- Fluid Mechanics by Cengel and Cimbala. (Typically Chapter 1)
- Online resources for unit conversions (e.g., NIST, Engineering Toolbox).

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Learning package
In
Fluid Definition and Properties
For
Students of First Year

By
Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students, particularly those beginning their studies in fluid mechanics, civil engineering, mechanical engineering, and chemical engineering.

1 / B – Rationale: A thorough understanding of fundamental fluid properties is essential for analyzing fluid behavior, designing fluid systems, and accurately solving problems in various engineering applications. Understanding static pressure and head is a foundational step before moving to fluid dynamics.

1 / C – Central Idea: Fluids, unlike solids, deform continuously under shear stress. Their unique behavior is characterized by distinct properties such as density, specific gravity, surface tension, compressibility, and viscosity. These properties, along with the concepts of static pressure and pressure head, form the bedrock for understanding fluid mechanics.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Define what a fluid is and distinguish it from a solid.
- Explain the concepts of density, specific weight, and specific gravity and calculate them for various fluids.
- Describe the phenomena of surface tension and capillarity and identify their practical implications.
- Understand the concept of compressibility and bulk modulus, and differentiate between compressible and incompressible fluids.
- Define viscosity and differentiate between Newtonian and non-Newtonian fluids, explaining its importance in fluid flow.
- Define static pressure and pressure head and calculate them for fluids at rest.

- Apply the knowledge of fluid properties and static pressure to solve basic fluid mechanics problems.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is the primary difference in behavior between a solid and a fluid when subjected to a force?
2. How is density defined? What are its common SI units?
3. Give an example of where surface tension is observed in everyday life.
4. Can water be considered incompressible under all conditions? Why or why not?
5. If a fluid has high viscosity, how would it flow compared to a fluid with low viscosity?
6. What is pressure, and how is it calculated in a static fluid at a certain depth?

3 / Lecture (Content):

Introduction: Fluid mechanics is the branch of science that deals with the behavior of fluids (liquids and gases) at rest or in motion. Before we delve into the complexities of fluid motion, it's crucial to understand what a fluid is and its fundamental properties. This lecture introduces the definition of a fluid and explores its key characteristics, along with the foundational concepts of static pressure and pressure head.

I. Definition of a Fluid

- A **fluid** is a substance that deforms continuously when subjected to a shear (tangential) stress, no matter how small that stress may be. This continuous deformation is called **flow**.
- **Distinction from Solids:**
 - **Solids:** Resist shear stress by deforming to a fixed shape, and the deformation is proportional to the applied stress (within the elastic limit). Once the stress is removed, they tend to return to their original shape.
 - **Fluids:** Do not resist shear stress (or offer very little resistance) and continuously deform as long as the stress is applied. They take the shape of their container.
- **Categories of Fluids:**

- **Liquids:** Have a definite volume but no definite shape. They form a free surface in a gravitational field.
- **Gases:** Have neither a definite volume nor a definite shape. They expand to fill the entire volume of their container.

II. Fluid Properties

A. Density (ρ), Specific Weight (γ), and Specific Gravity (SG)

1. Density (ρ):

- Definition: Mass per unit volume of a substance.
- Formula: $\rho = m/V$
 - m: mass
 - V: volume
- Units: kg/m³ (SI), lbm/ft³ (US Customary).
- Water density at 4°C: $\rho_{\text{water}} = 1000 \text{ kg/m}^3 = 1 \text{ g/cm}^3 = 62.4 \text{ lbm/ft}^3$.

2. Specific Weight (γ):

- Definition: Weight per unit volume of a substance. It's the force exerted by gravity on a unit volume of the fluid.
- Formula: $\gamma = W/V = \rho g$
 - W: weight
 - V: volume
 - g: acceleration due to gravity (approx. 9.81 m/s² in SI, 32.2 ft/s² in US Customary)
- Units: N/m³ (SI), lbf/ft³ (US Customary).
- Water specific weight at 4°C: $\gamma_{\text{water}} = 9.81 \text{ kN/m}^3 = 62.4 \text{ lbf/ft}^3$.

3. Specific Gravity (SG):

- Definition: The ratio of the density (or specific weight) of a substance to the density (or specific weight) of a standard reference substance (usually water at 4°C for liquids, and air for gases at specific temperature and pressure).
- Formula: $SG = \rho_{\text{water}}/\rho_{\text{fluid}} = \gamma_{\text{water}}/\gamma_{\text{fluid}}$
- Units: Dimensionless (no units).
- $SG > 1$: The fluid is denser than the reference substance.
- $SG < 1$: The fluid is less dense than the reference substance.

B. Surface Tension (σ)

- **Definition:** The cohesive forces between liquid molecules at the surface are stronger than those in the bulk of the liquid. This imbalance of forces creates a "skin-like" effect on the surface, causing it to resist external forces. It is the force per unit length along a line in the surface.
- **Formula:** Force due to surface tension ($F_\sigma = \sigma \times L$) (where L is the length of the line)
- **Units:** N/m (SI), lbf/ft (US Customary).
- **Effects:**
 - **Capillarity:** The rise or fall of a liquid in a narrow tube (capillary tube) due to the interplay between cohesive (within the liquid) and adhesive (liquid-solid) forces.
 - For water (wetting fluid): meniscus is concave, liquid rises.
 - For mercury (non-wetting fluid): meniscus is convex, liquid falls.
 - Capillary rise/fall height (h): $h = \frac{2\sigma \cos \theta}{\rho g R}$
 - θ : contact angle
 - R : radius of the tube
 - Formation of spherical droplets.
 - Flotation of small, dense objects (e.g., insects on water).

C. Compressibility and Bulk Modulus (K)

- **Compressibility:** A measure of the relative change in volume or density of a fluid in response to a change in pressure.
- **Bulk Modulus (K):** The reciprocal of compressibility. It quantifies a fluid's resistance to compression. A higher bulk modulus indicates lower compressibility.
- **Formula:** $K = -V \frac{dP}{dV} = \frac{\rho}{dp/d\rho}$
 - dP : change in pressure
 - dV : change in volume
 - V : original volume
 - dp : change in density
 - ρ : original density
- **Units:** Pa (SI), psi or psf (US Customary).
- **Incompressible Fluid:** A fluid whose density does not change significantly with changes in pressure. Liquids are generally considered incompressible in most engineering applications, especially at low to moderate pressures.

- **Compressible Fluid:** A fluid whose density changes significantly with changes in pressure (e.g., gases).

D. Viscosity (μ and ν)

- **Definition:** A measure of a fluid's resistance to shear deformation or flow. It represents the "internal friction" of the fluid.
- **Newton's Law of Viscosity:** For many common fluids (Newtonian fluids), the shear stress (τ) is directly proportional to the rate of shear strain (velocity gradient, du/dy).
 - $\tau = \mu \frac{du}{dy}$
 - τ : shear stress
 - μ : dynamic viscosity (absolute viscosity)
 - du/dy : velocity gradient (rate of shear deformation)
- **Dynamic Viscosity (μ):**
 - Units: $\text{Pa}\cdot\text{s}$ (Pascal-second) or $\text{N}\cdot\text{s}/\text{m}^2$ (SI), $\text{lbf}\cdot\text{s}/\text{ft}^2$ (US Customary).
 - Also commonly expressed in Poise (1 Poise = $0.1 \text{ Pa}\cdot\text{s}$) or centipoise (cP, $1 \text{ cP} = 0.001 \text{ Pa}\cdot\text{s}$). Water at 20°C has a dynamic viscosity of approximately 1 cP.
- **Kinematic Viscosity (ν):**
 - Definition: The ratio of dynamic viscosity to density. It describes how fast momentum diffuses through the fluid.
 - Formula: $\nu = \mu/\rho$
 - Units: m^2/s (SI), ft^2/s (US Customary).
 - Also commonly expressed in Stokes (St, $1 \text{ St} = 1 \text{ cm}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$) or centistokes (cSt).
- **Types of Fluids based on Viscosity:**
 - **Newtonian Fluids:** Fluids where viscosity is constant and independent of the shear rate (e.g., water, air, oil).
 - **Non-Newtonian Fluids:** Fluids where viscosity changes with the shear rate (e.g., blood, paint, toothpaste, polymer solutions).

III. Static Pressure and Pressure Head

A. Pressure (P)

- **Definition:** Force exerted perpendicularly on a unit area. In a static fluid, pressure at a point acts equally in all directions (Pascal's Law).
- **Formula:** $P = F/A$

- F: normal force
- A: area
- **Units:** Pa (Pascal) = N/m^2 (SI), psi (lbf/in²), psf (lbf/ft²) (US Customary). Other units: bar, atm, mmHg, inH₂O.
- **Absolute Pressure (P_{abs}):** Measured relative to a perfect vacuum (absolute zero pressure).
- **Gage Pressure (P_{gage}):** Measured relative to the local atmospheric pressure.
 - $P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}}$ (for pressures above atmospheric)
 - $P_{\text{abs}} = P_{\text{atm}} - P_{\text{vacuum}}$ (for pressures below atmospheric, vacuum pressure)
- **Atmospheric Pressure (P_{atm}):** Pressure exerted by the Earth's atmosphere. At sea level, standard $P_{\text{atm}} \approx 101.325 \text{ kPa} = 14.7 \text{ psi}$.

B. Pressure Variation in a Static Fluid

- For a fluid at rest, the pressure increases linearly with depth due to the weight of the fluid above it.
- **Formula for Pressure at Depth (h):** $P = P_0 + \rho gh = P_0 + \gamma h$
 - P: pressure at depth h
 - P₀: pressure at the free surface (often atmospheric pressure)
 - ρ : fluid density
 - g: acceleration due to gravity
 - h: vertical depth below the free surface
 - γ : specific weight of the fluid

C. Pressure Head (h)

- **Definition:** The height of a column of a specific fluid that would produce a given pressure. It's a way to express pressure in terms of a column height of a fluid.
- **Formula:** $h = \rho g P = \gamma P$
- **Units:** meters (m) or feet (ft) of a specific fluid (e.g., meters of water, feet of mercury).
- **Example:** A pressure of 100 kPa is equivalent to a head of approximately 10.2 meters of water ($100,000 \text{ Pa} / (1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2) \approx 10.2 \text{ m}$).

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. How would you visually distinguish between a highly viscous liquid and a low-viscosity liquid flowing down an inclined surface?
2. Calculate the specific weight and specific gravity of oil with a density of 850 kg/m^3 . (Assume $g=9.81 \text{ m/s}^2$ and $\rho_{\text{water}}=1000 \text{ kg/m}^3$).
3. Explain why water rises in a thin glass tube, while mercury falls. What property of fluids is responsible for this?
4. A pressure gage reads 50 kPa at the bottom of a water tank. If the atmospheric pressure is 101.3 kPa , what is the absolute pressure at the bottom of the tank?
5. What is the pressure head equivalent to a pressure of 20 psi for a fluid with a specific gravity of 0.8 ? (Given: $\gamma_{\text{water}}=62.4 \text{ lbf/ft}^3$, $1 \text{ ft}^2=144 \text{ in}^2$)

5 / Homework:

1. A liquid has a specific gravity of 0.9 .
 - a. What is its density in kg/m^3 and lbm/ft^3 ?
 - b. What is its specific weight in N/m^3 and lbf/ft^3 ?
2. A soap bubble has a diameter of 2 cm . If the surface tension of the soap solution is 0.025 N/m , what is the pressure difference across the soap film? (Hint: For a spherical bubble, $\Delta P=4\sigma/R$)
3. A steel ball of mass 0.01 kg and diameter 1 cm falls through oil. If the oil has a dynamic viscosity of $0.05 \text{ Pa}\cdot\text{s}$, and the ball reaches a terminal velocity of 0.1 m/s , calculate the drag force on the ball. (This problem might require more advanced concepts; focus on understanding the role of viscosity).
4. A scuba diver descends to a depth of 30 meters in freshwater ($\rho=1000 \text{ kg/m}^3$). What is the gage pressure and absolute pressure at this depth? (Assume atmospheric pressure is 101.3 kPa).
5. Express a pressure of 500 kPa as:
 - a. meters of water
 - b. meters of mercury (SG of mercury = 13.6)

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 2)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 2)
- Streeter, Wylie, and Bedford's Fluid Mechanics.

- Online resources for fluid properties data tables (e.g., engineering toolbox, academic databases).

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package
In
Static Pressure and Head

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, building foundational knowledge on pressure concepts in static fluids.

1 / B – Rationale: Pressure is a fundamental concept in fluid mechanics. A clear understanding of static pressure, its various forms (absolute, gauge, vacuum), its units, and its variation with depth, along with the concept of pressure head, is essential for analyzing fluid static problems and for understanding energy in flowing fluids (as seen in Bernoulli's equation).

1 / C – Central Idea: Static pressure in a fluid is the force exerted by the fluid per unit area. It increases linearly with depth due to the weight of the fluid above. This pressure can be expressed as an equivalent height of a fluid column, known as pressure head, which simplifies visualization and comparison of pressure levels.

- **1 / D – Performance Objectives:** Upon completion of this lecture, students will be able to:
 - Define pressure and state its units.
 - Differentiate between absolute, gauge, and vacuum pressure and convert between them.
 - Explain the concept of pressure head and convert between pressure and pressure head.
 - Derive and apply the hydrostatic equation to calculate pressure variation with depth in a static fluid.
 - Understand Pascal's Law and its implications for static fluid pressure.
 - Analyze simple pressure measurement systems like piezometers and U-tube manometers.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is the definition of pressure in terms of force and area?
2. What is the standard SI unit for pressure?
3. Why does the pressure at the bottom of a swimming pool increase if you add more water?
4. What is atmospheric pressure? Is it constant everywhere?
5. If a tire gauge reads 30textpsi, is that an absolute pressure or a gauge pressure? How do you know?

3 / Lecture (Content):

What is Pressure?

- **Pressure (P)** is the force applied per unit area.
- Equation:
- SI Unit: Pascal (Pa) = N/m^2

Types of Pressure:

- **Atmospheric Pressure (P_{atm}):** The pressure due to the weight of the atmosphere ($\sim 101325 \text{ Pa}$).
- **Gauge Pressure (P_g):** Pressure measured relative to atmospheric pressure.
- **Absolute Pressure (P_{abs}):** Total pressure including atmospheric pressure.

Pressure Head:

- Pressure can be expressed as an equivalent height (head) of a fluid column. where:
 - h = pressure head (m)
 - P = pressure (Pa)
 - ρ = fluid density (kg/m^3)
 - g = gravitational acceleration (9.81 m/s^2)

Example: Find the pressure head of water under a pressure of 9800 Pa.

Pascal's Law:

- Pressure applied to a confined fluid is transmitted equally in all directions.

Example: If you apply 100 N of force over a piston of 0.01 m^2 : transmitted equally.

Manometers:

- Devices used to measure pressure using fluid columns.

U-tube Manometer Equation:

Example Problem:

Problem: A mercury manometer shows a 0.2 m height difference. What is the pressure?

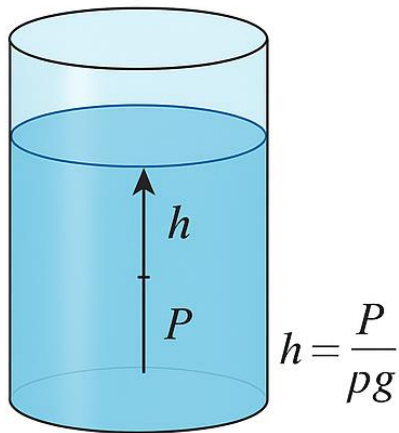


Figure 1 Pressure in a Static Fluid

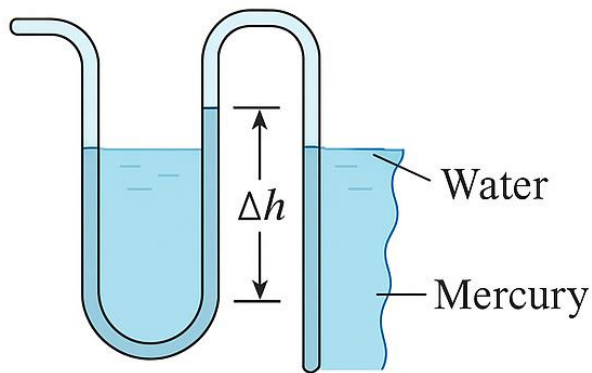


Figure 2 U-Tube Manometer

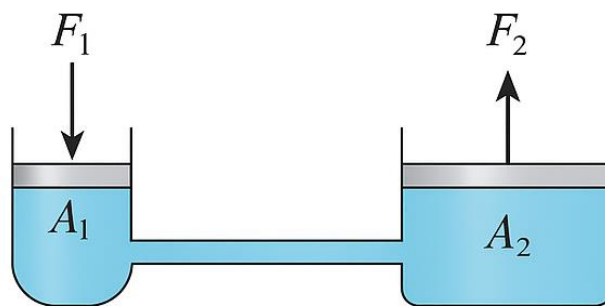


Figure 3 Pascal's Law

Homework:

1. Define gauge and absolute pressure.
2. A tank has a pressure of 150 kPa. What is the pressure head in meters of water?
3. Calculate the pressure difference shown by a U-tube manometer with a 0.15 m height difference and fluid density 1000 kg/m^3 .
4. A piston of area 0.02 m^2 is subjected to a pressure of 5000 Pa. Find the force.
5. Convert 2 atm to pascal.

Diagrams: Please refer to textbook illustrations for:

- U-tube manometer setup
- Pressure vs. depth diagram in static fluid
- Piston-cylinder system

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Buoyancy and Stability of Floating Bodies

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

• **1 / A – Target Population:** Undergraduate engineering students in disciplines such as civil, mechanical, naval architecture, and ocean engineering, as well as anyone interested in the principles governing floating objects.

1 / B – Rationale: Understanding buoyancy is fundamental to analyzing the behavior of submerged and floating objects. The concept of stability is crucial for designing safe and functional ships, submarines, buoys, and other structures that interact with fluids.

1 / C – Central Idea: Any object submerged or floating in a fluid experiences an upward buoyant force equal to the weight of the fluid it displaces (Archimedes' Principle). The stability of a floating body depends on the relative positions of its center of gravity and metacenter, determining whether it returns to its equilibrium position after a small disturbance.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- State and apply Archimedes' Principle to determine the buoyant force on submerged and floating bodies.
- Calculate the submerged volume of a floating object.
- Understand the concepts of center of gravity (G), center of buoyancy (B), and metacenter (M).
- Determine the stability of submerged and floating bodies based on the relative positions of G and B (for submerged) and G and M (for floating).
- Calculate the metacentric height (GM) and use it as a criterion for stability.
- Solve problems involving buoyancy and stability for various practical scenarios.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What causes an object to float or sink in water?
2. If you place a wooden block and a steel block of the same volume in water, which one displaces more water? Why?
3. What is the weight of water displaced by a floating object?
4. Why do ships typically have their cargo loaded as low as possible?
5. What happens if the center of gravity of a floating object is above its metacenter?

3 / Lecture (Content):

Introduction: Buoyancy is a force experienced by objects immersed in fluids, enabling ships to float and submarines to dive. The stability of these floating bodies is equally critical, ensuring they do not capsize under normal operating conditions. This lecture delves into Archimedes' Principle and the factors that govern the stability of submerged and floating objects.

I. Archimedes' Principle and Buoyant Force

- **Archimedes' Principle:** "Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."
- **Buoyant Force (FB):** The upward force exerted by a fluid on an immersed object.
 - Formula: $FB = \gamma_{\text{fluid}} \times V_{\text{displaced}}$
 - γ_{fluid} : Specific weight of the fluid (N/m³ or lbf/ft³)
 - $V_{\text{displaced}}$: Volume of the fluid displaced by the object (m³ or ft³). This is equal to the volume of the submerged part of the object.
- **Equilibrium of Submerged Bodies:**
 - If $FB > W_{\text{object}}$ (weight of object), the object accelerates upward (floats to surface).
 - If $FB < W_{\text{object}}$, the object accelerates downward (sinks).
 - If $FB = W_{\text{object}}$, the object is in neutral equilibrium (it floats at a constant depth).
- **Floating Bodies:**
 - For a body to float, FB must be equal to W_{object} .

- This implies that the weight of the floating object is exactly equal to the weight of the fluid it displaces.
- The object floats with a portion of its volume submerged.

II. Center of Buoyancy (B) and Center of Gravity (G)

- **Center of Buoyancy (B):** The centroid of the displaced volume of fluid. The buoyant force (FB) acts vertically upward through this point.
- **Center of Gravity (G):** The point through which the entire weight (Wobject) of the body acts vertically downward.

III. Stability of Submerged Bodies

- For a **fully submerged body**, stability depends on the relative positions of its center of gravity (G) and center of buoyancy (B).
 - **Stable Equilibrium:** If G is below B. When tilted, the buoyant force creates a restoring couple that brings the body back to its original position.
 - **Unstable Equilibrium:** If G is above B. When tilted, the buoyant force creates an overturning couple that causes the body to continue tilting and eventually flip.
 - **Neutral Equilibrium:** If G coincides with B. The body remains in its new tilted position after disturbance. (e.g., submarine trimmed for neutral buoyancy).

IV. Stability of Floating Bodies

- The stability of **floating bodies** is more complex because the center of buoyancy (B) shifts as the body tilts.
- **Metacenter (M):** The point of intersection of the line of action of the buoyant force (when the body is tilted slightly) with the body's original (untilted) vertical axis.
- **Metacentric Height (GM):** The distance between the center of gravity (G) and the metacenter (M). It is a crucial parameter for determining stability.
- **Criteria for Stability of Floating Bodies:**
 - **Stable Equilibrium:** If M is above G (GM is positive). When tilted, the buoyant force creates a restoring couple, bringing the body back to its original upright position. This is the desired condition for ships and floating structures.

- **Unstable Equilibrium:** If M is below G (GM is negative). When tilted, the buoyant force creates an overturning couple, causing the body to continue tilting and capsize.
- **Neutral Equilibrium:** If M coincides with G (GM is zero). The body remains in its new tilted position.
- **Calculating Metacentric Height (GM):**
 - The formula for metacentric height is given by: $GM = BM - BG$
 - $BM = \frac{V_{\text{displaced}} I}{V}$
 - I: Moment of inertia of the waterplane area about the longitudinal axis through the center of the waterplane (the axis about which tilting occurs). (m^4 or ft^4)
 - $V_{\text{displaced}}$: Volume of displaced fluid (m^3 or ft^3)
 - BG: Distance between the center of buoyancy (B) and the center of gravity (G).
 - A larger positive GM indicates greater initial stability.

V. Practical Implications

- **Ship Design:** Naval architects design ships to have a positive metacentric height to ensure stability, especially under various loading conditions (cargo, ballast water). Loading heavy cargo low in the ship helps to keep the center of gravity (G) low, increasing GM and stability.
- **Buoys:** Designed to remain upright and stable even in rough seas.
- **Submarines:** Achieve neutral buoyancy to remain submerged at a desired depth. Their stability mechanisms are different from surface vessels once fully submerged.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. A block of wood floats in water. If its volume is 0.5 m^3 and 60% of its volume is submerged, what is the weight of the block? (Density of water = 1000 kg/m^3 , $g = 9.81 \text{ m/s}^2$).
2. Explain why a submarine, when fully submerged, achieves neutral stability by adjusting its ballast tanks. Relate this to the positions of G and B.
3. Define the metacenter (M) and metacentric height (GM) for a floating body. What does a positive GM indicate?

4. A pontoon has a width of 4 m and a length of 6 m. Its center of gravity (G) is 1 m above its bottom. If the pontoon floats in water with a draft (submerged depth) of 0.5 m, calculate its initial metacentric height (GM) for rolling about its longitudinal axis. (Moment of inertia of a rectangle $I = \frac{1}{12} \text{width} \times \text{length}^3$ for pitching, and $I = \frac{1}{12} \text{length} \times \text{width}^3$ for rolling).
 - Waterplane area $A_{wp} = \text{length} \times \text{width}$.
 - Volume displaced $V_{\text{displaced}} = A_{wp} \times \text{draft}$.
 - Center of Buoyancy B is at half the draft from the bottom.
5. What are the consequences of a floating body having a negative metacentric height?

5 / Homework:

1. A solid block of material has dimensions of 0.2 m × 0.2 m × 0.1 m and weighs 50 N. Will it float or sink in water? If it floats, what percentage of its volume will be submerged?
2. A cylindrical buoy is 2 m long and has a diameter of 0.8 m. Its mass is 400 kg. It floats upright in seawater (SG = 1.025).
 - a. What is the submerged depth of the buoy?
 - b. What is the distance from the bottom of the buoy to its center of buoyancy (B)?
 - c. If the center of gravity (G) of the buoy is 1.0 m from its bottom, calculate the metacentric height (GM) for floating upright. (Moment of inertia of a circle $I = \frac{\pi R^4}{4}$). Is it stable?
3. Explain how filling or emptying ballast tanks affects the buoyancy and stability of a ship.
4. A piece of wood (SG = 0.7) has dimensions 1 m × 0.5 m × 0.3 m. It floats in oil (SG = 0.9). Calculate the volume of the wood submerged in the oil.
5. Discuss the difference between static stability and dynamic stability for a floating vessel.

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 2)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 2)
- Streeter, Wylie, and Bedford's Fluid Mechanics.
- Principles of Naval Architecture (for more in-depth stability analysis).

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Fluid Flow – Types of Flow

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students, particularly those in fluid mechanics, civil, mechanical, and aerospace engineering, beginning to study fluid dynamics.

1 / B – Rationale: Understanding the different classifications and characteristics of fluid flow is fundamental to selecting appropriate analytical methods, applying governing equations, and interpreting experimental results in fluid dynamics problems.

1 / C – Central Idea: Fluid motion, or flow, can be categorized based on various characteristics such as time dependency, spatial variation, compressibility, and the presence of viscosity. Distinguishing between these flow types (e.g., steady vs. unsteady, uniform vs. non-uniform, laminar vs. turbulent) is crucial for simplifying complex fluid problems and applying relevant theoretical models.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Differentiate between steady and unsteady flow, and uniform and non-uniform flow.
- Explain the concept of one-, two-, and three-dimensional flow.
- Distinguish between laminar and turbulent flow regimes and identify the significance of the Reynolds number.
- Understand the difference between compressible and incompressible flow.
- Define rotational and irrotational flow, and differentiate between viscous and inviscid flow.
- Identify flow lines: streamlines, pathlines, and streaklines.
- Classify a given fluid flow scenario based on its relevant characteristics.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. Imagine water flowing out of a garden hose. Is the flow likely to be smooth or chaotic?
2. What is the main difference between how a fluid moves in a slow, steady manner versus a fast, erratic manner?
3. If you trace the path of a single particle in a fluid over time, what is that path called?
4. Does the density of air change significantly when it flows at very high speeds, like in a jet engine?
5. What property of a fluid primarily dictates whether its flow is smooth or chaotic?

3 / Lecture (Content):

Introduction: After understanding fluid properties and static behavior, the next step in fluid mechanics is to analyze fluid motion, or "flow." Fluid flow is often complex, but by classifying it based on various characteristics, we can simplify its analysis and apply appropriate mathematical and physical models. This lecture introduces the primary classifications of fluid flow.

I. Description of Fluid Flow

A. Lagrangian vs. Eulerian Description

- **Lagrangian Description:** Focuses on following individual fluid particles as they move through space and time. It tracks the position, velocity, and acceleration of each particle. Useful for particle tracking and sometimes for solid-fluid interactions.
- **Eulerian Description:** Focuses on fluid properties at fixed points in space as a function of time. We observe what happens at a specific location rather than tracking individual particles. This is the more common approach in fluid mechanics for macroscopic analysis.

II. Types of Fluid Flow Classifications

A. Steady vs. Unsteady Flow

- **Steady Flow:** Fluid properties (velocity, pressure, density) at any point in the flow field do not change with respect to time. $\partial(\text{property})/\partial t = 0$.
 - Example: Water flowing through a pipe at a constant rate.
- **Unsteady Flow:** Fluid properties at any point in the flow field change with respect to time. $\partial(\text{property})/\partial t \neq 0$.
 - Example: Water flow during the opening or closing of a valve, or blood flow in arteries (pulsatile).

B. Uniform vs. Non-Uniform Flow

- **Uniform Flow:** The velocity vector is constant in magnitude and direction at every point within the flow field *at a given instant in time*. This implies that flow properties do not change with spatial coordinates along the flow direction ($\partial(\text{property})/\partial s = 0$, where s is the spatial coordinate along the flow).
 - Example: Flow through a long, straight pipe of constant diameter at a fixed flow rate, far from the entrance or exit effects.
- **Non-Uniform Flow:** The velocity vector changes in magnitude or direction (or both) from point to point within the flow field. Flow properties change with spatial coordinates ($\partial(\text{property})/\partial s \neq 0$).
 - Example: Flow through a converging nozzle, flow around an airfoil, flow near a pipe entrance.

C. One-, Two-, and Three-Dimensional Flow

- **One-Dimensional Flow:** Flow properties vary essentially in only one spatial direction. Assumed when cross-sectional velocity distribution is uniform or approximated by an average.
 - Example: Flow in a pipe (assuming average velocity over the cross-section).
- **Two-Dimensional Flow:** Flow properties vary in two spatial directions (e.g., x and y plane) but are constant in the third direction.
 - Example: Flow over a long weir or dam, flow around a very long cylinder.
- **Three-Dimensional Flow:** Flow properties vary in all three spatial directions (x , y , z). Most real-world flows are inherently three-dimensional.
 - Example: Flow around an airplane, flow in a complex pipe network.

D. Laminar vs. Turbulent Flow

- This is one of the most critical classifications, determined by the **Reynolds Number (Re)**.
- **Laminar Flow:** Fluid particles move in smooth, orderly, parallel layers or streamlines, with no significant mixing between layers. Occurs at low velocities and/or high viscosities.
 - Characteristics: Smooth, predictable flow; viscous forces dominate inertial forces.
 - Appearance: Dye injected into a laminar flow will remain as a distinct line.
- **Turbulent Flow:** Fluid particles move in highly chaotic, irregular, and random paths, resulting in intense mixing. Occurs at high velocities and/or low viscosities.
 - Characteristics: Random fluctuations in velocity and pressure; inertial forces dominate viscous forces; significant energy dissipation.
 - Appearance: Dye injected into a turbulent flow will rapidly disperse.
- **Reynolds Number (Re):** A dimensionless quantity that predicts the flow regime.
 - Formula: $Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$
 - ρ : fluid density
 - V : characteristic velocity
 - L : characteristic length (e.g., pipe diameter)
 - μ : dynamic viscosity
 - ν : kinematic viscosity
 - **Typical Regimes for Pipe Flow:**
 - $Re < 2000$ to 2300 : Laminar flow
 - 2000 to $2300 < Re < 4000$: Transition flow (can be laminar or turbulent)
 - $Re > 4000$: Turbulent flow
 - These values are indicative; the exact transition point can vary.

E. Compressible vs. Incompressible Flow

- **Incompressible Flow:** The density of the fluid remains essentially constant throughout the flow field.
 - $\rho \approx \text{constant}$

- Typically assumed for liquids and for gases moving at low speeds (Mach number < 0.3).
- **Compressible Flow:** The density of the fluid changes significantly during flow.
 - $\rho = \text{constant}$
 - Typically observed in gases, especially when velocities approach or exceed the speed of sound (high Mach numbers).
 - Mach Number (Ma): $Ma = V/c$ (where c is the speed of sound in the fluid).

F. Viscous vs. Inviscid Flow

- **Viscous Flow:** Flow where the effects of fluid viscosity are significant and cannot be neglected. All real fluids have viscosity.
 - Viscous effects are dominant near solid boundaries (boundary layers).
- **Inviscid Flow (Ideal Flow):** A theoretical simplification where viscosity is assumed to be zero.
 - Allows for simpler mathematical analysis but does not accurately represent real fluid behavior near boundaries.
 - Useful for analyzing high-speed external flows away from surfaces.

G. Rotational vs. Irrotational Flow

- **Rotational Flow:** Fluid particles rotate about their own axis as they move along the flow path. The fluid element has a net angular velocity.
- **Irrotational Flow:** Fluid particles do not rotate about their own axis; they only translate and deform. The net angular velocity of fluid elements is zero.
 - Many ideal (inviscid) flows are irrotational, especially away from boundaries. This simplifies calculations using potential flow theory.

H. Flow Lines (Visualizing Flow)

- **Streamline:** An imaginary line in a fluid flow field drawn such that the tangent to the line at any point gives the direction of the velocity vector at that point *at a given instant*. Streamlines cannot cross each other in steady flow.

- **Pathline:** The actual path traced by a single fluid particle over a period of time. It's like a time-exposure photograph of a marked particle.
- **Streakline:** The locus of all fluid particles that have passed through a specific fixed point in space *over a period of time*. It's what you see when you inject a continuous dye into a flow at a fixed point (e.g., smoke from a chimney).
- **Note:** For **steady flow**, streamlines, pathlines, and streaklines are identical. For unsteady flow, they are generally different.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. Describe a real-world example of steady, non-uniform flow.
2. Water flows through a 5 cm diameter pipe at an average velocity of 0.5 m/s. The kinematic viscosity of water is $1 \times 10^{-6} \text{ m}^2/\text{s}$. Calculate the Reynolds number. Is the flow laminar or turbulent?
3. Explain why air flowing at Mach 0.1 can be treated as incompressible, while air flowing at Mach 0.8 cannot.
4. If you release a tiny, neutrally buoyant bead into a flowing river and record its trajectory over 10 minutes, what kind of flow line are you tracing? If you continuously inject dye from a fixed point in the same river, what kind of flow line would you observe? Assume the river flow is unsteady.
5. What is the main advantage of assuming a fluid is "inviscid" in theoretical fluid mechanics? What are its limitations?

5 / Homework:

1. Classify the following flows as steady/unsteady, uniform/non-uniform, and compressible/incompressible (provide brief reasoning):
 - a. Water flowing from a tap at a constant rate into a sink.
 - b. Air flowing over a car as it accelerates from rest.
 - c. Blood flow in the aorta during one cardiac cycle.
 - d. Water flowing steadily through a pipe of changing cross-sectional area.
2. Oil with a dynamic viscosity of $0.1 \text{ Pa}\cdot\text{s}$ and density of 900 kg/m^3 flows in a pipe of diameter 10 cm. If the average velocity is 0.2 m/s, determine if the flow is laminar or turbulent.
3. Explain why laminar flow is generally more energy-efficient than turbulent flow for transporting fluids in pipes.

4. Consider an internal flow through a converging duct. Sketch typical streamlines and explain why the flow might be non-uniform.
5. Research and discuss one engineering application where distinguishing between rotational and irrotational flow is important.

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 1 & 4)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 1 & 4)
- Fox and McDonald's Introduction to Fluid Mechanics.
- White's Fluid Mechanics.

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Continuity Equation

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students, particularly those studying fluid mechanics, civil, mechanical, and chemical engineering, focusing on the fundamental laws governing fluid flow.

1 / B – Rationale: The Continuity Equation is a direct application of the principle of conservation of mass to fluid flow. It is one of the most fundamental and widely used equations in fluid mechanics, essential for analyzing flow rates, velocities, and cross-sectional areas in various fluid systems.

1 / C – Central Idea: For a fluid system, the Continuity Equation states that mass is conserved. In steady flow, this means the mass flow rate entering a control volume must equal the mass flow rate leaving it. For incompressible fluids, this simplifies to the volume flow rate remaining constant along a streamline or through a duct.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- State the principle of conservation of mass as it applies to fluid flow.
- Derive the one-dimensional continuity equation for steady, incompressible flow in a conduit.
- Apply the continuity equation to calculate velocities, areas, or flow rates at different sections of a pipe or channel.
- Understand the concept of mass flow rate and volume flow rate.
- Solve practical problems involving the continuity equation in various engineering scenarios, such as pipe expansions/contractions and branched systems.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What does the principle of "conservation of mass" mean in simple terms?
2. If water flows from a wide hose into a narrower nozzle, what happens to the speed of the water?
3. How do you calculate the volume of water passing through a pipe's cross-section per second?
4. Does the density of water change significantly as it flows through a typical household pipe?
5. Why might the flow speed increase in a narrower section of a river?

3 / Lecture (Content):

Introduction: The Continuity Equation is one of the cornerstone principles in fluid mechanics, stemming directly from the fundamental law of conservation of mass. It provides a powerful tool for analyzing fluid flow, particularly in situations where the flow area or velocity changes.

I. Principle of Conservation of Mass

- The **Principle of Conservation of Mass** states that mass can neither be created nor destroyed.
- In the context of fluid flow, this means that for any control volume, the net rate of mass flow into the volume must be equal to the net rate of change of mass within the volume.
- For **steady flow**, the mass within the control volume does not change with time. Therefore, the mass flow rate entering the control volume must equal the mass flow rate leaving the control volume.

II. Mass Flow Rate ($\dot{m}$)

- **Definition:** The mass of fluid flowing per unit time through a cross-sectional area.
- Formula: $\dot{m} = \rho AV$
 - ρ : fluid density (kg/m^3 or lbm/ft^3)
 - A : cross-sectional area perpendicular to flow (m^2 or ft^2)
 - V : average velocity of the fluid perpendicular to the area (m/s or ft/s)

- Units: kg/s (SI), lbm/s (US Customary).

III. Volume Flow Rate (Discharge, $\dot{V}$ or Q)

- **Definition:** The volume of fluid flowing per unit time through a cross-sectional area. Also known as discharge.
- Formula: $\dot{V} = AV$
 - A: cross-sectional area (m^2 or ft^2)
 - V: average velocity (m/s or ft/s)
- Units: m^3/s (SI), ft^3/s (US Customary), often expressed in L/s, GPM (gallons per minute).
- Relationship to mass flow rate: $\dot{m} = \rho \dot{V}$

IV. Derivation of the Continuity Equation

Consider a control volume (e.g., a section of a pipe) with an inlet (section 1) and an outlet (section 2).

- **General Form (for steady flow through a control volume):** The mass flow rate entering the control volume equals the mass flow rate leaving the control volume. $\sum \dot{m}_{in} = \sum \dot{m}_{out}$
- **For one-dimensional, steady flow through a single inlet and outlet:**
 $\dot{m}_1 = \dot{m}_2 \quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2$

This is the **general form of the Continuity Equation for steady flow**. It applies to both compressible and incompressible fluids.

- **For steady, Incompressible Flow (most common application in basic fluid mechanics):** If the fluid is incompressible, its density remains constant ($\rho_1 = \rho_2 = \rho$). Therefore, the Continuity Equation simplifies to: $A_1 V_1 = A_2 V_2$ Or, in terms of volume flow rate: $\dot{V}_1 = \dot{V}_2 = \text{constant}$

This means that for incompressible flow, the volume flow rate (or discharge) remains constant along a streamline or through a conduit, even if the cross-sectional area changes. If the area decreases, the velocity must increase, and vice versa.

V. Applications and Examples

A. Converging/Diverging Pipes:

- When flow moves from a larger cross-sectional area to a smaller one (converging nozzle), the velocity increases.
- When flow moves from a smaller cross-sectional area to a larger one (diverging diffuser), the velocity decreases.

Example 1: Flow through a pipe contraction

- **Question:** Water flows through a pipe that narrows from a diameter of 20 cm to 10 cm. If the velocity in the larger section is 2 m/s, what is the velocity in the narrower section?
- **Solution:** Given: $D_1=20\text{ cm}=0.2\text{ m} \Rightarrow A_1=\pi(0.2/2)^2=0.0314\text{ m}^2$ $V_1=2\text{ m/s}$ $D_2=10\text{ cm}=0.1\text{ m} \Rightarrow A_2=\pi(0.1/2)^2=0.00785\text{ m}^2$ Using Continuity Equation ($A_1V_1=A_2V_2$): $V_2=A_1V_1/A_2=0.0314\text{ m}^2 \times 2\text{ m/s} / 0.00785\text{ m}^2=8\text{ m/s}$ (Notice the velocity increased significantly due to the reduction in area)

B. Multiple Inlets/Outlets (Branched Systems):

- The total mass (or volume, for incompressible) flow rate entering a junction must equal the total mass (or volume) flow rate leaving the junction. $\sum \dot{V}_{in} = \sum \dot{V}_{out}$ (for incompressible flow)

Example 2: Water distribution system

- **Question:** A main pipe splits into two smaller pipes. If the main pipe has a flow rate of 0.5 m³/s and one of the smaller pipes carries 0.2 m³/s, what is the flow rate in the second smaller pipe?
- **Solution:** Let $\dot{V}_{main}=0.5\text{ m}^3/\text{s}$ Let $\dot{V}_{branch1}=0.2\text{ m}^3/\text{s}$ Let $\dot{V}_{branch2}$ be the unknown. Using Continuity Equation: $\dot{V}_{main}=\dot{V}_{branch1}+\dot{V}_{branch2}$ $0.5\text{ m}^3/\text{s}=0.2\text{ m}^3/\text{s}+\dot{V}_{branch2}$ $\dot{V}_{branch2}=0.5-0.2=0.3\text{ m}^3/\text{s}$

C. Open Channel Flow:

- The continuity equation also applies to open channels (rivers, canals). The cross-sectional area would be the flow area in the channel.

Important Considerations:

- The continuity equation is always valid, provided mass is conserved.

- For compressible flow (e.g., high-speed gas flow), density variations *must* be considered in $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$.
- The velocity used in the equation is the average velocity normal to the cross-sectional area.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. State the Continuity Equation for steady, incompressible flow.
2. Water flows through a 30 cm diameter pipe at a velocity of 1.5 m/s. Calculate the volume flow rate in m^3/s and L/s .
3. A rectangular duct carries air. At section 1, the duct is 0.5 m wide and 0.3 m high, and the air velocity is 5 m/s. At section 2, the duct narrows to 0.5 m wide and 0.15 m high. Assuming the air is incompressible, what is the velocity at section 2?
4. Why is it generally acceptable to use the simplified continuity equation ($A_1 V_1 = A_2 V_2$) for water flow, but not always for air flow?
5. A hose has an internal diameter of 2 cm. A nozzle is attached to the end, reducing the exit diameter to 0.5 cm. If water flows through the hose at a velocity of 1 m/s, what is the velocity of the water exiting the nozzle?

5 / Homework:

1. Oil ($\text{SG} = 0.85$) flows through a pipe. At section A, the pipe diameter is 15 cm, and the velocity is 2.5 m/s. At section B, the pipe diameter is 10 cm.
 - a. Calculate the volume flow rate in m^3/s and L/min .
 - b. Calculate the velocity at section B.
 - c. Calculate the mass flow rate in kg/s .
2. A large irrigation canal has a rectangular cross-section of 5 m width and 2 m depth. The water flows at an average velocity of 0.8 m/s. It then flows into a narrower section of the canal, which is 3 m wide and 1.5 m deep. What is the average velocity of the water in the narrower section?
3. A compressor draws in air at 1.2 kg/m^3 density, 0.5 m^2 area, and 10 m/s velocity. It discharges air at 2.0 kg/m^3 density through a 0.2 m^2 area. What is the discharge velocity of the air?
4. Explain the difference between mass flow rate and volume flow rate, and when it is appropriate to use one over the other.
5. A plumbing system has a main pipe (Diameter D) that branches into three smaller pipes, each with diameter $D/2$. If the average velocity in

the main pipe is V , what is the average velocity in each of the smaller pipes, assuming incompressible flow?

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 4)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 4)
- Fox and McDonald's Introduction to Fluid Mechanics.
- White's Fluid Mechanics.

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package
In
Energy of Flowing Fluid – Bernoulli's Theorem

For
Students of First Year

By
Halah Kadhim Mohsin

Dep. Of Chemical industries Techniques
2025

1/ Overview

1 / A – Target Population: Undergraduate engineering students, particularly those in fluid mechanics, civil, mechanical, and aerospace engineering, focusing on the energy conservation principle for fluid flow.

1 / B – Rationale: Bernoulli's Theorem is a cornerstone of fluid dynamics, providing a powerful tool for analyzing fluid flow where energy losses are negligible. It allows engineers to predict pressure, velocity, and elevation relationships in various applications, from pipe flow to aerodynamics.

1 / C – Central Idea: Bernoulli's Theorem is an expression of the conservation of energy for steady, incompressible, inviscid (frictionless) flow along a streamline. It states that the sum of pressure energy, kinetic energy, and potential energy per unit volume (or unit weight, or unit mass) remains constant along a streamline.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Understand the concept of energy conservation as applied to fluid flow.
- Identify the assumptions and limitations of Bernoulli's Theorem.
- Interpret the three terms of Bernoulli's Equation: pressure head, velocity head, and elevation head.
- Apply Bernoulli's Equation to solve problems involving steady, incompressible, and inviscid flow.
- Relate Bernoulli's Equation to practical scenarios like flow in varying cross-sectional areas, siphons, and simple orifices.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What are the three main forms of mechanical energy for a solid object?

2. If water flows faster, what generally happens to its pressure, assuming no external work or heat transfer?
3. Why does water squirt out further from a smaller hole in a bottle than from a larger one at the same height?
4. Can Bernoulli's equation be applied to highly viscous oil flowing through a very long, narrow pipe? Why or why not?
5. What happens to the total energy of a system if there's no friction or external work?

3 / Lecture (Content):

Introduction: Just as energy is conserved in mechanical systems (e.g., a ball rolling down a hill), it is also conserved in fluid systems. Bernoulli's Theorem, derived from Newton's second law and the work-energy principle, is a fundamental equation that expresses this conservation of mechanical energy for a specific class of fluid flows.

I. Forms of Energy in a Flowing Fluid

For a flowing fluid, mechanical energy can be expressed in three primary forms per unit mass, per unit weight, or per unit volume.

A. Per Unit Volume (Energy Density - Pressure Form):

1. **Pressure Energy:** Energy associated with the pressure of the fluid.
 - Term: P (Pressure)
 - Units: Pa or N/m^2
2. **Kinetic Energy:** Energy due to the motion of the fluid.
 - Term: $\frac{1}{2}\rho V^2$
 - Units: Pa or N/m^2 (similar to dynamic pressure)
3. **Potential Energy:** Energy due to the elevation of the fluid relative to a datum.
 - Term: $\rho g z$
 - Units: Pa or N/m^2

B. Per Unit Weight (Head Form): This is the most common form in hydraulics, where each term represents a "head" (height) of fluid.

1. **Pressure Head:** Represents the height of a column of fluid that would produce the given pressure.
 - Term: $\frac{P}{\gamma} = \frac{P}{\rho g}$

- Units: meters of fluid (m) or feet of fluid (ft)
- 2. **Velocity Head:** Represents the vertical distance through which a fluid would have to fall to attain the velocity V from rest.
 - Term: $\frac{V^2}{2g}$
 - Units: meters (m) or feet (ft)
- 3. **Elevation Head (Potential Head):** Represents the potential energy of the fluid due to its height above a chosen datum.
 - Term: z
 - Units: meters (m) or feet (ft)

C. Per Unit Mass:

1. Pressure Energy: $\frac{P}{\rho}$
2. Kinetic Energy: $\frac{V^2}{2}$
3. Potential Energy: gz

II. Bernoulli's Theorem (Equation)

For **steady, incompressible, inviscid (frictionless) flow along a streamline**, the sum of the pressure head, velocity head, and elevation head is constant:

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}$$

Or between two points (1 and 2) on the same streamline:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Assumptions and Limitations of Bernoulli's Equation:

1. **Steady Flow:** Fluid properties at any point do not change with time.
2. **Incompressible Flow:** Density of the fluid remains constant ($\rho = \text{constant}$). Applies well to liquids and gases at low Mach numbers ($Ma < 0.3$).
3. **Inviscid Flow (Frictionless):** No viscous effects (no shear stresses or internal friction). This is the most significant limitation, as all real fluids have viscosity. This means no energy loss due to friction.
4. **Flow Along a Streamline:** The equation applies only along a single streamline. While it can often be applied between two points in a pipe, strictly speaking, it's along one path particle takes.

5. **No Energy Addition/Removal:** No pumps (add energy), turbines (remove energy), or heat transfer to/from the fluid.
6. **No Shaft Work:** No work done by or on the fluid by external mechanical devices.

III. Practical Applications of Bernoulli's Equation

Despite its ideal assumptions, Bernoulli's equation is incredibly useful for a wide range of engineering problems, especially when viscous effects are minor or can be accounted for separately.

A. Converging/Diverging Flows (Venturi Effect):

- As fluid flows from a wider section to a narrower section (where A decreases, V increases by continuity), the pressure must decrease to conserve energy. This is the **Venturi effect**.
- **Example:** Venturi meters (for flow rate measurement), carburetors, diffusers.

Example 1: Venturi Meter

- **Question:** Water flows through a horizontal pipe. At section 1, the pipe diameter is 10 cm and the pressure is 200 kPa. At section 2, the pipe diameter narrows to 5 cm. What is the velocity at section 2 and the pressure at section 2? Assume flow is ideal.
- **Solution:** Given: $D_1=0.1 \text{ m} \Rightarrow A_1=\pi(0.1/2)^2=0.00785 \text{ m}^2$ $P_1=200 \text{ kPa}=200,000 \text{ Pa}$ $D_2=0.05 \text{ m} \Rightarrow A_2=\pi(0.05/2)^2=0.00196 \text{ m}^2$ $\rho_{\text{water}}=1000 \text{ kg/m}^3$, $g=9.81 \text{ m/s}^2$ Since horizontal, $z_1=z_2$. First, use Continuity Equation to find V_2 in terms of V_1 : $A_1V_1=A_2V_2 \Rightarrow V_2=V_1 \frac{A_1}{A_2}=V_1 \frac{0.00785}{0.00196}=4V_1$

Now, apply Bernoulli's Equation ($P_1/\rho g + V_1^2/2g = P_2/\rho g + V_2^2/2g$): Since we have two unknowns (V_1 and P_2), we usually need one of the velocities or a pressure difference from a manometer. Let's assume we have a manometer reading that gives $P_1 - P_2$. *Self-correction:* The problem statement doesn't give V_1 . Let's assume $V_1=1 \text{ m/s}$ for demonstrative purposes. If $V_1=1 \text{ m/s}$, then $V_2=4 \times 1=4 \text{ m/s}$. Now, solve for P_2 : $1000 \times 9.81 \frac{200,000}{1000} + \frac{2 \times 9.81 (1)^2}{2} = 1000 \times 9.81 \frac{P_2}{1000} + \frac{2 \times 9.81 (4)^2}{2}$
 $20.387 + 0.051 = 9.81 P_2 + 0.815$ $20.438 = 9.81 P_2 + 0.815$ $9.81 P_2 = 20.438 - 0.815 = 19.623$ $P_2 = 19.623 \times 9.81 = 192,492 \text{ Pa} \approx 192.5 \text{ kPa}$

(Pressure decreased as velocity increased, confirming the Venturi effect).

B. Siphons:

- Bernoulli's equation can be used to determine flow rates and pressures at different points in a siphon, where fluid flows over an elevation higher than the free surface due to atmospheric pressure.

C. Orifice Flow (Torricelli's Law):

- For flow from a large tank through a small orifice, Bernoulli's equation simplifies to give the efflux velocity: $V = \sqrt{2gh}$
 - This is Torricelli's Law, directly applicable when the tank surface velocity is negligible and the pressure at the surface and orifice exit are atmospheric.

Example 2: Orifice from a tank

- **Question:** A large tank is open to the atmosphere and contains water to a depth of 5 m. A small orifice is located at the bottom of the tank. What is the velocity of the water exiting the orifice?
- **Solution:** Apply Bernoulli's equation between point 1 (free surface of the tank) and point 2 (at the orifice exit). $P_1 = P_{atm}$, $V_1 \approx 0$ (large tank), $z_1 = 5$ m (datum at orifice). $P_2 = P_{atm}$ (jet exposed to atmosphere), $V_2 = ?$, $z_2 = 0$. $\rho g P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = \rho g P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$ $\rho g P_{atm} + 0 + \rho g 5 = \rho g P_{atm} + \frac{1}{2} \rho V_2^2 + 0$ $5 = \frac{1}{2} V_2^2$ $V_2 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 5} = 9.90 \text{ m/s}$

IV. Bernoulli's Equation Correction (for real fluids - Energy Loss)

- In reality, **friction (viscous effects)** and **minor losses** due to fittings (valves, bends, expansions, contractions) always exist. These cause a reduction in the total mechanical energy of the fluid.
- To account for these losses, Bernoulli's equation is modified to include a **head loss term (h_L)**:

$$\rho g P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = \rho g P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2 + \rho g h_L$$

- h_L represents the total head loss due to friction and minor losses between points 1 and 2. It is always positive and represents energy converted into thermal energy.
- This modified equation is often called the **Energy Equation** or the **Extended Bernoulli Equation**.
- Detailed calculation of h_L involves concepts like Darcy-Weisbach equation for major losses and loss coefficients for minor losses, which will be covered in a later lecture.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. List the three main assumptions required for the direct application of the ideal Bernoulli's Equation.
2. Water flows from a large diameter pipe into a smaller diameter pipe. According to Bernoulli's principle, what happens to the pressure of the water in the smaller pipe compared to the larger pipe? Explain why.
3. A fluid flows in a horizontal pipe. At one point, the pressure is 150 kPa and the velocity is 2 m/s. At another point along the same pipe, the pressure is 120 kPa. What is the velocity at the second point? (Assume water, $\rho=1000 \text{ kg/m}^3$).
4. Why is the term " h_L " added to the right side of Bernoulli's equation for real fluids? What does it represent?
5. A tank has a small hole at its bottom. If the water level above the hole is 2 meters, what is the theoretical efflux velocity of the water, neglecting friction?

5 / Homework:

1. Water flows from a faucet at the top floor (10 m above the ground) of a building with a velocity of 0.5 m/s and a pressure of 200 kPa. Calculate the total head at this point.
2. A large water storage tank (open to atmosphere) is used to supply water to a pipe at a level 15 m below the tank's free surface. The pipe diameter is constant. Neglecting friction, what is the velocity of water in the pipe?
3. A pitot tube is used to measure the velocity of air. If the stagnation pressure is 150 Pa (gage) and the static pressure is 50 Pa (gage), what is the velocity of the air? (Density of air = 1.2 kg/m^3).

4. Explain how Bernoulli's principle is applied in the design of an airplane wing (airfoil) to generate lift.
5. Water flows through a horizontal pipe section where the diameter changes from 6 cm to 3 cm. The pressure in the larger section is 180 kPa and the velocity is 1.5 m/s. Determine the pressure in the smaller section, assuming ideal flow.

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 3)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 3)
- Fox and McDonald's Introduction to Fluid Mechanics.
- White's Fluid Mechanics.

**Ministry of high Education and Scientific Research
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Department of Chemical Industries Techniques**



Learning package

In

**Practical Applications of Bernoulli's Equation & Energy
Losses in Fluid Systems**

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, civil, mechanical, and chemical engineering, focusing on real-world applications of energy principles and accounting for energy losses.

1 / B – Rationale: While the ideal Bernoulli's Equation provides a foundational understanding of energy conservation in fluids, real-world fluid systems always involve energy losses due to friction and other factors. This lecture bridges the gap between ideal theory and practical application by introducing the extended Bernoulli Equation (Energy Equation) and methods to quantify these losses.

1 / C – Central Idea: The extended Bernoulli Equation (or Energy Equation) is a powerful tool for analyzing practical fluid flow problems by accounting for pressure, velocity, and elevation changes, as well as energy additions (pumps) and removals (turbines), and crucially, energy losses due to friction (major losses) and components (minor losses). Understanding how to calculate these losses is key to accurate system design and analysis.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Apply the extended Bernoulli Equation (Energy Equation) to solve practical fluid flow problems, including those with pumps, turbines, and energy losses.
- Identify and calculate major losses (friction losses) in pipes using the Darcy-Weisbach equation and the Moody chart.
- Understand the concept of minor losses due to pipe fittings, valves, expansions, and contractions, and calculate them using loss coefficients.
- Select and use appropriate friction factors and loss coefficients from charts or tables.

- Solve comprehensive pipeline problems that involve both major and minor losses.
- Analyze systems involving energy input (pumps) and energy output (turbines).

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is the primary reason why the pressure at the end of a long pipe is often lower than predicted by ideal Bernoulli's equation?
2. What is the purpose of a pump in a fluid system according to energy principles?
3. How does the roughness of a pipe's inner surface affect fluid flow?
4. Why do pipe fittings like elbows and valves cause "losses" in a fluid system?
5. If water flows from a large tank, will the actual velocity out of a small hole be exactly $2gh$? Why or why not?

3 / Lecture (Content):

Introduction: The ideal Bernoulli's Equation is a theoretical framework, invaluable for understanding fundamental principles. However, in practical engineering applications, no fluid is truly inviscid, and no system is perfectly efficient. This lecture introduces the necessary corrections to Bernoulli's equation to account for real-world phenomena like friction and energy transfers, leading to the more general Energy Equation.

I. The Extended Bernoulli Equation (Energy Equation)

The Energy Equation is a modified form of Bernoulli's Equation that includes terms for pumps, turbines, and energy losses:

$$\rho g P_1 + 2gV_1^2 + z_1 + h_P = \rho g P_2 + 2gV_2^2 + z_2 + h_T + h_L$$

Where:

- $\rho g P$: Pressure Head
- $2gV^2$: Velocity Head
- z : Elevation Head
- h_P : Head added by a pump (energy input to the fluid)

- h_T : Head extracted by a turbine (energy removed from the fluid)
- h_L : Total head loss (energy dissipated due to friction and minor losses)
- Subscripts 1 and 2 refer to upstream and downstream sections, respectively.

II. Energy Losses in Pipes (h_L)

Total head loss (h_L) consists of two main components: $h_L = h_{L,major} + h_{L,minor}$

A. Major Losses ($h_{L,major}$): Friction Losses in Straight Pipes

- These losses occur due to friction between the fluid and the pipe walls, and internal fluid friction.
- They are calculated using the **Darcy-Weisbach Equation**: $h_{L,major} = fDL \frac{V^2}{2g}$ Where:
 - f : Darcy friction factor (dimensionless)
 - L : Length of the pipe section (m or ft)
 - D : Inside diameter of the pipe (m or ft)
 - V : Average velocity of flow in the pipe (m/s or ft/s)
 - g : Acceleration due to gravity (m/s² or ft/s²)
- **Darcy Friction Factor (f):**
 - **For Laminar Flow ($Re < 2300$):** $f = \frac{64}{Re}$ (exact)
 - **For Turbulent Flow ($Re > 4000$):** f depends on the Reynolds number (Re) and the relative roughness (ϵ/D) of the pipe.
 - **Relative Roughness (ϵ/D):** ϵ is the absolute roughness (height of pipe wall irregularities, from tables for different materials), D is the pipe diameter.
 - **Moody Chart:** This is a graphical representation used to find f for turbulent flow based on Re and ϵ/D .
 - **Colebrook Equation:** An implicit equation that the Moody Chart is based on, used for more precise numerical calculations (often solved iteratively): $\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{2.51}{Re \sqrt{f}} + \frac{\epsilon/D}{3.7} \right)$
 - **Swamee-Jain Equation:** An explicit approximation of the Colebrook equation, easier for direct calculation: $f = \frac{0.25}{\left[\log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}} \right) \right]^2}$

B. Minor Losses ($h_{L,minor}$): Losses in Fittings and Valves

- These losses occur due to flow separation, turbulence, and secondary flows caused by changes in flow direction or area (bends, elbows, valves, entrances, exits, expansions, contractions).
- They are typically expressed in terms of a **loss coefficient (KL)** times the velocity head: $h_{L, \text{minor}} = K_L \frac{V^2}{2g}$ Where:
 - **KL:** Loss coefficient (dimensionless), specific to each fitting type (from tables or manufacturer data).
 - **V:** Average velocity in the pipe associated with the fitting (usually the downstream velocity for sudden expansion, or upstream for sudden contraction).
- **Common Minor Loss Sources:**
 - **Entrance Loss:** KL varies from 0.04 (well-rounded) to 0.5 (re-entrant) to 0.8 (sharp-edged).
 - **Exit Loss:** $K_L \approx 1.0$ (when flow discharges into a large tank). All kinetic energy is dissipated.
 - **Sudden Contraction:** KL depends on area ratio (A_2/A_1).
 - **Sudden Expansion:** $K_L = (1 - A_2/A_1)^2$ (based on V_1).
 - **Bends/Elbows:** KL depends on bend angle, radius of curvature, and pipe diameter.
 - **Valves:** KL varies significantly with valve type and degree of opening. (e.g., gate valve, globe valve, check valve).
- **Equivalent Length Method:** Minor losses can also be expressed as an equivalent length of straight pipe (L_e) that would cause the same friction loss: $K_L \frac{V^2}{2g} = f \frac{D L_e}{4D} \frac{V^2}{2g} \Rightarrow L_e = f K_L D$ Then, total pipe length for friction calculation becomes $L_{\text{total}} = L_{\text{actual}} + \sum L_e$.

III. Pumps (hP) and Turbines (hT)

- **Pumps:** Devices that add mechanical energy to the fluid, increasing its pressure or elevation. The head added by the pump, h_P , is the useful energy transferred to the fluid per unit weight.
 - Power input to fluid: $P_{\text{fluid}} = \dot{m} g h_P = \rho g V \dot{h}_P = \gamma V \dot{h}_P$
 - Pump efficiency (η_P): $\eta_P = \frac{\text{Power delivered to fluid}}{\text{Electrical power input to pump}}$
- **Turbines:** Devices that extract mechanical energy from the fluid, converting it into shaft work (e.g., to generate electricity). The head extracted by the turbine, h_T , is the useful energy removed from the fluid per unit weight.
 - Power output from fluid: $P_{\text{out}} = \dot{m} g h_T = \rho g V \dot{h}_T = \gamma V \dot{h}_T$

- Turbine efficiency (η_T): $\eta_T = \frac{\text{Power extracted from fluid}}{\text{Electrical power output from turbine}}$

IV. Pipeline Problems (Applications of Energy Equation)

A. Determining Pressure Drop: Given flow rate, pipe geometry, and fluid properties, calculate pressure change between two points. **B. Determining Flow Rate:** Given pressure difference, pipe geometry, and fluid properties, calculate the flow rate (often requires iterative solution due to friction factor dependence on velocity/Reynolds number). **C. Determining Pipe Diameter:** Given flow rate, allowable pressure drop, and fluid properties, calculate the required pipe diameter (highly iterative).

Example 1: Pump in a system

- **Question:** Water is pumped from a lower reservoir (surface at $z_1=5$ m) to an upper reservoir (surface at $z_2=25$ m) through a 10 cm diameter pipe. The flow rate is $0.01 \text{ m}^3/\text{s}$. The total head loss in the pipe system (including friction and minor losses) is estimated to be 5 m. Calculate the head that must be added by the pump.
- **Solution:** Apply the extended Bernoulli Equation between the free surfaces of the two reservoirs (points 1 and 2). $P_1=P_2=P_{\text{atm}}$ (open to atmosphere), so $P_1/\rho g=P_2/\rho g=0$ (gage pressure). $V_1 \approx 0$, $V_2 \approx 0$ (large reservoirs, surface velocities are negligible). $z_1=5$ m $z_2=25$ m $h_L=5$ m $h_T=0$ (no turbine)

$$\rho g P_1 + 2g V_1^2 + z_1 + h_P = \rho g P_2 + 2g V_2^2 + z_2 + h_T + h_L \quad 0 + 0 + z_1 + h_P = 0 + 0 + z_2 + 0 + h_L \quad h_P = (z_2 - z_1) + h_L \quad h_P = (25 \text{ m} - 5 \text{ m}) + 5 \text{ m} \quad h_P = 20 \text{ m} + 5 \text{ m} = 25 \text{ m}$$

The pump must provide 25 meters of head to the water.

Example 2: Flow rate calculation with friction

- **Question:** Water flows from a large tank, through a 50 m long, 2 cm diameter smooth pipe, and exits to the atmosphere. The water level in the tank is 10 m above the pipe exit. Calculate the volume flow rate.
- **Solution:** Apply Energy Equation between point 1 (tank surface) and point 2 (pipe exit). $P_1=P_{\text{atm}}$, $V_1 \approx 0$, $z_1=10$ m (datum at exit). $P_2=P_{\text{atm}}$, $z_2=0$. $h_P=0$, $h_T=0$. $\rho g P_{\text{atm}} + 0 + 10 = \rho g P_{\text{atm}} + 2g V_2^2 + 0 + h_L \quad 10 = 2g V_2^2 + h_L$ Here, $h_L = f D L 2g V_2^2 + K_{L,\text{entrance}} 2g V_2^2$ (assuming sharp-edged entrance, $K_{L,\text{entrance}}=0.5$). $h_L = (f D L + K_{L,\text{entrance}}) 2g V_2^2$

$10 = (f \cdot 0.0250 + 0.5) 2gV^2$ This requires iteration as f depends on V (via Reynolds number).

1. **Assume Laminar Flow:** $Re = 2000$. $f = 64/2000 = 0.032$.
 $10 = (0.032 \cdot 0.0250 + 0.5) 2 \times 9.81 V^2 = (80 + 0.5) 19.62 V^2$
 $= 80.519.62 V^2$ $V^2 = 80.510 \times 19.62 = 2.437 \Rightarrow V = 1.56 \text{ m/s}$
 Check Re : $Re = vVD = 1 \times 10^{-6} \cdot 1.56 \times 0.02 = 31200$ (for water at typical temp, $v = 10^{-6} \text{ m}^2/\text{s}$). This is turbulent, so laminar assumption is wrong.
2. **Assume Turbulent Flow:** Need Moody Chart or Colebrook/Swamee-Jain. Let's assume a smooth pipe ($\epsilon/D = 0$). For $Re = 31200$ (from previous step), from Moody chart for smooth pipe, $f \approx 0.022$.
 $10 = (0.022 \cdot 0.0250 + 0.5) 19.62 V^2$
 $= (55 + 0.5) 19.62 V^2 = 55.519.62 V^2$ $V^2 = 55.510 \times 19.62$
 $= 3.535 \Rightarrow V = 1.88 \text{ m/s}$ Check Re : $Re = 1 \times 10^{-6} \cdot 1.88 \times 0.02 = 37600$. This new Re leads to a slightly different f . Iterate until V converges. (A few iterations would lead to $V \approx 1.88 \text{ m/s}$ and $f \approx 0.021$).
 Finally, $Q = AV$
 $= \pi(0.01)^2 \times 1.88 = 0.00059 \text{ m}^3/\text{s} = 0.59 \text{ L/s}$.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. Write down the full extended Bernoulli Equation and identify each term.
2. Water is pumped at a rate of $0.05 \text{ m}^3/\text{s}$ from a lower tank to an upper tank. If the elevation difference between the free surfaces is 10 m and the total head loss in the piping system is 2 m , what is the minimum head the pump must provide?
3. A 100 m long, 5 cm diameter commercial steel pipe ($\epsilon = 0.045 \text{ mm}$) carries water at 2 m/s . Calculate the major head loss due to friction. (For water, $v \approx 10^{-6} \text{ m}^2/\text{s}$). You may need a Moody chart or a suitable explicit friction factor correlation.
4. Explain why a sudden expansion in a pipe causes a minor loss, and how this loss is typically calculated.
5. If a fluid flows from a pipe into a large reservoir, what is the value of the exit loss coefficient (K_L)? Why?

5 / Homework:

1. Oil ($SG=0.88$, $\mu=0.01 \text{ Pa}\cdot\text{s}$) flows through a 200 m long, 8 cm diameter smooth pipe at a rate of $0.005 \text{ m}^3/\text{s}$. Calculate the major head loss due to friction.
2. A water pipe system includes a 90° standard elbow ($K_L=0.9$), a gate valve (fully open, $K_L=0.2$), and a sudden contraction from a 10 cm pipe to a 5 cm pipe ($K_L=0.3$ based on smaller pipe velocity). If the velocity in the 5 cm pipe is 3 m/s, calculate the total minor losses in this section.
3. Water flows from a reservoir A to reservoir B through a 200 m long, 15 cm diameter cast iron pipe ($\epsilon=0.26 \text{ mm}$). The elevation of reservoir A is 50 m, and reservoir B is 30 m. There is a gate valve ($K_L=0.2$) and a sudden expansion ($K_L=0.1$ at exit, based on pipe velocity) at the pipe exit into reservoir B. The entrance from A is sharp-edged ($K_L=0.5$). Calculate the flow rate in the pipe. (This will be an iterative problem. Start by assuming a friction factor or velocity).
4. A pump delivers $0.02 \text{ m}^3/\text{s}$ of water at 20°C from a tank to a discharge point 15 m higher. The suction pipe is 8 cm diameter and 5 m long. The discharge pipe is 6 cm diameter and 20 m long. All pipes are commercial steel. There is a sharp-edged entrance, a fully open gate valve in the discharge line, and a sharp-edged exit. Calculate the required pump power if the pump efficiency is 75%.
5. Discuss the significance of the Moody chart in engineering practice. What information do you need to use it effectively?

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapters 5, 6, & 7)
- Fluid Mechanics by Cengel and Cimbala. (Chapters 5, 6, & 8)
- Fox and McDonald's Introduction to Fluid Mechanics.
- White's Fluid Mechanics.
- Engineering Toolbox (for roughness values, K_L values).

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Bernoulli's Equation Correction (The Energy Equation)

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, transitioning from ideal fluid flow analysis to the more realistic consideration of energy transformations and losses in fluid systems.

1 / B – Rationale: While the ideal Bernoulli's Equation is fundamental, it does not account for energy losses due to viscosity, friction, turbulence, or energy additions/extractions by machinery (pumps, turbines). The "corrected" or "Extended" Bernoulli's Equation, often called the Energy Equation, is essential for analyzing real-world fluid systems.

1 / C – Central Idea: The Energy Equation is a more comprehensive form of Bernoulli's principle that includes terms for head added by pumps, head extracted by turbines, and head losses due to friction and minor components. It serves as the primary tool for analyzing energy conservation in practical fluid flow systems.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Identify the limitations of the ideal Bernoulli's Equation in real-world scenarios.
- State and explain each term in the Extended Bernoulli's Equation (the Energy Equation).
- Understand the physical meaning of head added by a pump (h_P), head removed by a turbine (h_T), and head loss (h_L).
- Apply the Energy Equation to solve basic problems involving pumps, turbines, and head losses in piping systems.
- Distinguish between ideal and real fluid behavior in terms of energy considerations.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is a key assumption made when using the ideal Bernoulli's Equation that is often not true in real fluid flow?
2. In a real pipe, if water flows from point A to point B, will the sum of pressure head, velocity head, and elevation head be conserved? Why or why not?
3. What is the purpose of a pump in a fluid system in terms of energy?
4. What is "friction" in fluid flow, and how does it relate to energy?
5. What does "head" mean in the context of fluid mechanics?

3 / Lecture (Content):

Introduction: In the previous lecture, we explored the ideal Bernoulli's Equation and its applications in simplified scenarios. While powerful for ideal cases, real fluids possess viscosity, leading to friction and turbulence. Furthermore, many engineering systems include devices like pumps that add energy or turbines that extract energy. To account for these real-world phenomena, we introduce the **Energy Equation**, which is essentially a more comprehensive form of Bernoulli's principle.

I. Limitations of the Ideal Bernoulli's Equation

The ideal Bernoulli's Equation ($\rho gP + 2gV^2 + z = \text{constant}$) relies on several key assumptions:

- **Inviscid flow:** No friction between fluid layers or between the fluid and solid boundaries.
- **Steady flow:** Flow properties do not change with time at a fixed point.
- **Incompressible flow:** Fluid density remains constant.
- **Flow along a streamline:** The equation is applied between two points on the same streamline.
- **No shaft work (pumps or turbines):** No external energy addition or extraction.
- **No heat transfer:** No energy is added or removed as heat.

In reality, all real fluids are viscous, and flow is rarely perfectly steady or truly incompressible over large pressure changes. The most significant departure from the ideal equation comes from **viscous effects (friction)** and the presence of **fluid machinery**.

II. The Energy Equation (Extended Bernoulli's Equation)

The Energy Equation extends Bernoulli's principle by incorporating terms for energy inputs (pumps), energy outputs (turbines), and energy losses (friction). It can be written as:

$$\rho g P_1 + 2gV_1^2 + z_1 + h_P = \rho g P_2 + 2gV_2^2 + z_2 + h_T + h_L$$

Where:

- **Subscripts 1 and 2** refer to the upstream and downstream sections of the flow, respectively.
- **$\rho g P$** : Pressure head (energy due to pressure).
- **$2gV^2$** : Velocity head (energy due to kinetic motion).
- **z** : Elevation head (energy due to potential height).

New Terms Explained:

1. **h_P (Head Added by a Pump):**

- Represents the energy (in terms of head or meters of fluid) that a pump adds to the fluid between points 1 and 2.
- This term is positive if a pump is present and adds energy to the flow.
- **Power of the pump transferred to the fluid (P_{fluid}):** $P_{\text{fluid}} = \dot{m} g h_P = \rho g Q h_P = \gamma Q h_P$
 - $\dot{m}$: Mass flow rate (kg/s)
 - Q : Volume flow rate (m^3/s)
 - γ : Specific weight of the fluid (ρg) (N/m^3)
- **Pump Efficiency (η_P):** Pumps are not 100% efficient; some input power is lost (e.g., to heat, vibration).
 - $\eta_P = P_{\text{input}} / P_{\text{fluid}} = P_{\text{input}} / \rho g Q h_P$

2. **h_T (Head Removed by a Turbine):**

- Represents the energy (in terms of head) that a turbine extracts from the fluid between points 1 and 2.
- This term is positive if a turbine is present and extracts energy from the flow.
- **Power extracted from the fluid by the turbine (P_{fluid}):** $P_{\text{fluid}} = \dot{m} g h_T = \rho g Q h_T = \gamma Q h_T$
- **Turbine Efficiency (η_T):** Turbines also have losses.
 - $\eta_T = P_{\text{fluid}} / P_{\text{output}} = \rho g Q h_T / P_{\text{output}}$

3. **h_L (Head Loss):**

- Represents the energy dissipated (lost) from the fluid due to viscous effects, friction, and turbulence as it flows from point 1 to point 2. This energy is typically converted into heat.
- This term is always positive, as it represents a loss from the fluid's mechanical energy.
- **Causes of Head Loss:**
 - **Major Losses ($h_{L,major}$):** Due to friction along the length of straight pipes. These depend on pipe length, diameter, roughness, and fluid velocity.
 - **Minor Losses ($h_{L,minor}$):** Due to flow separation and turbulence caused by pipe fittings (elbows, valves, tees), sudden expansions/contractions, entrances, and exits.
- **Total Head Loss:** $h_L = h_{L,major} + h_{L,minor}$
- *Detailed calculation methods for major and minor losses will be covered in the next lectures.*

III. Applying the Energy Equation: General Steps

1. **Select Two Points:** Choose an upstream (1) and downstream (2) point where conditions (pressure, velocity, elevation) are known or easily determinable. Often, these are free surfaces of tanks, pump/turbine inlets/outlets, or pipe sections.
2. **Draw a Sketch:** Clearly label points 1 and 2, flow direction, elevations (z_1, z_2), and any machinery (pumps/turbines).
3. **Write the Energy Equation:** $\rho g P_1 + 2gV_1^2 + z_1 + h_P = \rho g P_2 + 2gV_2^2 + z_2 + h_T + h_L$
4. **Simplify Terms:**
 - If a point is a free surface of a large tank, $V \approx 0$.
 - If pressures are atmospheric at both points, $P_1 = P_2 = P_{atm}$ (or use gauge pressure where $P = 0$).
 - If no pump, $h_P = 0$. If no turbine, $h_T = 0$.
 - Establish a common datum for elevations ($z = 0$).
5. **Calculate Velocities:** Use the continuity equation ($A_1 V_1 = A_2 V_2$) if pipe diameters change.
6. **Account for h_P, h_T, h_L :** Include these terms if present. For this lecture, h_L might be given or a basic understanding that it exists.
7. **Solve for the Unknown:** Isolate the desired variable (e.g., pressure, flow rate, required pump head).

Example: Water is pumped from a lower reservoir to an upper reservoir through a pipe.

- Point 1: Surface of the lower reservoir ($P_1 = P_{\text{atm}}, V_1 \approx 0, z_1 = 0$).
- Point 2: Surface of the upper reservoir ($P_2 = P_{\text{atm}}, V_2 \approx 0, z_2 = \Delta z$).
- A pump (h_P) is in the system, and there are head losses (h_L). Applying the Energy Equation: $0 + 0 + 0 + h_P = 0 + 0 + \Delta z + h_L$ $h_P = \Delta z + h_L$ This simple example shows that the pump must provide enough head to overcome the elevation difference and all system losses.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. Write down the full Energy Equation and identify each term.
2. If a fluid flows downhill in a pipe, why might its pressure actually decrease even if its elevation decreases?
3. A pump adds 50 m of head to water flowing at $0.01 \text{ m}^3/\text{s}$. What is the hydraulic power delivered by the pump to the water? (Assume $\rho = 1000 \text{ kg/m}^3, g = 9.81 \text{ m/s}^2$).
4. Explain the difference between h_P and h_T in the Energy Equation.
5. In a pipe connecting two reservoirs, if the surfaces are at the same elevation and there's no pump or turbine, what does the Energy Equation tell you about the head loss?

5 / Homework:

1. Water is pumped from a lake to a tank 20 m above the lake surface. The pump delivers $0.05 \text{ m}^3/\text{s}$ of water. If the total head loss in the piping system is 5 m, calculate the head that the pump must provide.
2. An old hydroelectric power plant utilizes a turbine at the base of a dam. Water flows from a reservoir surface (elevation 100 m) through a penstock to the turbine outlet (elevation 5 m). If the total head loss in the penstock is 8 m and the turbine extracts 80 m of head, what is the pressure at the turbine outlet if the flow velocity is 2 m/s ? (Assume reservoir surface velocity is negligible, and outlet discharges to atmosphere).
3. Explain why the efficiency of a pump is always less than 100%.
4. A pipe of constant diameter carries water from point A to point B. If the pressure at A is 200 kPa and at B is 150 kPa, and point B is 10 m higher

- than point A, what is the head loss between A and B? (Assume negligible velocity change).
5. Discuss two scenarios where ignoring head loss in the Energy Equation would lead to significantly incorrect results.

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 7)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 8)
- Fox and McDonald's Introduction to Fluid Mechanics. (Chapter 3 & 6)

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Energy Loss in Pipes – Minor Losses (Fittings and Valves)

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, focusing on the methods to calculate energy losses caused by pipe components other than straight sections.

1 / B – Rationale: While major losses dominate in long pipelines, minor losses can be significant or even predominant in systems with many fittings, valves, or sudden changes in pipe geometry. Accurate calculation of these losses is crucial for precise hydraulic design.

1 / C – Central Idea: Minor head losses are due to local disturbances in flow caused by fittings, valves, and sudden changes in pipe area. These losses are typically calculated using a loss coefficient (KL) multiplied by the velocity head, reflecting the energy dissipated through flow separation and turbulent mixing.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Define minor head loss and explain its causes.
- Apply the loss coefficient (KL) method to calculate minor losses for various pipe fittings and components.
- Identify common types of minor loss components (e.g., entrances, exits, elbows, valves, expansions, contractions).
- Distinguish when minor losses are significant compared to major losses.
- Calculate the total head loss in a piping system by combining major and minor losses.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. Name three common pipe components that would cause a "minor" loss.

2. Do minor losses refer to small magnitudes of loss, or a specific type of loss? Explain.
3. How does a sudden change in pipe diameter affect the fluid flow visually (e.g., streamlines)?
4. If a pipe system is very long, are minor losses generally more or less important than major losses?
5. What is the energy type that is converted into heat due to head losses?

3 / Lecture (Content):

Introduction: In the previous lecture, we focused on **major head losses**, which occur due to friction along the length of straight pipes. However, real piping systems contain numerous components such as valves, elbows, tees, expansions, and contractions. These components disrupt the smooth flow, causing turbulence, flow separation, and eddy formation, all of which lead to additional energy dissipation. These losses are termed **minor losses**.

I. Definition and Causes of Minor Losses

- **Definition:** Minor losses (or form losses) are the head losses that occur at localized pipe components where the flow path or cross-sectional area changes.
- **Causes:** They are primarily caused by:
 - **Flow Separation:** When the flow detaches from the pipe wall due to sharp corners or abrupt changes in geometry.
 - **Turbulent Mixing:** The formation of eddies and swirling flow regions that dissipate mechanical energy into heat.
 - **Changes in Velocity Magnitude or Direction:** Any component that forces the fluid to accelerate, decelerate, or change direction will induce minor losses.
- **Why "Minor"?** The term "minor" is historical. In very long pipe systems, the cumulative major losses often dwarf the minor losses. However, in short pipe systems (e.g., manifold systems, plumbing in a building) or systems with many fittings, minor losses can be equal to or even greater than major losses.

II. The Loss Coefficient Method (KL)

The most common method for calculating minor losses is using a **loss coefficient (KL)**, sometimes denoted as C:

$$h_{L, \text{minor}} = K_L \frac{V^2}{2g}$$

Where:

- $h_{L, \text{minor}}$: Minor head loss (in meters or feet of fluid) for a specific component.
- K_L : The dimensionless **loss coefficient** for that particular component.
- V : The average flow velocity in the pipe where the loss is occurring. It is crucial to use the correct velocity. For expansions/contractions, K_L values are often given with respect to the velocity in the *smaller* pipe (for contraction) or the *upstream* pipe (for expansion). For other fittings, it's typically the velocity in the pipe connected to the fitting.
- g : Acceleration due to gravity.

A. Typical Minor Loss Components and K_L Values

Loss coefficients (K_L) are typically determined experimentally and are usually found in tables in fluid mechanics textbooks or engineering handbooks. They depend on the specific geometry and type of fitting.

1. **Pipe Entrances:** Where fluid enters a pipe from a large reservoir.
 - **Sharp-edged entrance:** $K_L \approx 0.5$ (significant loss due to flow contraction and expansion).
 - **Reentrant entrance:** $K_L \approx 0.8$ (highest loss due to flow separation inside the pipe).
 - **Well-rounded entrance:** $K_L \approx 0.04$ (minimal loss, streamlines are smooth).
2. **Pipe Exits:** Where fluid discharges from a pipe into a large reservoir.
 - **Submerged exit/Free discharge:** $K_L \approx 1.0$. All the kinetic energy ($\frac{V^2}{2g}$) of the flow is dissipated as it mixes with the static fluid in the reservoir.
3. **Sudden Expansion:** A sudden increase in pipe diameter.
 - Formula (based on upstream velocity V_1): $K_L = \left(1 - \frac{A_2}{A_1}\right)^2 = \left(1 - \frac{D_2^2}{D_1^2}\right)^2$
 - Loss due to large eddies forming in the larger pipe.
4. **Sudden Contraction:** A sudden decrease in pipe diameter.
 - K_L depends on the area ratio A_2/A_1 or D_2/D_1 and typically ranges from 0.05 to 0.5. Values are usually tabulated.
 - Losses occur as the flow contracts at the entrance of the smaller pipe (vena contracta) and then expands to fill the smaller pipe.

5. **Bends and Elbows:** Components that change the direction of flow.
- KL depends on the angle of the bend, the radius of curvature (r/D ratio), and the type of elbow (e.g., standard, long-radius, mitered).
 - Example: A standard 90-degree threaded elbow typically has $KL \approx 0.9$. A long-radius elbow has a lower KL due to smoother turning.
6. **Valves:** Devices used to regulate flow.
- KL varies significantly with valve type (e.g., gate valve, globe valve, check valve, ball valve) and its degree of opening.
 - A fully open gate valve has a very small KL (e.g., 0.1–0.2), while a fully open globe valve can have a very high KL (e.g., 10). Closing a valve significantly increases KL.

B. Velocity Used in Minor Loss Calculations It's critical to use the appropriate velocity (V) in the $h_{L, \text{minor}} = KL \frac{\rho V^2}{2}$ formula. Generally, V refers to the average velocity in the pipe where the fitting is located. For components connecting two different pipe sizes, the KL value is usually referenced to the velocity in the *smaller* pipe (for contractions) or the *upstream* pipe (for expansions), or it's specified in the table. Always check the convention for the given KL value.

III. Equivalent Length Method (L_e/D)

An alternative (but less common for general analysis) method is the equivalent length method. Here, the minor loss of a fitting is expressed as an equivalent length of straight pipe that would produce the same major loss at the same flow conditions:

$$h_{L, \text{minor}} = f D L_e \frac{\rho V^2}{2}$$

Where:

- L_e : Equivalent length of straight pipe.
- f : Friction factor for the pipe (can be difficult to determine as it depends on flow regime).

From this, we can relate KL to L_e/D : $KL = f L_e/D$. This method is sometimes used in preliminary design or for complex systems where it's convenient to add all losses as an "effective total length." However, L_e/D values are not constant and depend on f , which itself depends on Reynolds number.

IV. Total Head Loss in a System

For a complete piping system, the total head loss (h_L) is the sum of all major and minor losses:

$$h_L = \sum h_{L,\text{major}} + \sum h_{L,\text{minor}}$$

Or, expanding the terms:

$$h_L = \sum (f D L 2g V^2) + \sum (K_L 2g V^2)$$

If the pipe diameter (and thus velocity) is constant throughout a significant portion of the system, this can often be simplified to:

$$h_L = (f D L_{\text{total}} + \sum K_L) 2g V^2$$

Where L_{total} is the total actual length of the straight pipes. This simplification assumes a constant V for all K_L values, which might not be strictly true if there are pipe size changes within the system.

Example: Water flows at 2 m/s through a 5 cm diameter smooth pipe ($\epsilon \approx 0$) which includes a sharp-edged entrance, 10 m of straight pipe, a standard 90-degree elbow, and a fully open gate valve. Calculate the total head loss. (Assume $\rho = 1000 \text{ kg/m}^3$, $\mu = 10^{-3} \text{ Pa}\cdot\text{s}$, $K_{L,\text{entrance}} = 0.5$, $K_{L,\text{elbow}} = 0.9$, $K_{L,\text{valve}} = 0.2$).

1. **Calculate Reynolds Number:** $Re = \mu \rho V D = 10^{-3} 1000 \times 2 \times 0.05 = 100,000$ (Turbulent).
2. **Calculate Major Loss (f):** Since it's a smooth pipe and turbulent, use a Moody Chart or Haaland equation (for smooth pipe $\epsilon/D \approx 0$): For $Re = 100,000$, $f \approx 0.018$. $h_{L,\text{major}} = f D L 2g V^2 = 0.018 \times 0.05 \times 10 \times 2 \times 9.81 \times 2^2 = 0.018 \times 200 \times 19.624 = 0.018 \times 200 \times 0.2039 = 0.734 \text{ m}$
3. **Calculate Minor Losses:**
 $h_{L,\text{entrance}} = K_{L,\text{entrance}} 2g V^2 = 0.5 \times 2 \times 9.81 \times 2^2 = 0.5 \times 0.2039 = 0.102 \text{ m}$
 $h_{L,\text{elbow}} = K_{L,\text{elbow}} 2g V^2 = 0.9 \times 2 \times 9.81 \times 2^2 = 0.9 \times 0.2039 = 0.184 \text{ m}$
 $h_{L,\text{valve}} = K_{L,\text{valve}} 2g V^2 = 0.2 \times 2 \times 9.81 \times 2^2 = 0.2 \times 0.2039 = 0.041 \text{ m}$
 $\sum h_{L,\text{minor}} = 0.102 + 0.184 + 0.041 = 0.327 \text{ m}$
4. **Total Head Loss:** $h_L = h_{L,\text{major}} + \sum h_{L,\text{minor}} = 0.734 + 0.327 = 1.061 \text{ m}$

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. List three types of pipe fittings that typically cause minor losses.
2. Explain the meaning of the loss coefficient (KL). Why is it dimensionless?
3. For a sudden expansion from diameter D1 to D2, if $D_2=2D_1$, what is the loss coefficient KL (based on V1)?
4. In what type of piping system (long or short) are minor losses likely to be more significant compared to major losses?
5. What happens to the kinetic energy of the fluid when it encounters a minor loss component like a sharp-edged entrance?

5 / Homework:

1. Water flows through a 3 cm diameter pipe system. The system includes a sharp-edged entrance, two standard 90-degree elbows, a fully open gate valve, and a submerged exit. The total length of the straight pipe is 25 m. If the water velocity is 1.8 m/s, calculate the total head loss in the system. (Use $\rho=1000 \text{ kg/m}^3$, $\mu=10^{-3} \text{ Pa}\cdot\text{s}$. Obtain KL values from typical tables or use the values provided in the lecture examples for general KL if not specified.)
2. Compare the head loss caused by a fully open gate valve ($KL=0.2$) with that of a fully open globe valve ($KL=10$) for the same pipe and flow conditions. Which valve is more restrictive to flow?
3. A 10 cm diameter pipe suddenly expands to a 20 cm diameter pipe. If the velocity in the 10 cm pipe is 3 m/s, calculate the head loss due to this sudden expansion.
4. Explain why a well-rounded pipe entrance has a much lower loss coefficient than a sharp-edged entrance.
5. A pumping system needs to deliver water from a lower reservoir to an upper reservoir. The total elevation difference is 15 m. The pipeline is 50 m long, 10 cm diameter, commercial steel. It includes a sharp-edged entrance, two 90-degree standard elbows, and a fully open gate valve. The flow rate required is $0.015 \text{ m}^3/\text{s}$. Calculate the total head loss (hL) in the system. (You will need to calculate Reynolds number and friction factor f as well as all minor losses. $\epsilon_{\text{steel}}=0.045 \text{ mm}$).

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 8)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 8)

- Fox and McDonald's Introduction to Fluid Mechanics. (Chapter 6)

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package
In
Pumps: Pump Types

For
Students of First Year

By
Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

- **1 / A – Target Population:** Undergraduate engineering students in fluid mechanics, mechanical, civil, chemical, and industrial engineering, interested in fluid machinery and system design.
- **1 / B – Rationale:** Pumps are critical components in countless engineering systems, responsible for moving fluids and increasing their energy. Understanding the various types of pumps, their operating principles, and their suitable applications is essential for effective system design, selection, and troubleshooting.
- **1 / C – Central Idea:** Pumps are mechanical devices that transfer energy to a fluid, enabling it to flow against resistance (like gravity or pressure differences). They are broadly classified into two main categories: dynamic (rotodynamic) pumps and positive displacement pumps, each operating on distinct principles and suitable for different fluid properties and flow conditions.
- **1 / D – Performance Objectives:** Upon completion of this lecture, students will be able to:
 - Define the function and purpose of a pump in a fluid system.
 - Classify pumps into their main categories: dynamic (rotodynamic) and positive displacement.
 - Describe the operating principles of common dynamic pumps, particularly centrifugal pumps, and their key components.
 - Describe the operating principles of common positive displacement pumps, such as reciprocating and rotary pumps.
 - Identify the typical applications and advantages/disadvantages of different pump types.
 - Understand key pump performance parameters like head, flow rate, power, and efficiency.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is the main function of a pump in a water supply system?
2. Can a pump create mass?
3. If a pump is described as "high head, low flow," what does that mean intuitively?
4. What is the difference between increasing the pressure of a fluid and increasing its velocity using a pump?
5. Name one type of pump you might find in a typical household.

3 / Lecture (Content):

Introduction: Pumps are ubiquitous in modern society, driving fluid movement in everything from municipal water supplies and oil pipelines to HVAC systems and industrial processes. They are energy conversion devices that transform mechanical energy (from a motor or engine) into fluid energy (increasing its pressure, velocity, or elevation). This lecture will introduce the fundamental principles of pumps and classify them into their main types.

I. Basic Function and Principle of a Pump

- **Function:** To add energy to a fluid to overcome resistance to flow (e.g., elevation changes, friction losses, pressure differences) and/or to move a fluid from one location to another.
- **Energy Addition:** In the Extended Bernoulli Equation, the pump adds a head (h_P) to the fluid: $\rho g P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 + h_P = \rho g P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2 + h_L + h_T$
- **Pump Power:** The power transferred to the fluid by the pump is given by: $P_{\text{fluid}} = \dot{m} g h_P = \rho g Q h_P = \gamma Q h_P$ Where: Q is the volume flow rate ($V \cdot$).
- **Pump Efficiency (η_P):** The ratio of hydraulic power delivered to the fluid to the mechanical/electrical power input to the pump. $\eta_P = \frac{P_{\text{fluid}}}{P_{\text{input}}}$

II. Main Classification of Pumps

Pumps are broadly classified based on the mechanism by which they transfer energy to the fluid:

A. Dynamic (Rotodynamic) Pumps

- These pumps use a rotating impeller (rotor) to impart kinetic energy to the fluid, which is then converted into pressure energy.

- They provide a continuous flow with no pulsations.
- Most common type for handling large flow rates with relatively low to moderate heads.
- **Key Characteristics:** Continuous flow, suitable for low viscosity fluids, sensitive to system resistance, efficiency varies with flow rate.

1. Centrifugal Pumps:

- **Principle:** Fluid enters the eye of the impeller, is accelerated radially outwards by centrifugal force, and then collects in a volute casing (or diffuser) where the high velocity is converted into high pressure.
- **Components:**
 - **Impeller:** The rotating part with vanes that imparts energy to the fluid.
 - **Casing (Volute or Diffuser):** Collects the fluid from the impeller and converts kinetic energy into pressure energy.
 - **Shaft & Bearings:** Support and transmit power to the impeller.
 - **Stuffing Box/Mechanical Seal:** Prevents leakage where the shaft exits the casing.
- **Types of Impellers:**
 - **Open:** Vanes attached only to the hub. Suitable for fluids with suspended solids.
 - **Semi-open:** Vanes attached to the hub and one shroud. For fluids with some solids.
 - **Closed:** Vanes enclosed by two shrouds. Most efficient, for clear liquids.
- **Applications:** Water supply, HVAC, chemical processing, wastewater treatment, irrigation, oil and gas.
- **Advantages:** Simple design, smooth (non-pulsating) flow, wide range of flow rates, relatively low cost.
- **Disadvantages:** Lower efficiency for high heads/low flows, prone to cavitation at high suction lifts, cannot handle high viscosity fluids well.

2. Axial Flow Pumps (Propeller Pumps):

- **Principle:** Fluid flows axially (parallel to the shaft) through an impeller, which acts like a propeller, adding energy to the fluid primarily by generating lift.
- **Characteristics:** High flow rates, very low heads.

- **Applications:** Drainage, flood control, large-volume circulation (e.g., power plant cooling water).

3. **Mixed Flow Pumps:**

- **Principle:** A hybrid between centrifugal and axial flow, with elements of both radial and axial flow.
- **Characteristics:** Intermediate head and flow characteristics.
- **Applications:** Irrigation, municipal water supply.

B. Positive Displacement Pumps

- These pumps trap a fixed volume of fluid and then force (displace) it into the discharge pipe.
- They provide a pulsating flow (unless multiple elements are used) and are capable of generating very high pressures.
- **Key Characteristics:** Self-priming, constant flow rate against varying pressure (within limits), suitable for high viscosity fluids, can be used for metering.

1. **Reciprocating Pumps:**

- **Principle:** Use a piston, plunger, or diaphragm that moves back and forth (reciprocates) to create a vacuum to draw fluid in and then push it out.
- **Types:**
 - **Piston/Plunger Pumps:** Use a solid cylinder (piston) or rod (plunger) to displace fluid.
 - **Diaphragm Pumps:** Use a flexible diaphragm to create the pumping action, useful for corrosive or abrasive fluids as fluid does not contact moving parts directly.
- **Characteristics:** High pressure, low to moderate flow rates, pulsating flow (can be reduced with multiple cylinders or dampeners), self-priming.
- **Applications:** High-pressure washing, metering pumps, hydraulic power, oil well pumping.
- **Advantages:** High pressure capability, good for high viscosity fluids, self-priming.
- **Disadvantages:** Pulsating flow, complex design, higher maintenance, limited flow rates.

2. **Rotary Pumps:**

- **Principle:** Use rotating elements (gears, lobes, screws, vanes) to trap and displace fluid. They have no reciprocating parts.

- **Types:**
 - **Gear Pumps (External/Internal):** Use meshing gears to trap and transport fluid.
 - **Lobe Pumps:** Similar to gear pumps but with fewer, larger lobes. Good for handling solids.
 - **Screw Pumps:** Use one or more screws to move fluid axially. Can handle very high viscosity fluids.
 - **Vane Pumps:** Use vanes that slide in and out of a rotor to create varying volumes.
 - **Peristaltic Pumps:** Squeeze a flexible tube to move fluid, good for sterile or corrosive fluids.
- **Characteristics:** Pulsation-free (or low pulsation), moderate to high pressure, self-priming, can handle viscous fluids.
- **Applications:** Lubrication systems, fuel injection, hydraulic systems, food processing, chemical transfer.
- **Advantages:** Smooth flow (many types), good for viscous fluids, self-priming, precise flow control (metering).
- **Disadvantages:** Close tolerances, can be damaged by solids, less forgiving if run dry.

III. Pump Selection Considerations

When selecting a pump, engineers consider:

- **Fluid Properties:** Viscosity, density, temperature, corrosiveness, presence of solids.
- **System Requirements:** Required flow rate, total head (elevation, pressure, friction losses), suction conditions.
- **Operational Factors:** Continuous or intermittent operation, efficiency, noise, maintenance, control requirements.
- **Cost:** Initial capital cost, operating cost (energy consumption), maintenance cost.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. Explain the fundamental difference in the energy transfer mechanism between a centrifugal pump and a positive displacement pump.
2. Identify a common application for a centrifugal pump and a common application for a reciprocating pump.

3. A pump is operating with a discharge of $0.02 \text{ m}^3/\text{s}$ and delivers a head of 30 m to water. Calculate the hydraulic power delivered to the water. (Assume $\rho=1000 \text{ kg/m}^3$, $g=9.81 \text{ m/s}^2$).
4. Why are positive displacement pumps generally preferred for handling highly viscous fluids, whereas centrifugal pumps are not?
5. What is cavitation in a pump, and why is it undesirable?

5 / Homework:

1. Draw a simple schematic of a centrifugal pump and label its main components. Briefly describe the function of each component.
2. A positive displacement pump is rated to deliver 0.5 L/s of fluid. If the discharge pressure is 500 kPa (gage) and the suction pressure is -20 kPa (gage), and the fluid has a density of 950 kg/m^3 , calculate the minimum head the pump must add to the fluid (assuming negligible velocity and elevation changes between suction and discharge points).
3. Discuss the advantages and disadvantages of using a diaphragm pump compared to a piston pump for pumping corrosive chemicals.
4. Research and explain the concept of Net Positive Suction Head (NPSH) for pumps. Why is it important in pump selection and system design?
5. A pump has a mechanical efficiency of 80% and is driven by a 10 kW electric motor. If the pump is used to lift water, what is the maximum head it can deliver at a flow rate of $0.01 \text{ m}^3/\text{s}$?

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 10)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 12)
- Hydraulic Machines by Jagdish Lal.
- Pump Handbook by Karassik, Messina, et al.
- Online resources from pump manufacturers (e.g., Grundfos, Sulzer, KSB).

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Pipeline Problems

For

Students of First Year

By

Halah Kadhim Mohsin

Dep. Of Chemical industries Techniques

2025

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, civil, mechanical, chemical, and industrial engineering, who need to apply fluid mechanics principles to practical piping system design and analysis.

1 / B – Rationale: Pipeline problems represent the core application of the Energy Equation (extended Bernoulli) and loss calculations. Engineers frequently encounter scenarios requiring the determination of flow rates, pressure drops, pump requirements, or pipe sizing in complex piping networks.

1 / C – Central Idea: Pipeline problems involve applying the principles of conservation of mass (Continuity Equation) and conservation of energy (Extended Bernoulli Equation), along with detailed calculations of major (friction) and minor (local) head losses, to analyze fluid flow in single pipes or interconnected piping networks. These problems often require iterative solutions due to the interdependence of flow parameters and friction factors.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Categorize common types of pipeline problems (Type 1: Head Loss/Pressure Drop; Type 2: Flow Rate; Type 3: Pipe Diameter).
- Systematically apply the Energy Equation to different sections of a piping system.
- Accurately calculate major head losses using the Darcy-Weisbach equation and the Moody chart/relevant correlations.
- Accurately calculate minor head losses for various fittings and components.
- Solve Type 1 and Type 2 pipeline problems using direct calculation or iterative methods.

- Understand the approach for solving Type 3 problems, recognizing their iterative nature.
- Analyze simple piping networks (pipes in series and parallel).

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is the primary equation used to account for energy changes and losses in a real piping system?
2. Name two factors that influence the friction factor in a pipe.
3. Why is the calculation of friction factor often an iterative process when solving for flow rate or pipe diameter?
4. If a pipe doubles in length, how does that theoretically affect the major head loss, assuming all else is constant?
5. What is the difference between pipes connected "in series" and "in parallel"?

3 / Lecture (Content):

Introduction: Solving pipeline problems is a fundamental skill for engineers in various disciplines. These problems involve applying the theoretical concepts of fluid flow, energy conservation, and head losses to design, analyze, and troubleshoot real-world piping systems. This lecture focuses on the methodology for solving common pipeline problems.

I. General Approach to Pipeline Problems

1. **Define Control Volume and Points:** Select two appropriate points (usually free surfaces, points of known pressure/velocity, or points where pump/turbine/major loss occurs) for applying the Energy Equation.
2. **Write Down the Energy Equation:** $\rho g P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 + h_P = \rho g P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2 + h_T + h_L$
3. **Identify Knowns and Unknowns:** Carefully list all given values and what needs to be determined.
4. **Simplify Terms:** Eliminate terms that are zero or negligible (e.g., atmospheric pressures if using gage pressure, surface velocities in large tanks).
5. **Calculate Velocities (using Continuity Equation):** If areas change, relate velocities using $A_1 V_1 = A_2 V_2$.

6. **Calculate Total Head Loss (h_L):** $h_L = \sum h_{L,\text{major}} + \sum h_{L,\text{minor}}$
 - $h_{L,\text{major}} = fDL/2gV^2$
 - $h_{L,\text{minor}} = K_L/2gV^2$
7. **Solve the Equation:** This often involves iterative solutions, especially when the flow rate or pipe diameter is unknown, because the friction factor (f) depends on the Reynolds number, which in turn depends on velocity or diameter.

II. Types of Pipeline Problems

Pipeline problems are broadly classified into three types based on the primary unknown:

Type 1: Determine Head Loss (or Pressure Drop)

- **Given:** Pipe geometry (L, D), fluid properties (ρ, μ, ϵ), and flow rate (Q or V).
- **To find:** Head loss (h_L) or pressure drop (ΔP).
- **Solution Method:** Direct calculation.
 1. Calculate average velocity $V = Q/A$.
 2. Calculate Reynolds number $Re = \rho VD/\mu$.
 3. Determine friction factor f :
 - If laminar ($Re < 2300$), $f = 64/Re$.
 - If turbulent ($Re > 4000$), use Moody chart or Colebrook/Swamee-Jain with Re and ϵ/D .
 4. Calculate $h_{L,\text{major}}$ and $h_{L,\text{minor}}$ (using given K_L values for fittings).
 5. Sum all losses. $\Delta P = \rho gh_L$.

Example 1: Pressure drop in a pipe

- **Question:** Water at 20°C ($\rho = 998 \text{ kg/m}^3, \mu = 1.0 \times 10^{-3} \text{ Pa}\cdot\text{s}$) flows through a horizontal, 100 m long, 5 cm diameter commercial steel pipe ($\epsilon = 0.045 \text{ mm}$) at a velocity of 1.5 m/s. The pipe includes two standard 90° elbows ($K_L = 0.9$ each) and a fully open gate valve ($K_L = 0.2$). Calculate the pressure drop between the inlet and outlet.
- **Solution:**
 1. **Velocity:** $V = 1.5 \text{ m/s}$ (Given).
 2. **Reynolds Number:** $Re = \rho VD/\mu = 1.0 \times 10^{-3} \times 998 \times 1.5 \times 0.05 = 74,850$. (Turbulent flow).

3. **Relative Roughness:** $\epsilon/D = 0.045 \text{ mm}/50 \text{ mm} = 0.0009$.
4. **Friction Factor (f):** Using Moody chart or Swamee-Jain equation for $Re = 74,850$ and $\epsilon/D = 0.0009$, we find $f \approx 0.021$.
5. **Major Head Loss:** $h_{L,\text{major}} = fDL/2gV^2 = 0.021 \times 100 \times 2 \times 9.81 / 2 \times 0.1146^2 = 0.021 \times 2000 \times 0.1146 = 4.81 \text{ m}$
6. **Minor Head Losses:**
 - Two elbows: $h_{L,\text{elbows}} = 2 \times K_L/2gV^2 = 2 \times 0.9 \times 0.1146 = 0.206 \text{ m}$
 - Gate valve: $h_{L,\text{valve}} = K_L/2gV^2 = 0.2 \times 0.1146 = 0.023 \text{ m}$
7. **Total Head Loss:** $h_L = h_{L,\text{major}} + h_{L,\text{elbows}} + h_{L,\text{valve}} = 4.81 + 0.206 + 0.023 = 5.039 \text{ m}$
8. **Pressure Drop:** $\Delta P = \rho gh_L = 998 \times 9.81 \times 5.039 = 49,360 \text{ Pa} \approx 49.4 \text{ kPa}$ (Note: For horizontal pipe, $z_1 = z_2$, and $V_1 = V_2$, so $\Delta P = \rho gh_L$ directly).

Type 2: Determine Flow Rate (or Velocity)

- **Given:** Pipe geometry (L, D), fluid properties (ρ, μ, ϵ), and pressure difference/elevation difference.
- **To find:** Flow rate (Q) or velocity (V).
- **Solution Method:** Iterative, because f depends on V.
 1. Write down the Energy Equation, isolating the terms involving V.
 2. Assume an initial value for f (e.g., from Moody chart assuming fully turbulent flow, or $f = 0.02$).
 3. Calculate V using the assumed f.
 4. Calculate Re using the calculated V.
 5. Find a new f from Moody chart/correlation using the new Re and ϵ/D .
 6. Compare the new f with the assumed f. If they are significantly different, go back to step 3 with the new f. Repeat until f (and V) converges.
 7. Once V is converged, calculate $Q = AV$.

Example 2: Flow rate calculation (Iterative)

- **Question:** Water flows from a large tank (open to atmosphere) through a 100 m long, 10 cm diameter cast iron pipe ($\epsilon = 0.26 \text{ mm}$) to an outlet 20 m below the tank surface. The entrance is sharp-edged ($K_L = 0.5$). Calculate the flow rate.

- **Solution:** Apply Energy Equation between tank surface (1) and pipe exit (2): $P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + z_2 + h_L$ $P_1 = P_2 = P_{atm}$, $V_1 \approx 0$, $z_1 = 20$ m, $z_2 = 0$. $20 = 2gV_2^2 + (fDL + K_{L,entrance} + K_{L,exit})2gV_2^2$
Assume exit loss $K_{L,exit} = 1.0$ (into atmosphere from pipe). $20 = 2gV_2^2(1 + f0.1100 + 0.5 + 1.0)$ $20 = 2gV_2^2(2.5 + 1000f)$
 $\epsilon/D = 0.26$ mm/100 mm = 0.0026. $\nu_{water} \approx 10^{-6}$ m²/s.

Iteration 1:

- Assume $f = 0.025$ (a common starting guess for turbulent flow).
- $20 = 2 \times 9.81 V_2^2 (2.5 + 1000 \times 0.025) = 19.62 V_2^2 (2.5 + 25) = 19.62 V_2^2 (27.5)$
- $V_2 = \sqrt{20 / 19.62 \times 27.5} = 14.265 \Rightarrow V = 3.777$ m/s
- $Re = \nu V D = 10^{-6} \times 3.777 \times 0.1 = 3.777 \times 10^5$.
- From Moody chart for $Re = 3.777 \times 10^5$ and $\epsilon/D = 0.0026$, new $f \approx 0.0255$. (Very close to assumed, indicates convergence)

Iteration 2 (optional, to confirm):

- Use $f = 0.0255$.
- $20 = 19.62 V_2^2 (2.5 + 1000 \times 0.0255) = 19.62 V_2^2 (28)$
- $V_2 = \sqrt{20 / 19.62 \times 28} = 14.01 \Rightarrow V = 3.743$ m/s. (Velocity has converged sufficiently).
- $Q = AV = \pi (0.05)^2 \times 3.743 = 0.0293$ m³/s.

Type 3: Determine Pipe Diameter

- **Given:** Length (L), fluid properties, flow rate (Q), and allowable head loss (ΔH or ΔP).
- **To find:** Pipe diameter (D).
- **Solution Method:** Highly iterative, as D affects V, Re, ϵ/D , and thus f.
 1. Assume a diameter D.
 2. Calculate V from Q and assumed D.
 3. Calculate Re and ϵ/D .
 4. Find f from Moody chart.
 5. Calculate h_L using f, D, and V.
 6. Compare calculated h_L with the allowed h_L . Adjust D and repeat until convergence. This is often done with specialized software or design tables.

III. Pipes in Series and Parallel

A. Pipes in Series:

- Flow through a sequence of pipes with different diameters, lengths, or roughness.
- **Continuity:** The volume flow rate (Q) is the same through all pipes.
- **Energy:** The total head loss is the sum of head losses in each individual pipe section and any fittings/minor losses along the path. $h_{L,\text{total}} = h_{L1} + h_{L2} + \dots + h_{Ln}$

B. Pipes in Parallel:

- A main pipe divides into two or more parallel pipes that then rejoin.
- **Continuity:** The total volume flow rate entering the parallel section is the sum of the flow rates in each parallel branch. $Q_{\text{main}} = Q_1 + Q_2 + \dots + Q_n$
- **Energy:** The head loss across each parallel branch is the same (i.e., the pressure drop between the two junction points is the same for all parallel pipes). $h_{L1} = h_{L2} = \dots = h_{Ln}$
- Solving parallel pipe problems often requires iteration or system of equations due to the interdependencies.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. Explain why Type 2 and Type 3 pipeline problems typically require an iterative solution process, while Type 1 problems do not.
2. A crude oil pipeline is 500 km long and has a diameter of 1 m. The flow rate is 5000 m³/hour. Assuming a given friction factor $f=0.015$, calculate the major head loss. (Density and viscosity of crude oil would be needed for a full solution, but just the formula application for this question).
3. Water flows from a tank through a 5 cm diameter pipe. The pipe has a sharp-edged entrance, a fully open globe valve ($K_L=10$), and a 90-degree bend ($K_L=0.9$). If the velocity in the pipe is 2 m/s, calculate the total minor head loss.
4. In a parallel pipe network, if pipe A has twice the length of pipe B (but same diameter and roughness), which pipe will have a higher flow rate, assuming they are both between the same two junctions? Explain.
5. What is the purpose of the "relative roughness" term in the Moody chart?

5 / Homework:

1. Water at 10°C ($\rho=1000\text{ kg/m}^3$, $\mu=1.307\times 10^{-3}\text{ Pa}\cdot\text{s}$) flows through a 300 m long, 20 cm diameter cast iron pipe ($\epsilon=0.26\text{ mm}$). If the flow rate is $0.05\text{ m}^3/\text{s}$, calculate the pressure drop due to friction alone.
2. A horizontal pipe system discharges water from a pump. The pipe is 150 m long, 8 cm diameter, and made of commercial steel. It includes a swing check valve ($K_L=2.0$) and three standard 90° elbows. If the desired flow rate is $0.01\text{ m}^3/\text{s}$, calculate the total head loss in the system.
3. A large elevated tank supplies water to a point 50 m below. The connecting pipe is 100 m long, 15 cm diameter. There is a sharp-edged entrance, and a gate valve (fully open) at the mid-point. Determine the maximum theoretical flow rate neglecting friction. Then, assuming the pipe is made of smooth plastic and accounting for friction and minor losses, calculate the actual flow rate. (This is an iterative problem, use $v_{\text{water}}=10^{-6}\text{ m}^2/\text{s}$).
4. Design a system to deliver $0.03\text{ m}^3/\text{s}$ of water from a lower reservoir to an upper reservoir 30 m higher. You have available 200 m of pipe. Suggest a suitable commercial steel pipe diameter if the maximum allowable head loss (excluding elevation change) is 5 m. (This is a Type 3 problem, requires iterative approach).
5. Two pipes are connected in parallel between two points. Pipe A is 100 m long, 10 cm diameter. Pipe B is 150 m long, 12 cm diameter. Both are smooth pipes. If the total flow rate between the points is $0.02\text{ m}^3/\text{s}$, calculate the flow rate through each pipe. (Hint: Head loss across both pipes is equal).

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 7)
- Fluid Mechanics by Cengel and Cimbala. (Chapter 8)
- Fox and McDonald's Introduction to Fluid Mechanics.
- White's Fluid Mechanics.
- Engineering Toolbox (for pipe roughness and loss coefficients).

**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Motion of Particles in Fluids

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, civil, environmental, chemical, and mechanical engineering, interested in processes involving particulate matter in fluids.

1 / B – Rationale: Understanding how particles move and settle in fluids is crucial for many engineering applications, including sedimentation, filtration, wastewater treatment, air pollution control, slurry transport, and even natural phenomena like erosion and deposition.

1 / C – Central Idea: The motion of a particle in a fluid is governed by a balance of forces: gravity, buoyancy, and drag. The interplay of these forces determines whether a particle settles, rises, or remains suspended, and if it settles, its terminal velocity is a critical parameter.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Identify the primary forces acting on a particle submerged or moving in a fluid.
- Define the concept of terminal velocity for a settling particle.
- Understand the difference between Stokes' Law and Newton's Law for drag, and their applicability based on the Reynolds number of the particle.
- Calculate the drag force on a spherical particle.
- Determine the terminal velocity of a spherical particle settling in a fluid.
- Apply the principles of particle motion to basic engineering problems such as sedimentation or classification.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. Why does sand settle to the bottom of a glass of water, while clay particles might stay suspended for a long time?
2. What is the main upward force acting on a submerged object?
3. Does the shape of a particle affect how fast it settles?
4. If a small rock falls through water, does it continue to accelerate indefinitely? Why or why not?
5. What is "drag" in the context of fluid flow?

3 / Lecture (Content):

Introduction: The interaction between solid particles and fluids is a vast and critical area in many engineering disciplines. From designing settling tanks in water treatment plants to predicting the dispersion of pollutants in the atmosphere, understanding the motion of particles in fluids is essential. This lecture will introduce the fundamental forces involved and how they determine particle behavior.

I. Forces Acting on a Particle in a Fluid

When a particle is suspended or moving in a fluid, three primary forces act upon it:

1. **Gravitational Force (Weight, W):** Acts vertically downward.
 - $W = m_p g = \rho_p V_p g$
 - m_p : mass of the particle
 - ρ_p : density of the particle material
 - V_p : volume of the particle
2. **Buoyant Force (FB):** Acts vertically upward, equal to the weight of the fluid displaced by the particle (Archimedes' Principle).
 - $F_B = m_{\text{fluid}} g = \rho_f V_p g$
 - ρ_f : density of the fluid
 - V_p : volume of the particle (same as displaced fluid volume)
3. **Drag Force (FD):** Acts in the direction opposite to the particle's motion relative to the fluid. It arises from the fluid's resistance to the particle's movement.
 - $F_D = \frac{1}{2} C_D \rho_f A_p V^2$
 - C_D : Drag coefficient (dimensionless, depends on particle shape and particle Reynolds number)
 - ρ_f : density of the fluid

- A_p : projected frontal area of the particle perpendicular to the flow direction
- V : relative velocity between the particle and the fluid

II. Terminal Velocity (V_t)

- When a particle is released in a fluid, it initially accelerates due to the net gravitational force (weight minus buoyancy).
- As its velocity increases, the drag force also increases.
- Eventually, the drag force becomes large enough to balance the net gravitational force (weight - buoyancy). At this point, the net force on the particle becomes zero, and it stops accelerating, moving at a constant velocity called the **terminal velocity (V_t)**.
- For a settling particle, at terminal velocity: $W = F_B + F_D$
 $\rho_p V_p g = \rho_f V_p g + 21 C_D \rho_f A_p V_t^2$ $(\rho_p - \rho_f) V_p g = 21 C_D \rho_f A_p V_t^2$

III. Drag Coefficient (C_D) and Particle Reynolds Number (Re_p)

The drag coefficient C_D is not constant; it depends heavily on the shape of the particle and the **particle Reynolds number (Re_p)**.

- **Particle Reynolds Number:** Measures the ratio of inertial forces to viscous forces for the particle's motion. $Re_p = \frac{\mu_f \rho_f V_t D_p}{\mu_f} = \frac{v_f V_t D_p}{\nu_f}$
 - D_p : characteristic dimension of the particle (e.g., diameter for a sphere)
 - V_t : terminal velocity
 - ρ_f : fluid density
 - μ_f : fluid dynamic viscosity
 - ν_f : fluid kinematic viscosity
- **For a Sphere:**
 - Volume: $V_p = \frac{1}{6} \pi D_p^3$
 - Projected Area: $A_p = \frac{1}{4} \pi D_p^2$

A. Stokes' Law Regime (Laminar Flow around Particle)

- Applicable when $Re_p < 1$ (or sometimes up to $Re_p \approx 0.1 - 0.2$). This occurs for very small particles or highly viscous fluids.
- In this regime, viscous forces dominate.
- **Stokes' Drag Law for a Sphere:** $F_D = 3\pi \mu_f D_p V_t$

- **Stokes' Law for Terminal Velocity of a Sphere:** By equating this drag force with the net gravitational force: $(\rho_p - \rho_f) \frac{4}{3} \pi D_p^3 g = 3 \pi \mu_f D_p V_t$
 $V_t = \frac{1}{18} \mu_f (\rho_p - \rho_f) g D_p^2$

B. Intermediate/Transition Regime

- $1 < Re_p < 1000$ (approx).
- Neither viscous nor inertial forces fully dominate.
- C_D is complex and often given by empirical correlations for spheres:

$$C_D = \frac{24}{Re_p} + \frac{3}{Re_p^{0.34}} \quad (\text{often cited})$$

C. Newton's Law Regime (Turbulent Flow around Particle)

- Applicable when $Re_p > 1000$ (or sometimes up to $Re_p \approx 500,000$). This occurs for larger particles or low viscosity fluids.
- In this regime, inertial forces dominate, and C_D becomes nearly constant for spheres.
- For spheres, $C_D \approx 0.44$.
- **Newton's Law Terminal Velocity for a Sphere:** $V_t = \sqrt{\frac{4}{3} \frac{(\rho_p - \rho_f) g D_p}{C_D}}$ (using $C_D = 0.44$)

IV. General Procedure for Calculating Terminal Velocity

Since V_t is needed to calculate Re_p , which in turn is needed to find C_D , an iterative approach is often required:

1. Calculate $(\rho_p - \rho_f)g$ and D_p .
2. Assume a value for V_t or C_D or Re_p to start.
 - Often, assume Stokes' Law applies ($Re_p < 1$) and calculate V_t using Stokes' Law.
3. Calculate Re_p using the assumed/calculated V_t .
4. Determine C_D based on the calculated Re_p and the appropriate regime (Stokes, Intermediate, Newton) using chart or correlation.
5. Calculate a new V_t using the general force balance equation: $V_t = \sqrt{\frac{4}{3} \frac{(\rho_p - \rho_f) g D_p}{C_D}}$ (substitute V_p and A_p for sphere: $V_t = \sqrt{\frac{4}{3} \frac{(\rho_p - \rho_f) g D_p}{C_D}}$)
6. Compare the new V_t with the assumed/previous V_t . If not converged, use the new V_t and repeat from step 3.

V. Practical Applications

- **Sedimentation Tanks:** Designing basins to allow particles to settle out of water (e.g., wastewater treatment, water purification).
- **Centrifuges and Cyclones:** Separating particles based on density and size using centrifugal forces.
- **Filtration:** Understanding how particles are removed from a fluid stream.
- **Dust Collection:** Designing systems to remove particulate matter from air.
- **Slurry Transport:** Predicting how solid particles are transported in a liquid medium.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. List the three main forces acting on a spherical particle settling in a fluid.
2. Define terminal velocity. Why doesn't a particle in a fluid accelerate indefinitely under gravity?
3. Under what conditions is Stokes' Law applicable for calculating drag force and terminal velocity?
4. A small glass bead (diameter 0.1 mm, $\rho_p=2500 \text{ kg/m}^3$) settles in water ($\rho_f=1000 \text{ kg/m}^3, \mu_f=10^{-3} \text{ Pa}\cdot\text{s}$). Calculate its terminal velocity using Stokes' Law. Is the use of Stokes' Law justified?
5. What happens to the drag coefficient of a spherical particle when its Reynolds number increases from 0.01 to 1000?

5 / Homework:

1. Derive the general equation for terminal velocity (V_t) for a spherical particle settling in a fluid, starting from the force balance equation.
2. A lead sphere (diameter 1 mm, $\rho_p=11340 \text{ kg/m}^3$) falls through oil ($\rho_f=900 \text{ kg/m}^3, \mu_f=0.1 \text{ Pa}\cdot\text{s}$). Determine its terminal velocity. (You will need to iterate or check the Reynolds number to select the correct drag correlation/law).
3. Compare the settling behavior of two spherical particles of the same material and density ($\rho_p=2000 \text{ kg/m}^3$) in water ($\rho_f=1000 \text{ kg/m}^3, \mu_f=10^{-3} \text{ Pa}\cdot\text{s}$). Particle A has a diameter of 0.05 mm, and Particle B has a diameter of 1 mm.

4. Explain how the shape of a particle (e.g., a flat plate vs. a sphere) would affect its terminal velocity, assuming the same mass and density.
5. A fine sand particle (diameter 0.08 mm, SG=2.65) is in water (SG=1.0, $\mu=10^{-3}$ Pa·s). Calculate its terminal settling velocity. Will it settle by Stokes' Law?

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 9)
 - Fluid Mechanics by Cengel and Cimbala. (Chapter 11)
 - Unit Operations of Chemical Engineering by McCabe, Smith, and Harriott (for more detailed particle mechanics).
 - Introduction to Environmental Engineering and Science by Gilbert M. Masters.
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**Ministry of high Education and Scientific Research
Southern Technical University
Technological institute of Basra
Department of Chemical Industries Techniques**



Learning package

In

Fluid Flow Through Packed Beds

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students (especially chemical and environmental engineering) in fluid mechanics, focusing on the unique challenges and models for fluid flow through porous media, specifically packed beds.

1 / B – Rationale: Packed beds are ubiquitous in many industrial processes (e.g., catalysis, filtration, adsorption) and natural phenomena (e.g., groundwater flow). Understanding fluid dynamics in these systems is crucial for designing and optimizing chemical reactors, separation processes, and filtration units.

1 / C – Central Idea: Fluid flow through a packed bed is characterized by high resistance due to the tortuous path and large surface area of the particles. The pressure drop across the bed is a key design parameter, which can be accurately predicted using empirical correlations like the Ergun equation, accounting for both viscous and inertial effects.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

Define a packed bed and identify its key characteristics (particle size, voidage).

- Explain the concept of superficial and interstitial velocities.
- Identify the flow regimes (laminar and turbulent) in packed beds based on the Reynolds number.
- Apply the Ergun equation to calculate the pressure drop across a packed bed for various flow conditions.
- Understand the contribution of viscous and inertial terms in the Ergun equation.

- Recognize the limitations and practical considerations in applying packed bed correlations.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. What is a porous medium? Give an example.
2. Imagine pushing water through a sand filter. Would you expect the pressure drop to be high or low compared to flowing through an empty pipe? Why?
3. What is the "void" space in a bed of particles?
4. How might the size of the particles affect the flow resistance?
5. What are some industrial applications where fluids flow through solid particles?

3 / Lecture (Content):

Introduction: So far, we have primarily discussed fluid flow in open channels or through empty pipes. However, many engineering applications involve fluid flowing through a bed of solid particles, known as a **packed bed**. Examples include catalytic converters in chemical reactors, filtration systems, adsorption columns, and heat exchangers. Naturally occurring examples include groundwater flow through soil and rock formations. Understanding the pressure drop and flow behavior in these complex geometries is essential for their design and operation.

I. Characteristics of Packed Beds

A packed bed consists of a collection of solid particles (which can be uniform spheres, irregular granules, or specially shaped packings) contained within a vessel, through which a fluid (liquid or gas) flows.

Key characteristics that define a packed bed:

1. **Particle Size (D_p):** This is the characteristic diameter of the packing material. For non-spherical particles, an equivalent spherical diameter is often used, based on the particle's volume or surface area.
2. **Bed Voidage (Porosity, ϵ or ϕ):**
 - **Definition:** The fraction of the total bed volume that is occupied by voids (empty spaces) where the fluid can flow.

- $\epsilon = \frac{\text{Total bed volume} - \text{Volume of solids}}{\text{Total bed volume}}$
 - Typical values for randomly packed spheres usually range from 0.3 to 0.5. Voidage significantly affects flow resistance.
3. **Specific Surface Area (a_v):** Total surface area of the particles per unit volume of the bed (m^2/m^3). It represents the effective area for fluid-solid interaction.

II. Flow Velocities in Packed Beds

When dealing with flow through a packed bed, two types of velocities are commonly used:

1. **Superficial Velocity (V_s):**
 - This is the velocity calculated as if the bed were empty, i.e., the volumetric flow rate (Q) divided by the total cross-sectional area of the empty vessel (A_c).
 - $V_s = \frac{Q}{A_c}$
 - This is the velocity commonly used in packed bed correlations because it is easily measurable.
2. **Interstitial Velocity (V_i or V_a):**
 - This is the actual average velocity of the fluid within the void spaces of the bed. Since the fluid only flows through the voids, this velocity is higher than the superficial velocity.
 - $V_i = \epsilon V_s$
 - This velocity is more representative of the true fluid speed experienced by the particles.

III. Flow Regimes and Reynolds Number for Packed Beds

Similar to pipe flow, flow in packed beds can be laminar, transitional, or turbulent. The regime is determined by a modified Reynolds number for packed beds (Re_p):

$$Re_p = \frac{\mu(1-\epsilon)\rho V_s D_p}{\mu}$$

- **Laminar Flow:** $Re_p < 10$ (or sometimes up to 20). Viscous forces dominate.
- **Transitional/Turbulent Flow:** $Re_p > 1000$. Inertial forces dominate.
- **Intermediate/Transitional Region:** $10 < Re_p < 1000$. Both viscous and inertial effects are significant.

IV. Pressure Drop in Packed Beds: The Ergun Equation

Predicting the pressure drop (ΔP) across a packed bed is critical for designing pumps, blowers, and bed dimensions. The most widely used correlation that covers both laminar and turbulent regimes in packed beds is the **Ergun Equation**:

$$L\Delta P = \frac{\epsilon^3 D_p^2}{150 \mu V_s (1-\epsilon)^2} + \frac{\epsilon^3 D_p}{1.75 \rho V_s^2 (1-\epsilon)}$$

Where:

- $L\Delta P$: Pressure drop per unit length of the bed (Pa/m or psi/ft).
- μ : Fluid dynamic viscosity (Pa·s).
- V_s : Superficial velocity (m/s).
- ϵ : Bed voidage (dimensionless).
- D_p : Particle diameter (m).
- ρ : Fluid density (kg/m³).

Analysis of the Ergun Equation Terms:

The Ergun equation is essentially a sum of two terms:

1. **Viscous (Laminar) Term:** $\frac{\epsilon^3 D_p^2}{150 \mu V_s (1-\epsilon)^2}$
 - This term dominates at low Reynolds numbers (laminar flow, $Re_p < 10$).
 - It is directly proportional to the superficial velocity (V_s) and fluid viscosity (μ).
 - This term is related to **Darcy's Law** for flow through porous media: $V_s = -\frac{\mu K L \Delta P}{\mu L \Delta P}$ Where K is the permeability of the porous medium. The Ergun equation's laminar term effectively predicts the permeability. The Kozeny-Carman equation specifically applies to the laminar region.
2. **Inertial (Turbulent) Term:** $\frac{\epsilon^3 D_p}{1.75 \rho V_s^2 (1-\epsilon)}$
 - This term dominates at high Reynolds numbers (turbulent flow, $Re_p > 1000$).
 - It is proportional to the square of the superficial velocity (V_s^2) and the fluid density (ρ).
 - This term accounts for kinetic energy losses due to turbulence and eddies. It's sometimes associated with the Burke-Plummer equation for turbulent flow.

V. Practical Considerations and Limitations:

- **Particle Shape:** The Ergun equation typically assumes spherical particles. For non-spherical particles, a "shape factor" or "sphericity" (often ϕ_s) is introduced to modify D_p in the equation (e.g., D_p is replaced by $\phi_s D_p$).
- **Uniformity of Packing:** The packing structure can vary, leading to deviations from theoretical predictions.
- **Wall Effects:** Near the walls of the containing vessel, the voidage tends to be higher. This "wall effect" can lead to lower pressure drops than predicted, especially in beds where the particle diameter is a significant fraction of the bed diameter ($D_p/D_{bed} > 0.1$).
- **Fluidization:** If the upward superficial velocity of the fluid becomes high enough, the pressure drop can equal the weight of the packed bed, causing the particles to lift and suspend in the fluid. This is known as fluidization, a separate and important area of study.

Example Calculation: Air (density $\rho = 1.2 \text{ kg/m}^3$, viscosity $\mu = 1.8 \times 10^{-5} \text{ Pa}\cdot\text{s}$) flows through a packed bed of spherical catalyst particles with diameter $D_p = 3 \text{ mm}$. The bed has a voidage $\epsilon = 0.4$ and a length $L = 2 \text{ m}$. If the superficial velocity of the air is $V_s = 0.5 \text{ m/s}$, calculate the pressure drop across the bed.

1. **Calculate Rep:** $Re_p = \mu(1-\epsilon)\rho V_s D_p$
 $= 1.8 \times 10^{-5} \times (1-0.4) \times 1.2 \times 0.5 \times (3 \times 10^{-3}) = 1.08 \times 10^{-5}$
 $1.8 \times 10^{-3} = 166.67$
 (This is in the intermediate/transitional regime, so both terms of Ergun equation are important).
2. **Calculate Pressure Drop using Ergun Equation:** $L\Delta P = \epsilon^3 D_p^2 \frac{150\mu V_s(1-\epsilon)}{K_{150}} + \frac{\epsilon^3 D_p^3}{K_{180}} \frac{\rho V_s^2(1-\epsilon)}{K_{180}}$
 - **Laminar Term:** L_{term}
 $= (0.4)^3 \times (3 \times 10^{-3})^2 \times \frac{150 \times (1.8 \times 10^{-5}) \times 0.5 \times (1-0.4)}{K_{150}}$
 $= 0.064 \times 9 \times 10^{-6} \times \frac{150 \times 1.8 \times 10^{-5} \times 0.5 \times (0.6)}{K_{150}}$
 $= 5.76 \times 10^{-7} \times \frac{150 \times 1.8 \times 10^{-5} \times 0.5 \times 0.36}{K_{150}} = 5.76 \times 10^{-7} \times 48.6$
 $= 843.75 \text{ Pa/m}$
 - **Turbulent Term:** T_{term}
 $= (0.4)^3 \times (3 \times 10^{-3})^3 \times \frac{1.75 \times 1.2 \times (0.5)^2 \times (1-0.4)}{K_{180}}$
 $= 0.064 \times 3 \times 10^{-3} \times \frac{1.75 \times 1.2 \times 0.25 \times 0.6}{K_{180}} = 1.92 \times 10^{-4} \times 70.315$
 $= 1640.625 \text{ Pa/m}$
 - **Total Pressure Drop per unit length:** $L\Delta P$
 $= 843.75 + 1640.625 = 2484.375 \text{ Pa/m}$

- **Total Pressure Drop (ΔP):** $\Delta P = L \Delta P$
 $\times L = 2484.375 \text{ Pa/m} \times 2 \text{ m} = 4968.75 \text{ Pa}$ or approximately 4.97 kPa.

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. Define bed voidage (ϵ) and explain why it is important for flow through packed beds.
2. What is the difference between superficial velocity (V_s) and interstitial velocity (V_i)? Which one is used directly in the Ergun equation?
3. Write down the Ergun equation and identify the terms responsible for viscous and inertial pressure drop.
4. For a very slow, highly viscous flow through a packed bed, which term in the Ergun equation would dominate?
5. If you double the particle diameter (D_p) while keeping other parameters constant, how would it affect the pressure drop according to the Ergun equation (qualitatively)?

5 / Homework:

1. Water ($\rho = 1000 \text{ kg/m}^3, \mu = 0.001 \text{ Pa}\cdot\text{s}$) flows through a packed bed of 2 mm diameter spherical glass beads. The bed has a voidage of 0.38 and is 1.5 m long. If the volumetric flow rate is $0.005 \text{ m}^3/\text{s}$ through a bed with a cross-sectional area of 0.1 m^2 , calculate the pressure drop across the bed.
2. A packed bed of catalyst particles (average diameter $D_p = 5 \text{ mm}$, voidage $\epsilon = 0.42$) is used in a reactor. Air ($\rho = 1.2 \text{ kg/m}^3, \mu = 1.8 \times 10^{-5} \text{ Pa}\cdot\text{s}$) flows through the bed at a superficial velocity of 0.08 m/s . If the pressure drop per unit length is 100 Pa/m , what is the estimated length of the bed?
3. Discuss how the shape of particles (e.g., irregular sand grains vs. perfect spheres) might affect the pressure drop in a packed bed, even if they have the same equivalent spherical diameter.
4. Explain the phenomenon of fluidization. Under what conditions does it occur?
5. Consider a packed bed where the Reynolds number (Re_p) is 5. Which term in the Ergun equation (viscous or inertial) contributes more to the total pressure drop? Calculate the ratio of the inertial term to the viscous term.

6 / Reference:

- Unit Operations of Chemical Engineering by McCabe, Smith, and Harriott. (Chapter on Fluid Flow in Packed and Fluidized Beds)
- Transport Phenomena by Bird, Stewart, and Lightfoot. (Chapter on Flow through Porous Media)
- Fluid Mechanics for Chemical Engineers by Noel de Nevers. (Chapter on Flow through Porous Media)

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Learning package

In

Fluid Flow Between Tanks

For

Students of First Year

By

Halah Kadhim Mohsin

**Dep. Of Chemical industries Techniques
2025**

1/ Overview

1 / A – Target Population: Undergraduate engineering students in fluid mechanics, focusing on the application of the Extended Bernoulli (Energy) Equation to analyze and design systems involving fluid transfer between reservoirs or tanks.

1 / B – Rationale: Many real-world engineering systems, such as municipal water distribution networks, industrial fluid transfer lines, and drainage systems, involve fluid flowing from one tank or reservoir to another. Analyzing these systems requires a comprehensive understanding of energy conservation, including all forms of head and losses.

1 / C – Central Idea: The Energy Equation provides a robust framework for balancing energy in fluid systems. By applying it between the free surfaces of tanks (or other strategic points), accounting for elevation differences, pump/turbine work, and all major and minor head losses in the connecting pipe network, we can determine unknown parameters like flow rates, required pump sizes, or pipe diameters.

1 / D – Performance Objectives: Upon completion of this lecture, students will be able to:

- Apply the Extended Bernoulli (Energy) Equation to systems involving flow between two or more tanks.
- Identify and correctly incorporate all major and minor head losses within the connecting piping system.
- Calculate required pump head or power for transferring fluid between tanks.
- Determine the flow rate between tanks given system parameters.
- Understand the basic principles for sizing pipes for inter-tank flow (though iterative solutions might be required).
- Utilize concepts of Hydraulic Grade Line (HGL) and Energy Grade Line (EGL) as visual aids.

2 / Pretest: (Students should answer these questions before the lecture to assess their prior knowledge)

1. Write down the full Extended Bernoulli (Energy) Equation.
2. What are the two main types of head loss, and what causes each?
3. How is the head added by a pump (h_P) related to its power output?
4. If a large open tank is connected to a pipe, what can you assume about the velocity and pressure at the free surface of the fluid in the tank?
5. What is the purpose of a pump in a system that transfers fluid from a lower to a higher elevation?

3 / Lecture (Content):

Introduction: Analyzing fluid flow between tanks or reservoirs is a common problem in various engineering disciplines. Whether it's designing a water supply system for a city, transferring chemicals in a process plant, or understanding the drainage of a storm system, the principles of fluid mechanics, particularly the Energy Equation, are indispensable. This lecture will guide you through the methodology for solving such problems.

I. Fundamental Principles and the Energy Equation

The cornerstone for analyzing fluid flow between tanks is the **Extended Bernoulli Equation**, also known as the **Energy Equation**:

$$\rho g P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 + h_P = \rho g P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2 + h_T + h_L$$

Where:

- Subscripts 1 and 2 refer to the upstream and downstream points, respectively.
- $\rho g P$: Pressure head.
- $\frac{1}{2} \rho V^2$: Velocity head.
- z : Elevation head.
- h_P : Head added by a pump (energy input to fluid).
- h_T : Head removed by a turbine (energy extracted from fluid).
- h_L : Total head loss (energy dissipated due to friction and minor losses).

Other Essential Equations:

- **Continuity Equation:** For incompressible flow in a single pipe or system with constant density: $Q=A_1V_1=A_2V_2$.
- **Major Losses (Friction):** $h_{L,major}=fDL^2gV^2$
- **Minor Losses (Fittings):** $h_{L,minor}=KL^2gV^2$
- **Total Head Loss:** $h_L=\sum h_{L,major}+\sum h_{L,minor}$

II. Methodology for Solving Flow Between Tanks Problems

A systematic approach is key to solving these problems:

1. **Sketch the System:** Draw a clear diagram of the entire system, including tanks, pipes, pumps, turbines, valves, and any other components. Label all known dimensions, elevations, and pressures.
2. **Select Control Points (1 and 2):**
 - For flow between two tanks, the most common and often convenient points to choose are the **free surfaces of the reservoirs/tanks**.
 - At the free surface of a large tank, the velocity (V) is typically negligible ($V\approx 0$) because the surface area is much larger than the pipe's cross-sectional area.
 - If the tank is open to the atmosphere, the pressure (P) at the free surface is atmospheric ($P=P_{atm}$). For gauge pressure calculations, $P=0$. If the tank is pressurized, use the specified gauge pressure.
3. **Establish a Datum:** Choose a convenient horizontal datum ($z=0$) for elevation measurements. This is often the lowest point in the system, or the centerline of the lower tank.
4. **Write the Energy Equation:** Apply the full Energy Equation between the two chosen control points (e.g., between the surface of Tank 1 and the surface of Tank 2).
5. **Simplify Terms:**
 - Cancel out terms that are zero or negligible (e.g., V_1, V_2 for large tank surfaces; P_1, P_2 for open tanks at atmospheric pressure).
 - Set $h_P=0$ if no pump is present, and $h_T=0$ if no turbine is present.
6. **Calculate Total Head Loss (h_L):**
 - Sum all major losses (friction along straight pipes) and all minor losses (due to entrance, exit, elbows, valves, sudden expansions/contractions).
 - Remember that the velocity (V) used in the h_L formulas must correspond to the velocity in the specific pipe section where that

loss occurs. If the pipe diameter changes, velocities will change via continuity.

- If the friction factor f depends on velocity (as in turbulent flow), the solution may require iteration.

7. **Solve for the Unknown:** Rearrange the simplified Energy Equation to solve for the desired unknown (e.g., flow rate Q , required pump head h_P , or unknown tank elevation z).

III. Common Problem Types

1. Type 1: Determine Flow Rate (Q)

- **Given:** Tank levels (z_1, z_2), pipe dimensions (L, D), all fittings, fluid properties.
- **Challenge:** f depends on Re , which depends on V . h_L also depends on V^2 . This typically requires an iterative solution:
 - Assume an initial f (e.g., from the fully turbulent zone on Moody chart) or calculate h_L in terms of V^2 .
 - Solve for V from the Energy Equation.
 - Calculate Re and find a new f .
 - Repeat until f (or V) converges.
- **Simpler Case:** If the fluid is highly viscous (laminar flow), $f = 64/Re$, and the equation becomes directly solvable.

2. Type 2: Determine Required Pipe Diameter (D)

- **Given:** Flow rate (Q), tank levels, pipe length, fittings, fluid properties.
- **Challenge:** D appears in V , Re , f , K_L (for expansions/contractions), and the L/D term. Highly iterative. Often, engineers will select standard pipe sizes and then calculate the resulting flow or pump head.

3. Type 3: Determine Required Pump Head (h_P) or Turbine Power (P_T)

- **Given:** Flow rate (Q), tank levels, pipe dimensions, fittings, fluid properties.
- **Solution:** Calculate V , Re , f , and all h_L terms directly. Substitute into the Energy Equation and solve for h_P or h_T .
- **Pump Power (P_{input}):** $P_{input} = \eta_P \rho g Q h_P$
- **Turbine Power (P_{output}):** $P_{output} = \eta_T \rho g Q h_T$

IV. Visual Aids: Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

These are graphical representations that help visualize the energy balance in pipe systems:

- **Hydraulic Grade Line (HGL):** Represents the sum of pressure head and elevation head ($P/\rho g + z$). If an open manometer were attached to the pipe, the fluid would rise to the HGL level.
- **Energy Grade Line (EGL):** Represents the total mechanical energy head ($P/\rho g + V^2/2g + z$). It is always above the HGL by the amount of the velocity head ($V^2/2g$).

Characteristics of HGL and EGL:

- **Slope:** Both lines slope downwards in the direction of flow, reflecting head loss. The vertical distance between EGL and HGL is $V^2/2g$.
- **Pumps:** A pump causes a sudden upward jump in both EGL and HGL.
- **Turbines:** A turbine causes a sudden downward drop in both EGL and HGL.
- **Reservoirs:** At the free surface of a reservoir, HGL and EGL coincide (since $V \approx 0$). The EGL at the surface of the upstream reservoir is the starting point for total energy.
- **Atmospheric Pressure:** Where the pipe is open to the atmosphere (e.g., free jet exit), the HGL coincides with the centerline of the jet, and the EGL is $V^2/2g$ above it. If the pipe pressure drops below atmospheric, the HGL drops below the pipe centerline.

V. Example Problem:

Water ($\rho = 1000 \text{ kg/m}^3, \mu = 10^{-3} \text{ Pa}\cdot\text{s}$) flows by gravity from an upper reservoir to a lower reservoir through a 10 cm diameter commercial steel pipe ($\epsilon = 0.045 \text{ mm}$). The vertical distance between the free surfaces of the two reservoirs is 10 m. The total length of the pipe is 100 m. The system includes a sharp-edged entrance ($K_L = 0.5$), two standard 90-degree elbows ($K_L = 0.9$ each), and a submerged exit ($K_L = 1.0$). Calculate the flow rate of water through the pipe.

Solution Steps:

1. **Sketch & Points:** Let Point 1 be the surface of the upper reservoir, and Point 2 be the surface of the lower reservoir.
2. **Energy Equation (1 to 2):** $\rho g P_1 + 2g V_1^2 + z_1 = \rho g P_2 + 2g V_2^2 + z_2 + h_L$
3. **Simplify:**

- $P_1 = P_2 = P_{atm} \Rightarrow \rho g P_1 = \rho g P_2 = 0$ (gauge pressure).
 - $V_1 \approx 0, V_2 \approx 0$ (large reservoir surfaces).
 - No pump ($h_P = 0$), no turbine ($h_T = 0$).
 - Set $z_2 = 0$ (datum at lower reservoir surface), so $z_1 = 10$ m.
 - Equation becomes: $z_1 = h_L$
 - So, $10 \text{ m} = h_L$
4. **Calculate Total Head Loss (h_L):** $h_L = h_{L, \text{major}} + \sum h_{L, \text{minor}}$ $h_L = f D L \frac{V^2}{2g} + (K_{L, \text{entrance}} + 2K_{L, \text{elbow}} + K_{L, \text{exit}}) \frac{V^2}{2g}$ $h_L = f 0.11002 \frac{V^2}{2g} + (0.5 + 2 \times 0.9 + 1.0) \frac{V^2}{2g}$ $h_L = f(1000) \frac{V^2}{2g} + (0.5 + 1.8 + 1.0) \frac{V^2}{2g}$ $h_L = (1000f + 3.3) \frac{V^2}{2g}$
5. **Iterative Solution for V (and then Q):**
- Substitute $h_L = 10$ m: $10 = (1000f + 3.3) \frac{V^2}{2g}$
 - **Iteration 1 (Assume f):** Assume $f \approx 0.02$ (typical for turbulent flow in steel pipes). $10 = (1000 \times 0.02 + 3.3) \frac{V^2}{2 \times 9.81}$ $10 = (20 + 3.3) \frac{V^2}{19.62}$ $19.62 \times 10 = 23.3 V^2$ $V^2 = \frac{19.62 \times 10}{23.3}$ $V = 2.903 \text{ m/s}$
 - **Check Reynolds Number:** $Re = \frac{\rho V D}{\mu} = \frac{1000 \times 2.903 \times 0.1}{0.0001} = 290,300$ (Turbulent)
 - **Check Relative Roughness:** $\epsilon/D = 0.045 \text{ mm} / 100 \text{ mm} = 0.00045$
 - **Find new f (from Moody Chart/Haaland):** For $Re = 290,300$ and $\epsilon/D = 0.00045$, $f \approx 0.018$.
 - **Iteration 2 (Using new f=0.018):** $10 = (1000 \times 0.018 + 3.3) \frac{V^2}{2g}$ $10 = (18 + 3.3) \frac{V^2}{19.62}$ $19.62 \times 10 = 21.3 V^2$ $V^2 = \frac{19.62 \times 10}{21.3}$ $V = 3.035 \text{ m/s}$
 - **Check Reynolds Number:** $Re = \frac{1000 \times 3.035 \times 0.1}{0.0001} = 303,500$
 - **Find new f:** For $Re = 303,500$ and $\epsilon/D = 0.00045$, f is still approximately 0.018 (or very close, further iteration would show negligible change). So, the solution converges.
6. **Calculate Flow Rate (Q):**
 $Q = AV = (\pi D^2 / 4) V = (\pi (0.1)^2 / 4) \times 3.035 = 0.007854 \times 3.035 = 0.0238 \text{ m}^3/\text{s}$

4 / Post-test: (Students should answer these questions after the lecture to assess their understanding)

1. When applying the Energy Equation between two reservoirs, why are the velocity head terms often considered negligible at the free surfaces?
2. Describe a scenario where a pump's head (h_P) would be needed in the Energy Equation for flow between tanks.
3. If you are designing a system to transfer a specific flow rate of water between two tanks, and you need to determine the required pipe

diameter, what key challenge would you face in solving the Energy Equation?

4. Explain what the Hydraulic Grade Line (HGL) represents graphically. How does it differ from the Energy Grade Line (EGL)?
5. Consider a system where water flows from a higher tank to a lower tank. If the water level in the upper tank drops, how would this affect the flow rate (assuming all other parameters remain constant)?

5 / Homework:

1. Oil ($\rho=880 \text{ kg/m}^3, \mu=0.01 \text{ Pa}\cdot\text{s}$) is to be pumped from a lower tank to an upper tank whose surface is 25 m above the lower tank's surface. The pipe is 50 m long, 8 cm diameter, made of commercial steel ($\epsilon=0.045 \text{ mm}$). The system includes a reentrant entrance ($K_L=0.8$), three standard 90-degree elbows ($K_L=0.9$ each), and a submerged exit ($K_L=1.0$). If the desired flow rate is $0.008 \text{ m}^3/\text{s}$, calculate the required head that the pump must provide to the oil.
2. Water flows by gravity from Tank A to Tank B. Tank A is open to the atmosphere and its water level is 5 m above the pipe entrance. Tank B is also open to the atmosphere. The pipe is 20 m long, 5 cm diameter, smooth plastic. The system includes a well-rounded entrance ($K_L=0.04$) and a fully open gate valve ($K_L=0.2$). The pipe discharges freely into Tank B. Calculate the height of the water level in Tank B above the pipe exit if the measured flow rate is $0.003 \text{ m}^3/\text{s}$.
3. Draw the HGL and EGL for the example problem solved in the lecture. Assume the lower reservoir surface is at $z=0$.
4. Discuss why problems to determine the flow rate or pipe diameter often require iterative solutions when using the Darcy-Weisbach equation for turbulent flow.
5. A siphon is used to drain a tank. The highest point of the siphon is 2 m above the liquid surface in the tank. The exit of the siphon is 3 m below the liquid surface. The siphon pipe has a diameter of 2 cm and a total length of 8 m. Assume the pipe is smooth and neglect minor losses except for the entrance ($K_L=0.5$) and exit ($K_L=1.0$). If the fluid is water, calculate the volumetric flow rate through the siphon.

6 / Reference:

- Fundamentals of Fluid Mechanics by Munson, Young, and Okiishi. (Chapter 8)

- Fluid Mechanics by Cengel and Cimbala. (Chapter 8)
- Fox and McDonald's Introduction to Fluid Mechanics. (Chapter 6)