

K-Map (Karnaugh Map)

In many digital circuits and practical problems, we need to find expressions with minimum variables. We can minimize Boolean expressions of 3, 4 variables very easily using K-map without using any Boolean algebra theorems.

Steps to Solve Expression using K-map

1. Select the K-map according to the number of variables.
2. Identify minterms or maxterms as given in the problem.
3. For SOP put 1's in blocks of K-map respective to the minterms (0's elsewhere).
4. For POS put 0's in blocks of K-map respective to the max terms (1's elsewhere).
5. Make rectangular groups containing total terms in power of two like 2,4,8 ..(except 1) and try to cover as many elements as you can in one group.
6. From the groups made in step 5 find the product terms and sum them up for SOP form.

2 Variable Truth Table and K-Map

A logical specification is often created using a truth table. A truth table is a list of the inputs (A, B) on the left and the corresponding output (F) on the right. See Figure 1 showing a 2 variable truth table and corresponding K-Map.

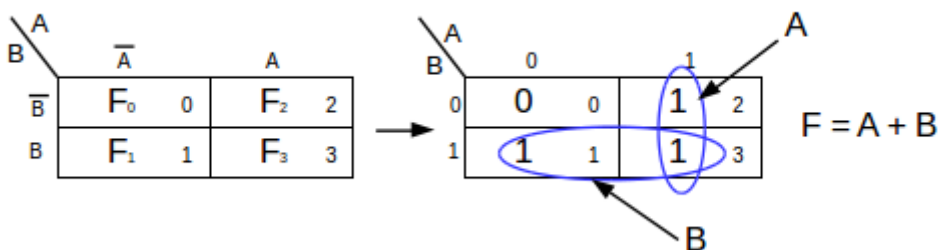
2 Variable Truth Table

	A	B	F
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1



		K-Map	
		A	B
B	A	0	1
0	0	0	1
1	0	0	2
1	1	1	3

Each cell of the K-Map represents an input state (A, B). The value of each cell represents the output function (F). In order to find the minimum logic function, it is necessary to identify matching adjacent cells. Once these matches are found, an expression can be written.



3 Variable Truth Table and K-Map

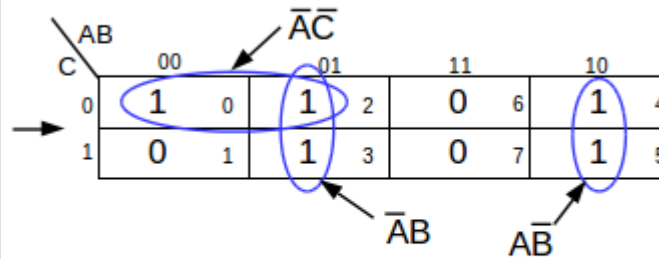
Below is an example of a 3 variable K-Map. Notice that the cells are ordered in the K-Map to ensure only one bit changes on any adjacent cell. From left to right instead of 0, 1, 2, 3, 4, 5, 6, 7, the cell ordering is 0, 1, 2, 3, 6, 7, 4, 5.

000	010	110	100
001	011	111	101

3 Variable Truth Table

	A	B	C	F
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

K-Map



Note : $\bar{A}B + A\bar{B} = A \oplus B$ (Exclusive OR)

$$F = \bar{A}\bar{C} + \bar{A}B + A\bar{B}$$

$$F = \bar{A}\bar{C} + A \oplus B$$

4 Variable Truth Table and K-Map

Below is an example of a 4 variable K-Map. Notice that the cells are ordered in the K-Map to ensure only one bit changes on any adjacent cell.

4-Variables K-map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Example :

Four-Variable K-Maps

<table border="1"> <tr><td>CD</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>AB</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>00</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>01</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>11</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>10</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> </table> $f = \sum(0,8) = \bar{B} \cdot \bar{C} \cdot \bar{D}$	CD	00	01	11	10	AB	00	01	11	10	00	1	0	0	0	01	0	0	0	0	11	0	0	0	0	10	1	0	0	0	<table border="1"> <tr><td>CD</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>AB</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>00</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>01</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>11</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>10</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </table> $f = \sum(5,13) = B \cdot \bar{C} \cdot D$	CD	00	01	11	10	AB	00	01	11	10	00	0	0	0	0	01	0	1	0	0	11	0	1	0	0	10	0	0	0	0	<table border="1"> <tr><td>CD</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>AB</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>00</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>01</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>11</td><td>0</td><td>1</td><td>1</td><td>0</td></tr> <tr><td>10</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </table> $f = \sum(13,15) = A \cdot B \cdot D$	CD	00	01	11	10	AB	00	01	11	10	00	0	0	0	0	01	0	0	0	0	11	0	1	1	0	10	0	0	0	0	<table border="1"> <tr><td>CD</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>AB</td><td>00</td><td>01</td><td>11</td><td>10</td></tr> <tr><td>00</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>01</td><td>1</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>11</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>10</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </table> $f = \sum(4,6) = \bar{A} \cdot B \cdot \bar{D}$	CD	00	01	11	10	AB	00	01	11	10	00	0	0	0	0	01	1	0	0	1	11	0	0	0	0	10	0	0	0	0
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Question:

1. Four chairs A,B,C,D are placed in a circle. Each chair may be occupied ("1") or empty ("0"). A Boolean function F is "1" if and only if there are two or more adjacent chairs that are empty.

- Give the truth table defining the Boolean function F.
- Simplify the function F and draw the logic circuit.