Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Computer Networks
and Software Techniques

Learning package

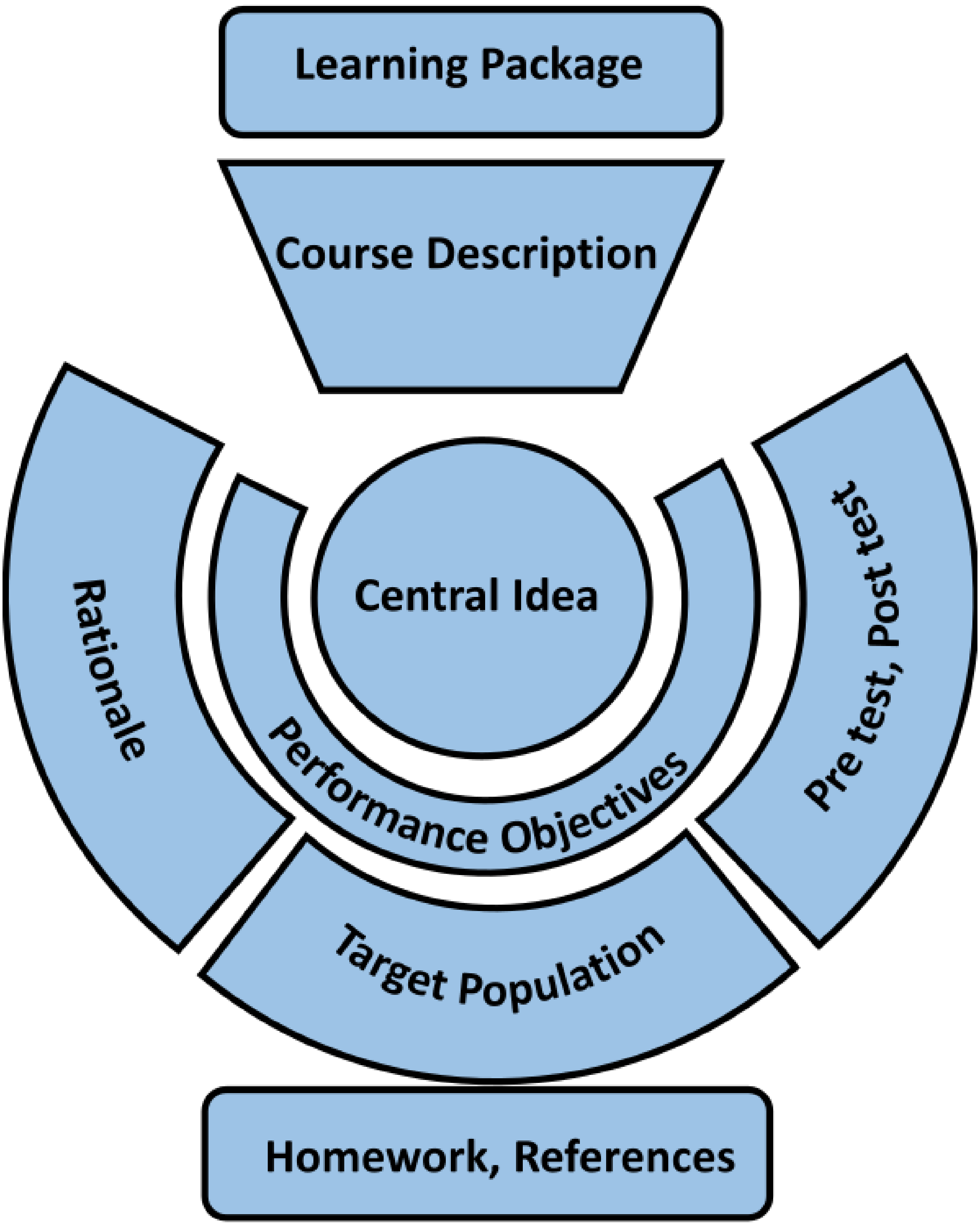
Mathematics and Numerical Analysis

For

First year students

By

Mazin Salih Kadhim
Assistant Lecturer
Dep. Of Computer Networks
And Software Techniques
2025


Learning Package
Course Description
Central Idea
Rationale
Performance Objectives
Pre test, Post test
Target Population
Homework, References

Course Description

Course Name:
Mathematics and Numerical Analysis
Course Code:
Semester / Year:
Semester
Description Preparation Date:
08 / 07 / 2025
Available Attendance Forms:
Attendance only
Number of Credit Hours (Total) / Number of Units (Total)
60 hours / 2 hour weekly
Course administrator's name (mention all, if more than one name)
Name: Mazin Salih Kadhim Email: mazin.s.kadhim@stu.edu.iq
Course Objectives

1. **Understand fundamental concepts in mathematics and numerical methods such as algebra, calculus, equations, and approximation techniques.**
2. **Apply mathematical and numerical skills to solve practical and engineering problems.**
3. **Use numerical methods to find approximate solutions when exact analytical solutions are difficult or impossible.**
4. **Develop logical and analytical thinking in building models and designing algorithms.**
5. **Analyze the accuracy and stability of numerical solutions and identify sources of error.**

Teaching and Learning Strategies

1. Cooperative Concept Planning Strategy.

2. Brainstorming Teaching Strategy.

3. Note–taking Sequence Strategy.

Course Structure

Weeks	Hours	Required Learning Outcomes	Unit or subject name	Learning method	Evaluation method
1-2	2hours	The concept of matrices, their types and how to find their ranks.	The concept of matrices.	Solve different types of problems to build skill and confidence.	Daily exam and home work

3-4	2hours	The equality of matrices and the operations on them (addition, subtraction and multiplication)	The equality of matrices	Solve different types of problems to build skill and confidence.	
5	2hours	The determinant of matrix and its relation with their rank, sarus method to find the value of determinant.	The determinant of matrix	Connect math to real-life applications to make learning meaningful.	
6-7	2hours	The inverse matrix and its relation with rank, cofactors method to find the inverse matrix, Solving the system of linear equations simultaneously using the inverse matrix of the coefficients.	The inverse matrix	Connect math to real-life applications to make learning meaningful.	
8-9	2hours	Differentiation rules of the algebraic, trigonometric,	Differentiation rules of the algebraic,	Collaboration & Discussion	

		exponential and logarithmic functions, Derivative of a composite function “chain rule”, implicit differential and partial derivatives. .	trigonometric, exponential		
10	2hours	The approximate real root of non-linear equation in some interval applying the iteration and newton-Raphson methods.	The approximate real root of non-linear equation	Collaboration & Discussion	
11-12	2hours	Integration rules of algebraic, trigonometric, exponential and logarithmic functions, Integration by parts and integration by partial fractions.	Integration rules of algebraic, trigonometric, exponential	Learn with others, ask questions, and explain ideas to reinforce your knowledge.	

13-14	2hours	The concept of sequence and infinite series and their types, ratio and root tests of their convergence and divergence.	The concept of sequence and infinite series	Learn with others, ask questions, and explain ideas to reinforce your knowledge.	

Course Evaluation	
Distribution as follows: 20 points for Midterm Theoretical Exams for the first semester, 20 points for Midterm Practical Exams for the first semester, 10 points for Daily Exams and Continuous Assessment, and 50 points for the Final Exam.	
Learning and Teaching Resources	
Required textbooks (curricular books, if any)	calculus
Main references (sources)	Stewart, J. (2016). *Calculus: Early Transcendentals* (8th ed.). Cengage Learning.
Recommended books and references (scientific journals, reports…)	Stewart, J., Redlin, L., & Watson, S. (2011). *Precalculus: Mathematics for Calculus* (6th ed.). Brooks/Cole, Cengage Learning.
Electronic References, Websites	https://www.mathsisfun.com/calculus

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1/ Overview

1 / A –Target population :-

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Technological institute of Basra
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1 / B –Rationale :-

Understanding matrices is foundational for linear algebra, which underpins many applications in engineering, science, and computer science.

1 / C –Central Idea :-

1 – Matrices organize data and operations

2 – rank determines the system's solvability and structure.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

1- Define and identify different types of matrices.

2- Calculate the rank of a given matrix using row operations and determinants.

3- Classify matrices based on their properties.

2/ Pretest :

1. Define a matrix and give two examples of its types.
2. What is the rank of the matrix [1 2]
   [3 4]
3. True/False: A zero matrix can have a rank greater than zero.

3/ Concept of Matrices:

1.1 Concept of Matrices

matrix, a set of numbers arranged in rows and columns so as to form a rectangular array. The numbers are called the elements, or entries, of the matrix. Matrices have wide applications in engineering, physics, economics, and statistics as well as in various branches of mathematics. Matrices also have important applications in computer graphics, where they have been used to represent rotations and other transformations of images.

A matrix with m rows and n columns is called an m × n matrix. Each element in the matrix is denoted by a_ij, where i is the row number and j is the column number.

For example, the dimension of the matrix below is 2 × 3 (read as “tow” by “three”), because there are two rows and three columns , and the second one example is 4 × 3 (read as “Four” by “three”) because there are four rows and three columns:

$$A_{2\times3}= \begin{bmatrix} 2 & 3 & 5 \\ 4 & 5 & 8 \end{bmatrix} \qquad B_{4\times3}= \begin{bmatrix} 3 & 5 & 0 \\ 1 & 3 & 2 \\ 4 & 7 & 8 \\ 0 & 9 & 2 \end{bmatrix}$$

(2 rows, 3 columns) (4 rows , 3 columns)

$$D_{2\times1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad C_{3\times5}= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 6 & 7 & 9 & 1 \\ 3 & 7 & 8 & 2 & 0 \end{bmatrix}$$

(2 rows, 1 column) (3 rows, 5 columns)

1. **Type of matrix:-**

a) **Row matrix**: A matrix having a single row:

$$R_{1\times4} = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$ Row matrix (1 Row, 4 Columns)

b) **Column matrix:** A matrix having a single column:

$$C_{3\times1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ column matrix (3 rows, 1 column)

c) **Square matrix**: A square matrix has the same number of rows as columns.

$$S_{2\times2} = \begin{bmatrix} 2 & 0 \\ 1 & 8 \end{bmatrix}$$

A square matrix (2 rows, 2 columns)

$$S_{3\times3} = \begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \\ 3 & 0 & 7 \end{bmatrix}$$

Also a square matrix (3 rows, 3 columns)

d) **Identity matrix**: An identity matrix has 1s on the main diagonal and 0s everywhere else.

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ identity matrix (3 Rows, 3 columns)

Zero matrix (3 rows, 3 columns)

e) **Diagonal matrix**: A diagonal matrix has zero anywhere not on the main diagonal.

$$D_{3\times3}=\begin{bmatrix}2 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 1\end{bmatrix}$$

A diagonal matrix (3 rows, 3 columns)

f) **Scalar matrix**: A scalar matrix has all main diagonal entries the same, with zero everywhere else.

$$S_{3\times3}=\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}$$

A scalar matrix (3 rows, 3 columns)

g) **Triangular matrix:**

Lower triangular is when all entries above the main diagonal are zero.

$$L_{3\times3}=\begin{bmatrix}5 & 0 & 0\\ 2 & 1 & 0\\ 7 & 6 & -3\end{bmatrix}$$

A lower triangular matrix (3 rows, 3 columns)

Upper triangular is when all entries below the main diagonal are zero.

$$U_{3\times3}=\begin{bmatrix}2 & -2 & 7\\ 0 & 4 & 11\\ 0 & 0 & 5\end{bmatrix}$$

An upper triangular matrix (3 rows, 3 columns)

h) **Zero matrix (Null Matrix)**: zeroes just everywhere.

$$Z_{3\times3}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

i) **Symmetric**: in a symmetric matrix matching entries side of the main diagonal are **equal,** like this:

$$S_{4\times4}=\begin{bmatrix} 3 & 2 & 11 & 5 \\ 2 & 9 & -1 & 6 \\ 11 & -1 & 0 & 7 \\ 5 & 6 & 7 & 9 \end{bmatrix}$$

Symmetric matrix (4 rows, 4 columns)

Note: Symmetric matrix must be square, and is equal to its own transpose A = A^T

2.1 Rank of a Matrix

The rank of a matrix is the maximum number of linearly independent rows or columns. It indicates the dimension of the row space or column space.

Methods to Find Rank:

- Method 1: Row Echelon Form (REF)

1. Convert the matrix to row echelon form using elementary row operations.
2. Count the number of non-zero rows. That number is the rank.

Example:

A = [[1 2 3], [2 4 6], [3 6 9]]

After row reduction:

[[1 2 3], [0 0 0], [0 0 0]]

Rank = 1

- Method 2: Determinant (for square matrices)

Find the largest order of any non-zero minor (determinant of a submatrix). That order is the rank.

Key Notes:
- Rank ≤ min(number of rows, number of columns)
- If all rows/columns are independent → Rank = number of rows/columns
- Rank is important in solving linear systems and matrix invertibility.

4/ Posttest :-

1. Define a diagonal matrix and give an example.
2. Find the rank of the matrix [2 4]
   [0 0].
3. Is every square matrix also a row matrix? Explain.

5/ HomeWorks: -

1 -List all types of matrices mentioned in your notes and provide an example for each.

2 -Create a 3x3 zero matrix and a 2x2 identity matrix.

3 -Explain the difference between row matrix and column matrix.

4 -Find the rank of [1 2]
[2 4].

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1/ Overview

1 / A –Target population :-

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1 / B –Rationale :-

Mastery of matrix operations is crucial for solving systems of equations and modeling real-world problems.

1 / C –Central Idea :-

Matrix equality and operations (addition, subtraction, multiplication) are essential tools for manipulating data structures.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

1- Determine when two matrices are equal.

2- Perform addition, subtraction, and multiplication of matrices.

3- Apply matrix operations to solve basic problems.

2/Pretest :

1. When are two matrices considered equal?
2. Compute: [1 2] + [3 4].
3. Is matrix multiplication commutative? Explain.

3/operations on Matrices:

1. Equality of Matrices
Two matrices A and B are said to be equal (A = B) if they have the same dimensions (same number of rows and columns), and their corresponding elements are equal.

Example:
A = [[1, 2], [3, 4]], B = [[1, 2], [3, 4]] → A = B
But if A = [[1, 2], [3, 4]] and B = [[1, 2], [4, 3]] → A ≠ B

2. Matrix Addition
Matrix addition is performed by adding the corresponding elements of two matrices of the same size.

- If A and B are both $m \times n$ matrix, we form $A + B$ by adding corresponding entries (it means that the only way to adding matrices, both of matrix must have the same number of rows and columns).

Properties of matrix addition:

- Commutative: $A+B = B+A$
- Associative : $(A+B) + C = A+(B+C)$
- $A+0 = 0+A = A$

Example1:

$$A_{2\times2}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} + B_{2\times2}\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix} = \begin{bmatrix}1+4 & 2+5\\ 3+3 & 4+4\end{bmatrix} = \begin{bmatrix}5 & 7\\ 6 & 8\end{bmatrix}$$

So we can add these tow matrix because both of them having the same number of rows & columns 2×2

Example 2:

$$\boldsymbol{C}_{3\times3} = \begin{bmatrix}1 & 2 & 6\\ -2 & -1 & 4\\ 0 & 2 & 3\end{bmatrix} + \boldsymbol{D}_{3\times3}\begin{bmatrix}1 & 3 & 8\\ 0 & 9 & -1\\ 2 & 4 & 0\end{bmatrix} = \begin{bmatrix}1+1 & 2+3 & 6+8\\ (-2)+0 & (-1)+9 & 4+(-1)\\ 0+2 & 2+4 & 3+0\end{bmatrix} = \begin{bmatrix}2 & 5 & 14\\ -2 & 8 & 3\\ 2 & 6 & 3\end{bmatrix}$$

Example 3:

$$I_{2\times3} = \begin{bmatrix}0 & 1 & 2\\ 2 & 3 & 4\end{bmatrix} + F_{2\times3}\begin{bmatrix}2 & -1 & 2\\ 4 & 3 & 7\end{bmatrix} = \begin{bmatrix}0+2 & 1+(-1) & 2+2\\ 2+4 & 3+3 & 4+7\end{bmatrix} = \begin{bmatrix}2 & 0 & 4\\ 6 & 6 & 11\end{bmatrix}$$

Also we can add these two matrix because the number of rows & columns are equal 2×3

Example 4:

$$R_{3\times2} = \begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix} + S_{2\times3}\begin{bmatrix}2 & 4 & 6\\ 8 & 10 & 12\end{bmatrix} = xxx$$

We can not add them because they have a different number of rows and columns

Q1: $A_{2\times2} = \begin{bmatrix} 4 & 0 \\ 2 & -3 \end{bmatrix}$ $B_{2\times2} = \begin{bmatrix} -3 & 6 \\ 0 & 1 \end{bmatrix}$ $C_{2\times2} = \begin{bmatrix} 11 & 8 \\ 2 & 6 \end{bmatrix}$

Find (A+B)+C ?

3. Matrix Subtraction

Matrix subtraction is done by subtracting the corresponding elements of two matrices of the same size.

Let A = [[a_{11}, a_{12}], [a_{21}, a_{22}]] and B = [[b_{11}, b_{12}], [b_{21}, b_{22}]], then:
A - B = [[a_{11} - b_{11}, a_{12} - b_{12}], [a_{21} - b_{21}, a_{22} - b_{22}]]

Tow matrices may be subtracted only if they have the same dimension, that is, they must have the same number of rows and columns.

- **Properties of matrix subtraction:**
  a) Commutative property A-B ≠ B-A
  b) Associative property (A-B)-C ≠ A-(B-C)
  c) Identity property A-0 ≠ 0-A

Example1:

$$A_{3\times1} = \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix} - B_{3\times1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4-1 \\ 6-2 \\ 3-3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

SO we can subtracted these tow matrix because they have the same number of rows and columns

Example 2:

$$C_{2\times3} = \begin{bmatrix} 0 & -3 & 11 \\ 4 & 10 & -1 \end{bmatrix} - D_{2\times3} \begin{bmatrix} 4 & 2 & 3 \\ 1 & -9 & 1 \end{bmatrix} = \begin{bmatrix} 0-4 & (-3)-2 & 11-3 \\ 4-1 & 10-(-9) & (-1)-1 \end{bmatrix} = \begin{bmatrix} -4 & -5 & 8 \\ 3 & 19 & -2 \end{bmatrix}$$

Both of matrix have the same number of rows and columns 2× 3

Example 3:

$$S_{2\times2} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - T_{3\times2} \begin{bmatrix} 1 & 4 \\ 5 & 6 \\ 0 & 10 \end{bmatrix} = xxx$$

We can not subtract these two matrix because they have a different number of rows and columns, the first one is 2×2, and the second is 3×2

4 .Matrix Multiplication

Matrix multiplication is different from addition and subtraction. It is defined only when the number of columns in the first matrix equals the number of rows in the second matrix. It is a binary operation that produces a matrix from two matrices.

The resulting matrix, known as matrix product, has the number of rows of the first matrix and the number of columns of the second matrix.

If A is $m \times n$ and B is $n \times p$, then the product AB is an $m \times p$ matrix.

Each element of the resulting matrix is computed as :
$(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$

Example:
A = [[1, 2], [3, 4]], B[[2 ,1] ,[0 ,2]] =
AB = [[1*2 + 2*1, 1*0 + 2*2], [3*2 + 4*1, 3*0 + 4*2[[8 ,10] ,[4 ,4]] = [[

- **Properties of matrix multiplication:**

property	Example
1-commutative property of multiplication Does not hold!	$AB \neq BA$
2-Associative property	$(AB)C = A(BC)$
3-Distributative property	• $A(B+C) = AB + AC$ • $(B+C)A = BA + CA$
4-Multiplicative identity property	$IA = A$ and $AI = A$
5-Multiplicative property of zero	$0A = 0$ and $A0 = 0$
6-Dimension property	The product of an $(m\times n)$ matrix and $(n\times k)$ matrix is an $(m\times k)$ matrix

EXAMPLE 1:

$$\begin{bmatrix} 3 & 6 & 2 \\ -1 & 7 & 0 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 1 & 3 & 8 \\ 5 & 12 & 11 \\ 0 & 2 & 0 \end{bmatrix}_{3\times3} = \begin{bmatrix} 33 & 85 & 90 \\ 34 & 81 & 85 \end{bmatrix}_{2\times3}$$

They are **defined (number of columns in the first matrix is match to the number of rows of the second matrix)** so we can multiply

(3×1)+(6×5)+(2×0)=33 , (3×3)+(6×12)+(2×2)=85 , (3×8)+(6×11)+(2×0)=90

Q1: A=$\begin{bmatrix} 2 & 9 \\ 7 & 5 \end{bmatrix}$, K=3

Find (KA)?

$k \times A = 3\times\begin{bmatrix} 2 & 9 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 27 \\ 21 & 15 \end{bmatrix}$

(3×2)=6, (3×9) =27, (3×7) =21, (3×5) =15

Q2: B=$\begin{bmatrix} 4 & 6 \\ 0 & -12 \end{bmatrix}$, R=1/2, FIND RB?

RB = (½)×$\begin{bmatrix} 4 & 6 \\ 0 & -12 \end{bmatrix} = \begin{bmatrix} 4/2 & 6/2 \\ 0/2 & -12/2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & -6 \end{bmatrix}$

Q3: A=$\begin{bmatrix} 1 & -1 \\ 2 & 5 \\ 3 & -3 \end{bmatrix}_{3\times2}$, B=$\begin{bmatrix} -3 & 4 & 15 \\ 0 & -1 & 24 \end{bmatrix}_{2\times3}$, C=$\begin{bmatrix} -5 & 8 & 0 & 1 \\ 3 & 5 & 0 & 2 \\ 3 & 6 & 4 & -1 \end{bmatrix}_{3\times4}$

FIND (A$\times B) \times C$?

$$A\times B = \begin{bmatrix} 1 & -1 \\ 2 & 5 \\ 3 & -3 \end{bmatrix}_{3\times2} \times \begin{bmatrix} -3 & 4 & 15 \\ 0 & -1 & 24 \end{bmatrix}_{2\times3} = \begin{bmatrix} -3 & 5 & -9 \\ -6 & 3 & 150 \\ -9 & 15 & -27 \end{bmatrix}_{3\times3} \times C$$

$$= \begin{bmatrix} -3 & 5 & -9 \\ -6 & 3 & 150 \\ -9 & 15 & -27 \end{bmatrix}_{3\times3} \times \begin{bmatrix} -5 & 8 & 0 & 1 \\ 3 & 5 & 0 & 2 \\ 3 & 6 & 4 & -1 \end{bmatrix}_{3\times4}$$

$$= \begin{bmatrix} 3 & 55 & -36 & 16 \\ 489 & 867 & 600 & -150 \\ 9 & -159 & -108 & 48 \end{bmatrix}_{3\times4}$$

Q4: $A=\begin{bmatrix} 2 & 3 \\ 11 & -7 \end{bmatrix}_{2\times2}$, $B=\begin{bmatrix} 0 & 2 \\ 9 & -1 \\ 3 & 4 \end{bmatrix}_{3\times2}$, **Find A×B ?**

Answer:

We can not find the multiplication of A and B , because they are(undefined) , the number of columns in the first matrix does not match to the number of rows in the second matrix

4/ Posttest :-

1 - When are two matrices considered equal? Provide an example.

2 - Compute: [2 3] + [1 4]

3 - If A is a 2×3 matrix and B is a 3×2 matrix, can you compute A+B? Why or why not?

5/ HomeWorks: -

1 - Give an example where matrix multiplication is not commutative.

2 - Write the properties of matrix addition and give examples for each.

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1/ Overview

1 / A –Target population :-

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1 / B –Rationale :-

Determinants are key in understanding matrix invertibility and the solution of linear systems.

1 / C –Central Idea :-

The determinant provides information about a matrix’s invertibility and the linear independence of its rows/columns.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

1 - Calculate determinants for 2x2 and 3x3 matrices.

2 - Relate determinant value to matrix rank and invertibility.

3 - Apply Sarrus' method for 3x3 determinants.

2/Pretest :

1 -Find the determinant of [2314][2134].

2 -What does it mean if a matrix's determinant is zero?

3 -Briefly describe Sarrus' method.

3/ Determinants:

Determinants

It is a numerical value that can be calculated from a strictly square matrix and is symbolized by the symbol (det).

1-To calculate the determinant of a matrix of degree (2 × (2), subtract the product of the elements of the main diagonal from product of the elements of the secondary diagonal.

EX: $A = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$

$$\det(A) = 7 \times 1 - 3 \times 2$$

$$= 7 - 6 = 1$$

Ex: $A = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}$ find matrix Determinants?

$$\det(A) = 1(0) - 2(5)$$

$$= 0 - 10 = -10$$

2- Calculate the determinant of a matrix of degree 3 × 3

EX1:

$$\begin{vmatrix} 1 & 3 & -2 \\ 5 & 1 & 4 \\ 2 & -3 & 2 \end{vmatrix}$$

First, we have to rewrite the first two columns of the matrix to its right:

And then we perform the products of the Sarrus' formula:

$$\left|\begin{matrix} 1 & 3 & -2 \\ 5 & 1 & 4 \\ 2 & -3 & 2 \end{matrix}\right| \begin{matrix} 1 & 3 \\ 5 & 1 \\ 2 & -3 \end{matrix} = \begin{array}{l} 1\cdot 1\cdot 2 + 3\cdot 4\cdot 2 + (-2)\cdot 5\cdot(-3) \\ -2\cdot 1\cdot(-2) - (-3)\cdot 4\cdot 1 - 2\cdot 5\cdot 3 \end{array}$$

$$= 2 + 24 + 30 + 4 + 12 - 30$$

$$= \mathbf{42}$$

EX2: $\begin{vmatrix} 5 & 0 & 2 \\ 1 & 3 & 4 \\ -1 & 1 & 0 \end{vmatrix}$

$$\left|\begin{matrix} 5 & 0 & 2 \\ 1 & 3 & 4 \\ -1 & 1 & 0 \end{matrix}\right| \begin{matrix} 5 & 0 \\ 1 & 3 \\ -1 & 1 \end{matrix} = \begin{array}{l} 5\cdot 3\cdot 0 + 0\cdot 4\cdot(-1) + 2\cdot 1\cdot 1 \\ -(-1)\cdot 3\cdot 2 - 1\cdot 4\cdot 5 - 0\cdot 1\cdot 0 \end{array}$$

$$= 0 + 0 + 2 + 6 - 20 + 0$$

$$= \mathbf{-12}$$

4/ Posttest :-

1. Find the determinant of $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$.
2. What does it mean if a matrix's determinant is zero?

5/ HomeWorks: -

1 - Use Sarrus' method to find the determinant of a given 3x3 matrix.

2 - Explain with examples how the determinant helps in finding the rank.

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1/ Overview

1 / A –Target population :-

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1 / B –Rationale :-

Inverse matrices are fundamental for solving linear systems and understanding transformations.

1 / C –Central Idea :-

A matrix’s inverse allows for the solution of linear equations and deeper exploration of linear transformations.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

1- Find the inverse of 2x2 and 3x3 matrices.

2- Use the inverse to solve systems of linear equations.

3- Verify solutions using matrix multiplication.

2/Pretest :

1 - Solve the system: $x+2y=5x+2y=5$
$3x+4y=113x+4y = 11$ using matrices.

2 - What condition must be met for a matrix to have an inverse?

3/ Inverse matrices

Inverted Matrix:
The inverse of the matrix is the multiplicative inverse where the product of the matrix and its inverse is equal to the unit matrix.

The inverse of matrix can be found from the following formula:

$$A^{-1} = \frac{1}{|A|}\, adj(A)$$

$|A|$: Is the determinant of the matrix.

adj(A): Is the bound matrix.

1-Finding the Inverse of a 2×2 Matrix
The following rule provides a simple way for finding the inverse of a 2 x 2 matrix, when it exists. For larger matrices there is a more general procedure for finding inverses, which we consider later in this section.

First step: To find $|A|$, we should find the product of the elements of the main diagonal minus from the product of the element of the secondary diagonal , like this:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \rightarrow |A| = ((a_{11} \times a_{22}) - (a_{12} \times a_{21}))$$

Second step: To find adj(A) we should invert the position of the main diagonal element and change the signals of the secondary diagonal elements , like this:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \rightarrow adj(A) = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Third step: and finally we can apply the formula:

$$A^{-1} = \frac{1}{|A|}\, adj(A)$$

Example 1: when A=$\begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{11} & \mathbf{7} \end{bmatrix}$,Find A^{-1}?

Solution:

$\boldsymbol{A^{-1}}$= = $\frac{1}{|A|}\, adj(A)$

First step: find $|A|$, **A=**$\begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{11} & \mathbf{7} \end{bmatrix} \rightarrow \ |\boldsymbol{A}| = ((\mathbf{2} \times \mathbf{7}) - (\mathbf{3} \times \mathbf{11}))$

$$|A| = -19$$

Second step: find $adj(A)$, **A=**$\begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{11} & \mathbf{7} \end{bmatrix} \rightarrow$ **A=**$\begin{bmatrix} \mathbf{7} & \mathbf{-3} \\ \mathbf{-11} & \mathbf{2} \end{bmatrix}$

Third step: $\boldsymbol{A^{-1}}$= = $\frac{1}{|A|}\, adj(A) \rightarrow \ \boldsymbol{A^{-1}}$= $\frac{1}{-19}\begin{bmatrix} \mathbf{7} & \mathbf{-3} \\ \mathbf{-11} & \mathbf{2} \end{bmatrix}$

$$=\begin{bmatrix} \frac{\mathbf{7}}{\mathbf{-19}} & \frac{\mathbf{-3}}{\mathbf{-19}} \\ \frac{\mathbf{-11}}{\mathbf{-19}} & \frac{\mathbf{2}}{\mathbf{-19}} \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{7}}{\mathbf{-19}} & \frac{\mathbf{3}}{\mathbf{19}} \\ \frac{\mathbf{11}}{\mathbf{19}} & \frac{\mathbf{2}}{\mathbf{-19}} \end{bmatrix}$$

Example 2: when B=$\begin{bmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{1} & \mathbf{4} \end{bmatrix}$, Find B^{-1}?

Solution:

First step: find $|A|$ $\rightarrow$ **A=**$\begin{bmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{1} & \mathbf{4} \end{bmatrix} \rightarrow \ |\boldsymbol{A}| = ((\mathbf{3} \times \mathbf{4}) - (\mathbf{5} \times \mathbf{1}))$

$$|A| = 7$$

Second step: find $adj(A)$, **A=** $\begin{bmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{1} & \mathbf{4} \end{bmatrix} \rightarrow$ **A=**$\begin{bmatrix} \mathbf{4} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{3} \end{bmatrix}$

Third step: $\boldsymbol{A^{-1}}$= = $\frac{1}{|A|}\, adj(A) \rightarrow \ \boldsymbol{A^{-1}}$= $\frac{1}{7}\begin{bmatrix} \mathbf{4} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{3} \end{bmatrix}$

$$=\begin{bmatrix} \frac{\mathbf{4}}{\mathbf{7}} & \frac{\mathbf{-5}}{\mathbf{7}} \\ \frac{\mathbf{-1}}{\mathbf{7}} & \frac{\mathbf{3}}{\mathbf{7}} \end{bmatrix}$$

Remark:
The quantity ad - bc that appears in the rule for calculating the inverse of a 2 × 2 matrix is called the determinant of the matrix. If the determinant is 0, then the matrix does not have an inverse (since we cannot divide by 0).

2- Finding the Inverse of a 3 × 3 Matrix

When the matrix is $A_{3\times3}$ we can find the inverse of it by this steps:

Step 1: we can find $|A|$, like this:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \rightarrow |A| = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

$$|A| = ((a_{11}\times a_{22}\times a_{33}) + (a_{12}\times a_{23}\times a_{31}) + (a_{13}\times a_{21}\times a_{32})) - ((a_{12}\times a_{21}\times a_{33}) + (a_{11}\times a_{23}\times a_{32}) + (a_{13}\times a_{22}\times a_{31}))$$

Step 2: To find adj(A), we must do this:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \rightarrow adj(A) = \begin{vmatrix} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \\ \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix} \\ a_{12} \quad a_{13} & a_{11} \quad a_{13} & a_{11} \quad a_{12} \end{vmatrix}$$

$$\text{adj(A)}=\begin{bmatrix} (a_{22}\times a_{33}-a_{23}\times a_{32}) & (a_{21}\times a_{33}-a_{23}\times a_{31}) & (a_{21}\times a_{32}-a_{22}\times a_{31}) \\ (a_{12}\times a_{33}-a_{13}\times a_{32}) & (a_{11}\times a_{33}-a_{13}\times a_{31}) & (a_{11}\times a_{32}-a_{12}\times a_{31}) \\ (a_{12}\times a_{23}-a_{13}\times a_{22}) & (a_{11}\times a_{23}-a_{13}\times a_{21}) & (a_{11}\times a_{22}-a_{12}\times a_{21}) \end{bmatrix}$$

adj(A)= $\begin{bmatrix} +a_{11} & -a_{12} & +a_{13} \\ -a_{21} & +a_{22} & -a_{23} \\ +a_{31} & -a_{32} & +a_{33} \end{bmatrix}$ **(we changes the signals of the matrix elements)**

adj(A)= $\begin{bmatrix} +a_{11} & -a_{21} & +a_{31} \\ -a_{12} & +a_{22} & -a_{32} \\ +a_{13} & -a_{23} & +a_{33} \end{bmatrix}$ **(we transpose the matrix)**

Step 3: and finally, we can apply the formula:

$$A^{-1} = \frac{1}{|A|}\ adj(A)$$

= now we will transpose the matrix $\begin{bmatrix} 0 & 0 & -2 \\ 5 & -3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$

Step 3: now we will apply the formula:

$$A^{-1} = \frac{1}{|A|}\ adj(A)$$

$$= \frac{1}{-2}\begin{bmatrix} 0 & 0 & -2 \\ 5 & -3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$$

$= \begin{bmatrix} \frac{0}{-2} & \frac{0}{-2} & \frac{-2}{-2} \\ \frac{5}{-2} & \frac{-3}{-2} & \frac{-5}{-2} \\ \frac{-4}{-2} & \frac{2}{-2} & \frac{4}{-2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{5}{-2} & \frac{3}{2} & \frac{5}{2} \\ 2 & -1 & -2 \end{bmatrix}$ **this is the final solution**

Note: to ensure that your solution is correct, multiply the result of inverse matrix by the original matrix and you will get the unit matrix.

Solving linear equation using inverted matrix

We can make a step to solving linear equation using inverted matrix: -

1-Extract a matrix from equations
2-Find the inverted matrix A^{-1}
3-Apply the formula: $[X] = A^{-1} \times [B]$

Example 1: solve these linear equation using inverted matrix:

$2X_1 + 3X_2 + X_3 = 2$

$-5X_1 + 4X_2 + 2X_3 = 10$

$3X_1 + X_2 + 5X_3 = 9$

Solution:

Step 2: find the inverse matrix A^{-1} as we learned earlier:

$$|A| = \begin{bmatrix} 2 & 3 & 1 \\ -5 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{matrix} 2 & 3 \\ -5 & 4 \\ 3 & 1 \end{matrix}$$

$|A|=((2\times4\times5)+(3\times2\times3)+(1\times(-5)\times1) - ((3\times4\times1)+(1\times2\times2)+(5\times(-5)\times3))$

$|A|=$ 1

adj(A)

$$\begin{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} & \begin{bmatrix} -5 & 2 \\ 3 & 5 \end{bmatrix} & \begin{bmatrix} -5 & 4 \\ 3 & 1 \end{bmatrix} \\ \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix} & \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} & \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} \\ \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} & \begin{bmatrix} 2 & 1 \\ -5 & 2 \end{bmatrix} & \begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix} \end{bmatrix}$$

$$\text{Adj(A)}=\begin{bmatrix} (4\times5-2\times1) & ((-5)\times5-2\times3) & ((-5)\times1-4\times3) \\ (3\times5-1\times1) & (2\times5-1\times3) & (2\times1-3\times3) \\ (3\times2-1\times4) & (2\times2-1\times(-5)) & (2\times4-3\times(-5)) \end{bmatrix}$$

$$\text{Adj(A)}=\begin{bmatrix} 18 & -31 & -17 \\ 14 & 7 & -7 \\ 2 & 9 & 23 \end{bmatrix} \rightarrow \begin{bmatrix} 18 & 31 & -17 \\ -14 & 7 & 7 \\ 2 & -9 & 23 \end{bmatrix} \rightarrow \begin{bmatrix} 18 & -14 & 2 \\ 31 & 7 & -9 \\ -17 & 7 & 23 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$= \frac{1}{112} \begin{bmatrix} 18 & -14 & 2 \\ 31 & 7 & -9 \\ -17 & 7 & 23 \end{bmatrix}$$

Step 3: apply the final formula:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = A^{-1} \times [B]$$

$$= \frac{1}{112} \begin{bmatrix} 18 & -14 & 2 \\ 31 & 7 & -9 \\ -17 & 7 & 23 \end{bmatrix} \times \begin{bmatrix} 2 \\ 10 \\ 9 \end{bmatrix}$$

$$X_1 = \frac{(18\times2)+(-14\times10)+(2\times9)}{112} = \frac{-43}{56}$$

$$X_2 = \frac{(31\times2)+(7\times10)+(-9\times9)}{112} = \frac{51}{112}$$

$$X_3 = \frac{(-17\times2)+(7\times10)+(23\times9)}{112} = \frac{243}{112}$$

NOTE: if you want to ensure that the solution is correct you can apply the final produces of (X) in the equations, and it must to be the same result of the original equations.

4/ Posttest :-

1 - State the formula for the inverse of a 2x2 matrix.

2 - Find the inverse of [2 3]
[1 4].

5/ HomeWorks: -

1 -Give an example of a matrix that does not have an inverse and explain why.

2 -For a 3x3 matrix, outline the steps to find its inverse.

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Computer Networks
and Software Techniques

Learning package

Mathematics and Numerical Analysis

For

First year students

(5)

By

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Dep. Of Computer Networks
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2025

1/ Overview

1 / A –Target population :-

For First year students

Technological institute of Basra

Dep. Of Computer

Networks and Software Techniques

1 / B –Rationale :-

Differentiation is a core concept in calculus, crucial for modeling change in engineering and science.

1 / C –Central Idea :-

Differentiation rules enable the calculation of rates of change for various functions.

1 / D – Performance Objectives

After studying the first unit, the student will be able to: -

1 - Apply differentiation rules to algebraic, trigonometric, exponential, and logarithmic functions.

2 - Compute derivatives using chain, product, and quotient rules.

3 - Solve problems involving implicit and partial derivatives.

2/Pretest :

1. Differentiate $f(x)=x^2+3x$.
2. What is the derivative of $\sin(x)$?
3. Define implicit differentiation.

3/Differentiation

Definition:

These functions can be defined as ratio between the sides of a right triangle containing that angle, more generally, coordinates on the unite circle. when referring to triangles, the triangle is often referred to as a flat surface. this can always the sum of 180°.

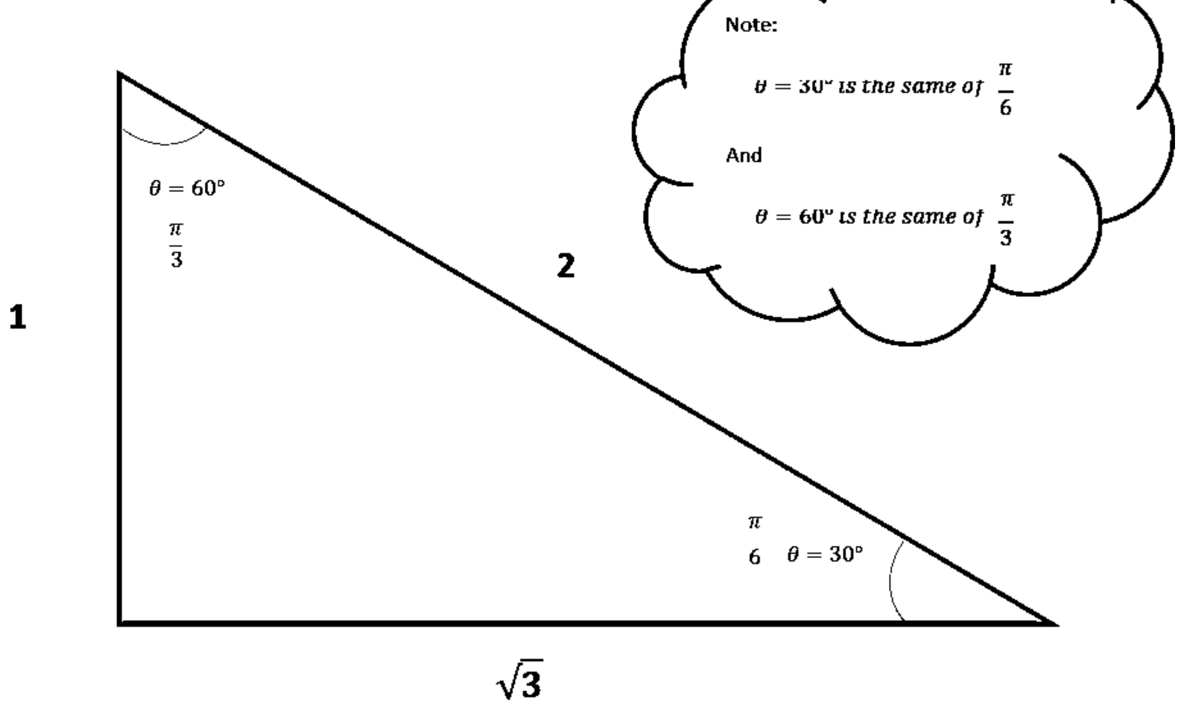
There are six functions that are core of trigonometry. There are three primary ones that you need to understand completely:

- **Sine (sin)**
- **Cosine (cos)**
- **Tangent (tan)**

The other three are not used as often and can be derived from the three primary functions. Because they can easily be derived, calculators and spreadsheets do not usually have them.

- **Secant (sec)**
- **Cosecant (csc)**
- **Cotangent(cot)**

when $\theta = 30^\circ$ and $\theta = 60^\circ$



The rules of trigonometric functions

$$\sin x = \frac{o}{h} = \frac{\text{المقابل}}{\text{الوتر}}$$

$$\cos x = \frac{a}{h} = \frac{\text{المجاور}}{\text{الوتر}}$$

$$\tan x = \frac{o}{a} = \frac{\text{المقابل}}{\text{المجاور}}$$

$$\sec x = \frac{1}{\cos x} = \frac{\text{الوتر}}{\text{المجاور}}$$

$$\csc x = \frac{1}{\sin x} = \frac{\text{الوتر}}{\text{المقابل}}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{\text{المجاور}}{\text{المقابل}} = \frac{1}{\tan x}$$

And here is an example when $\theta = 30° \ or \ \frac{\pi}{6}$ and when $\theta = 60° \ or \ \frac{\pi}{3}$

Ex 1: find the solution of :

$\cos\frac{\pi}{3}$, $\tan\frac{\pi}{6}$, $\sec\frac{\pi}{6}$, $\csc\frac{\pi}{3}$, $\sin\frac{\pi}{6}$, $\cot\frac{\pi}{3}$ **?**

Solution:

$\cos\frac{\pi}{3} = \frac{\text{المجاور}}{\text{الوتر}} = \frac{1}{2}$

$\tan\frac{\pi}{6} = \frac{\text{المقابل}}{\text{المجاور}} = \frac{1}{\sqrt{3}}$

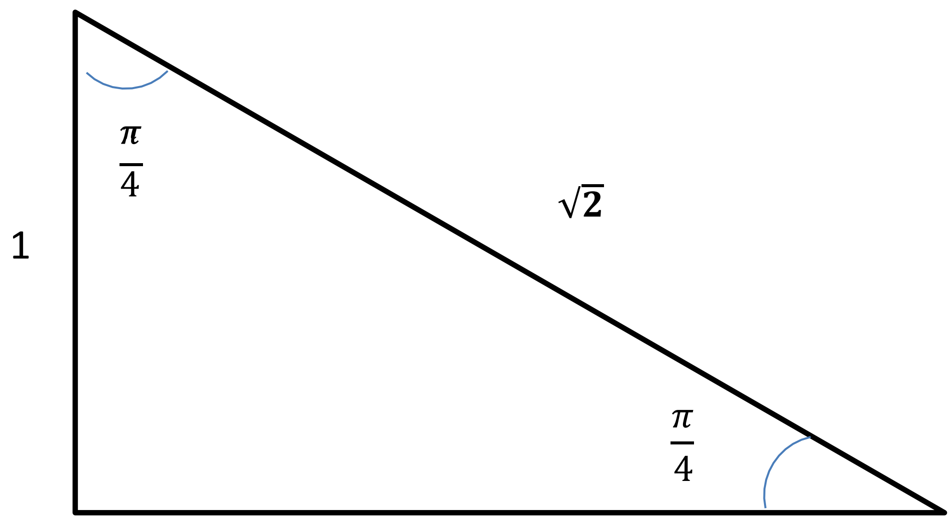
$\sec\frac{\pi}{6} = \frac{\text{الوتر}}{\text{المجاور}} = \frac{2}{\sqrt{3}}$

$\csc\frac{\pi}{3} = \frac{\text{الوتر}}{\text{المقابل}} = \frac{2}{\sqrt{3}}$

$\sin\frac{\pi}{6} = \frac{\text{المقابل}}{\text{الوتر}} = \frac{1}{2}$

$\cot\frac{\pi}{3} = \frac{\text{المجاور}}{\text{المقابل}} = \frac{1}{\sqrt{3}}$

When $\theta = 45°$ or $\frac{\pi}{4}$



Ex2:- find the solution of these triangles?

$$\text{Sin } \frac{\pi}{4}$$

$$\text{cot } \frac{\pi}{}$$

$$\text{Tan } \frac{\pi}{4}$$

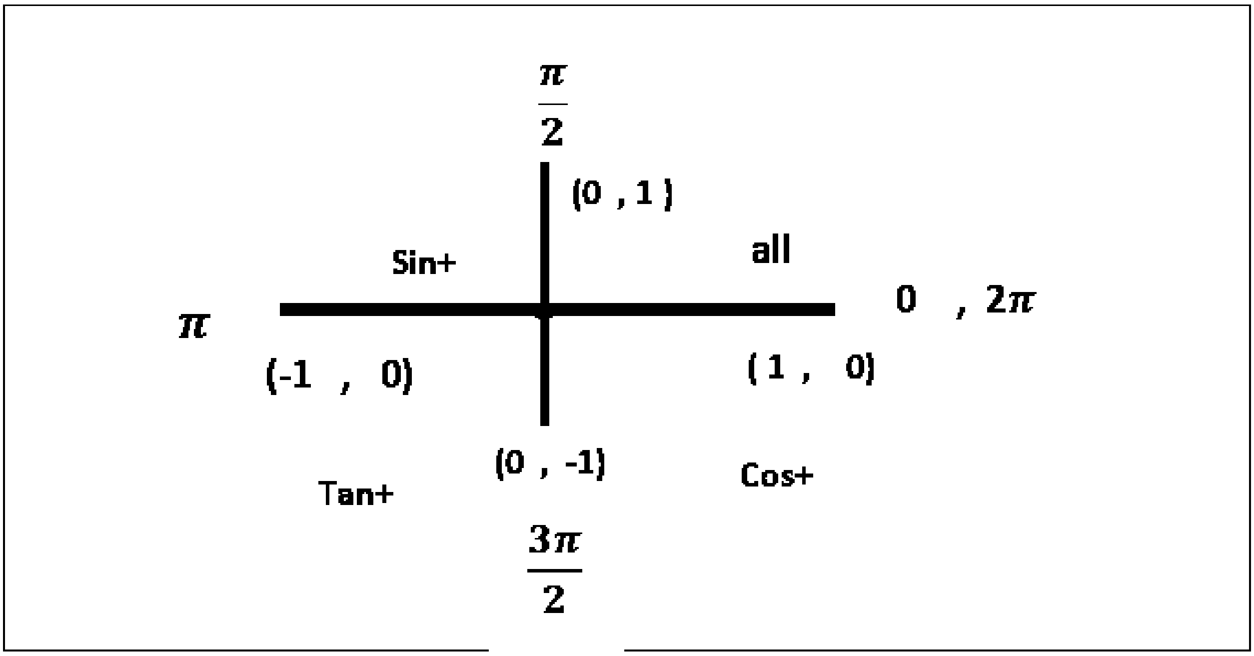
Solution:

$$\sin\frac{\pi}{4} = \frac{\text{المقابل}}{\text{الوتر}} = \frac{1}{\sqrt{2}}$$

$$\cot\frac{\pi}{4} = \frac{\text{المجاور}}{\text{المقابل}} = 1$$

$$\tan\frac{\pi}{4} = \frac{\text{المقابل}}{\text{المجاور}} = 1$$

Unit circle



Note:-

Cos : x-axis

Sin : y-axis

$\frac{\pi}{2}$ **= 90°** , π **= 180°**

$\frac{3\pi}{2}$ **= 270°** , 2π **= 360°**

-The Derivative:

Definition 1:- If $f: x \to y$ is a function, the derivative of a function f at a point x_0 written $f'(x_0)$; is given by
$f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$,
If this limit exists and finite.
Remark: The geometric meaning of the derivative function at a point, is the presence of tangent to the function at that point; and the derived value of the function at the point, is the value of the angle tan made by the tangent with the axis-x at the point.

Derivatives of usual functions

Below you will find a list of the most important derivatives. Although these formulas can be formally proven, we will only state them here. We recommend you learn them by heart.

	Name of function	The function	Derivative
1	The Constant Function	$f(x) = k$	$f'(x) = 0$
2	Constant Multiple Rule	$g(x) = kf(x)$	$g'(x) = k\, f'(x)$
3	Power Rule	$f(x) = x^n$	$f'(x)=n\, x^{n-1}$
4	Sum and Difference Rule	$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$
5	Product Rule	$h(x) = f(x).g(x)$	$h'(x) = f\ (x)g'(x)+g(x)f'(x)$
6	Quotient Rule	$h(x) = \frac{f(x)}{g(x)}$	$h'(x)=\frac{g(x)f'(x)-f\ (x)g'(x)}{(g\ (x))^2}$
7	Chain Rule for composite function	$h(x) = f\big(g(x)\big)$ $g(x) = (f(x))^n$	$h'(x) = f'(g(x))g'(x)$ $g'(x)=n(f(x))^{n-1}\, f'(x)$

Examples:

1- $f(x) = \frac{1}{x} + \frac{3}{x^2}$

Solution: $f(x) = x^{-1} + 3x^{-2}$

$f'(x) = -1x^{-2} - (2)(3)x^{-3} = -\frac{1}{x^2} - \frac{6}{x^3}$

2- $f(x) = 2\sqrt{x} + 6\sqrt[3]{x}$

Solution: $f(x) = 2(x)^{\frac{1}{2}} + 6(x)^{\frac{1}{3}}$

$f'(x) = 2\left(\frac{1}{2}\right)x^{-\frac{1}{2}} + 6\left(\frac{1}{3}\right)x^{-\frac{2}{3}} = \frac{1}{\sqrt{x}} + \frac{2}{\sqrt[3]{x^2}}$.

3- $f(x) = (x^2 + 5x - 3)^3$

$f'(x) = 3(x^2 + 5x - 3)^2(2x + 5)$

4- $f(x) = \frac{x^2}{x^3+2x}$

$f'(x) = \frac{(x^3+2x)(2x)-x^2(3x^2+2)}{(x^3+2x)^2}$

$= \frac{2x^4+4x^2-3x^4-2x^2}{(x^3+2x)^2}$

$= \frac{-x^4+2x^2}{(x^3+2x)^2}$

5- Find all the derivatives of the following function

$f(x) = (x + 3)^3$

$f'(x) = 3(x + 3)^2 . 1$

$f'' = 6(x + 3)$

$f'''(x) = 6$

$f^{(4)}(x) = 0$

3.2. Chain Rule.

If y is a function that can be derived by u and u is a function that is derived by the x, then

$\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$

Examples :

1- Let $y = u^3$ and $u = 4x^2 - 2x + 5$, find $\frac{dy}{dx}$

Solution: $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx} = 3u^2 . (8x - 2)$

$= 3(4x^2 - 2x + 5)^2(8x - 2)$

2- $y = \sqrt{2 - \sqrt{x}}$, find y'.

Solution: Suppose $u = \sqrt{x}$, $t = 2 - \sqrt{x} = 2 - u$, $y = \sqrt{t}$

$\frac{dy}{dx} = \frac{dy}{dt} . \frac{dt}{du} . \frac{du}{dx}$

$$= \frac{1}{2} t^{-\frac{1}{2}}.(-1)\frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{4\sqrt{x}\sqrt{t}} = -\frac{1}{\sqrt{x(2-\sqrt{x})}}$$

3- $y = \sqrt[3]{(2x^2 + x - 1)^2}$ find y' by chain rule

Solution: Suppose $u = (2x^2 + x - 1)$, $y = \sqrt[3]{u^2} = u^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{2}{3}u^{-\frac{1}{3}}.(4x+1)$$

$$= \frac{2}{3\sqrt[3]{u}}(4x+1)$$

$$= \frac{8x+2}{3\sqrt[3]{2x^2+x-1}}$$

4- If $y = n^2 + 2n + 2$, $n = 2x + 1$, $find\ \frac{dy}{dx}, when\ x = 2.$

Solution: $\frac{dy}{dx} = \frac{dy}{dn}.\frac{dn}{dx} = (2n+3).2 = 2(2n+3) = 2(2(2x+1)+3) = 2(4x+5).$

When $x = 2$, $\quad \frac{dy}{dx} = 2(4 \times 2 + 5) = 2(13) = 26$

3.3. The Implicit derivative

When we cannot write y in terms x, we are implicit the derivative to obtain y'.

Examples:

1- $y = xy^2 + 2x^2$

Solution: $y' = x(2yy') + y^2.1 + 4x$

$y' - 2xyy' = y^2 + 4x$

$y'(1 - 2xy) = y^2 + 4x$

$y' = \frac{y^2+4x}{(1-2xy)}$.

2- $x = \frac{x-y}{x+y}$

$x(x + y) = x - y$

$x^2 + xy = x - y$

$2x + xy' + y.1 = 1 - y'$

$2x + y = 1 - y' - xy'$

$y' + xy' = 1 - 2x - y$

$y'(1 + x) = 1 - 2x - y$

$y' = \frac{1-2x-y}{1+x}$.

4- $x = \frac{\sqrt{y}+1}{xy+x}$.

Solution: $x(xy + x) = \sqrt{y} + 1$

$x^2y + x^2 = \sqrt{y} + 1$
$x^2y' + y(2x) + 2x = \frac{1}{2\sqrt{y}}.y'$
$x^2y' + 2xy + 2x = \frac{y'}{2\sqrt{y}}$
$2x(1 + y) = \frac{y'}{2\sqrt{y}} - x^2y'$
$2x(1 + y) = y'\left(\frac{1}{2\sqrt{y}} - x^2\right)$
$y' = \frac{2x(1+y)}{\left(\frac{1}{2\sqrt{y}}-x^2\right)}$.

5- If $y = (x^2 - 3)^4$, find y' by the chain rule

Solution : $u = x^2 - 3, y = (u)^4$

$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

$= 4(u)^3.2x = 8x(u)^3 = 8x\ (x^2 - 3)^3$

6- Given $x^2 + y^2 = 25\ \ find\ \frac{d^2y}{dx^2}$.

Solution: 2x+2y$y' = 0\quad \rightarrow 2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x \rightarrow\ \frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$.

$\frac{d^2y}{dx^2} = -\frac{(y).1-(x)\frac{dy}{dx}}{y^2} = \frac{-y+x\left(-\frac{x}{y}\right)}{y^2} = \frac{-y-\frac{x^2}{y}}{y^2} = \frac{-y^2-y^2}{y^3}$.

3.4. Derivative of the Trigonometric Functions

	The function	Its derivative
1	$\sin(x)$	$\cos(x)$
2	$\cos(x)$	$-\sin(x)$
3	$\tan(x)$	$sec^2(x)$
4	$\cot(x)$	$-csc^2(x)$
5	$sex(x)$	$\sec(x)\tan(x)$
6	$\csc(x)$	$-\csc(x)\cot(x)$

Examples :

1- Find y' if $y = \cos(3x-2)$.

Solution: $y' = -\sin(3x-2).3 = -3\sin(3x-2)$

2- Find $\frac{dy}{dx}$ by using the chain rule, $y = \cos(\cot(\sec(2x)))$

Solution : Let $u = 2x,\quad t = \sec(2x) = \sec(u)\,,\quad v = \cot(\sec(2x)) = \cot(t)$.

$\therefore y = \cos(v) = \cos(\cot(\sec(2x)))$

$\frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{dt}.\frac{dt}{du}.\frac{du}{dx}$

$= (-\sin(v)).(-csc^2(t)).(\sec(u)\tan(u)).(2)$

$= 2\sin(v)\,csc^2(t)\sec(u)\tan(u)$

$\frac{dy}{dx} = 2\sin(cot(sec(2x))).csc^2(\sec(2x)).\sec(2x)\tan(2x)$

3- Find $\boldsymbol{y'}\ \boldsymbol{if}\ \ \boldsymbol{y = sin(x^2)cos(x^2)}$.

Solution: $y' = (\sin(x^2).-\sin(x^2).2x) + (\cos(x^2).\cos(x^2).2x)$

$= -2x\sin^2(x^2) + 2x\cos^2(x^2)$

$= 2x(\cos^2(x^2) - sin^2(x^2))$

4- Find $\frac{dy}{dx}$ where $y = 2\sin(9x^2 + 3x^2 + 1)$

Solution: $\frac{dy}{dx} = 2cos\,(9x^2 + 3x^2 + 1)(27x^2 + 6x)$

$= 2(27x^2 + 6x)cos\,(9x^2 + 3x^2 + 1)$

3.5. The Logarithm and exponential functions

1. Exponential Function

Definition 2: The Exponential function with base a is defined for all real number x by

$f(x) = a^x,\ where\ a > 1,\ a \neq 1$

Definition 3 :(The Natural Exponential Function)
The natural exponential function is the exponential function
f(x) $= e^x$ with **e** ≈ **2.71828**
with base e . It is often referred to as the exponential function .

2. Logarithm Function

Definition 4: Let a be a positive number with $a \neq 0$, then the logarithmic function with base a, denoted by log_a is defined by

$log_a(x) = y \leftrightarrow a^y = x$

3. Properties of the logarithmic function

1- $log_a(AB) = log_a(A) + log_a(B)$

2- $log_a\left(\frac{A}{B}\right) = log_a(A) - log_a(B)$

3- $log_a(A^c) = clog_a(A)$

When $A, B, and\ c$ are real numbers.

4- $log_a 1 = 0$

5- $log_a a = 1$

6- $log_a a^x = x$

7- $a^{log_a x} = x$

8- $log\left(\frac{1}{a}\right) = -\log(a)$.

Example:

1- $log_5 1 = 0$ 2- $log_5 5 = 1$ 3- $log_5 5^8 = 8$ 4- $5^{log_5 12} = 12$ 5- $log_6 6^5 = 5$

-Examples

Logarithm Form **Exponential Form**

1- $log_8 8 = 1 \quad \leftrightarrow \quad 8^1 = 8$

2- $log_8 4 = \frac{2}{3} \quad \leftrightarrow \quad 8^{\frac{2}{3}} = 4$

3- $log_8 512 = 3 \quad \leftrightarrow \quad 8^3 = 512$

4- $log_4 2 = \frac{1}{2} \quad \leftrightarrow \quad 4^{\frac{1}{2}} = 2$

5- $log_8\left(\frac{1}{8}\right) = -1 \quad \leftrightarrow \quad 8^{-1} = \frac{1}{8}$

6- $log_8 \frac{1}{64} = -2 \quad \leftrightarrow \quad 8^{-2} = \frac{1}{64}$

7- $\log_4 \frac{1}{16} = -2 \quad \leftrightarrow \quad 4^{-2} = \frac{1}{16}$

Definition 5: (Common Logarithm)

The logarithm with base **10** is called the common logarithm and is denoted by omitting the base:

$\mathbf{log\ x = \ log_{10}\, x}$

Definition 6: (Natural Logarithms)

The logarithm with base e is called the natural logarithm and is denoted by ln:

Ln x $= \ \mathbf{log}_e\, x$

Remark:

1- The natural logarithmic function **y** $=$ **Ln x** is the inverse function of the natural exponential function **y**$=$ e^x. By the definition of inverse functions we have

Ln x $=$ **y** $\Leftrightarrow e^y = $ **x**

2- $ln1 = 0$

3- $\ln(e) = 1$

4- $lne^x = 1$

5- $e^{\ln(x)} = x$

6- $log_a x = \frac{\ln(x)}{\ln(a)}$

For example :- $lne^8 = 8$, $\ln\left(\frac{1}{e^2}\right) = \ln(e^{-2}) = -2$, $log_2(4) = \frac{\ln(4)}{ln2} = \frac{ln(2)^2}{ln2} = 2$.

The Derivative of the Logarithm and exponential functions

1- **Derivative of $f(x) = a^x$**

The derivative of $f(x) = a^x$, $a > 0, a \neq 0$, is $f'(x) = a^x lna$. That is $\frac{d}{dx}a^x = a^x lna$.

Example: Find the derivative $f(x) = 2^x$,

$f'(x) = 2^x ln2$

2- Derivative of $f(x) = lnx$

If $f(x) = lnx$, then $f' = \frac{1}{x}$. that is $\frac{d}{dx}lnx = \frac{1}{x}$.

Example: Find the derivative of $f(x) = x^3 lnx$

Solution : $f' = x^3\frac{1}{x} + lnx\,.(3x^2) = x^2 + 3x^2 lnx = x^2(1 + 3lnx)$

3- **Derivative of $ln\, g(x)$**

The formula for finding the derivative of the composite function $ln\, g(x)$, where g is differentiable function, is $\frac{d}{dx}ln\,(g(x) = \frac{\frac{d}{dx}g(x)}{g(x)}$.

Examples:

1- $f(x) = ln(x^3 + 2x^2 + 1)$

Solution: $f'(x) = \frac{\frac{d}{dx}(x^3+2x^2+1)}{(x^3+2x^2+1)} = \frac{3x^2+4x}{(x^3+2x^2+1)}$

2- $f(x) = (\ln x)^2$

Solution: The function $f(x)$ is $\ln x$ raised to the power 2. We use the power rule. Then

$f'(x) = 2\ln x\,(\frac{d}{dx}\ln x) = \frac{2\ln x}{x}$.

4- **Derivative of $f(x) = log_a x$**

If $f(x) = log_a x = \frac{\ln(x)}{\ln(a)}$, then $f'(x) = \frac{1}{xlna}$. That is $\frac{d}{dx}log_a x = \frac{1}{xlna}$

Example: find the derivative of: $f(x) = log_2 x$

$f'(x) = \frac{1}{xln2}$

5- Derivative of $f(x) = e^x$

The derivative of exponential function $f(x) = e^x$ is e^x . That is $\frac{d}{dx}e^x = e^x$.

Examples :

1- $f(x) = x^2 + e^x$

Solution: $f'(x) = 2x + e^x$

2- $f(x) = x^2e^x$

Solution : $f'(x) = x^2e^x + 2xe^x = xe^x(x+2)$

3- $f(x) = \frac{e^x}{x^3}$

Solution : $f'(x) = \frac{x^3e^x - 3x^2e^x}{(x^3)^2} = \frac{x^2e^x(x-3)}{x^6} = \frac{e^x(x-3)}{x^4}$.

4- Find y' if $e^{2y} = ln\left(x + e^{3x^2+1}\right)$

Solution: $e^{2y}.(2y') = \frac{1}{\left(x+e^{3x^2+1}\right)}.\left(1 + e^{3x^2+1}.(6x)\right)$

$$2y'e^{2y} = \frac{1+6x\,e^{3x^2+1}}{\left(x+e^{3x^2+1}\right)}$$

$$y' = \frac{1+6x\,e^{3x^2+1}}{2e^{2y}\left(x+e^{3x^2+1}\right)}$$

5- $e^{\sec x} = y$

Solution: $y' = e^{\sec x}.sec(x)\tan(x)$

6- Derivative of $y = e^{g(x)}$

The derivative of composite function $y = e^{g(x)}$, where g is a differentiable function, is $\frac{d}{dx}e^{g(x)} = e^{g(x)}.\frac{d}{dx}g(x)$.

Example : Find the derivative of $f(x) = x^3e^{x^2}$

Solution: $f'(x) = x^3e^{x^2}(2x) + 3x^2e^{x^2} = x^2e^{x^2}(2x^2+3)$.

Partial Derivatives

A partial derivative is when you take the derivative of a function with more than one variable but focus on just one variable at a time, treating the others as constants.

Find all of the first order partial derivatives for the following functions.

(a) $f(x,y) = x^4 + 6\sqrt{y} - 10$

(b) $w = x^2y - 10y^2z^3 + 43x - 7\tan(4y)$

(c) $h(s,t) = t^7 \ln(s^2) + \dfrac{9}{t^3} - \sqrt[7]{s^4}$

(d) $f(x,y) = \cos\left(\dfrac{4}{x}\right) \mathbf{e}^{x^2y-5y^3}$

Solution

(a) $f(x,y) = x^4 + 6\sqrt{y} - 10$

Let's first take the derivative with respect to x and remember that as we do so all the y's will be treated as constants. The partial derivative with respect to x is,

$$f_x(x,y) = 4x^3$$

(b) $w = x^2y - 10y^2z^3 + 43x - 7\tan(4y)$

With this function we've got three first order derivatives to compute. Let's do the partial derivative with respect to x first. Since we are differentiating with respect to x we will treat all y's and all z's as constants. This means that the second and fourth terms will differentiate to zero since they only involve y's and z's.

This first term contains both x's and y's and so when we differentiate with respect to x the y will be thought of as a multiplicative constant and so the first term will be differentiated just as the third term will be differentiated.

Here is the partial derivative with respect to x.

$$\frac{\partial w}{\partial x} = 2xy + 43$$

Let's now differentiate with respect to y. In this case all x's and z's will be treated as constants. This means the third term will differentiate to zero since it contains only x's while the x's in the first term and the z's in the second term will be treated as multiplicative constants. Here is the derivative with respect to y.

$$\frac{\partial w}{\partial y} = x^2 - 20yz^3 - 28\sec^2(4y)$$

Finally, let's get the derivative with respect to z. Since only one of the terms involve z's this will be the only non-zero term in the derivative. Also, the y's in that term will be treated as multiplicative constants. Here is the derivative with respect to z.

$$\frac{\partial w}{\partial z} = -30y^2z^2$$

(c) $h(s,t) = t^7 \ln(s^2) + \dfrac{9}{t^3} - \sqrt[7]{s^4}$

With this one we'll not put in the detail of the first two. Before taking the derivative let's rewrite the function a little to help us with the differentiation process.

$$h(s,t) = t^7 \ln(s^2) + 9t^{-3} - s^{\frac{4}{7}}$$

Now, the fact that we're using s and t here instead of the "standard" x and y shouldn't be a problem. It will work the same way. Here are the two derivatives for this function.

$$h_s(s,t) = \frac{\partial h}{\partial s} = t^7\left(\frac{2s}{s^2}\right) - \frac{4}{7}s^{-\frac{3}{7}} = \frac{2t^7}{s} - \frac{4}{7}s^{-\frac{3}{7}}$$
$$h_t(s,t) = \frac{\partial h}{\partial t} = 7t^6 \ln(s^2) - 27t^{-4}$$

Remember how to differentiate natural logarithms.

$$\frac{d}{dx}\Big(\ln(g)(x)\Big) = \frac{g'(x)}{g(x)}$$

(d) $f(x,y) = \cos\left(\frac{4}{x}\right) \mathbf{e}^{x^2 y - 5y^3}$

Now, we can't forget the product rule with derivatives. The product rule will work the same way here as it does with functions of one variable. We will just need to be careful to remember which variable we are differentiating with respect to.

Let's start out by differentiating with respect to x. In this case both the cosine and the exponential contain x's and so we've really got a product of two functions involving x's and so we'll need to product rule this up. Here is the derivative with respect to x.

$$\begin{aligned} f_x(x,y) &= -\sin\left(\frac{4}{x}\right)\left(-\frac{4}{x^2}\right)\mathbf{e}^{x^2y-5y^3} + \cos\left(\frac{4}{x}\right)\mathbf{e}^{x^2y-5y^3}(2xy) \\ &= \frac{4}{x^2}\sin\left(\frac{4}{x}\right)\mathbf{e}^{x^2y-5y^3} + 2xy\cos\left(\frac{4}{x}\right)\mathbf{e}^{x^2y-5y^3} \end{aligned}$$

Do not forget the chain rule for functions of one variable. We will be looking at the chain rule for some more complicated expressions for multivariable functions in a later section. However, at this point we're treating all the y's as constants and so the chain rule will continue to work as it did back in Calculus I.

Also, don't forget how to differentiate exponential functions,

$$\frac{d}{dx}\left(\mathbf{e}^{f(x)}\right) = f'(x)\,\mathbf{e}^{f(x)}$$

4/ Posttest :-

1. Differentiate $f(x) = 3x^2 + 2x + 1$.
2. Use the chain rule to differentiate $f(x) = (2x+1)^3$.
3. Define implicit differentiation and give an example.

5/ HomeWorks: -

1- $y = (3x \quad - 5)_2,$ Find y

2- $y = \frac{5}{(4x-3)^2},$ Find y'

3- $y = tan\frac{1}{x^2},$ Find y''

4- $y = \sin\sqrt{x},$ find y'''

5- $y = sec(x^2 + 5x),$ find y'

6- $y = cos^2\frac{x}{9},$ find y'

7- $y = \frac{\sec x}{1+tan^2 x},$ find $\frac{dy}{dx}$

8- $y = \frac{secx+tanx}{sexx-tanx},$ find $\frac{dy}{dx}$

9- $y = \frac{(lnx)^2}{x},$ find y'

10- $y = \frac{e^{x+1}}{x^2+2},$ find y'

11- $y = \sqrt{e^x sinx},$ find y'

12- $y = \frac{1}{\sqrt{x^3}\ \ln(x)},$ find y'

13- $y = (e^x + 1)sinx$ find y'

14- $e^x + \sin x = 2\sqrt{xy},$ find y'

15- $tan\sqrt{x} + ycosx = 13,$ find y'

16- $y = x^3 log_3 x$ find y'

17- $y = x + 4^x,$ find y'

18- $y = \frac{e^{-2x}}{x^2}$

19- $y = x^3.3^x$ find y'

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Computer Networks
and Software Techniques

Learning package

Mathematics and Numerical Analysis

For

First year students

(6)

By

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And Software Techniques
2025

1/ Overview

1 / A –Target population :-

For First year students

Technological institute of Basra

Dep. Of Computer Networks and Software Techniques

1 / B –Rationale :-

Numerical methods are essential when analytical solutions are difficult or impossible.

1 / C –Central Idea :-

Iterative methods such as Newton-Raphson efficiently approximate roots of nonlinear equations.

1 / D – Performance Objectives

After studying the fifth unit, the student will be able to: -

1 - Apply the Newton-Raphson and iteration methods to find approximate roots.

2 - Analyze convergence and error in iterative solutions.

3 - Solve practical root-finding problems.

2/Pretest :

1. What is the Newton-Raphson formula?
2. Apply one iteration of Newton-Raphson to $f(x)=x^2-2$, starting at $x_0=1.5$.

3/Approximate Roots of Nonlinear Equations :

The approximate real root of non-linear

Newton Raphson Method or Newton Method is a powerful technique for solving equations numerically. It is most commonly used for .approximation of the roots of the real-valued functions

Newton-Raphson Method is a numerical technique for approximating .the roots of real-valued functions

It starts with initial guess of root and iteratively refines the result .using a formula that involves derivative of the function

Compared to other root-finding methods like bisection and secant methods, the Newton-Raphson method stands out due to its significantly faster convergence rate (quadratic while other have .linear)

Newton Raphson method requires computation of derivative and preferred over other methods when this computation easier and we can find good estimate of root.

Step 1: *Start with an initial guess x0.*

Step 2: *Use the formula, xn+1=xn−f(xn)f′(xn)xn+1=xn−f′(xn)f(xn) to find the next approximation, where f′(xn) is the derivative of f(x) at xn.*

Step 3: *Repeat the iteration until the change between xn and xn+1 is smaller than a predefined tolerance.*

Newton Raphson Method Example

Let's consider the following example to learn more about the process of finding the root of a real-valued function.

Example 1: For the initial value $x0 = 3$, approximate the root of $f(x)=x3+3x+1$.

Solution:

Given, $x0 = 3$ and $f(x) = x3+3x+1$
$f'(x) = 3x2+3$
$f'(x0) = 3(9) + 3 = 30$
$f(x0) = f(3) = 27 + 3(3) + 1 = 37$

Using Newton Raphson method:
$x1 = x0 - f(x0)f'(x0)x1 = x0 - f'(x0)f(x0)$
$= 3 - 37/30$
$= 1.767$

Example 2: For the initial value $x0 = 1$, approximate the root of $f(x)=x2-5x+1$.

Solution:

Given, $x0 = 1$ and $f(x) = x2-5x+1$
$f'(x) = 2x-5$
$f'(x0) = 2 - 5 = -3$
$f(x0) = f(1) = 1 - 5 + 1 = -3$

Using Newton Raphson method:
$\Rightarrow x1 = 1 - (-3)/-3$
$\Rightarrow x1 = 1 -1$
$\Rightarrow x1 = 0$

Problem 3: For the initial value $x0 = 2$, approximate the root of $f(x)=x3-6x+1$.

Solution:

Given, $x0 = 2$ and $f(x) = x3-6x+1$
$f'(x) = 3x2 - 6$
$f'(x0) = 3(4) - 6 = 6$
$f(x0) = f(2) = 8 - 12 + 1 = -3$

Using Newton Raphson method:
⇒ $x1 = 2 - (-3)/6$
⇒ $x1 = 2 + 1/2$
⇒ $x1 = 5/2 = 2.5$

Problem 4: For the initial value x0 = 3, approximate the root of f(x)=x2-3.

Solution:

Given, $x0 = 3$ and $f(x) = x2-3$
$f'(x) = 2x$
$f'(x0) = 6$
$f(x0) = f(3) = 9 - 3 = 6$

Using Newton Raphson method:
⇒ $x1 = 3 - 6/6$
⇒ $x1 = 2$

Problem 5: Find the root of the equation f(x) = x3 - 5x + 3 = 0, if the initial value is 3.

Solution:

Given $x0 = 3$ and $f(x) = x3 - 5x + 3 = 0$
$f'(x) = 3x2 - 5$
$f'(x0 = 3) = 3 \times 9 - 5 = 22$
$f(x0 = 3) = 27 - 15 + 3 = 15$

Using Newton Raphson method
⇒ $x1 = 3 - 15/22$
⇒ $x1 = 2.3181$

Using Newton Raphson method again:
$x2 = 1.9705$
$x3 = 1.8504$
$x4 = 1.8345$
$x5 = 1.8342$
Therefore, the root of the equation is approximately $x = 1.834$.

4/ Posttest :-

1 - State the Newton-Raphson formula.

2 - Why might we need to approximate roots numerically?

3 - Give an example where the Newton-Raphson method fails.

5/ HomeWorks: -

1 - Use the Newton-Raphson method to approximate a root for $f(x)=x^3 -4x + 1$, starting at $x_0=1$.

2 - Compare the results of two different initial guesses.

3 - Discuss sources of error in numerical root-finding.

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Mathematics and Numerical Analysis

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1/ Overview

1 / A –Target population :-

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Technological institute of Basra

Dep. Of Computer Networks and Software Techniques

1 / B –Rationale :-

Integration is vital for calculating areas, volumes, and solving differential equations in engineering.

1 / C –Central Idea :-

Integration rules and techniques allow the evaluation of a wide range of definite and indefinite integrals.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

1 - Integrate algebraic, trigonometric, exponential, and logarithmic functions.

2 - Use integration by parts and partial fractions.

3 - Solve applied integration problems.

2/Pretest :

1 - Integrate f(x)=3x2$f(x)=3x2$.

2 - What is integration by parts?

3 - Find ∫01xdx.

3/Integration Rules and Techniques

Integration

first: the indefinite integral

if $f(x)$ function defined at some interval, let $F(x)$ be another function such that

$$F'(x) = f(x)$$

is $F(x)$ called an indefinite integral of $f(x)$ and is written as the following form,

$$\int f(x)dx = F(x)) + C$$

Properties of indefinite integration:

1) $\int d\big(f(x)\big) = f(x) + C$
2) $\int k\, f(x)dx = k\, \int f(x)dx$
3) $\int \{f(x) \mp g(x)\}\, dx = \int f(x)dx \mp \int g(x)dx$ $\quad + C \quad , \quad n \neq -1$
4) $\int k\, dx = kx + C$

Exampels: Calculate the following integrals

1) $\int (x-1)\sqrt{x^2 - 2x + 1}\ dx$

$= \int (x-1)(x^2 - 2x + 1)^{\frac{1}{2}}\, dx$

$= \frac{1}{2}\int (2x-2)(x^2 - 2x + 1)^{\frac{1}{2}}\, dx$

$= \frac{1}{2} * \frac{(x^2-2x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{1}{3}(x^2 - 2x + 1)^{\frac{3}{2}} + C$

2) $\int (x-1)(x+1)^4\, dx$

let $u = x + 1 \Longrightarrow du = dx,\ \ x = u - 1$

$\therefore \int (x-1)(x+1)^4 = \int (u - 1 - 1)u^4 du$

$\int (u-2)u^4 du = \int u^5 - 2u^4 du$

$= \frac{u^6}{6} - 2\frac{u^5}{5} + C$

$= \frac{(x+1)^6}{6} - 2\frac{(x+1)^5}{5} + C$

$Second: definite\ integral$

$if\ f(x) a\ continuous\ function\ over\ the\ closed\ interval\ [a,b], let\ F(x) be\ another\ function\ such\ that\ \ F'(x) = f(x), the\ function\ F(x)\ is\ called\ a\ definite\ integral\ from\ point\ a\ (the\ lower\ bound)\ to\ the\ point\ b\ (\ the\ upper\ bound)\ of\ the\ function\ f(x) and\ written\ in\ the\ following\ from:$

$$\int_a^b f(x)dx = |F(x)|_a^b = F(b) - F(a)$$

$Definite\ integral\ properties:$

1) $\int_a^a f(x)dx = 0$
2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3) $\int_a^b k\ f(x)dx = k\int_a^b f(x)dx$
4) $\int_a^b \big(f(x) \mp g(x)\big)dx = \int_a^b f(x)dx \mp \int_a^b g(x)dx$
5) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,\ a < c < b$

$Exampels: Calculate\ the\ following\ integrals$

1) $\int_1^2 \sqrt{x} + \frac{1}{\sqrt{x}}\ dx$

$= \int_1^2 x^{\frac{1}{2}} + x^{\frac{-1}{2}}\ dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{3}}\right]_1^2$

$= \frac{4}{3}\sqrt{2} + 2\sqrt{2} - \frac{2}{3} - 2$

$= \frac{10\sqrt{2}-8}{3}$

2) $\int_{-1}^2 |x-1|dx$

$|x-1| = \begin{cases} x-1 & if\ x \geq 1 \\ -(x-1) & if\ x < 1 \end{cases}$

$\int_{-1}^2 |x-1|dx = -\int_{-1}^1 (x-1)dx + \int_1^2 (x-1)dx$

$= \left[\frac{-(x-1)^2}{2}\right]_{-1}^1 + \left[\frac{(x-1)^2}{2}\right]_1^2$

$= -(0-2) + (\frac{1}{2} - 0)$

$= \frac{5}{2}$

3) $\int_{-2}^{2} |x^2 - 1|\, dx$

$$|x^2 - 1| = \begin{cases} x^2 - 1 & if\ x^2 - 1 \geq 0 \\ -(x^2 - 1) & if\ x^2 - 1 < 0 \end{cases}$$

$= \int_{-2}^{-1} (x^2 - 1)dx + \int_{-1}^{1} -(x^2 - 1)dx + \int_{1}^{2} (x^2 - 1)dx$

$= \left[\frac{(x)^3}{3} - x\right]_{-2}^{-1} - \left[\frac{x^3}{3} - x\right]_{-1}^{1} + \left[\frac{x^3}{3} - x\right]_{1}^{2}$

$= 4$

$Integrals\ of\ Trigonometric\ Function$:

1) $\int \sin(u)\, du = -\cos(u) + C$

2) $\int \cos(u)\, du = \sin(u) + C$

3) $\int sec^2(u) du = \tan(u) + C$

4) $\int \sec(u)\tan(u)\, du = \sec(u) + C$

5) $\int csc^2(u) du = -\cot(u) + C$

6) $\int \csc(u)\cot(u)\, du = -\csc(u) + C$

$Exampels$: $Calculate\ the\ following\ integrals$

1) $\int cos^2(x) dx = \int \frac{(1+\cos(2x))}{2}\, dx$

$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)\, dx$

$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

2) $\int \frac{\sin(3x)}{cos^2(3x)} dx = \int cos^{-2}(3x)\sin(3x)\, dx$

$= \frac{-1}{3}\int -3cos^{-2}(3x)\sin(3x) dx$

$= \frac{-1}{3}\frac{(\cos(3x))^{-2+1}}{-2+1} + C$

$$= \frac{1}{3\cos(3x)} + C$$

$$\int \frac{\sin(3x)}{cos^2(3x)}\, dx \quad = \frac{1}{3}\sec(3x) + C$$

it is possible to solve example number 2 in another way

$$\int \frac{\sin(3x)}{cos^2(3x)} dx = \int \frac{\sin(3x)}{\cos(3x)} * \frac{1}{\cos(3x)} dx$$

$$= \int \tan(3x) * \sec(3x)\, dx$$

$$= \frac{1}{3}\sec(3x) + C$$

3) $\int (\sec(x) - \tan(x))^2\, dx = \int (sec^2(x) - 2\sec(x)\tan(x) + tan^2(x))\, dx$

$$= \int (sec^2(x) - 2\sec(x)\tan(x) + sec^2(x) - 1) dx$$

$$= 2\int sec^2(x) dx - 2\int \sec(x)\tan(x)\, dx - \int dx$$

$$= 2\tan(x) - 2\sec(x) - x + C$$

4) $\int \frac{\sin(2t)}{\sqrt{2-\cos(2t)}}\, dt = \int \sin(2t)\,(2-\cos(2t))^{\frac{-1}{2}} dt$

$$u = 2 - \cos(2t) \Longrightarrow du = 2\sin(2t)\, dt \Longrightarrow \frac{du}{2} = \sin(2t)\, dt$$

$$\therefore \int \frac{\sin(2t)}{\sqrt{2-\cos(2t)}} dt = \frac{1}{2}\int u^{\frac{-1}{2}} du$$

$$= \frac{1}{2}\ \frac{u^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C$$

$$= \sqrt{2 - \cos(2t)} + C$$

5) $\int \frac{dx}{1+\sin(x)} = \int \frac{dx}{1+\sin(x)} * \frac{1-\sin(x)}{1-\sin(x)}$

$$= \int \frac{1-\sin(x)}{1-sin^2(x)} dx$$

$$= \int \frac{1-\sin(x)}{cos^2(x)} dx$$

$$= \int \frac{1}{cos^2(x)} dx - \int \tan(x)\sec(x)\, dx$$

$$= \tan(x) - \sec(x) + C$$

6) $\int sin^3(x) dx = \int \sin(x)\, sin^2(x) dx$

$$= \int \sin(x)\left(1 - cos^2(x)\right) dx$$

$$= \int \sin(x)\,dx + \int -\sin(x)\,cos^2(x)dx$$

$$= -\cos(x) + \frac{cos^3(x)}{3} + C$$

$The\ integrals\ of\ the\ natural\ logarithm\ function$:

$$\int \frac{du}{u} = \ln|u| + C$$

$Exampels$: $Calculate\ the\ following\ integrals$

1) $\int \frac{x^2}{x^3+5}dx = \frac{1}{3}\int \frac{3x^2}{x^3+5}dx$

$$= \frac{1}{3}ln|x^3 + 5| + C$$

2) $\int \frac{\cos(3x)}{1+\sin(3x)}dx = \frac{1}{3}\int \frac{3\cos(3x)}{1+\sin(3x)}dx$

$$= \frac{1}{3}ln|1 + \sin(3x)| + C$$

3) $\int \frac{dx}{\sqrt{x}(1-2\sqrt{x})} = \int \frac{\frac{dx}{-\sqrt{x}}}{(1-2\sqrt{(x)})}$

$$= -ln\left|1 - 2\sqrt{x}\right| + C$$

4) $\int \csc(x)\,dx = \int \csc(x) * \frac{\csc(x)+\cot(x)}{\csc(x)+\cot(x)}dx$

$$= -\int -\frac{(csc^2(x)+\csc(x)\cot(x))}{\csc(x)+\cot(x)}dx$$

$$= -ln|\csc(x) + \cot(x)| + C$$

5) $\int \sec(x)\,dx = \int \sec(\mathrm{x})\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}dx$

$$= \int \frac{sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}dx$$

$$= ln|\sec(x) + \tan(x)| + C$$

6) $\int tan^3(x)dx = \int \tan(x)\,tan^2(x)dx$

$$= \int \tan(x)\,(sec^2(x) - 1)\,dx$$

$$= \int(\tan(x)\,sec^2(x) - \tan(x))dx$$

$$= \int \tan(x)\,sec^2(x)dx + \int -\frac{\sin(x)}{\cos(x)}dx$$

$$= \frac{tan^2(x)}{2} + ln|\cos(x)| + C$$

7) $\int \frac{dx}{xln(x)} = ln|\ln(x)| + C$

8) $\int \frac{x^3+2x^2-x+1}{x+2} dx = \int \left(x^2 - 1 + \frac{3}{x+2}\right) dx$

$$= \frac{x^3}{3} - x + 3ln|x+2| + C$$

Exponential function integrals:

$$\int e^u du = e^u + C$$

Exampels: 1) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2\int \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx$

$$= 2e^{\sqrt{x}} + C$$

2) $\int e^{3x} (sec^2(e^{3x}))\, dx = \frac{1}{3}\int sec^2(3x) * (3e^{3x}) dx$

$$= \frac{1}{3}\tan(e^{3x}) + C$$

3) $\int \frac{dx}{1+e^{-x}} = \int \frac{dx}{1+\frac{1}{e^x}}$

$$= \int \frac{dx}{\frac{e^x+1}{e^x}}$$

$$= \int \frac{e^x dx}{e^x+1}$$

$$= \ln|e^x + 1| + C$$

4) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \int \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} dx$

$$\Longrightarrow \int \frac{e^{2x}-1}{e^{2x}+1}\,dx = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+1}\,dx - \frac{1}{2}\int \frac{2dx}{e^{2x}+1}$$

$$\Longrightarrow \frac{1}{2}\ln|e^{2x}+1| + \frac{1}{2}\ln|1+e^{-2x}| + C$$

Integration method :

1) $fragmentation\ integration\ method (udv)$:
$if\ u\ and\ v\ are\ differentiable\ and\ integral\ \ function\ \ d(u*v) = udv + vdu$
$take\ the\ integration\ of\ both\ sides\ \ \int d(uv) = \int udv + \int vdu$

$$\therefore \int udv = uv - \int vdu$$

$Exampels$: 1) $\int x^2 \sin(2x)\,dx$
$let\ u = x^2 \Longrightarrow du = 2x\,dx$
$dv = \sin(2x)dx \Longrightarrow v = \frac{-1}{2}\cos(2x)$
$\int udv = uv - \int vdu$
$\int x^2 \sin(2x)\,dx = \frac{-x^2}{2}\cos(2x) + \underbrace{\int x\cos(2x)dx}$
$let\ u = x \Longrightarrow du = dx$
$dv = \cos(2x)\,dx \Longrightarrow v = \frac{1}{2}\sin(2x)$
$\int x^2 \sin(2x)\,dx = \frac{-x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \int \frac{1}{2}\sin(2x)\,dx$
$= \frac{-x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$

2) $\int \sin(ln(x))\,dx$
$let\ u = \sin(ln(x)) \Longrightarrow du = \frac{\cos(ln(x))}{x}$
$dv = dx \Longrightarrow v = x$
$\int \sin(ln(x))\,dx = x\sin(\ln(x)) - \int \underbrace{\cos(ln(x))\,dx}$
$let\ u = \cos(ln(x)) \Longrightarrow du = \frac{-\sin(ln(x))}{x}$
$dv = dx \Longrightarrow v = x$
$\therefore\ \int \sin(ln(x))\,dx = x\sin(ln(x)) - [x\cos(\ln(x)) + \int \sin(ln(x))\,dx]$
$\int \sin(ln(x))\,dx + \int \sin(ln(x))\,dx = x[\sin(ln(x)) - \cos(ln(x))] + C$

$\int \sin(ln(x))\,dx = \frac{x}{2}[\sin(ln(x)) - \cos(ln(x))] + C$

3) $\int e^x \cos(3x)\,dx$

$let\ u = \cos(3x) \Longrightarrow du = -3\sin(3x)$

$dv = e^x dx \Longrightarrow v = e^x$

$\int e^x \cos(3x)\,dx = e^x \cos(3x) + \int \underbrace{e^x \sin(3x)\,dx}$

$let\ u = \sin(3x) \Longrightarrow du = 3\cos(3x)$

$dv = e^x dx \Longrightarrow v = e^x$

$\int e^x \cos(3x)\,dx = e^x \cos(3x) + 3[e^x \sin(3x) - \int 3e^x \cos(3x)\,dx]$

$\int e^x \cos(3x)\,dx = e^x \cos(3x) + 3e^x \sin(3x) - 9\int e^x \cos(3x)\,dx$

$\int e^x \cos(3x)\,dx + 9\int e^x \cos(3x)\,dx = e^x \cos(3x) + 3e^x \sin(3x)$

$\int e^x \cos(3x)\,dx = \frac{1}{10}[e^x \cos(3x) + 3e^x \sin(3x)] + C$

2) $Trigonometric\ Compensation\ Method: if\ the\ integral\ in\ the\ numerator\ or\ denomi$
$contains\ the\ following\ r,elations$

$i)\ \sqrt{a^2 - b^2x^2}$ $\quad ii)\ \sqrt{a^2 + b^2x^2}$ $\quad iii)\ \sqrt{b^2x^2 - a^2}$

$we\ use\ the\ following\ relations:$

$cos^2\theta = 1 - sin^2\theta$

$sec^2\theta = 1 + tan^2\theta$

$tan^2\theta = sec^2\theta - 1$

$i)\ if\ the\ integral\ contains\ relation\ \sqrt{a^2 - b^2x^2}$

$let\ x = \frac{a}{b} sin\theta$

$$\sqrt{a^2 - b^2x^2} = \sqrt{a^2 - b^2\left(\frac{a^2}{b^2}\ sin^2\theta\right)} = a\sqrt{1 - sin^2\theta} = a\ cos\theta$$

$Example: \int \frac{x^2}{\sqrt{3-4x^2}}\ dx$

$let\ x = \frac{a}{b} sin\theta \Longrightarrow x = \frac{\sqrt{3}}{2}\ sin\theta \Longrightarrow dx = \frac{\sqrt{3}}{2}\ cos\theta\ d\theta\ \left(a = \sqrt{3}, b = 2\right)$

$$\sqrt{3 - 4x^2} = \sqrt{3 - 4\left(\frac{3}{4}\ sin^2\theta\right)} = \sqrt{3}\ cos\theta$$

$$\therefore \int \frac{x^2}{\sqrt{3-4x^2}} dx = \int \frac{\frac{3}{4} sin^2\theta}{\sqrt{3}cos\theta}\ \frac{\sqrt{3}}{2} cos\theta\ d\theta$$

$\frac{3}{8}\int sin^2\theta\ d\theta = \frac{3}{8}\int \frac{1-cos\theta}{2}\ d\theta$

$\frac{3}{8}\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right] + C$

$\frac{3}{8}\left[\frac{1}{2}\theta - \frac{1}{4}sin\theta cos\theta\right] + C$

$x = \frac{\sqrt{3}}{2}sin\theta \Longrightarrow sin\theta = \frac{2}{\sqrt{3}}x \Longrightarrow \theta = sin^{-1}\frac{2}{\sqrt{3}}x$

$\frac{3}{8}\left[\frac{1}{2}sin^{-1}\frac{2}{\sqrt{3}} - \frac{1}{2}\frac{2x}{\sqrt{3}}\ \frac{\sqrt{3-4x^2}}{\sqrt{3}}\right] + C$

$ii)\ if\ the\ integral\ \ contains\ relation\ \sqrt{a^2+b^2x^2}$

$let\ x = \frac{a}{b}tan\theta$

$\sqrt{a^2+b^2x^2} = \sqrt{a^2+b^2\left(\frac{a^2}{b^2}\ tan^2\theta\right)} = a\sqrt{1+tan^2\theta} = a\ sec\theta$

$Example\text{:}\ \int\frac{dx}{\sqrt{4x^2+4x+17}}$

$\sqrt{4x^2+4x+17} = \sqrt{4\left(x^2+x+\frac{17}{4}\right)}$

$\sqrt{4(x^2+x+\frac{1}{4}-\frac{1}{4}+\frac{17}{4})} = 2\sqrt{(x+\frac{1}{2})^2+4}$

$\frac{1}{2}\int\frac{dx}{\sqrt{(x+\frac{1}{2})^2+4}} \Longrightarrow let\ u = x+\frac{1}{2}$

$\frac{1}{2}\int\frac{du}{\sqrt{u^2+4}} \Longrightarrow \frac{1}{2}\int\frac{du}{\sqrt{4+u^2}}$

$let\ u = \frac{2}{1}tan\theta \Longrightarrow tan\theta = \frac{u}{2} \Longrightarrow du = sec^2\theta\ d\theta$

$\frac{1}{2}\int\frac{du}{\sqrt{4+u^2}} = \frac{1}{2}\int\frac{2sec^2\theta\ d\theta}{\sqrt{4+(\frac{2}{1}tan\theta)^2}}$

$\frac{1}{2}\int\frac{2sec^2\theta\ d\theta}{2\ sec\theta} = \frac{1}{2}\int sec\theta\ d\theta$

$\frac{1}{2}\int sec\theta\ \frac{sec\theta+tan\theta}{sec\theta+tan\theta}\ d\theta = \frac{1}{2}\ln|sec\theta+tan\theta| + C$

$\frac{1}{2}\ln\left|\frac{\sqrt{4+u^2}}{2}+\frac{u}{2}\right| + C = \frac{1}{2}\ln\left|\frac{\sqrt{4+(x+\frac{1}{2})^2}}{2}+\frac{x+\frac{1}{2}}{2}\right| + C$

$iii)\ if\ the\ integral\ contains\ relation\ \sqrt{b^2x^2-a^2}$

$let\ x = \frac{a}{b}sec\theta \Longrightarrow tan^2\theta = sec^2\theta - 1$

$$\sqrt{b^2x^2-a^2}=\sqrt{b^2\ \frac{a^2}{b^2}\ sec^2\theta-a^2}=a\sqrt{sec^2\theta-1}=a\ tan\theta$$

$Example:\ \int\frac{5x^2-3}{x}\ dx$

$let\ x=\frac{a}{b}sec\theta\Longrightarrow x=\frac{\sqrt{3}}{\sqrt{5}}sec\theta\Longrightarrow dx=\frac{\sqrt{3}}{\sqrt{5}}sec\theta tan\theta\ d\theta\Longrightarrow sec\theta=\frac{\sqrt{5}}{\sqrt{3}}x\Longrightarrow\theta=sec^{-1}\frac{\sqrt{5}}{\sqrt{3}}x$

$$\sqrt{5x^2-3}=\sqrt{5\ \frac{3}{5}x^2-3}=\sqrt{3}\sqrt{sec^2\theta-1}=\sqrt{3}\,tan\theta$$

$$\int\frac{5x^2-3}{x}\ dx=\int\frac{\sqrt{3}tan\theta\frac{\sqrt{3}}{\sqrt{5}}sec\theta tan\theta\ d\theta}{\sqrt{\frac{3}{5}}sec\theta}$$

$$\sqrt{3}\int tan^2\theta\ d\theta=\sqrt{3}\ \int(sec^2\theta-1)d\theta$$

$$\sqrt{3}tan\theta-\theta+C=\sqrt{3}\left[\frac{\sqrt{5x^2-3}}{\sqrt{3}}-sec^{-1}\frac{\sqrt{5}}{\sqrt{3}}x+C\right]$$

the numerator and denominator are polynomial, so that they cannot be integrated by the pervious methods, then we will divde this function as the sum of two or more fractional functions so that the integration of each of these fractions is easy.

i) if the denominator of a rational function contains the expression $(ax+b)^n$ *we will divide the integration function as follows,*

$$\frac{?}{(ax+b)^n}=\frac{A_1}{(ax+b)}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_n}{(ax+b)^n}$$

such that $A_1, A_2, \dots, A_n$ *are constant*

Remark: n must always be a positive number and the power of x must be an integer, and the number of partial fractions must equal the number of parentheses in the denominator of the function.

$ii) if\ the\ denominator\ of\ a\ rational\ function\ contains\ the\ expression\ (ax^2+bx+c)^n$ $we\ will\ divide\ the\ integration\ function\ as\ follows,$

$$\frac{?}{(ax^2+bx+c)^n}=\frac{A_1x+B_1}{(ax^2+bx+c)}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

$iii) if\ the\ denominator\ of\ a\ rational\ function\ contains\ a\ common\ case\ of\ i\ and\ ii, then\ we\ divide\ this\ function, each\ according\ to\ its\ own\ method$

$Example$: $\int \frac{3x+5}{(x-1)^2(x+1)}dx=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}$

$=\frac{(x-1)^2A+B(x+1)+C(x-1)^2}{(x-1)^2(x+1)}$

$3x+5=(A+C)x^2+(B-2C)x+(B-A+C)$

$A+C=0 \Longrightarrow A=-C$

$B-2C=3 \Longrightarrow B=3+2C$

$B-A+C=5 \Longrightarrow 3+2C+C+C=5 \Longrightarrow C=\frac{1}{2}$

$A=\frac{-1}{2}, B=4$

$\int \frac{3x+5}{(x-1)^2(x+1)}dx=\int \frac{\frac{-1}{2}}{(x-1)}dx+\int \frac{4dx}{(x-1)^2}+\int \frac{\frac{1}{2}dx}{(x+1)}$

$=\frac{1}{2}\ln|x-1|-\frac{4}{(x-1)}+\frac{1}{2}\ln|x+1|+C$

$h.w$: 1) $\int \frac{x+1}{x^3+x^2-6x}dx$ 2) $\int \frac{x^2+2}{4x^3-x^2+4x-1}dx$

4/ Posttest :-

1 - Integrate $f(x) = 2x$.

2 - What is integration by parts? Provide the formula.

3 - Integrate f(x)=sin x

5/ HomeWorks: -

1 - Integrate the following functions:

a) f(x) =x3

b) g(x) =*ex*

2 - Solve at least one definite and one indefinite integral.

Ministry of Higher Education and Scientific Research
Southern Technical University
Technological Institute of Basra
Department of Computer Networks
and Software Techniques

Learning package

Mathematics and Numerical Analysis

For

First year students

(8)

By

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2025

1/ Overview

1 / A –Target population :-

For First year students

Technological institute of Basra

Dep. Of Computer Networks and Software Techniques

1 / B –Rationale :-

Sequences and series are fundamental for understanding convergence, summation, and mathematical modeling.

1 / C –Central Idea :-

Analyzing sequences and series helps in understanding infinite processes and their applications.

1 / D – Performance Objectives

After studying the first unit, the student will be able to:-

1 -Define and classify sequences and series.

2 -Test for convergence/divergence using ratio and root tests.

3 -Calculate sums of geometric and arithmetic series.

2/ Pretest:

1. Define a sequence and give an example.
2. What is the sum of the first 5 terms of the sequence an=2n*an*=2*n*?

3/Sequences and Series:

Sequences And Summation Notation

A sequence is an infinite list of numbers. The numbers in the sequence are often written as a_1 , a_2, a_3, ... The dots mean that the list continues forever.
A simple example is the sequence

$$\begin{array}{cccccc} 5, & 10, & 15, & 20, & 25, & \dots \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ \boldsymbol{a_1} & \boldsymbol{a_2} & \boldsymbol{a_3} & \boldsymbol{a_4} & \boldsymbol{a_5} & \dots \end{array}$$

We can describe the pattern of the sequence displayed above by the following formula :

$$\boldsymbol{a_n} = 5n$$

You go from one number to the next by adding **5** . This natural way of describing the sequence is expressed by the recursive formula:

$$a_n = \boldsymbol{a_{n-1}} + 5$$

starting with $\boldsymbol{a_1} = 5$. Try substituting n = 1, 2, 3, . . . in each of these formulas to see how they produce the numbers in the sequence .

Definition (Sequence)

A sequence is a function $\boldsymbol{a}$ whose domain is the set of natural numbers. The terms of the sequence are the function values
$a(1)$, $a(2)$, $a(3)$, . . . , $a(n)$, . . .
We usually write an instead of the function notation a(n) . So the terms of the sequence are written as

$$a_1, a_2, a_3, \dots, a_n \quad \dots$$

The number a_1 is called the first term, , a_2 is called the second term, and in general, a_n is called the nth term.

Here is a simple example of a sequence:

2, 4, 6, 8, 10, . . .

This sequence consists of even numbers . This can be done by giving a formula for the nth term a_n of the sequence. In this case ,
$\boldsymbol{a_n = 2n}$ and the sequence can be written as

2 ,	*4 ,*	*6 ,*	*8 ,*	*. . .*	*2n ,*	*. . .*
1st term	*2nd term*	*3rd term*	*4th term*		*nth term*	

Notice how the formula $\boldsymbol{a_n}$ ***=2n*** *gives all the terms of the sequence. For instance, substituting* ***1, 2, 3,*** *and* ***4*** *for* ***n*** *gives the first four terms:*

$\boldsymbol{a_1} = 2 . 1 = 2$ $\quad$ $\boldsymbol{a_2} = 2 . 2 = 4$ $\quad$ $\boldsymbol{a_3} = 2 . 3 = 6$ $\quad$ $\boldsymbol{a_4} = 2 . 4 = 8$

To find the 103rd term of this sequence, we use *n* = 103 to get
$\boldsymbol{a_{103}} = 2 . 103 = 206$

Example 1 ■ **Finding the Terms of a Sequence**

Find the first five terms and the 100th term of the sequence defined by each formula

(a) a_n = 2n −1 (b) $c_n = n^2 - 1$ (c) $t_n = \frac{n}{n+1}$ (d) $r_n = \frac{(-1)^n}{2^n}$

Solution : To find the first five terms, we substitute **n = 1, 2, 3, 4,** and 5 in the formula for the nth term. To find the 100th term, we substitute n = 100. This gives the following.

nth term	First five terms	100th term
(a) a_n = 2n -1	1 3, 5 , 7 , 9	199
(b) $c_n = n^2$ - 1	0 , 3 , 8 , 15 , 24	9999
(c) $t_n = \frac{n}{n+1}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$	$\frac{100}{101}$
(d) $r_n = \frac{(-1)^n}{2^n}$	$-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}$	$\frac{1}{2^{100}}$

Remark

In Example 1(d) the presence of $(-1)^n$ in the sequence has the effect of making successive terms alternately negative and positive.

It is often useful to picture a sequence by sketching its graph.

Since a sequence is a function whose domain is the natural numbers, we can draw its graph in the Cartes

For instance, the graph of the sequence

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{n}, \ldots$ is show in figure **1**

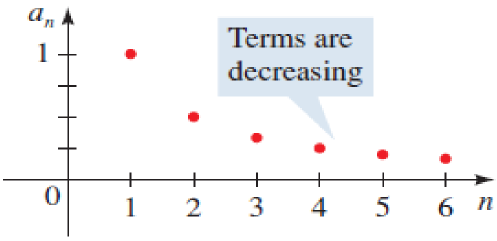

FIGURE 1

Compare the graph of the sequence shown in Figure **1** to the graph of

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots, \frac{(-1)^n}{n}, \ldots$

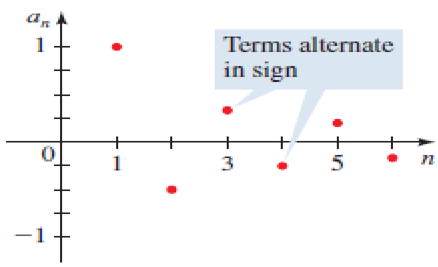

FIGURE 2

Homework :

Find the first four terms and the 100th term of the sequence whose nth term is given.

1) $\boldsymbol{a_n}$ = n -3 2) $\boldsymbol{a_n} = \frac{1}{2n+1}$ 3) $\boldsymbol{a_n} = 5^n$ 4) $\boldsymbol{a_n} = \frac{(-1)^n}{n^2}$

Example 2 ■ Finding the nth Term of a Sequence

Find the nth term of a sequence whose first several terms are given .

a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$ b) -2 , 4 , $-$ 8 , 16 ,$-$ 32 , . . .

Solution

(**a**) We notice that the numerators of these fractions are the odd numbers and the denominators are the even numbers. Even numbers are of the form **2n**, and odd numbers are of the form **2n-1** (an odd number differs from an even number by **1**). So a sequence that has these numbers for its first four terms is given by

$\boldsymbol{a_n} = \frac{2n-1}{2n}$

(**b**) These numbers are powers of 2, and they alternate in sign, so a sequence that agrees with these terms is given by

$\boldsymbol{a_n} = (-1)^n 2^n$

Homework : ■ *n*th term of a Sequence

Find the *n*th term of a sequence whose first several terms are given.

1) 2 , 4 ,8 , 16 , . . . 2) 1 , $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \ldots$

■ Recursively Defined Sequences

Some sequences do not have simple defining formulas like those of the preceding example. The nth term of a sequence may depend on some or all of the terms preceding it. A sequence defined in this way is called recursive.

Example 3 ■ Finding the Terms of a Recursively Defined Sequence

A sequence is defined recursively by$\boldsymbol{a_1}$**= 1** and $\boldsymbol{a_n}$= **3**($\boldsymbol{a_{n-1}}$ + 2)

Find the first five terms of the sequence.

Solution :The defining formula for this sequence is recursive. It allows us to find the **nth** term $\boldsymbol{a_n}$ if we know the preceding term $\boldsymbol{a_{n-1}}$. Thus we can find the **second term** from the **first term**, the third term from the second term, the **fourth term** from the **third term**, and so on. Since we are given the first term$\boldsymbol{a_1}$**=1**, we can proceed as follows.

$\boldsymbol{a_2}$ = 3 ($\boldsymbol{a_1}$ +2) = 3(1+2) = 9

$\boldsymbol{a_3}$ = 3($\boldsymbol{a_2}$ + 2) = 3(9 + 2) = 33

$\boldsymbol{a_4}$ = 3 ($\boldsymbol{a_3}$ + 2) = 3(33 + 2) =105

$\boldsymbol{a_5}$ = 3($\boldsymbol{a_4}$ + 2) = 3(105 + 2) = 321

Thus the first five terms of this sequence are

1, 9, 33, 105, 321, . . .

Homework

A sequence is defined recursively by the given formulas. Find the first five terms of the sequence.

1) a_n = 2(a_{n-1} + 3) and a_1= 4

2) Find the first ten terms of the sequence $a_n = \frac{1}{a_{n-1}}$ a_1 = 2

Example 4 ■ The Fibonacci Sequence

Find the first 11 terms of the sequence defined recursively by F1 = 1, F2 = 1

$F_n = F_{n-1} + F_{n-2}$

Solution : To find F_n, we need to find the two preceding terms, F_{n-1} and F_{n-2}.

Since we are given F_1 and F_2 , we proceed as follows.

$F_3 = F_1 + F_2 = 1 + 1 = 2$

$F_4 = F_3 + F_2 = 2 + 1 = 3$

$F_5 = F_4 + F_3 = 3 + 2 = 5$

It's clear what is happening here. Each term is simply the sum of the two terms that precede it, so we can easily write down as many terms as we please. Here are the first 11 terms. (You can also find these using a graphing calculator.)

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , . . .

Homework:

sequence is defined recursively by the given formulas. Find the first five terms of the sequence .

$a_n = a_{n-1} + a_{n-2}$ and a_1 = 1 a_2 = 1

▪ Definition (The Partial Sums of a Sequence)

For the sequence

$a_1 , a_2 , a_3 , a_4 , . . . , a_n , . . .$

the partial sums are

$$s_1 = a_1$$

$s_2 = a_1 + a_2$

$s_3 = a_1 + a_2 + a_3$

$s_4 = a_1 + a_2 + a_3 + a_4$

$$\vdots$$

$s_n = a_1 + a_2 + a_3 + a_4 + . . . + a_n$

$$\vdots$$

s_1 is called the first partial sum, s_2 is the second partial sum, and so on. s_n is called the nth partial sum. The sequence $s_1, s_2, s_3, . . . , s_n, . . .$ is called the sequence of partial sums.

Example 5 ■ Finding the Partial Sums of a Sequence

Find the first four partial sums , 10th term and the nth partial sum of the sequence given by

$a_n = \frac{1}{2^n}$

Solution The terms of the sequence are

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

The first four partial sums are

$s_1 = a_1 = \frac{1}{2} \qquad = \frac{1}{2}$

$s_2 = a_1 + a_2 = \frac{1}{2} + \frac{1}{4} \qquad = \frac{3}{4}$

$s_3 = a_1 + a_2 + a_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \qquad = \frac{7}{8}$

$s_4 = a_1 + a_2 + a_3 + a_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$

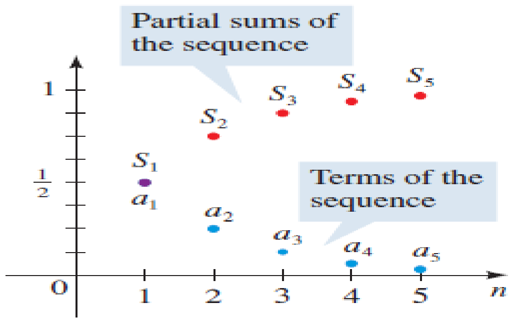

FIGURE 7 Graph of the sequence a_n and the sequence of partial sums S_n

Notice that in the value of each partial sum, the denominator is a power of 2 and the numerator is one less than the denominator. In general, the *n*th partial sum is

$s_n = \frac{2^n - 1}{2^n} = 1 - \frac{1}{2^n}$

Example 6 ■ Finding the Partial Sums of a Sequence

Find the first four partial sums and the *n*th partial sum of the sequence given by

$$a_n = \frac{1}{n} - \frac{1}{n+1}$$

Solution The first four partial sums are

$s_1 = 1 - \frac{1}{2} \qquad = 1 - \frac{1}{2}$

$s_2 = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) \qquad = 1 - \frac{1}{3}$

$s_3 = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) \qquad = 1 - \frac{1}{4}$

$s_4 = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) = 1 - \frac{1}{5}$

The *n*th partial sum is

$$s_n = 1 - \frac{1}{n+1}$$

Homework

Find the first four partial sums and the *n*th partial sum of the sequence a_n .

1) $a_n = \frac{2}{3^n}$

2) $a_n = \sqrt{n} - \sqrt{n+1}$

(b) Consider the arithmetic sequence

9, 4, − 1, -6, − 11, . . .

Here the common difference is d = − 5. The terms of an arithmetic sequence decrease if the common difference is negative. The nth term is a_n = 9 − 5(n− 1)

(c) The graph of the arithmetic sequence a_n = 1 + 2(n− 1) is shown in **Figure 1**. Notice that the points in the graph lie on the straight line y = 2x − 1, which has slope d = 2

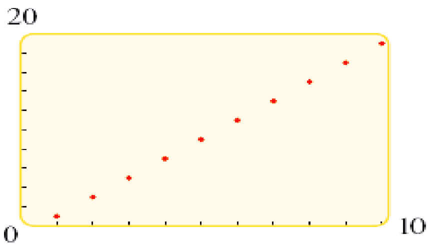

FIGURE 1

Remark:

An arithmetic sequence is determined completely by the first term **a** and the common difference **d**. Thus if we know the first two terms of an arithmetic sequence, then we can find a formula for the nth term, as the next example shows.

Example 2 ■ **Finding Terms of an Arithmetic Sequence**

Find the common difference, the first six terms, the nth term, and the 300th term of the arithmetic sequence

13, 7, 1, − 5, . . .

Solution Since the first term is **13**, we have **a = 13**. The common difference is **d = 7−13 = − 6** . Thus the **nth** term of this sequence is

a_n = 13− 6(n− 1)

From this we find the first six terms:

13, 7, 1, − 5, − 11, − 17, . . .

The 300th term is

a_{300} = 13− 6(300− 1) =− 1781

Homework

1) The **nth** term of an arithmetic sequence is given. (a) Find the first five terms of the sequence. (b) What is the common difference **d** ?

a_n = 7 + 3(n− 1)

2) Find the **nth**term of the arithmetic sequence with given first term **a** and common difference **d** . What is the 10th term?

a = 9 , d = 4

3) The first four terms of a sequence are given. Can these terms be the terms of an arithmetic sequence? If so, find the common difference .

11 , 17 , 23 , 29 , . . .

4) Determine the common difference, the fifth term, the **nth** term, and the **100th** term of the arithmetic sequence.

4 , 10 , 16 , 22 , . . .

- Infinite Series

Definition: (infinite Series)

An expression of the form $\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + a_4 + \ldots$ is called an infinite series

Remark : The dots mean that we are to continue the addition indefinitely. What meaning can we attach to the sum of infinitely many numbers? It seems at first that it is not possible to add infinitely many numbers and arrive at a finite number. But consider the following problem. You have a cake, and you want to eat it by first eating half the cake, then eating half of what remains, then again eating half of what remains. This process can continue indefinitely because at each stage, some of the cake remains. (See Figure 3.)

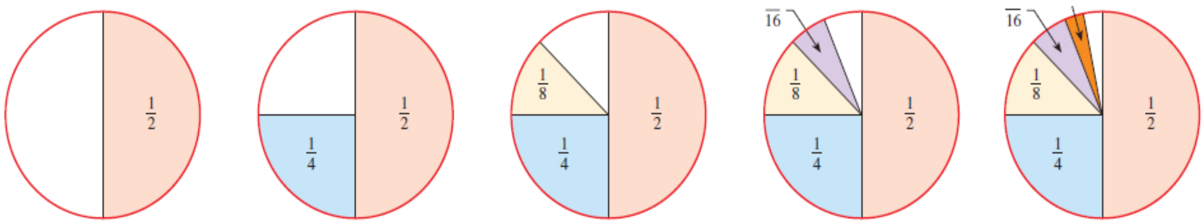

FIGURE 3

Does this mean that it's impossible to eat all of the cake? Of course not. Let's write down what you have eaten from this cake :

$\sum_{k=1}^{\infty}(\frac{1}{2^k}) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + . . .$

This is an infinite series, and we note two things about it: First, from Figure 3 it's clear that no matter how many terms of this series we add , the total will never exceed **1**. Second, the more terms of this series we add , the closer the sum is to **1** (see Figure **3**). This suggests that the number **1** can be written as the sum of infinitely many smaller numbers :

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + . . . + \frac{1}{2^n} + . . .$$

To make this more precise, let's look at the partial sums of this series :

$s_1 = \frac{1}{2} \qquad = \frac{1}{2}$

$s_1 = \frac{1}{2} + \frac{1}{4} \qquad = \frac{3}{4}$

$s_1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \qquad = \frac{7}{8}$

$s_1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \quad = \frac{15}{16}$

and, in general ,

$s_n = 1 - \frac{1}{2^n}$

As **n** gets larger and larger, we are adding more and more of the terms of this series. Intuitively, as **n** gets larger$\boldsymbol{s_n}$, gets closer to the sum of the series. Now notice that as **n** gets large**,** $\boldsymbol{\frac{1}{2^n}}$, gets closer and closer to **0** . Thus$\boldsymbol{s_n}$gets close to **1 - 0 = 1** . we can write

$\boldsymbol{s_n} \rightarrow 1 \qquad \text{as} \quad n \rightarrow \infty$

In general , if $\boldsymbol{s_n}$gets close to a finite number **S** as **n** gets large, we say that the infinite series converges (or is convergent). The number **S** is called the sum of the infinite series. If an infinite series does not converge, we say that the series diverges (or is divergent).

Definition ■ (Infinite Geometric Series)

An **infinite geometric series** is a series of the form

$$a + ar + ar^2 + ar^3 + ar^4 + \ . \ . \ . \ + ar^{n-1} + . . .$$

We can apply the reasoning used earlier to find the sum of an infinite geometric series. The **nth** partial sum of such a series is given by the formula

$s_n = \dfrac{a(1 - r^n)}{(1 - r)} \quad r \neq 1$

It can be shown that if $|r| < 1$, then r^n gets close to **0** as n gets large (you can easily convince yourself of this using a calculator). It follows that s_n gets close to $\dfrac{a}{(1-r)}$ as n gets large, or

$s_n \rightarrow \dfrac{a}{(1-r)} \quad$ as $\quad n \rightarrow \infty$

Thus the sum of this infinite geometric series is $\dfrac{a}{(1-r)}$

Definition (Sum of an Infinite Geometric Series)

if $|r| < 1$ then the infinite geometric series

$\sum_{k=1}^{n} ar^{k-1} = a + ar + ar^2 + ar^3 + ar^4 + \ . \ . \ .$

converges and has the sum

$\dfrac{a}{(1-r)}$ if $|r| \geq 1$, the series diverges

Example6 ■ Infinité Series

Determine whether the infinite geometric series is convergent or divergent. If it is convergent , find its sum .

a) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + . . .$ B) $1 + (\frac{7}{5}) + (\frac{7}{5})^2 + (\frac{7}{5})^3 + . . .$

Solution

(a) This is an infinite geometric series with a=2 and r = $\frac{1}{5}$. Since $|r| = \left|\frac{1}{5}\right| < 1$, the series converges. By the formula for the sum of an infinite geometric series we have

$$S = \frac{2}{(1 - \frac{1}{5})} = \frac{5}{2}$$

(b) This is an infinite geometric series with a = 1 and r = $\frac{7}{5}$. Since $|r| = \left|\frac{7}{5}\right| > 1$, the series diverges.

Homework :

Determine whether the infinite geometric series is convergent or divergent. If it is convergent , Find its sum

a) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + . . .$ **b)** $1 + (\frac{3}{2}) + (\frac{3}{2})^2 + (\frac{3}{2})^3 + . . .$ (C) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

(d) $\frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \frac{1}{4} + . . .$ **(e)** $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + . . .$

<u>Example 7</u> ■ Writing a Repeated Decimal as a Fraction

Find the fraction that represents the rational number 2. 3**<u>51</u>** .

<u>Solution</u> This repeating decimal can be written as a series:

$$\frac{23}{10}+\frac{51}{1000}+\frac{51}{100{,}000}+\frac{51}{10{,}000{,}000}+\frac{51}{1{,}000{,}000{,}000}\ \ldots$$

After the first term, the terms of this series form an infinite geometric series with

a = $\mathbf{\frac{51}{1000}}$ and r = $\mathbf{\frac{1}{100}}$

Thus the sum of this part of the series is

$$\mathbf{S} = \frac{\frac{51}{1000}}{(1-\frac{1}{100})}=\frac{\frac{51}{1000}}{(\frac{99}{100})} = \frac{51}{1000}\cdot\frac{100}{99} = = \frac{51}{990}$$

2. 3**<u>51</u>** = $\frac{23}{10}+\frac{51}{990}=\frac{2328}{990}=\frac{388}{165}$

<u>Homework</u> Express the repeating decimal as a fraction

(1) 0. 2**<u>53</u>**(2)0.0303030…….. (3) 2.11**<u>25</u>**

- **Definition (Arithmetic – Geometric Series)**

A series is said to be an Arithmetic – Geometric Series if its each terms is formed by multiplying the corresponding term of an arithmetic sequence and geometric sequence

For example **1 + 3x + 5x^2 + 7x^3 + . . .**

Here **1 , 3 , 5 , 7 , . . .** Are in arithmetic sequence and **1 , x , x^2 , x^3 . . .** Are in geometric series .

❑ Sum of n – terms an Arithmetic – Geometric series

Let s_n = a + (a + d)r + (a + 2d) r^2 + . . . +[a + (n – 1)d]r^{n-1} … (1)

Multiplying both sides of (1) by common ratio r and write as follows .

rs_n = 0 + ar + (a + d)r^2 + . . . + [a + (n- 1)d]r^n … (2)

Subtracting (2) from (1) , we get

s_n (1-r) = a + [dr + d r^2 + . . . + dr^{n-1}] − [a + (n -1)d] r^n

s_n (1-r) = a + d[r + r^2 + . . . + r^{n-1}] − [a + (n -1)d] r^n

= a + $\frac{dr\,(1-r^{n-1})}{1-r}$ − [a + (n -1)d] r^n

$$s_n = \frac{a}{1-r}+\frac{dr\,(1-r^{n-1})}{(1-r)^2}-\frac{[a+(n-1)d\,]\,r^n}{(1-r)} \quad \ldots (3)$$

<u>Remark :</u>

The above result (3) is not used

❑ Sum of infinity

If $|r| < 1$, i . e. $-1 < r < 1$ and n → ∞ then $\lim_{n\to\infty} r^n = 0$

∴ $s_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^n}$

Example 1 : find the sum of the series

1 + 2x + 3x^2 + 4 x^3. . .

(a) To n terms **(b) to infinity**

Solution : (a)Let sum of **n** terms of the series is denoted by s_n

Then $s_n = 1 + 2x + 3x^2 + 4x^3 + \ldots + (n-1)x^{n-2} + n$... **(1)**

$xs_n = x + 2x^2 + 3x^3 + \ldots + (n-1)x^{n-1} + nx^n$ ... **(2)**

Subtracting **(2)** from **(1)** , we get

(1 − x) $s_n = 1 + 2x - x + 3x^2 - 2x^2 + 4x^3 - 3x^3 + (n-1)x^{n-2}$ _ $(n-2)x^{n-2} + nx^{n-1}$ $-(n-1)x^{n-1} - nx^{n-1}$

(1 − x) $s_n = 1 + x + x^2 + x^3 + \ldots + x^{n-1} - nx^n$

$= \frac{1(1-x^n)}{(1-x)} - nx^n$

$$s_n = \frac{(1-x^n)}{(1-x)^2} - \frac{nx^n}{(1-x)}$$

(b) $s_\infty = 1 + 2x + 3x^2 + 4x^3 + \ldots \infty$... **(i)**

∴ $xs_\infty = x + 2x^2 + 3x^3 + \ldots + \infty$... **(ii)**

Subtracting **(i)** from **(ii)** , we get

(1 − x)$s_\infty = x + x^2 + x^3 + \ldots \infty$

(1 − x)$s_\infty = \frac{1}{(1-x)}$, ∴ $s_\infty = \frac{1}{(1-x)^2}$

Example 2 : **If sum of infinity of the series**

1+ 4x + 7x^2 + 10 x^3 + . . . Is $\frac{35}{16}$then find x

Solution : Let $s_\infty = 1 + 4x + 7x^2 + 10x^3 + \ldots \infty$ . . . (1)

$xs_\infty = x + 4x^2 + 7x^3 + \ldots + \infty$ . . . (2)

Subtracting **(2)** from **(1)** , we get

(1 − x)$s_\infty = 1 + 3x + 3x^2 + 3x^3 + \ldots \infty$

(1 − x)$s_\infty = 1 + \frac{3x}{(1-x)}$

(1 − x)$\frac{35}{16} = \frac{1+2x}{(1-x)} \Rightarrow 35(1-x)^2 = 16 + 32x$

$\Rightarrow 35x^2 - 102x + 19 = 0$

$\Rightarrow (7x - 19)(5x - 1) = 0$

5/ HomeWorks: -

Homework

1-Write the sum using sigma notation.

a- $1^2 + 2^2 + 3^2 + \ldots + 10^2$

b- $\frac{\sqrt{1}}{1^2} + \frac{\sqrt{2}}{2^2} + \frac{\sqrt{3}}{3^2} + \ldots + \frac{\sqrt{n}}{n^2}$

c- $1 + x + x^2 + x^3 + \ldots + x^n$

d- $1 - 2x + 3x^2 - 4x^3 + 5x^4 + \ldots - 100x^{99}$

e- $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} \ldots + \frac{1}{999\times 1000}$

2- Find the **nth** term of the arithmetic sequence with given first term a and common difference **d.** What is the **10th** term**?**

a) $a = 9$, $d = 4$ **(b)** $a = -5$, $d = 4$

(c) $a = \frac{5}{2}$, $d = -\frac{1}{2}$

3) Determine the common difference, the **fifth term**, the **nth** term, and the **100th** term of the arithmetic sequence

(a) $29, 11, -7, -25 \ldots$ **(b)** $-t$, $-t+3$, $-t+6$, $-t+9$, $\ldots$,

(c) $\frac{7}{6}$, $\frac{5}{3}$, $\frac{13}{6}$, $\frac{8}{3}$, $\ldots$

(4) The 50th term is **1000**, and the common difference is **6**. Find nth , the first and second terms.

(5) The 100th term is **− 750**, and the common difference is **− 20.** Find the fifth term.

(6) The fourteenth term is **23** , and the ninth term is **14** . Find the first term and the nth term.

(7) The first term is **25**, and the common difference is **18** . Which term of the sequence is **601 ?**

(**8)** Find the partial sum s_n of the arithmetic sequence that satisfies the given conditions.

(a) $a=3$, $d=5$ n=20 (b) $a=-2$, $d=23$ n=25

(9) A partial sum

of an arithmetic sequence is given. Find the sum.

(a) $1 + 5 + 9 + \ldots + 401$ (b) $89 + 85 + 81 + \ldots + 13$

(10) Find the number of terms of the arithmetic sequence with the given description that must be added to get a value of **2700**. (a) The first term is **5**, and the common difference is **2 .**

(11) A drive-in theater has spaces for **20** cars in the first parking row, **22** in the second, **24** in the third, and so on. If there are **21** rows in the theater, find the number of cars that can be parked.

(12) Find the **nth** term of the geometric sequence with given first term a and common ratio **r**. What is the fourth term?

(a) $a = 7$, $r = 4$ *(b)* $a = \frac{5}{2}$, $r = -\frac{1}{2}$

(13) Find the first five terms of the sequence, and determine whether it is geometric. If it is geometric, find the common ratio, and express the nth term of the sequence in the standard form $a_n = ar^{n-1}$.

(a) $a_n = 2(3)^n$ (b) $a_n = \frac{1}{4^n}$

(14)Find the sum. **(a)** $\sum_{k=1}^{5} 3(\frac{1}{2})^{k-1}$ **(b)** $\sum_{k=16} 64(\frac{3}{2})^{k-1}$

(15) The fourth term is **12** and the seventh term is $\frac{32}{9}$. Find the first and nth terms..

(16) The third term is **−18** and the sixth term is **9216**. Find the first and nth terms.