Heat Transfer

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Chapter1:Dimensions and units

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Useful conversion factors

Physical quantity	Symbol	SI to English conversion	English to SI conversion
Length	L	1 m = 3.2808 ft	1 ft = 0.3048 m
Area	A	1 m^2 = 10.7639 ft^2	1 ft^2 = 0.092903 m^2
Volume	V	1 m^3 = 35.3134 ft^3	1 ft^3 = 0.028317 m^3
Velocity	v	1 m/s = 3.2808 ft/s	1 ft/s = 0.3048 m/s
Density	ρ	1 kg/m^3 = 0.06243 lb_m/ft^3	1 lb_m/ft^3 = 16.018 kg/m^3
Force	F	1 N = 0.2248 lb_f	1 lb_f = 4.4482 N
Mass	m	1 kg = 2.20462 lb_m	1 lb_m = 0.45359237 kg
Pressure	p	1 N/m^2 = 1.45038×10^{-4} lb_f/in^2	1 lb_f/in^2 = 6894.76 N/m^2
Energy, heat	q	1 kJ = 0.94783 Btu	1 Btu = 1.05504 kJ
Heat flow	q	1 W = 3.4121 Btu/h	1 Btu/h = 0.29307 W
Heat flux per unit area	q/A	1 W/m^2 = 0.317 Btu/h · ft^2	1 Btu/h · ft^2 = 3.154 W/m^2
Heat flux per unit length	q/L	1 W/m = 1.0403 Btu/h · ft	1 Btu/h · ft = 0.9613 W/m
Heat generation per unit volume	$\dot{q}$	1 W/m^3 = 0.096623 Btu/h · ft^3	1 Btu/h · ft^3 = 10.35 W/m^3
Energy per unit mass	q/m	1 kJ/kg = 0.4299 Btu/lb_m	1 Btu/lb_m = 2.326 kJ/kg
Specific heat	c	1 kJ/kg · °C = 0.23884 Btu/lb_m · °F	1 Btu/lb_m · °F = 4.1869 kJ/kg · °C
Thermal conductivity	k	1 W/m · °C = 0.5778 Btu/h · ft · °F	1 Btu/h · ft · °F = 1.7307 W/m · °C
Convection heat-transfer coefficient	h	1 W/m^2 · °C = 0.1761 Btu/h · ft^2 · °F	1 Btu/h · ft^2 · °F = 5.6782 W/m^2 · °C
Dynamic		1 kg/m · s = 0.672 lb_m/ft · s	
Viscosity	μ	= 2419.2 lb_m/ft · h	1 lb_m/ft · s = 1.4881 kg/m · s
Kinematic viscosity and thermal diffusivity	ν, α	1 m^2/s = 10.7639 ft^2/s	1 ft^2/s = 0.092903 m^2/s

$$1\ \text{Btu} = 778.16\ \text{lb}_f \cdot \text{ft}$$

$$1\ \text{Btu} = 1055\ \text{J}$$

$$1\ \text{kcal} = 4182\ \text{J}$$

$$1\ \text{lb}_f \cdot \text{ft} = 1.356\ \text{J}$$

$$1\ \text{Btu} = 252\ \text{cal}$$

$$^\circ\text{F} = \tfrac{9}{5}\,^\circ\text{C} + 32$$

$$^\circ\text{R} = ^\circ\text{F} + 459.69$$

$$\text{K} = ^\circ\text{C} + 273.16$$

$$^\circ\text{R} = \tfrac{9}{5}\text{K}$$

Multiplier factors for SI units.

Multiplier	Prefix	Abbreviation
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-18}	atto	a

SI quantities used in heat transfer.

Quantity	Unit abbreviation
Force	N (newton)
Mass	kg (kilogram mass)
Time	s (second)
Length	m (meter)
Temperature	°C or K
Energy	J (joule)
Power	W (watt)
Thermal conductivity	$W/m \cdot °C$
Heat-transfer coefficient	$W/m^2 \cdot °C$
Specific heat	$J/kg \cdot °C$
Heat flux	W/m^2

Chapter2

Heat Transfer Mechanisms

- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.
- Heat can be transferred in three different modes:

1. conduction
2. convection
3. and radiation.

- All modes of heat transfer require the existence of a temperature difference.

2.1 Conduction Heat Transfer

- The transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.
- In gases and liquids: conduction is due to the collisions and diffusion of the molecules during their random motion.
- In solids: it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.

- The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.

$$\text{Rate of heat conduction} \propto \frac{(\text{Area})(\text{Temperature difference})}{\text{Thickness}}$$

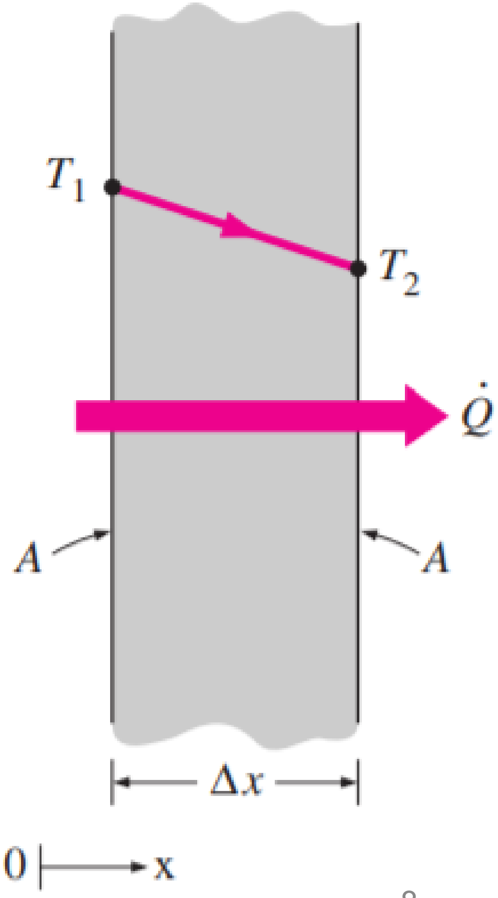
or,

$$\dot{Q}_{cond} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad (W)$$

In the limiting case of $\Delta x \to 0$, the equation above reduces to the differential form

$$\dot{Q}_{cond} = -kA\frac{dT}{dx} \qquad (W)$$

→ **Fourier's law of heat conduction**



Fourier's law of heat conduction

$$Q_{cond} = -kA\frac{dT}{dx}$$

where, Q = rate of heat transfer in W,

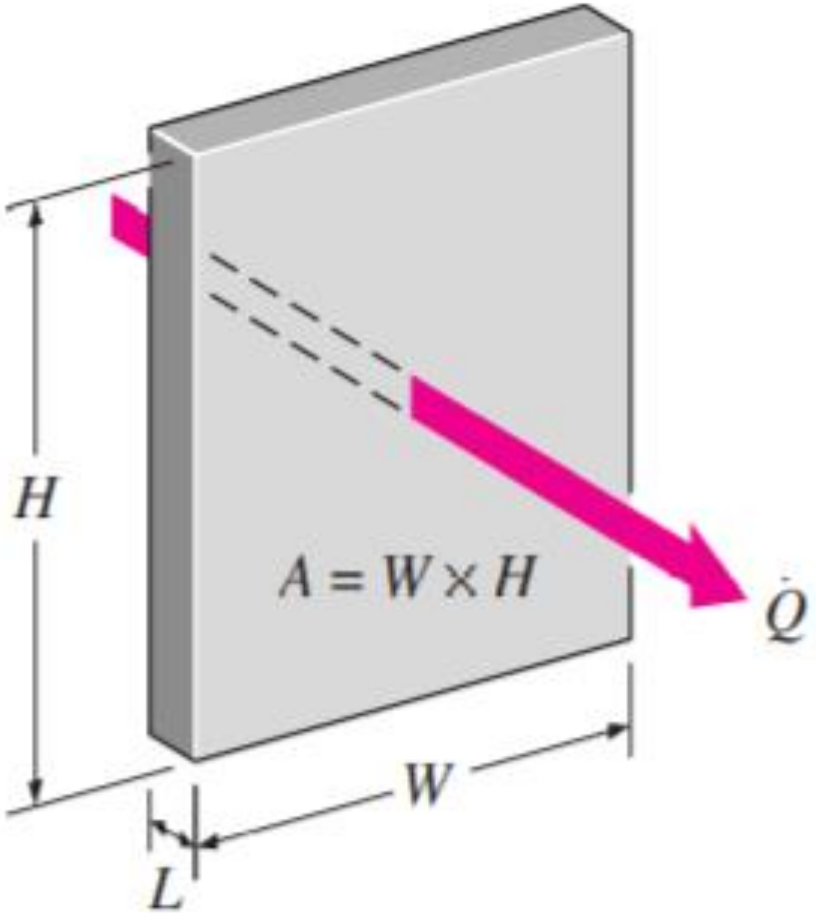
A = heat transfer area in m^2 ; normal to direction of heat flow,

$\frac{dT}{dx}$ = temperature gradient in °C/m ; slope of temperature curve on T–x diagram,

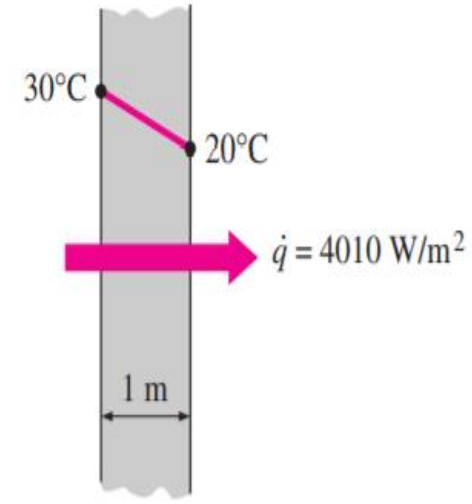
k = constant of proportionality, called the *thermal conductivity* of material in W/m·°C or W/m.K.

- Heat transfer area

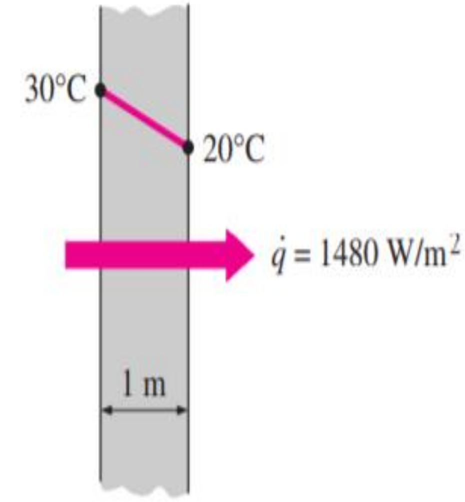
- In heat conduction analysis, A represents the area normal to the direction of heat transfer.
- Note that the thickness of the wall has no effect o[n]
A



- Thermal conductivity (K) : A measure of the ability of a material to conduct heat
- temperature gradient dT/dx: The slope of the temperature curve on a T-x diagram.
- Heat is conducted in the direction of decreasing temperature,
- the temperature gradient becomes negative when temperature decreases with increasing x.
- The negative sign in Equation ensures that heat transfer in the positive x direction is a positive quantity.



(a) Copper (k = 401 W/m·°C)



(b) Silicon (k = 148 W/m·°C)

Example 1. The wall of a furnace is constructed from 15 cm thick fire brick having constant thermal conductivity of 1.6 W/m.K. The two sides of the wall are maintained at 1400 K and 1100 K, respectively. What is the rate of heat loss through the wall which is50 cm × 3 m on a side?

Solution

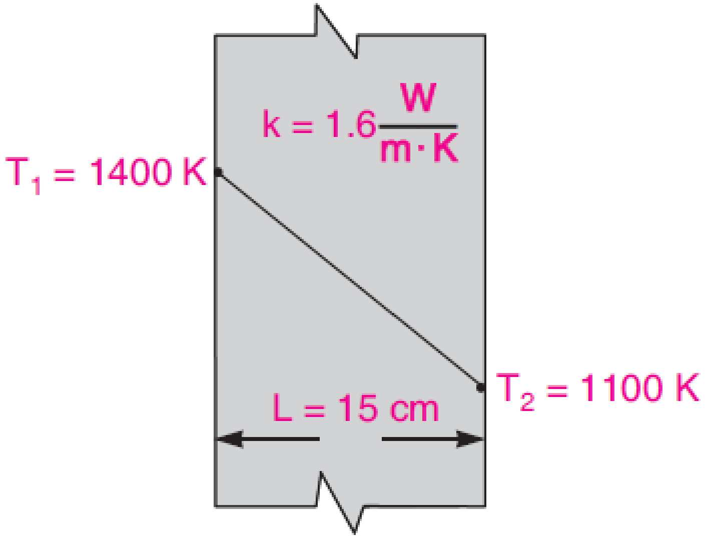
Given : A furnace wall with

T1 = 1400 K, T2 = 1100 K

A = 50 cm × 3 m = 0.5 × 3 = 1.5 m2

k = 1.6 W/m.K

L = 15 cm = 0.15 m



To find : Heat loss through the wall.

Assumptions :

1. Steady state conditions.
2. One dimensional heat conduction through the wall.
3. Constant properties.

Analysis : According to Fourier law of heat conduction, equation (1.9)

$$Q = kA\frac{(T_1 - T_2)}{L}$$

Using numerical values

$$Q = \frac{(1.6 \text{ W/m.K}) \times (1.5 \text{ m}^2) \times (1400 \text{ K} - 1100 \text{ K})}{(0.15 \text{ m})}$$

= **4800 W.** **Ans.**

- H.W.1 One face of a copper plate 3 cm thick is maintained at 500◦C, and the other face is maintained at 200◦C. How much heat is transferred through the plate?

- H.W2: a household oven is modeled to be a hollow rectangular box having inside dimensions of 46cm *64cm*76cm and outside dimensions of 51*66*81cm. if the heat losses through the corners and edges one ignored, the inside wall temperature is 204 C, the outside wall temp. is 38C and the wall material is asbestos estimate the power input in watts necessary to maintainvthis steady state condition. given k= 0. 1660w/ m. K.

Thermal conductivity

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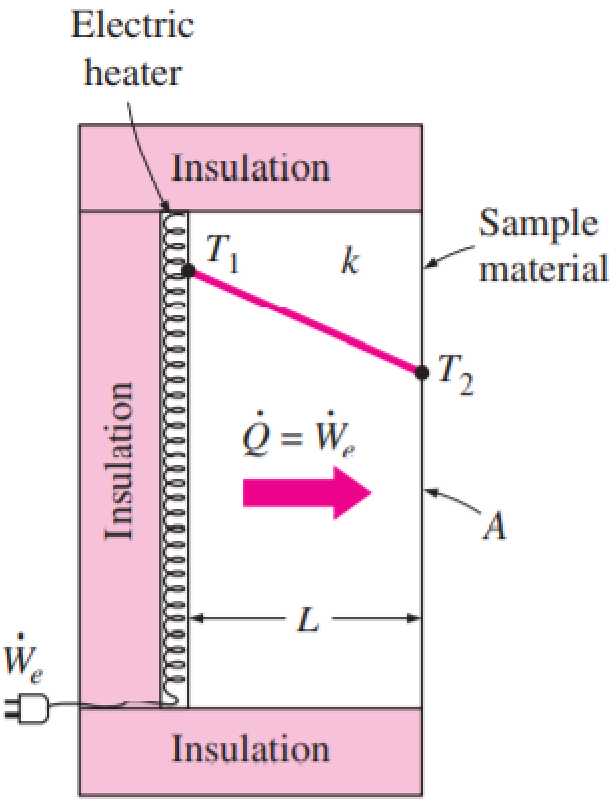
✓A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator.

✓A simple experimental setup to determine the thermal conductivity of a material.



$$k = \frac{L}{A(T_1 - T_2)}\dot{Q}$$

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

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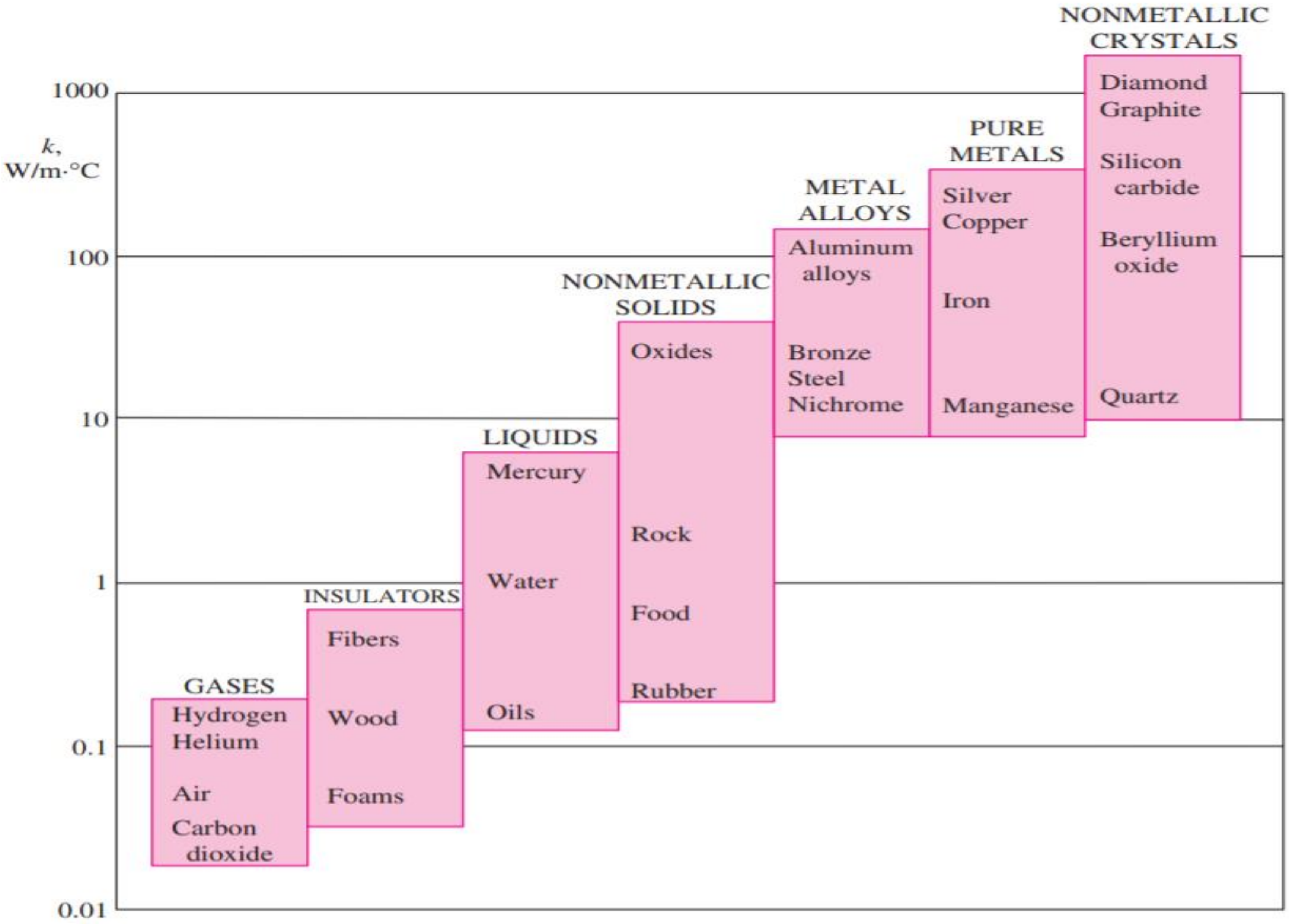

The range of thermal conductivity of various materials at room temperature.

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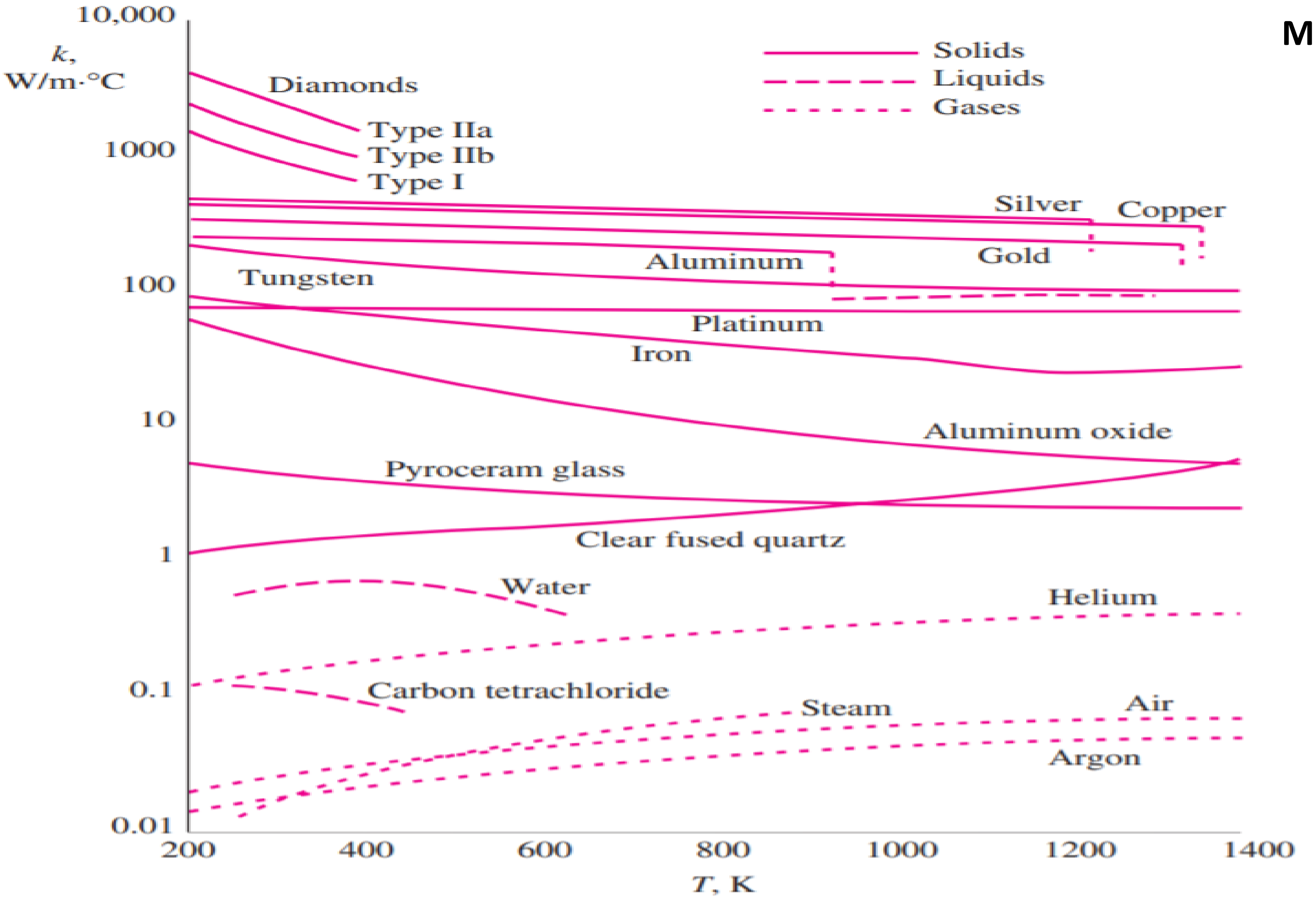

The variation of the thermal conductivity of various solids, liquids, and gases with temperature

Thermal Diffusivity

- Cp: specific heat, J/kg · °C: heat capacity per unit mass
- ρCp: heat capacity, J/m^3 · °C, heat capacity per unit volume
- α : thermal diffusivity, m^2/s : represents how fast heat diffuses through a material

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p} \qquad (\text{m}^2/\text{s})$$

- A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity.
- The larger the thermal diffusivity, the faster the propagation of heat into the medium.
- A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat will be conducted further.

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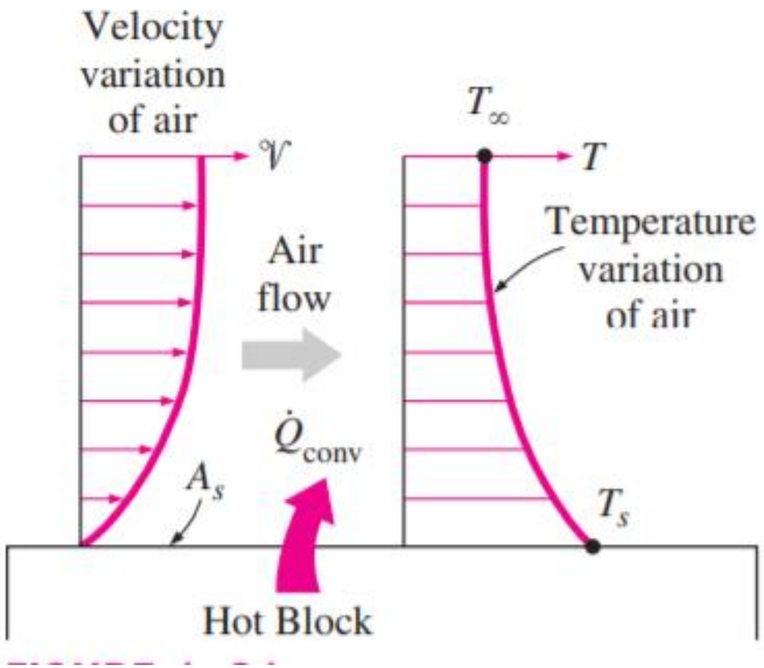
2.2 Convection Heat Transfer

- Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.
- The faster the fluid motion, the greater the convection heat transfer.
- In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.

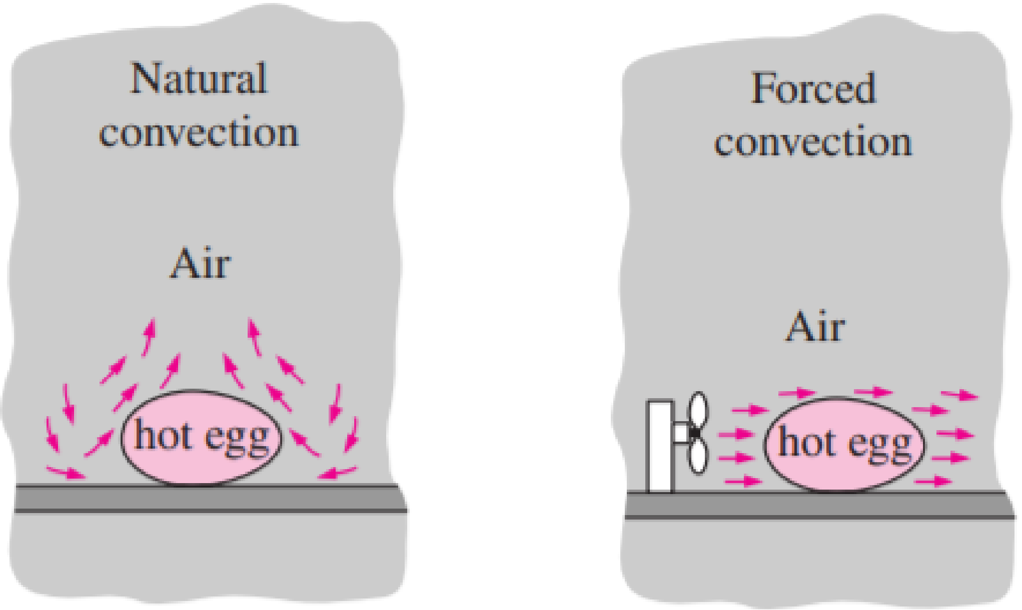


- Natural (or free) convection

if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.

- Forced convection

if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.



The cooling of a boiled egg by forced and natural convection.

Newton's law of cooling

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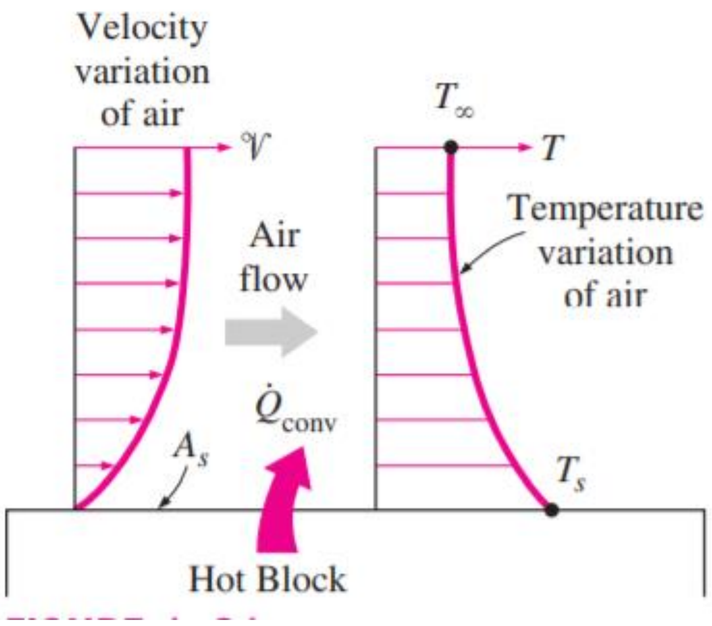
$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$ (W) Newton's law of cooling



where h is the convection heat transfer coefficient in W/m^2 · °C or Btu/h · ft^2 · °F,
As is the surface area through which convection heat transfer takes place,
Ts is the surface temperature,

And T_∞ is the temperature of the fluid sufficiently far from the surface.

- The convection heat transfer coefficient h is not a property of the fluid.
- It is an experimentally determined parameter whose value depends on all the variables influencing convection such as:
- the surface geometry .
- the nature of fluid motion.
- the properties of the fluid.
- and the bulk fluid velocity.

Typical values of convection heat transfer coefficient

Type of convection	h, W/m^2 · °C*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

Example: Hot air at 150°C flows over a flat plate maintained at 50°C. The forced convection heat transfer coefficient is 75 W/m^2.K. Calculate the heat gain rate by the plate through an area of 2 m^2.

Solution

Given : Flow of hot air over a flat plate

$T_\infty = 150°C$, $T_s = 50°C$

$h = 75\ W/m^2.K$, $A = 2\ m^2$.

To find : Heat transfer rate by air to plate.

Assumptions :

(*i*) Steady state conditions,

(*ii*) Constant properties,

(*iii*) Heat is transferred by forced convection only.

Analysis : According to Newton's law of cooling

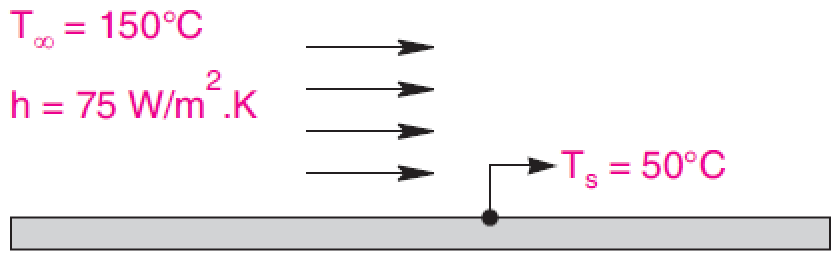


Fig. 1.6. Flow over a flat plate

$$Q = hA_s(T_\infty - T_s)$$

$$= (75\ W/m^2.K) \times (2\ m^2) \times (150 - 50)\ (°C\ or\ K)$$

$$= 15 \times 10^3\ W = \mathbf{15\ kW}.$$ **Ans.**

- H.W. : Air at 22◦C blows over a hot plate 50 by 75 cm maintained at 260◦C. The convection heat-transfer coefficient is 25 W/m2 · ◦C. Calculate the heat transfer.

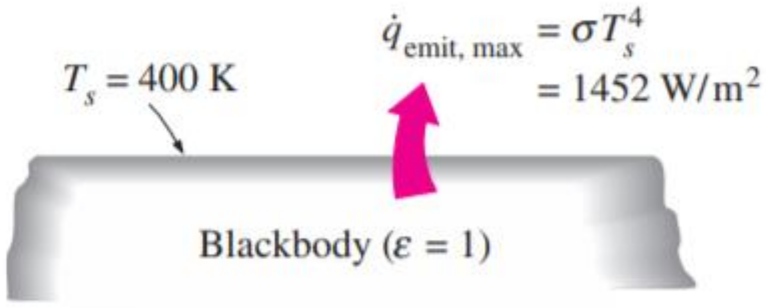
2.3 Radiation Heat Transfer

- The energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules.
- Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an intervening medium.
- Radiation is a volumetric phenomenon, and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees. However, radiation isusually considered to be a surface phenomenon for solids that are opaque to thermal radiation such as metals, wood, and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface.

- The maximum rate of radiation that can be emitted from a surface at an absolute temperature Ts (in K or R) is given by the Stefan–Boltzmann law as

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \qquad \text{(W)}$$

where $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$ or $0.1714 \times 10^{-8}\ \text{Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4$ ⟵ Stefan–Boltzmann constant



Blackbody : The idealized surface that emits radiation at this maximum rate.

- The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4 \qquad (W)$$

- Emissivity Ɛ: a measure of how closely a surface approximates a blackbody for which Ɛ=1.

$$0 \leq \varepsilon \leq 1$$

Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

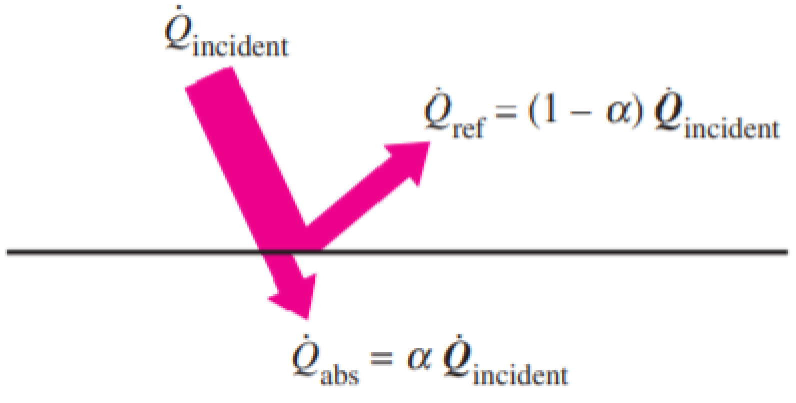
- Absorptivity α:the fraction of the radiation energy incident on a surface that is absorbed by the surface. $0 \leq \alpha \leq 1.$
- A blackbody absorbs the entire radiation incident on it. α=1

- Kirchhoff's law of radiation states that the emissivity and the absorptivity of a surface at a given temperature and wavelength are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity.

- The rate at which a surface absorbs radiation is determined from

$$\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident} \qquad \text{(W)}$$

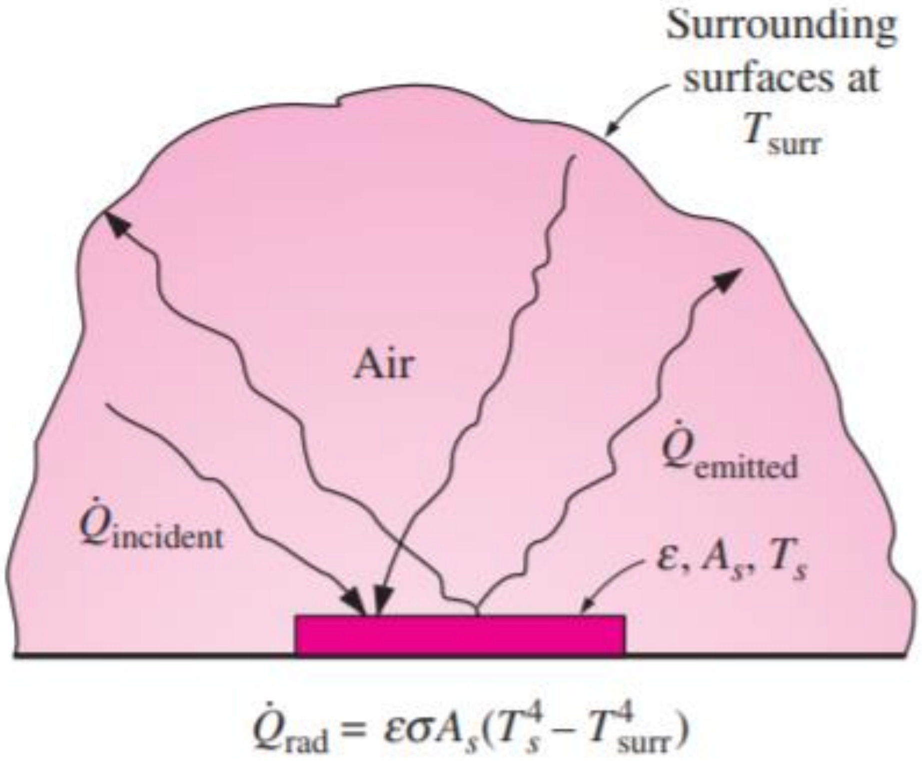
For opaque (nontransparent) surfaces, the portion of incident radiation not absorbed by the surface is reflected back.



- Net radiation heat transfer: the difference between the rates of radiation emitted by the surface and the radiation absorbed.
- Radiation is usually significant relative to conduction or natural convection, but negligible relative to forced convection.

- When a surface is completely enclosed by a much larger (or black) surface at absolute temperature Tsurr separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) \qquad \text{(W)}$$



Example *The surface temperature of a central heating radiator is 60°C. What is the net black body radiation heat transfer unit surface area between the radiator and its surroundings at 20° ?*

Take $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$

Solution

Given : Central heating radiator as black body with

$$T_s = 60°C = 333\ K, \quad T_\infty = 20°C = 293\ K.$$

To find : Radiation heat transfer

Analysis : The black body radiation heat transfer rate per unit area between radiator surface and its surroundings is expressed as ;

$$q = \frac{Q}{A} = \sigma\,(T_s^4 - T_\infty^4)$$

$$= (5.67 \times 10^{-8}\ W/m^2.K^4) \times [(333\ K)^4 - (293\ K)^4]$$

$$= 5.67 \times 10^{-8} \times 4.9263 \times 10^9$$

$$= \mathbf{279.32\ W/m^2}.$$ **Ans.**

- H.W.3 Two infinite black plates at 800◦C and 300◦C exchange heat by radiation. Calculate the heat transfer per unit area

Chapter3:

Heat Transfer in rectangular, cylinderical and spherical walls

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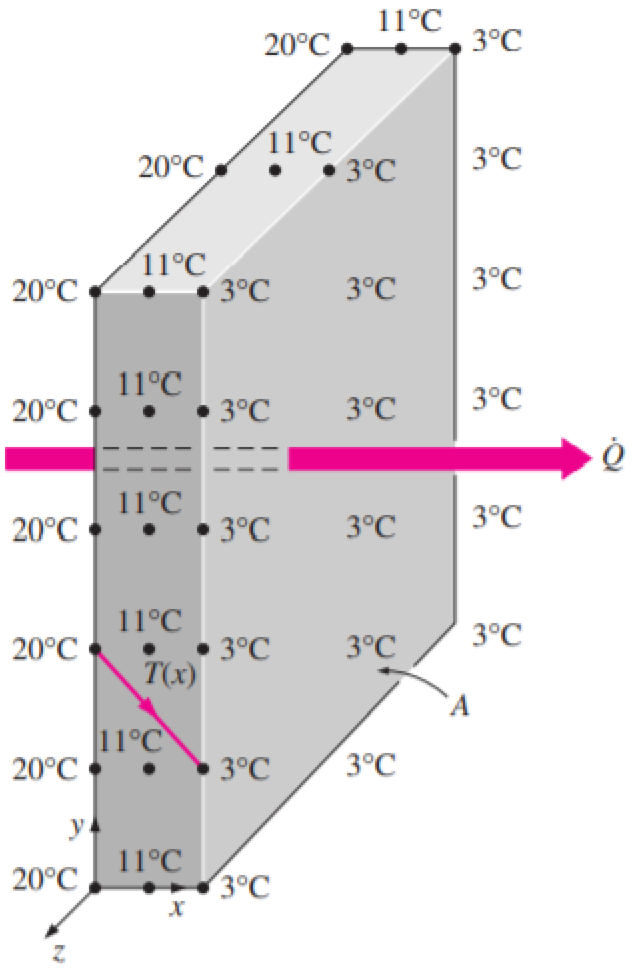
3.1 Steady heat conduction in planes wall

- Heat flow through a wall is onedimensional when the temperature of the wall varies in one direction only
- The temperature of the wall in this case will depend on one direction only (say the x-direction) and can be expressed as T(x).



$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

$dE_{wall}/dt = 0$ for *steady* operation,

- since there is no change in the temperature of the wall with time at any point. Therefore, the rate of heat transfer into the wall must be equal to the rate of heat transfer out of it. In other words, the rate of heat transfer through the wall must be constant, Q· cond, wall constant.
- Consider a plane wall of thickness L and average thermal conductivity k. The two surfaces of the wall are maintained at constant temperatures of T1 and T2. For one-dimensional steady heat conduction through the wall, we have T(x) Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{cond,\ wall} = -kA\frac{dT}{dx} \qquad (W)$$

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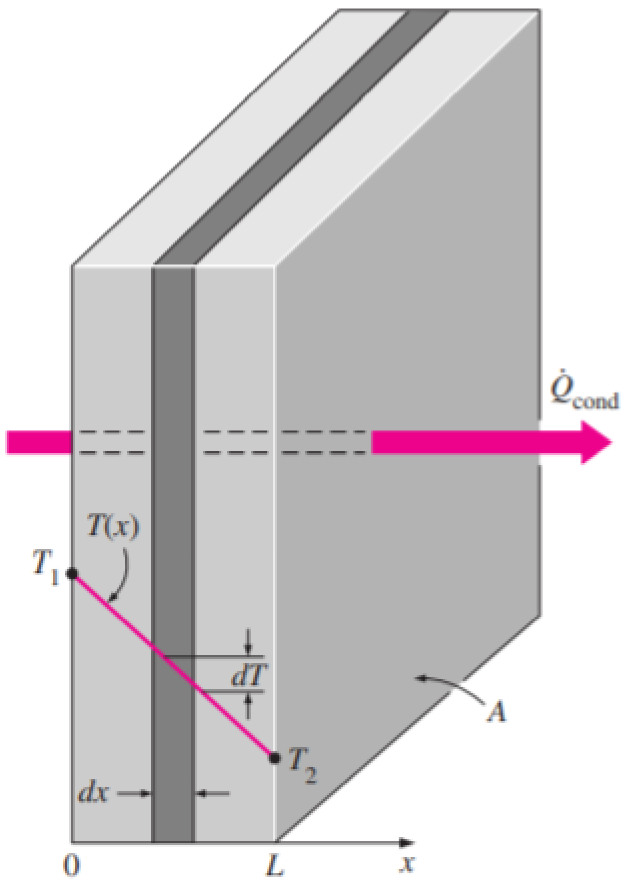
$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

- Separating the variables in the above equation and integrating from x= 0, where T(0)= T1, to x =L, where T(L) =T2, we get

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

- once the rate of heat conduction is available, the temperature T(x) at any location x can be determined by replacing T2 in above equation by T, and L by x



Under steady conditions, the temperature distribution in a plane wall is a straight line.

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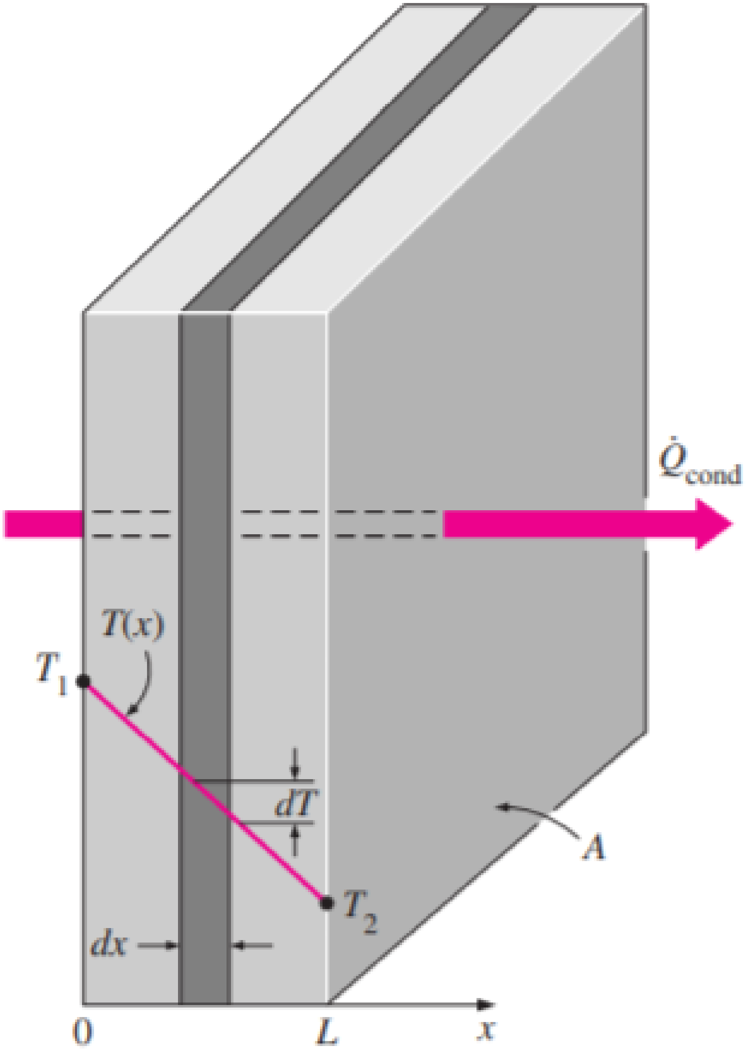
Thermal Resistance Concept

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)} \qquad R_{\text{wall}} = \frac{L}{kA} \qquad (°\text{C/W})$$

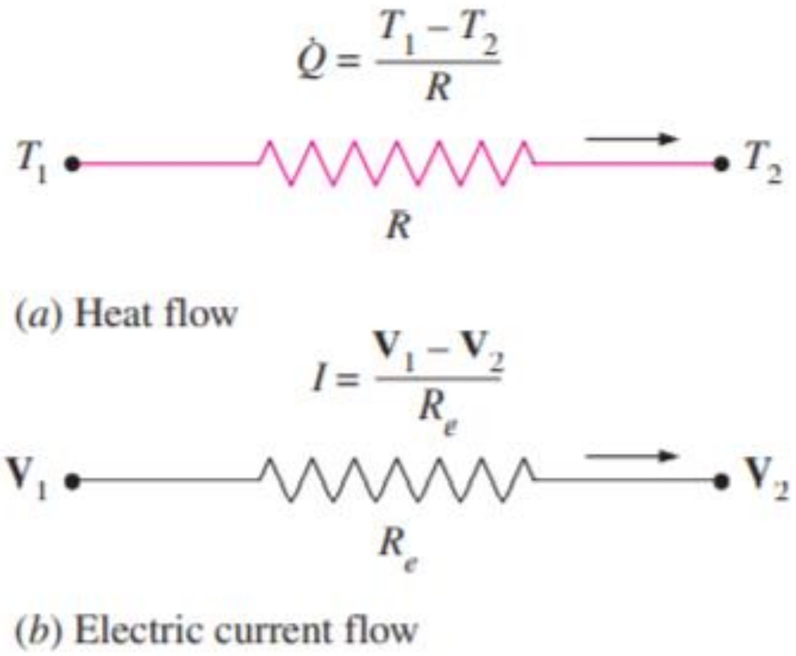


Conduction resistance of the wall: the thermal resistance of the wall against heat conduction.

Thermal resistance of a medium depends on the geometry and the thermal properties of the medium.

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

$$I = \frac{V_1 - V_2}{R_e}$$

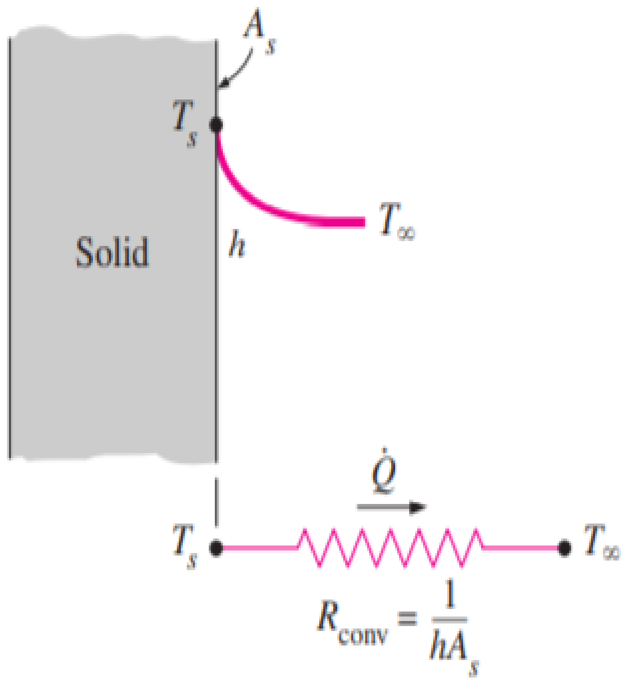


The rate of heat transfer through a layer corresponds to the electric current, the thermal resistance corresponds to electrical resistance, and the temperature difference corresponds to voltage difference across the layer

- convection resistance of the surface

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \quad \text{(W)} \qquad R_{conv} = \frac{1}{hA_s} \quad (°\text{C/W})$$



- ➢ Convection resistance of the surface: the thermal resistance of the surface against heat convection

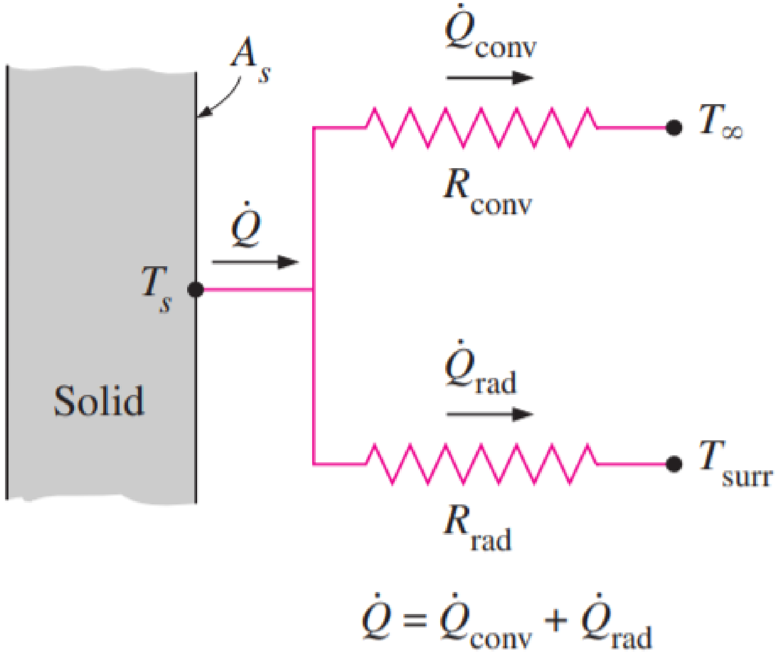
- ➢ Note that when the convection heat transfer coefficient is very large the convection resistance becomes zero and $T_s \approx T_\infty$. That is, the surface offers no resistance to convection, and thus it does not slow down the heat transfer process.

- radiation resistances at a surface

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad} A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \qquad \text{(W)}$$

where

$$R_{rad} = \frac{1}{h_{rad} A_s} \qquad \text{(K/W)}$$



radiation resistance : thermal resistance of a surface against radiation

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \qquad (\text{W/m}^2 \cdot \text{K})$$

h_{rad}. The definition of the radiation heat transfer coefficient

Thermal Resistance Network

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$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

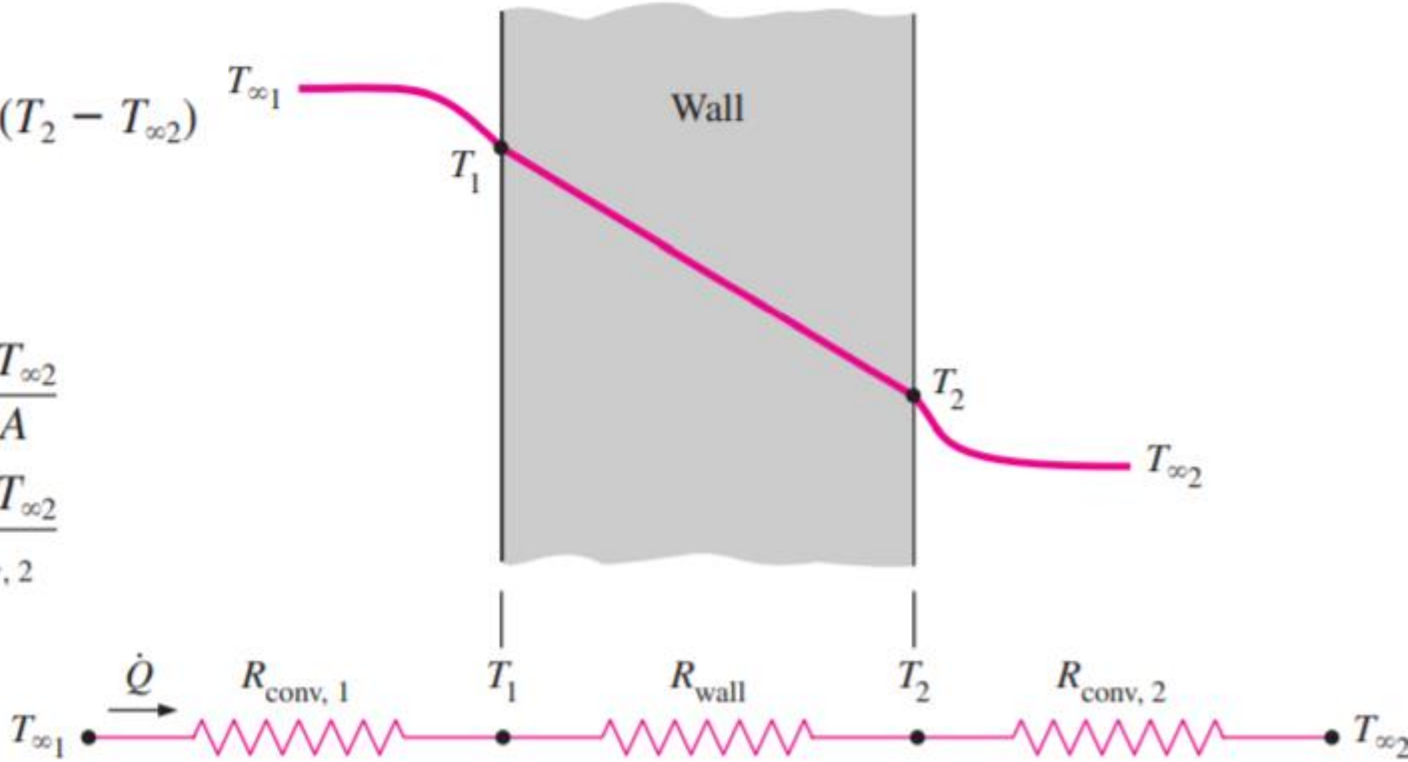
$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}}}$$



$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°\text{C/W})$$

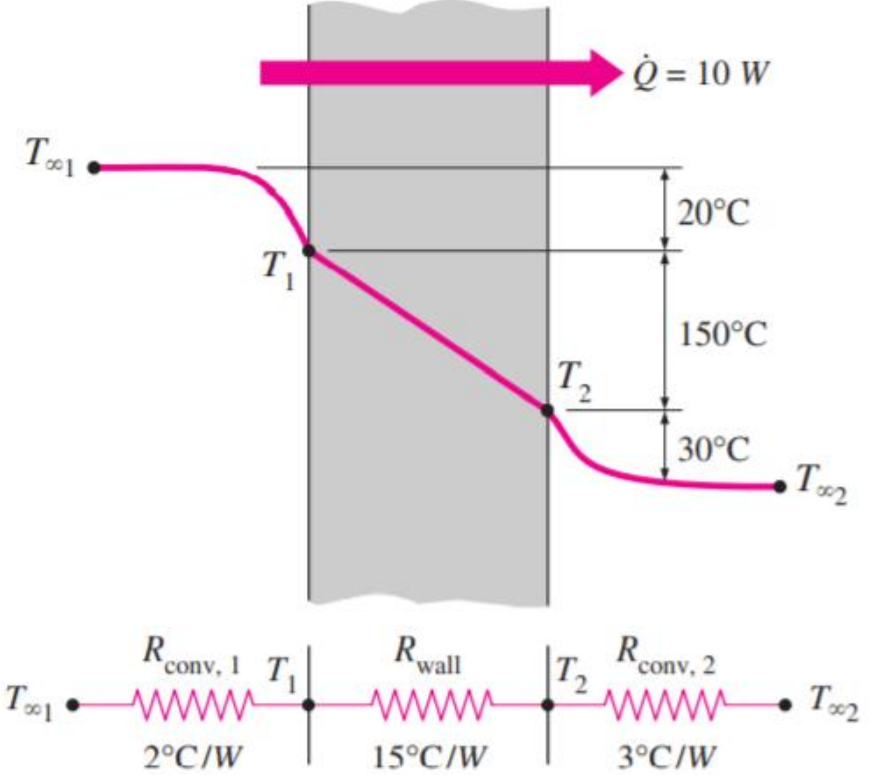
- The temperature drop across a layer is proportional to its thermal resistance.

$$\dot{Q} = \frac{T_{\infty} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)} \qquad \Delta T = \dot{Q} R$$

- It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

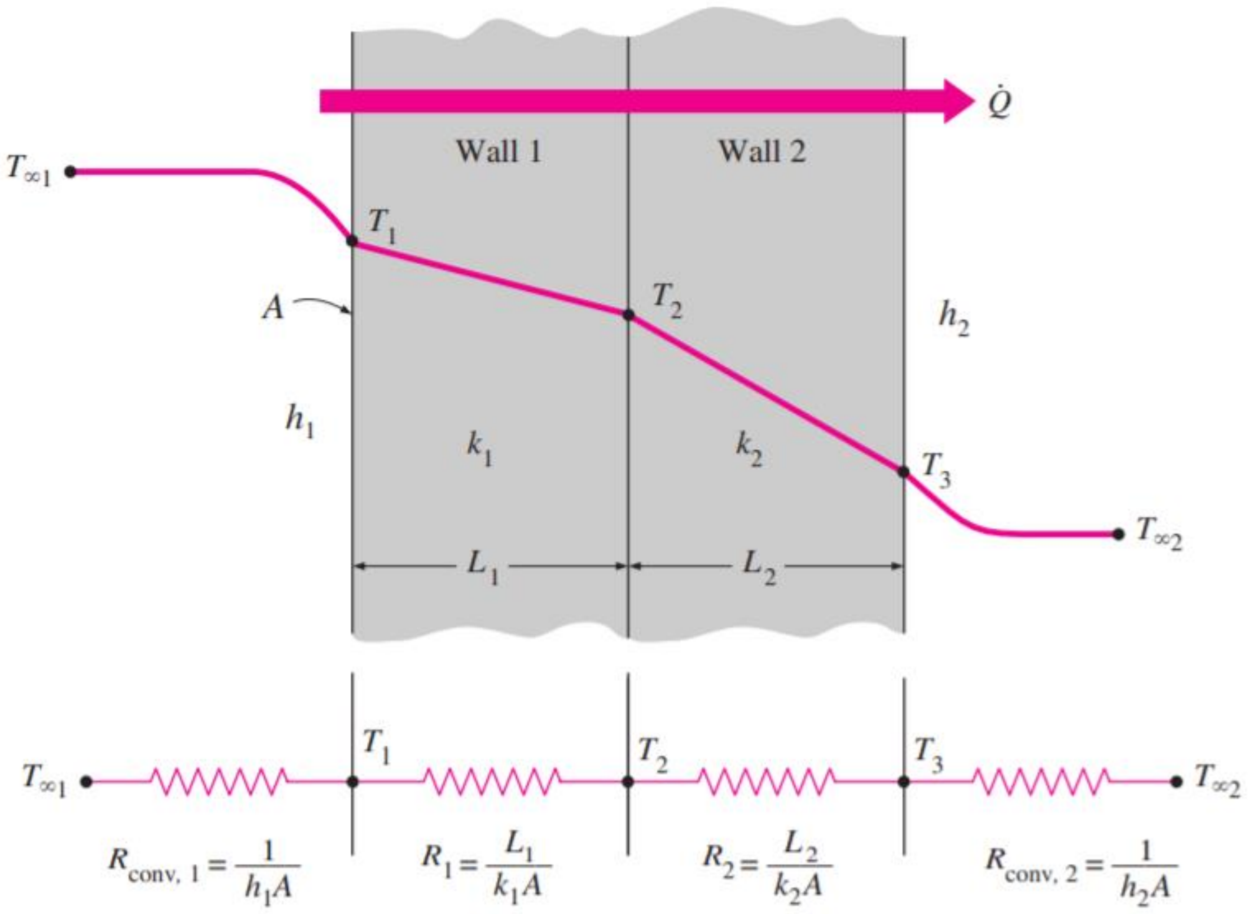
$$\dot{Q} = UA\,\Delta T \qquad \text{(W)} \qquad UA = \frac{1}{R_{\text{total}}}$$

U is the overall heat transfer coefficient.



Multilayer Plane Walls

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$



- Once Q is known, an unknown surface temperature Tj at any surface or interface j can be determined from

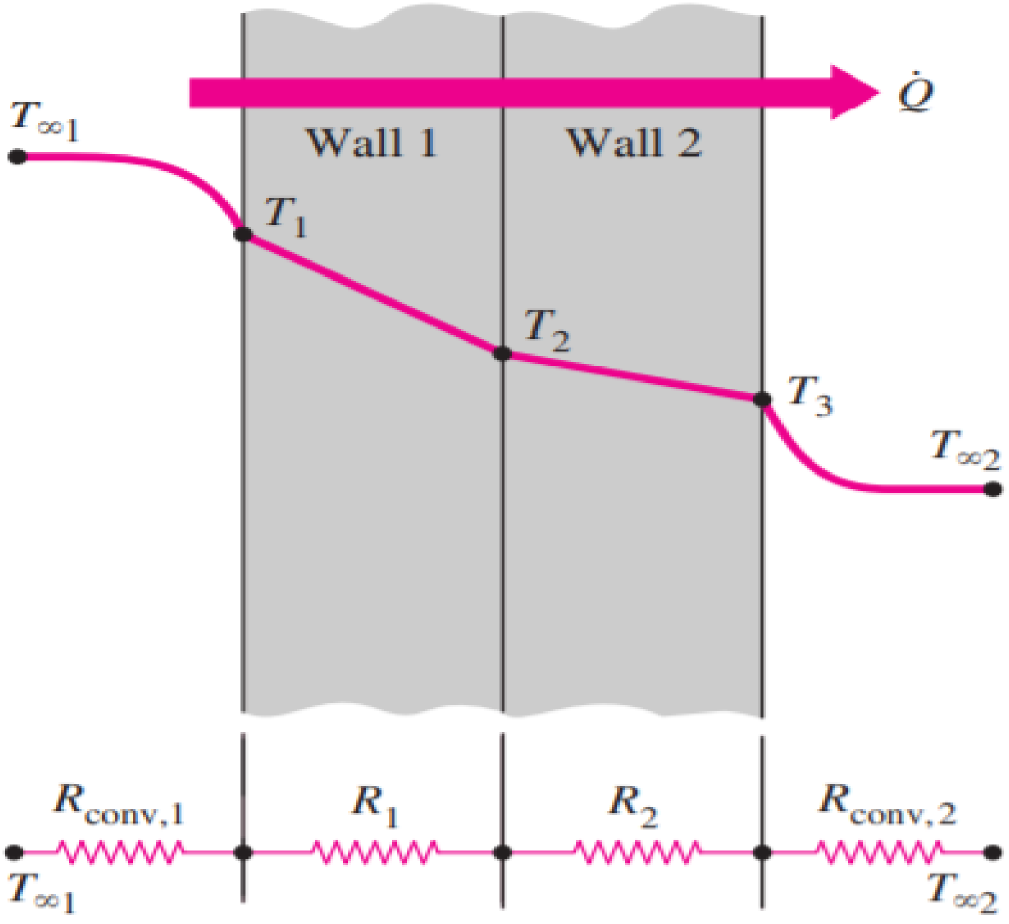
$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total}, i-j}}$$

To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



Thermal resistance network for two parallel layers:

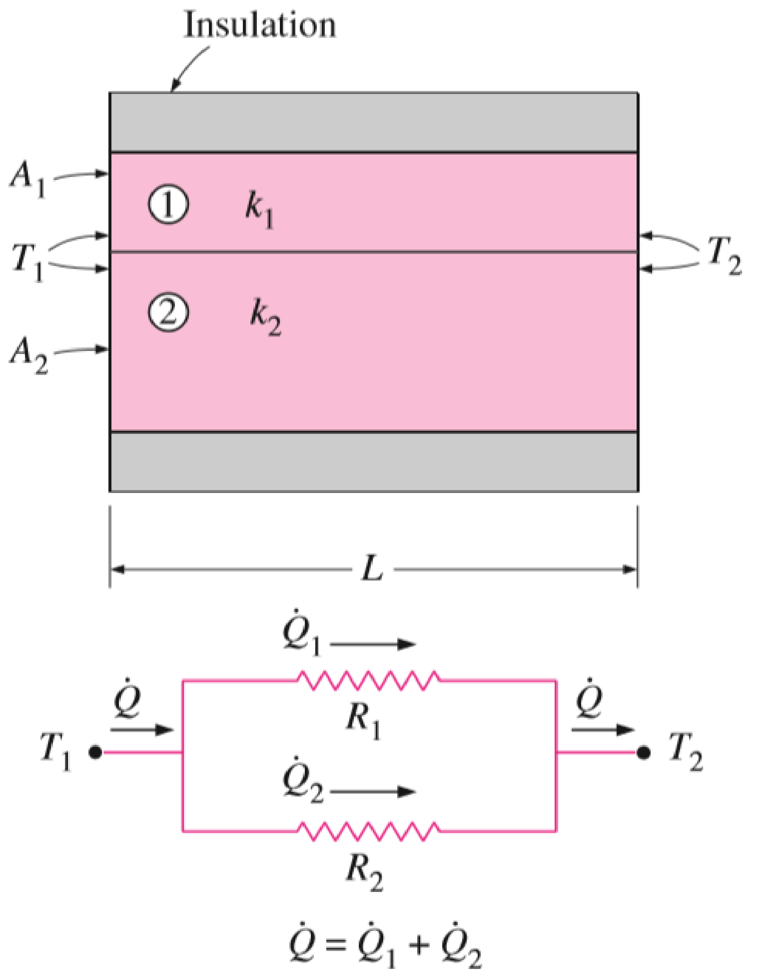
Noting that the total heat transfer is the sum of the heat transfers through each layer

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

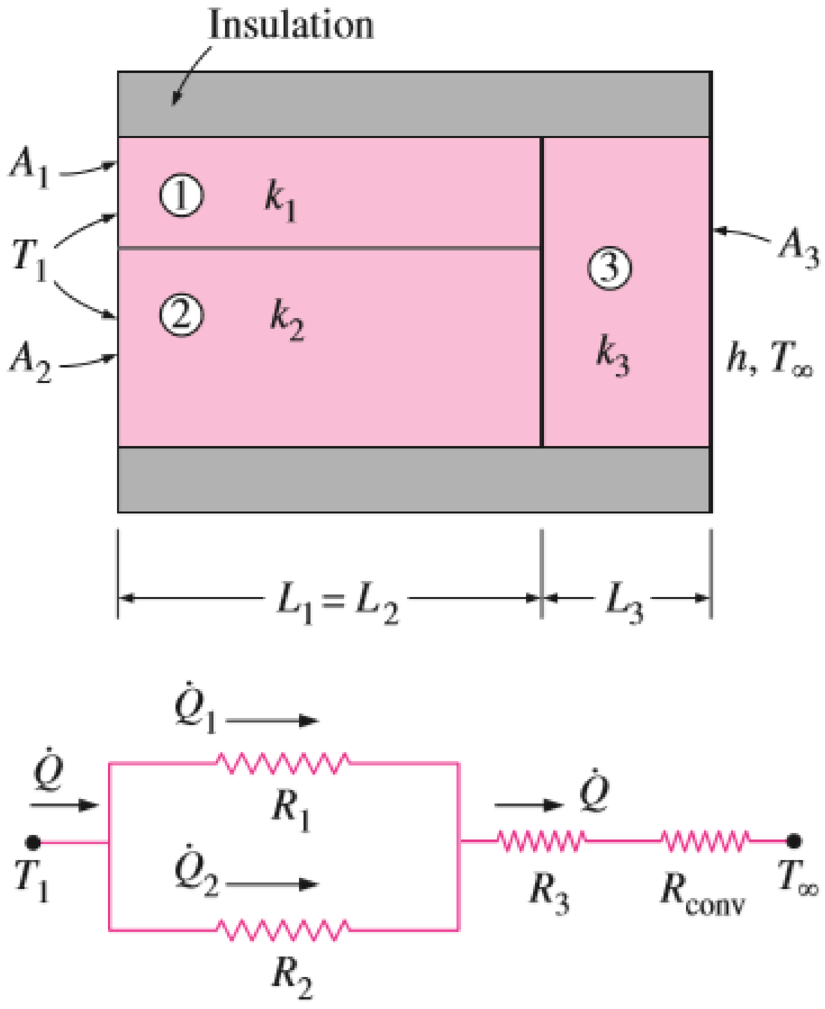
$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

since the resistances are in **parallel.**



Now consider the combined **series-parallel** arrangement shown in Fig.

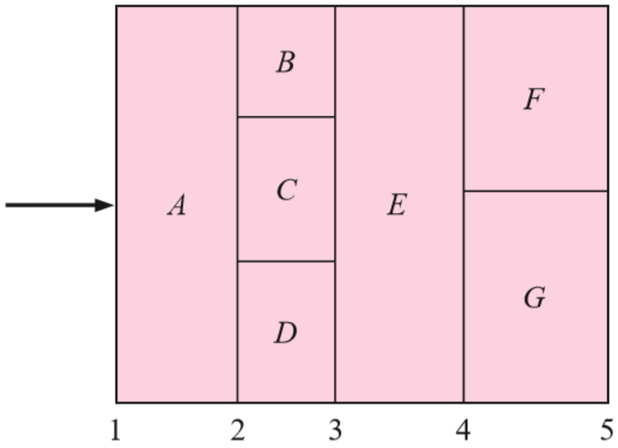


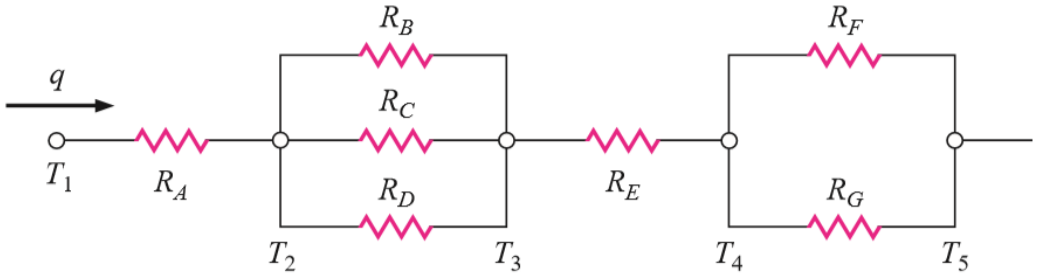
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

$$R_1 = \frac{L_1}{k_1 A_1}, \qquad R_2 = \frac{L_2}{k_2 A_2}, \qquad R_3 = \frac{L_3}{k_3 A_3}, \qquad R_{conv} = \frac{1}{hA_3}$$

Example:





Example:The composite wall of an oven consists of three materials, two of them are of known thermal conductivity, kA = 20 W/m.K and kC = 50 W/m.K and known thickness LA = 0.3 m and LC = 0.15 m. The third material B, which is sandwiched between material A and C is of known thickness, LB = 0.15 m, but of unknown thermal conductivity kB. Under steady state operating conditions, the measurement reveals an outer surface temperature of material C is 20°C and inner surface of A is 600°C and oven air temperature is 800°C. The inside convection coefficient is 25 W/m2.K. What is the value of kB ?

Solution

Given : A composite wall of an oven with

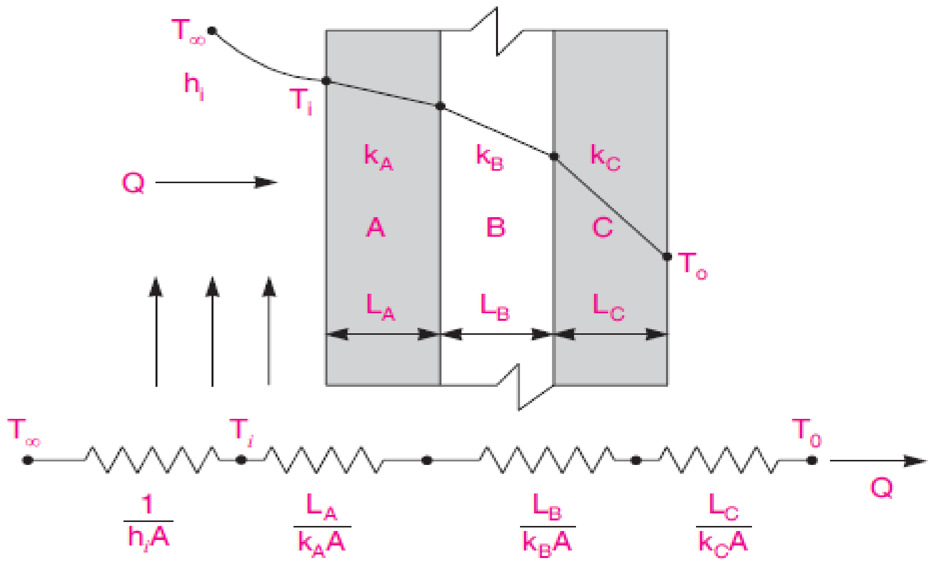
$k_A = 20$ W/m.K, $k_C = 50$ W/m.K

$L_A = 0.3$ m, $L_C = 0.15$ m

$L_B = 0.15$ m, $T_i = 600°C$

$T_o = 20°C$, $T_\infty = 800°C$

$h_i = 25$ W/m^2.K



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To find : The thermal conductivity k_B.

Assumptions :

(*i*) Steady state heat conduction in axial direction only.

(*ii*) Constant properties.

Analysis : The heat transfer rate per unit area in the slab can be calculated by considering convection at inner side.

$$\frac{\dot{Q}}{A} = h_i (T_\infty - T_i) = 25 \times (800 - 600)$$

$$= 5000 \text{ W/m}^2$$

Further this heat is conducted through composite wall, therefore ;

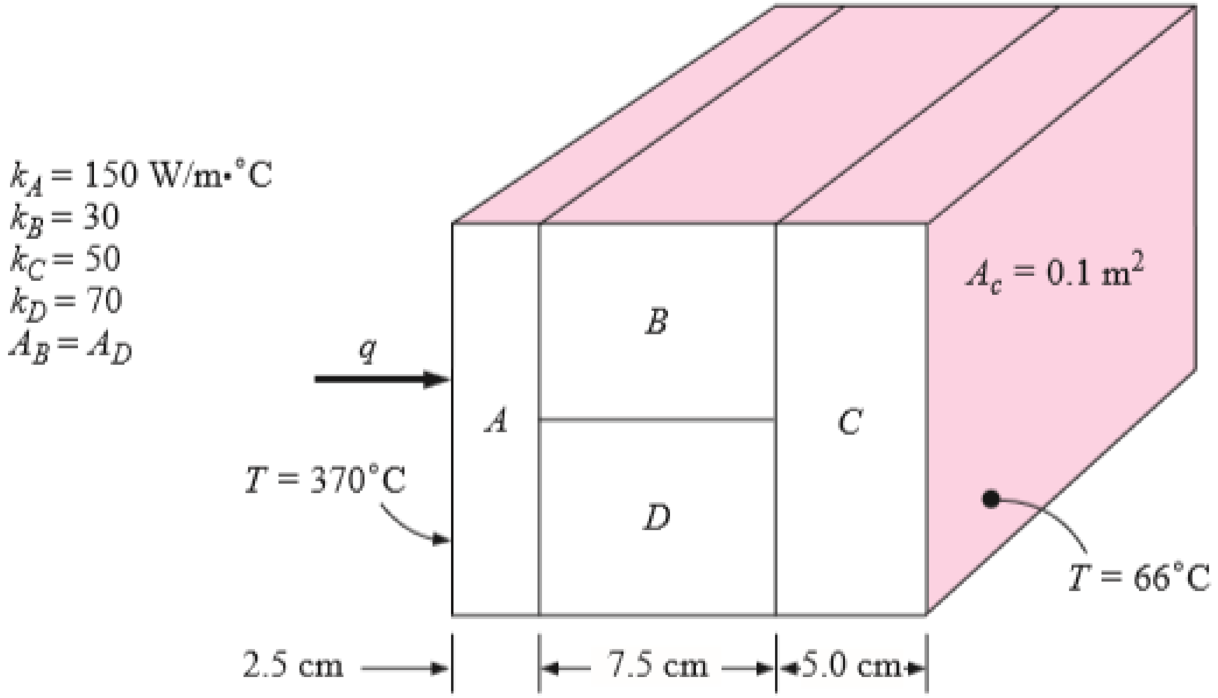
$$\frac{\dot{Q}}{A} = \frac{T_i - T_o}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}}$$

or $$5000 = \frac{600 - 20}{\dfrac{0.3}{20} + \dfrac{0.15}{k_B} + \dfrac{0.15}{50}}$$

or $$0.018 + \frac{0.15}{k_B} = 0.116$$

or $$k_B = \mathbf{1.53 \text{ W/m.K.}}$$ **Ans.**

H.W: Find the heat transfer per unit area through the composite wall in Figure . Assume one-dimensional heat flow.



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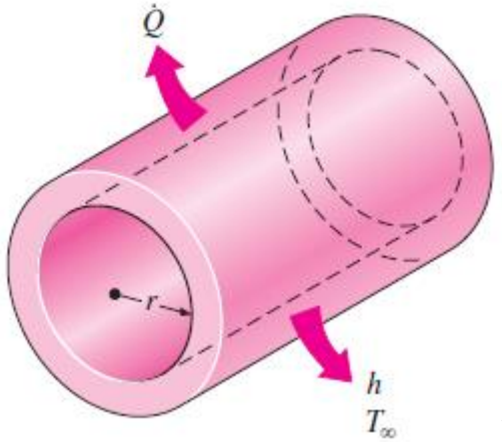
3.2 Heat conduction in cylinders and spheres

- Consider steady heat conduction through a hot water pipe. Heat is continuously lost to the outdoors through the wall of the pipe,
- The wall of the pipe, whose thickness is rather small, separates two fluids at different temperatures, and thus the temperature gradient in the radial direction will be relatively large.
- Further, if the fluid temperatures inside and outside the pipe remain constant, then heat transfer through the pipe is steady.



- Thus heat transfer through the pipe can be modeled as **steady** and one-dimensional.
- The temperature of the pipe in this case will depend on one direction only (the radial r-direction) and can be expressed as T = T(r).
- The temperature is independent of the azimuthal angle or the axial distance. This situation is approximated in practice in **long cylindrical** pipes and spherical containers.
- In steady operation, there is no change in the temperature of the pipe with time at any point.
- the rate of heat transfer into the pipe must be equal to the rate of heat transfer out of it. In other words, heat transfer through the pipe must be constant

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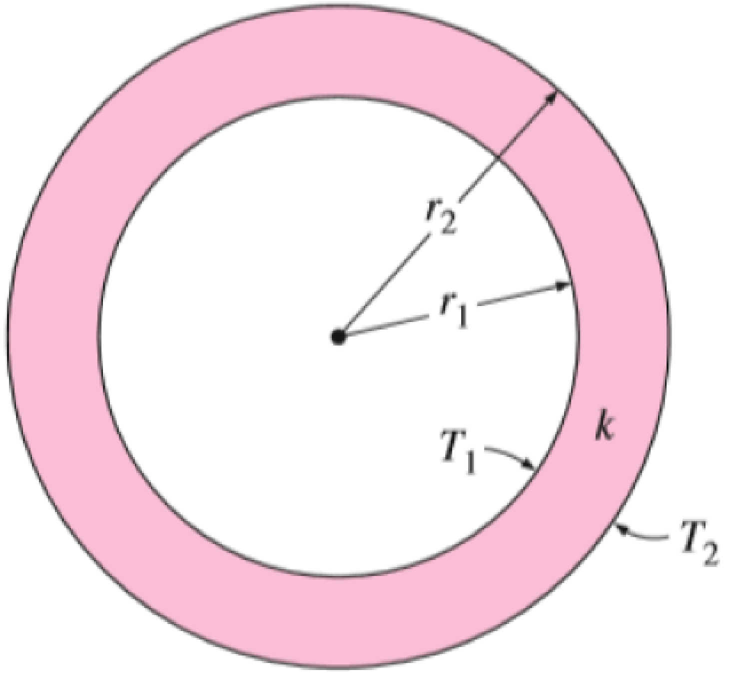
- A long cylindrical layer (such as a circular pipe) of inner radius r1, outer radius r2, l
- The two surfaces of the cylindrical layer are maintained at constant temperatures T1 and T2.
- Heat transfer in one direction (r-direction)
- Steady-state conditions.
- Constant thermal conductivity.
- No heat generated through this cylinder.



$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

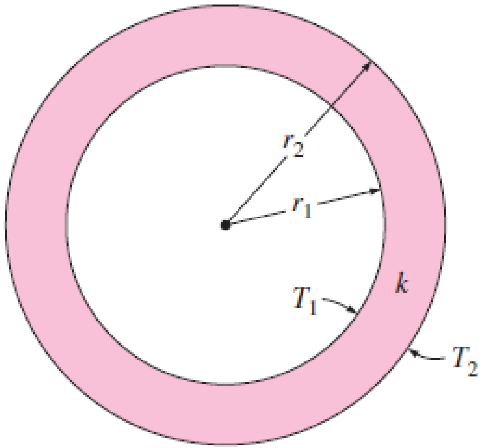
$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\, dr = -\int_{T=T_1}^{T_2} k\, dT$$

Substituting $A = 2\pi\, r\, L$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk \frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)}$$

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)}$$

❑ Conduction resistance of the cylinder layer.



$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})}$$

is the thermal resistance of the cylindrical layer against heat conduction, or simply the conduction resistance of the cylinder layer.

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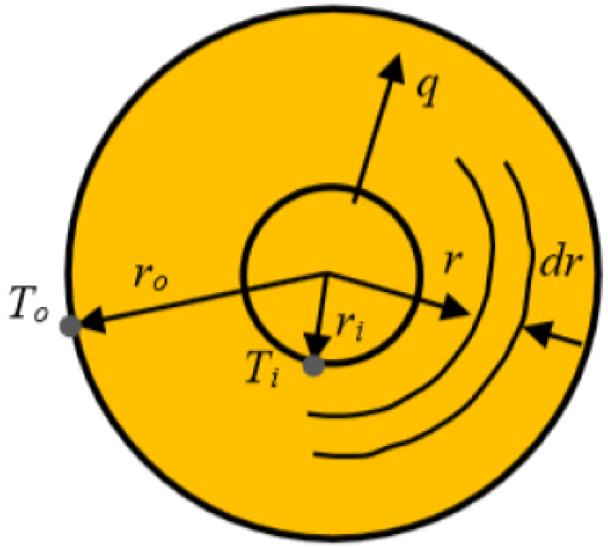
Conduction through a sphere body

- Heat transfer in one direction (r-direction)
- Steady-state conditions.
- Constant thermal conductivity.
- No heat generated through this sphere.



$$q = -kA_r \frac{dT}{dr}$$

$$A_r = 4\pi r^2$$

$$q = -4\pi k r^2 \frac{dT}{dr}$$

$$\frac{q}{4\pi k} \frac{dr}{r^2} = -dT$$

Note: Here q = Q

$$\frac{q}{4\pi k}\int_{r_i}^{r_o} r^{-2}\,dr = -\int_{T_i}^{T_o} dT$$

$$\frac{q}{4\pi k}\left[\frac{r^{-1}}{-1}\right]_{r_i}^{r_o} = -[T]_{T_i}^{T_o}$$

$$\frac{q}{4\pi k}\int_{r_i}^{r_o} r^{-2}\,dr = -\int_{T_i}^{T_o} dT$$

$$\frac{q}{4\pi k}\left[\frac{r^{-1}}{-1}\right]_{r_i}^{r_o} = -[T]_{T_i}^{T_o}$$

$$-\frac{q}{4\pi k}\left[\frac{1}{r}\right]_{r_i}^{r_o} = -[T_o - T_i]$$

$$-\frac{q}{4\pi k}\left[\frac{1}{r_o} - \frac{1}{r_i}\right] = +[T_i - T_o]$$

$$+\frac{q}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r_o}\right] = +[T_i - T_o]$$

$$q = \frac{4\pi k(T_i - T_o)}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}$$

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❑ Conduction resistance of the spherical layer.

Note: Here q = Q

$$q = \frac{(T_i - T_o)}{\frac{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}{4\pi k}}$$

By comparing the above equation with $q = \frac{\Delta T}{R}$

Hence, the thermal resistance, $R = \frac{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}{4\pi k}$

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

where

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

- Now consider steady one-dimensional heat flow through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T1$ and $T2$ with heat transfer coefficients $h1$ and $h2$, respectively, as shown in Fig.The thermal resistance network in this case consists of one conduction and two convection resistances in series, just like the one for the plane wall, and the rate of heat transfer under steady conditions can be expressed as

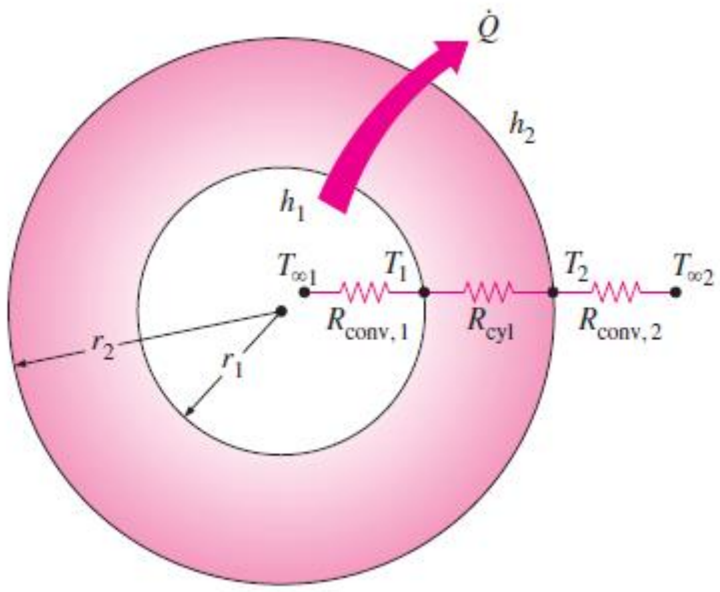
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

where

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$

for a *cylindrical* layer, and

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{sph}} + R_{\text{conv, 2}} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

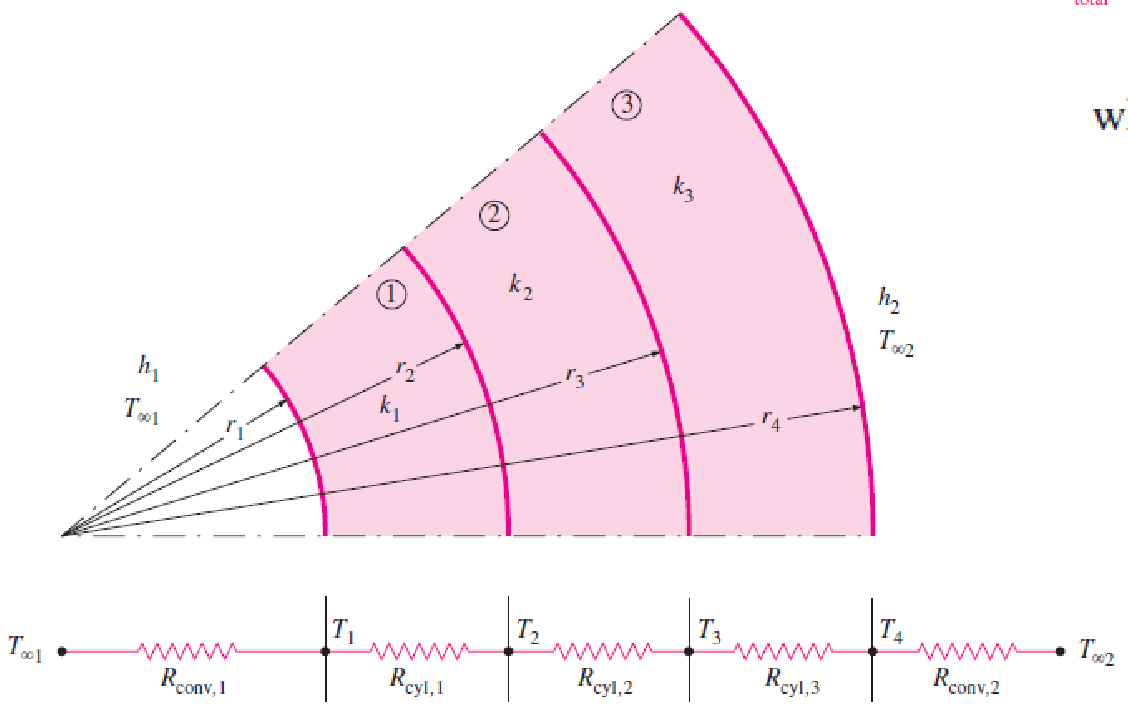


$R_{\text{total}} = R_{\text{conv},1} + R_{\text{cyl}} + R_{\text{conv},2}$

Multilayered Cylinders and Spheres

- Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane walls discussed earlier by simply adding an *additional resistance* in series for each *additional layer.* For example, the steady heat transfer rate through the three-layered composite cylinder of length L shown in Fig. with convection on both sides can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$



where R_{total} is the *total thermal resistance,* expressed as

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

where $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$.

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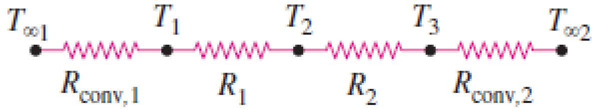
$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{cyl, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

We could also calculate T_2 from

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv, 2}}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$$

$$= \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$$

$$= \frac{T_1 - T_3}{R_1 + R_2}$$

$$= \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}}$$

EXAMPLE 3–7 Heat Transfer to a Spherical Container

A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at $T_{\infty 1}$ = 0°C. The tank is located in a room whose temperature is $T_{\infty 2}$ = 22°C. The walls of the room are also at 22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are h_1 = 80 W/m² · °C and h_2 = 10 W/m² · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

$$A_1 = \pi D_1^2 = \pi(3 \text{ m})^2 = 28.3 \text{ m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04 \text{ m})^2 = 29.0 \text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

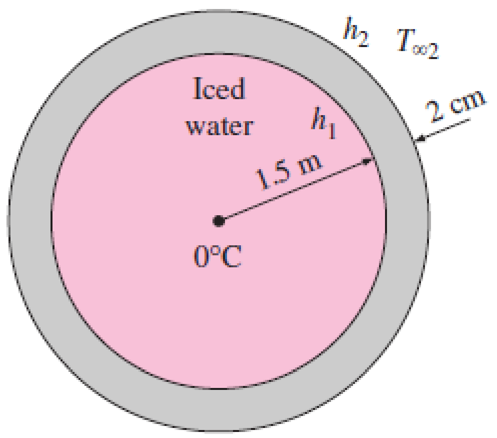
We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking T_2 = 5°C = 278 K, the radiation heat transfer coefficient is determined to be

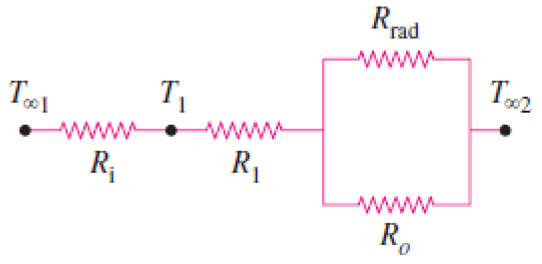
$$h_{rad} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(295 \text{ K})^2 + (278 \text{ K})^2][(295 + 278) \text{ K}]$$
$$= 5.34 \text{ W/m}^2 \cdot \text{K} = 5.34 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

Then the individual thermal resistances become

$$R_i = R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{(80 \text{ W/m}^2 \cdot {}^\circ\text{C})(28.3 \text{ m}^2)} = 0.000442^\circ\text{C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50) \text{ m}}{4\pi (15 \text{ W/m} \cdot {}^\circ\text{C})(1.52 \text{ m})(1.50 \text{ m})}$$
$$= 0.000047^\circ\text{C/W}$$





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$$R_o = R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{(10\ W/m^2 \cdot {}^\circ C)(29.0\ m^2)} = 0.00345^\circ C/W$$

$$R_{rad} = \frac{1}{h_{rad} A_2} = \frac{1}{(5.34\ W/m^2 \cdot {}^\circ C)(29.0\ m^2)} = 0.00646^\circ C/W$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{equiv}} = \frac{1}{R_o} + \frac{1}{R_{rad}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7\ W/^\circ C$$

which gives

$$R_{equiv} = 0.00225^\circ C/W$$

Now all the resistances are in series, and the total resistance is determined to be

$$R_{total} = R_i + R_1 + R_{equiv} = 0.000442 + 0.000047 + 0.00225 = 0.00274^\circ C/W$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(22 - 0)^\circ C}{0.00274^\circ C/W} = 8029\ W \qquad (\text{or } \dot{Q} = 8.027\ kJ/s)$$

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equiv}} \longrightarrow T_2 = T_{\infty 2} - \dot{Q}R_{equiv}$$
$$= 22^\circ C - (8029\ W)(0.00225^\circ C/W) = 4^\circ C$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.

(*b*) The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\,\Delta t = (8.029\ \text{kJ/s})(24 \times 3600\ \text{s}) = 673{,}700\ \text{kJ}$$

Noting that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the amount of ice that will melt during a 24-h period is

$$m_{\text{ice}} = \frac{Q}{h_{if}} = \frac{673{,}700\ \text{kJ}}{333.7\ \text{kJ/kg}} = 2079\ \text{kg}$$

Therefore, about 2 metric tons of ice will melt in the tank every day.

EXAMPLE 3–8 Heat Loss through an Insulated Steam Pipe

Steam at $T_{\infty 1} = 320°C$ flows in a cast iron pipe ($k = 80$ W/m · °C) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with $k = 0.05$ W/m · °C. Heat is lost to the surroundings at $T_{\infty 2} = 5°C$ by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m² · °C. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m² · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

$$A_1 = 2\pi r_1 L = 2\pi(0.025\ \text{m})(1\ \text{m}) = 0.157\ \text{m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575\ \text{m})(1\ \text{m}) = 0.361\ \text{m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.157\ \text{m}^2)} = 0.106^\circ\text{C/W}$$
$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80\ \text{W/m}\cdot{}^\circ\text{C})(1\ \text{m})} = 0.0002^\circ\text{C/W}$$
$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05\ \text{W/m}\cdot{}^\circ\text{C})(1\ \text{m})} = 2.35^\circ\text{C/W}$$
$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A_3} = \frac{1}{(18\ \text{W/m}^2\cdot{}^\circ\text{C})(0.361\ \text{m}^2)} = 0.154^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = 121\ \text{W} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121\ \text{W})(0.0002^\circ\text{C/W}) = 0.02^\circ\text{C}$$
$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121\ \text{W})(2.35^\circ\text{C/W}) = 284^\circ\text{C}$$

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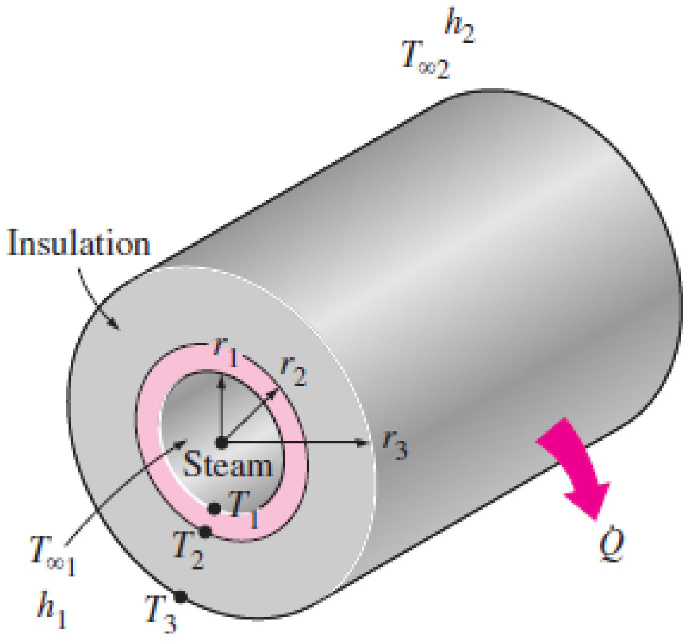

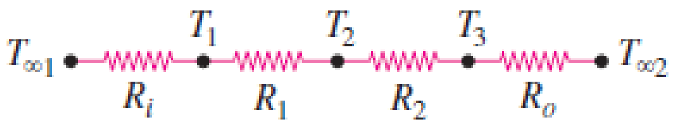

The Overall Heat-Transfer Coefficient, U

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- Consider the double pipe heat exchanger shown in Figure, where the hot fluid flows inside the inner pipe while the cold fluid flows outside of the pipe. **Note: Here $q = Q$**

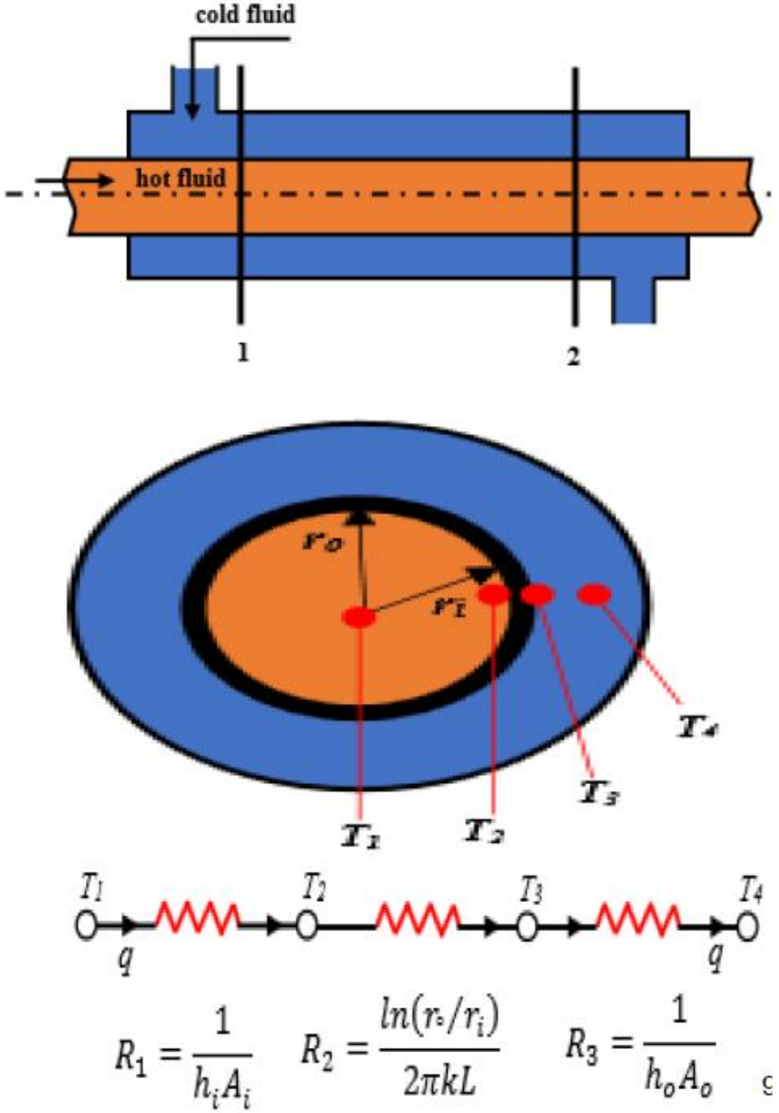
$$q = h_i A_i (T_1 - T_2) = \frac{(T_2 - T_3)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}} = h_o A_o (T_3 - T_4)$$

$$q = h_i A_i (T_1 - T_2)$$

$$q = \frac{(T_2 - T_3)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L}}$$

$$q = h_o A_o (T_3 - T_4)$$

$$(T_1 - T_2) = \frac{1}{h_i A_i} q$$



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$$(T_2 - T_3) = \frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi kL}\, q$$

$$(T_3 - T_4) = \frac{1}{h_o A_o} q$$

$$T_1 - T_2 + T_2 - T_3 + T_3 - T_4 = \frac{1}{h_i A_i}\, q + \frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi kL}\, q + \frac{1}{h_o A_o} q$$

$$T_1 - T_4 = q\left[\frac{1}{h_i A_i} + \frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{h_o A_o}\right]$$

$$T_1 - T_4 = q\, R_{overall}$$

$$U = \frac{1}{R_{overall}\, A} \Rightarrow R_{overall} = \frac{1}{U\, A}$$

$$T_1 - T_4 = q\, \frac{1}{UA} \Rightarrow q = UA\,(T_1 - T_4)$$

U: Overall heat transfer coefficient, W/m^2. °C, W/m^2. K

$$q = U_i A_i\,(T_1 - T_4)$$

$$q = U_o A_o\,(T_1 - T_4)$$

$$R_{overall} = \frac{1}{U_i\, A_i} \Rightarrow \frac{1}{U_i} = A_i\, R_{overall} = A_i \left[\frac{1}{h_i A_i} + \frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{h_o A_o}\right]$$

$$\frac{1}{U_i} = \left[\frac{A_i}{h_i A_i} + \frac{A_i\, ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{A_i}{h_o A_o}\right]$$

$$A_i = \pi D_i L$$

$$A_o = \pi D_o L$$

$$\frac{1}{U_i} = \left[\frac{1}{h_i} + \frac{\pi D_i L\, ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{\pi D_i L}{h_o \pi D_o L}\right]$$

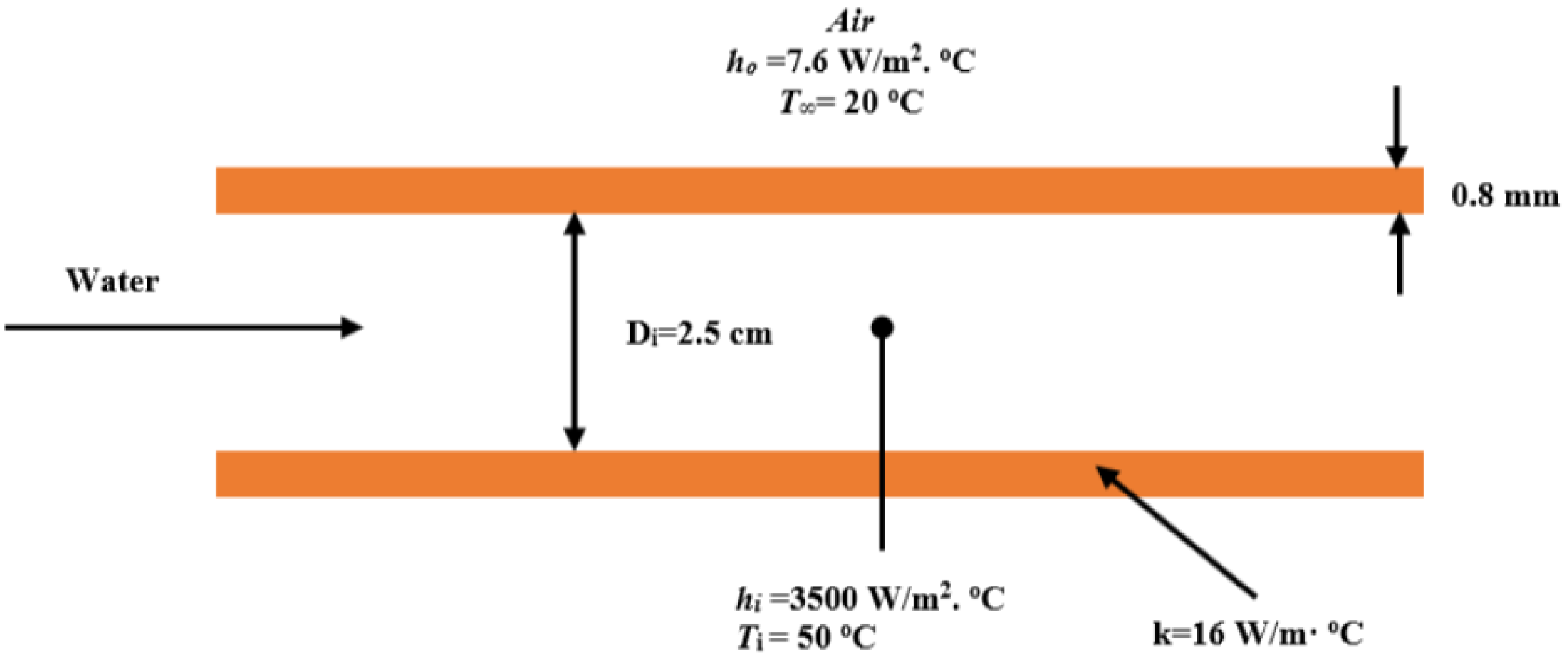
$$\frac{1}{U_i} = \left[\frac{1}{h_i} + \frac{D_i\, ln\left(\frac{r_o}{r_i}\right)}{2k} + \frac{D_i}{h_o D_o}\right]$$

$$\frac{1}{U_i} = \left[\frac{1}{h_i} + \frac{D_i \, ln\left(\frac{D_o/2}{D_i/2}\right)}{2k} + \frac{D_i}{h_o D_o}\right]$$

$$\frac{1}{U_i} = \left[\frac{1}{h_i} + \frac{D_i \, ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{D_i}{h_o D_o}\right]$$

$$\frac{1}{U_o} = \left[\frac{D_o}{h_i D_i} + \frac{D_o \, ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{h_o}\right]$$

- Example 1: Water flows at 50∘C inside a 2.5 cm-inside-diameter tube such that hi =3500 W/m2. C. The tube has a wall thickness of 0.8 mm with thermal conductivity of 16 W/m C. The outside of the tube loses heat by free convection with ho =7.6 W/m2 C. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20 C.



$$R_1 = \frac{1}{h_i A_i} \qquad R_2 = \frac{ln(D_\circ/D_i)}{2\pi kL} \qquad R_3 = \frac{1}{h_o A_o}$$

$$R_1 = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ °C/W$$

$$D_\circ = D_i + 2(\Delta x) = 0.025 + 2(0.0008) = 0.0266\ m$$

$$R_2 = \frac{ln(D_\circ/D_i)}{2\pi kL} = \frac{ln(0.0266/0.025)}{2\pi(16)(1)} = 0.00062\ °C/W$$

$$R_3 = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1)} = 1.575\ °C/W$$

$$q = U_i A_i\ \Delta T_{overall} = U_o A_o\ \Delta T_{overall}$$

$$\frac{1}{U_o} = \left[\frac{D_o}{h_i D_i} + \frac{D_o\ ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{1}{h_o}\right]$$

$$\frac{1}{U_o} = \left[\frac{0.0266}{3500(0.025)} + \frac{0.0266\,ln\left(\frac{0.0266}{0.025}\right)}{2(16)} + \frac{1}{7.6}\right] \Rightarrow U_o = 7.6\,W/m^2.°C$$

$$\frac{1}{U_i} = \left[\frac{1}{h_i} + \frac{D_i\,ln\left(\frac{D_o}{D_i}\right)}{2k} + \frac{D_i}{h_o D_o}\right]$$

$$\frac{1}{U_i} = \left[\frac{1}{3500} + \frac{0.025\,ln\left(\frac{0.0266}{0.025}\right)}{2(16)} + \frac{0.025}{7.6(0.0266)}\right] \Rightarrow U_i = 8.1\,W/m^2.°C$$

$$q = U_o A_o\,\Delta T_{overall} = U_o(\pi D_{\circ} L)\,\Delta T_{overall} = 7.6(\pi)(0.0266)(1)(50-20) = 19\,W$$

$$q = U_i A_i\,\Delta T_{overall} = U_i(\pi D_i L)\,\Delta T_{overall} = 8.1(\pi)(0.0250)(1)(50-20) = 19\,W$$

Examples

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1. *The walls of a house, 4 m high, 5 m wide and 0.3 m thick are made from brick with thermal conductivity of 0.9 W/m.K. The temperature of air inside the house is 20°C and outside air is at –10°C. There is a heat transfer coefficient of 10 W/m2.K on the inside wall and 30 W/m2.K on the outside wall. Calculate the inside and outside wall temperatures, heat flux and total heat transfer rate through the wall.*

2. *A thermopane window consists of two 5 mm thick glass (k = 0.78 W/m.K) sheets separated by 10 mm stagnant air gap (k = 0.025 W/m.K). The convection heat transfer coefficient for inner and outside air are 10 W/m2.K and 50 W/m2.K, respectively. (a) Determine the rate of heat loss per m2 of the glass surface for a temperature difference of 60°C between the inside and outside air.*
(b) Compare the result with the heat loss, if the window had only a single sheet of glass of thickness 5 mm instead of thermopane. (c) Compare the result with the heat flow, if window has no stagnant air (i.e., a sheet of glass, 10 mm thick).

3. *A wall is constructed of several layers. The first layer consists of brick (k = 0.66 W/m.K), 25 cm thick, the second layer 2.5 cm thick mortar (k = 0.7 W/m.K), the third layer 10 cm thick limestone (k = 0.66 W/m.K) and outer layer of 1.25 cm thick plaster (k = 0.7 W/m.K). The heat transfer coefficients on interior and exterior of the wall fluid layers are 5.8 W/m2.K and 11.6 W/m2.K, respectively. Find :*
(i) Overall heat transfer coefficient,
(ii) Overall thermal resistance per m2,
(iii) Rate of heat transfer per m2, if the interior of the room is at 26°C while outer air is at – 7°C,
(iv) Temperature at the junction between mortar and limestone.

4. The composite wall of an oven consists of three materials, two of them are of known thermal conductivity, kA = 20 W/m.K and kC = 50 W/m.K and known thickness LA = 0.3 m and LC = 0.15 m. The third material B, which is sandwiched between material A and C is of known thickness, LB = 0.15 m, but of unknown thermal conductivity kB. Under steady state operating conditions, the measurement reveals an outer surface temperature of material C is 20°C and inner surface of A is 600°C and oven air temperature is 800°C. The inside convection coefficient is 25 W/m2.K. What is the value of kB ?

. The wall of a cold storage consists of three layers: an outer layer of ordinary bricks, 25 cm thick, a middle layer of cork, 10 cm thick and an inner layer of cement, 6 cm thick. The thermal conductivities of the materials are 0.7, 0.043 and 0.72 W/m.K, respectively. The temperature of the outer surface of the wall is 30°C and that of inner is – 15°C. Calculate :

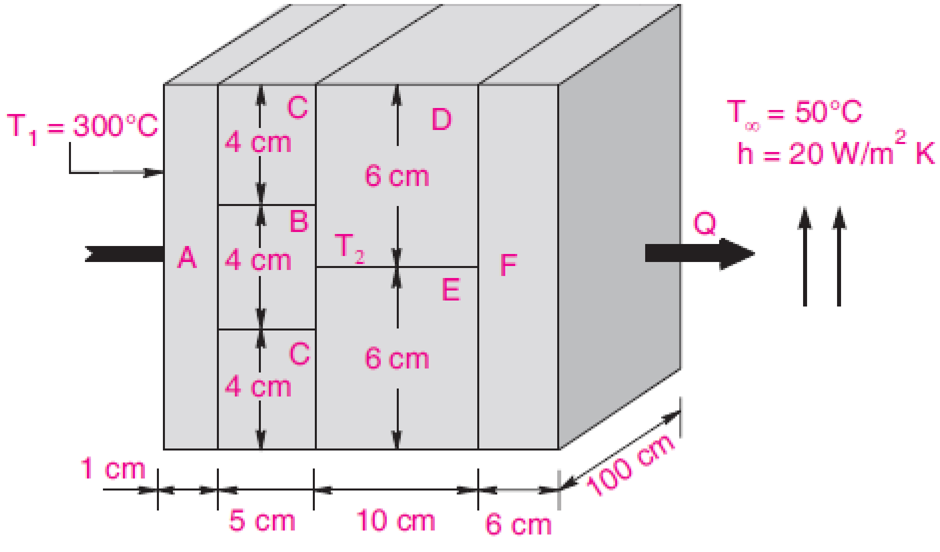
(a) Steady state rate of heat gain per unit area, (b) Temperature at the interfaces of composite wall,

(c) The percentage of total heat resistance offered by individual layers, and

(d) What additional thickness of cork should be provided to reduce the heat gain 30% less than the present value ?

6. Consider a 5 m high and 8 m long and 0.22 m thick wall whose representation is shown in Figure. The thermal conductivity of various materials used are kA = kF = 2, kB = 8, kC = 20, kD = 15, and kE = 35 W/m.K. The left surface of the wall is maintained at uniform temperatures of 300°C. The right surface is exposed to convection environment at 50°C with h = 20 W/m2.K. Determine (a) one dimensional heat transfer rate through the wall, (b) temperature at the point where section B, D and E meet, and (c) temperature drop across the section F.

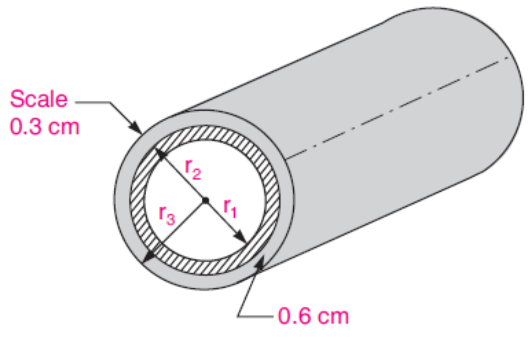
7. A long hollow cylinder (k = 50 W/m.K) has an inner radius of 10 cm, and outer radius of 20 cm. The inner surface is heated uniformly at constant rate of 1.16 × 105 W/m2 and outer surface is maintained at 30°C. Calculate the temperature of inner surface.



8. A hollow cylinder with inner radius 30 mm and outer radius 50 mm is heated at the inner surface at a rate of 105 W/m2 and dissipated heat by convection from outer surface into a fluid at 80°C with heat transfer coefficient of 400 W/m2.K. There is no energy generation and thermal conductivity of the material is constant at 15 W/m.K. Calculate the temperatures of inside and outside surfaces of the cylinder

9. A steam pipe, 10 cm in outer diameter is covered with two layers of insulation material each 2.5 cm thick, one having thermal conductivity thrice the other. Show that the effective thermal conductivity of two layers is approximately 15% less when better insulation material is placed as inside layer, than when it is on the outside.

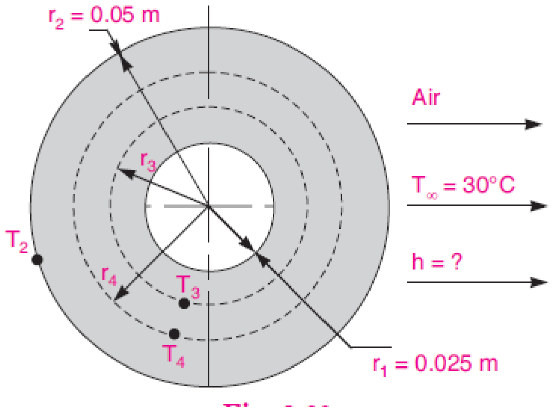
10. Air at 90°C flows in a copper tube (k = 384 W/m.K) of 4 cm inner diameter and with 0.6 cm thick walls which are heated from the outside by water at 125°C. A scale of 0.3 cm thick is deposited on outer surface of the tube whose thermal conductivity is 1.75 W/m.K. The air and water side heat transfer coefficients are 221 and 3605 W/m2.K, respectively. Find (a) overall heat transfer coefficient on the outside area basis (b) water to air heat transfer (c) temperature drop across the scale deposit.



(a) Schematic

11. *A spherical thin walled metallic container is used to store liquid nitrogen at 77 K. The container has a diameter of 0.5 m and is covered with an evacuated reflective insulation system composed of silica powder (k = 0.0017 W/m.K). The insulation is 25 mm thick and its outer surface is exposed to ambient air at 300 K. The convective coefficient is known to be 20 $W/m^2.K$. The latent heat of vaporization and density of liquid nitrogen are 2×10^5 J/kg and 804 kg/m^3, respectively. What is the rate of heat transfer to the liquid nitrogen ?*

12. *A hollow spherical form is used to determine thermal conductivity of an insulating material. The inner diameter is 50 mm and outer diameter is 100 mm. A 40 W heater is placed inside and under steady state conditions, the temperature at 32 and 40 mm radii were found to be 100°C and 70°C, respectively. Determine the thermal conductivity of the material. Also calculate the outside temperature of sphere. If surrounding air is at 30°C, calculate convection heat transfer coefficient over the surface.*



H.W: A hollow sphere is constructed of aluminum with an inner diameter of 4 cm and an outer diameter of 8 cm. The inside temperature is 100◦C and the outer temperature is 50◦C. Calculate the heat transfer.


Thank you