Engineering & Cadastral Surveying

Introducing engineering survey, work areas, engineering units

- Engineering survey is that branch of surveying that seeks to provide the best ways to solve the problems facing the surveyor in engineering projects that are implemented in the manner of (ground survey).
- The interest of this type of surveys appears “clearly” in the calculations (the areas and volumes of the vast lands, determining the paths of roads, including horizontal and vertical curves, as well as many construction surveys that are usually attributed to the surveyor).

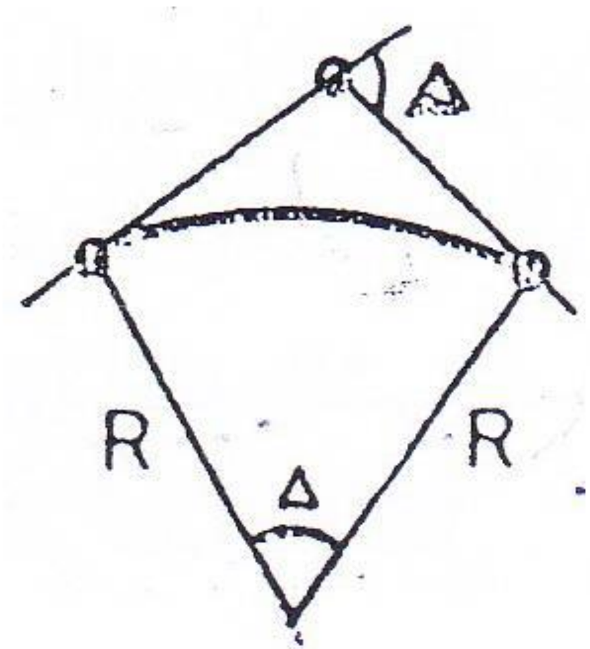
work fields

- 1- Areas
- This part deals with calculating regular and irregular areas, whether they are on the ground or on the map. It is also concerned with the different methods of calculating areas (mathematically) or using devices and tools such as (the planometer), and software may be used in this field.
- 2- Volumes
- The same applies to volumes, some of which are regular and irregular, and we mainly emphasize the volumes resulting from the earthworks of road surveys and their longitudinal sections and cross-sections, in addition to the volumes resulting from contour maps, which are represented in cutting heights and filling in depressions.

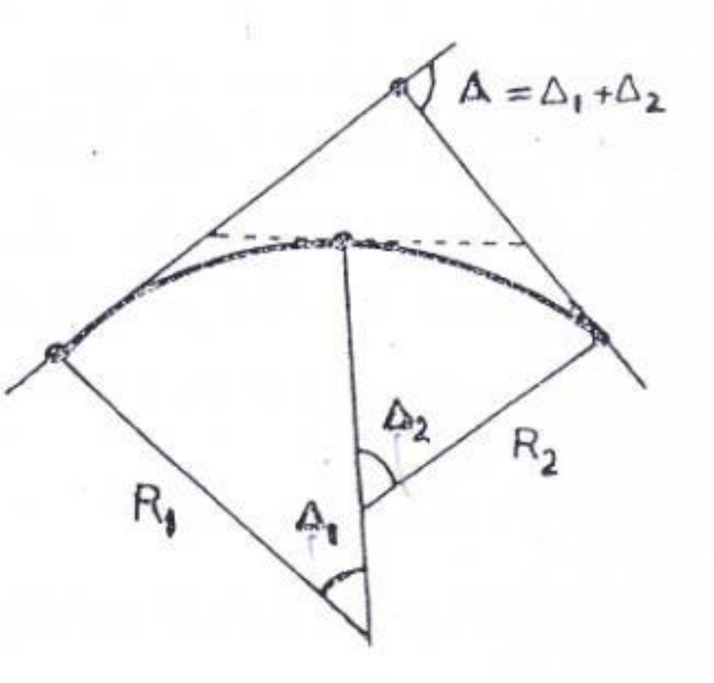
- 3- Road survey:

- Road survey works are numerous and extensive, and may overlap with road and bridge engineering works. In all, they are of two types; The first: the technical works related to the road such as (traffic volume, number of lanes, acceleration and deceleration lanes, shoulders and lateral inclination to overcome the central force when bending, construction costs ... and others) and the second type is specific to road planning and engineering trajectory identification; Such as (determining the coordinates of the track, vertical curves, horizontal curves...), where the surveyor’s work is concentrated in the second type based on the data of the first type (artworks).
- 4- Vertical curves: They are of two types (convex and concave) and are represented in bridges and tunnels, as well as their use in plains and slopes on the one hand, and hills and heights on the other. It will be discussed in detail later. ..
- 5- Horizontal curves: • They are of three types: (simple, compound, and transitional), as shown in the figure.

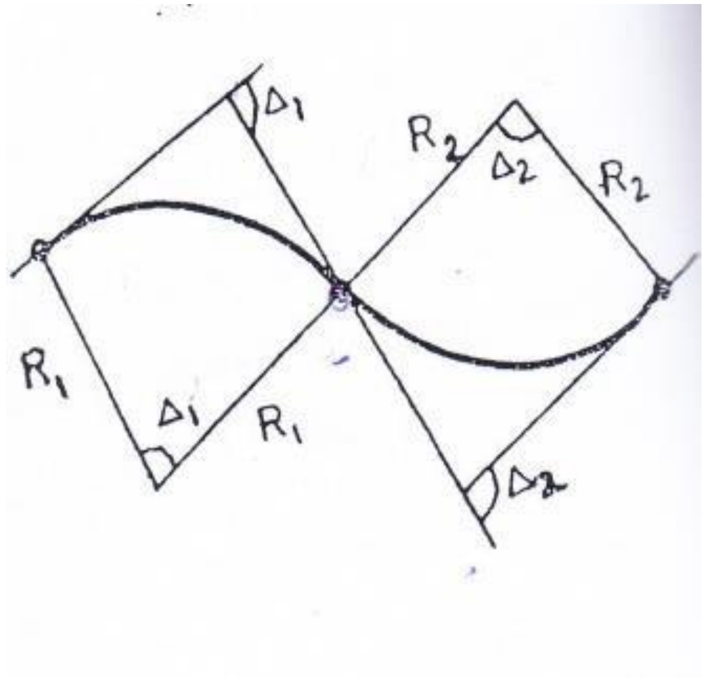
a- Simple.



b- Compound.



c- Reverse.



- Structural survey: It deals with the following
  - locating and aligning the buildings.
  - Verticality of columns (electricity, buildings...)
  - Barrages, water and sewage pipes.
  - locating and aligning bridges.
  - locating and aligning the electrical and telephone lines and determine their integrity.
  - And other construction works.




Courses contain

- first Chapter :-Areas Measurement
- Second Chapter :- Volumes Measurements
- Third Chapter:-Vertical curve
- Fourth Chapter:-Horizontal curve
- Fifth Chapter:-Construction Surveying
- Sixth Chapter:-Angles and Direction
- Seventh Chapter:-Unknown Measurements
- Eighth Chapter:-Forward & Backward Intersection
- Ninth Chapter :-Subdividing of Lands
- Tenth chapter:- Resection
- Eleventh chapter :- Project

Reference:- المصــادر:-

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- 5- Surveying Vol. 1 & Vol. 2) / B.C. Punmi a/Standard Book House, Delhi, India. 1978.
- 6- Engineering Surveying (Vol. I & Vol.2)/ W. Scho field / Newness - Butter Woths/ London / Britain. 1978.
- 7- Surveying for Engineers / J. Uren. & W.F. Price / Macmillan / London/ Britain. 1985

Area units commonly used

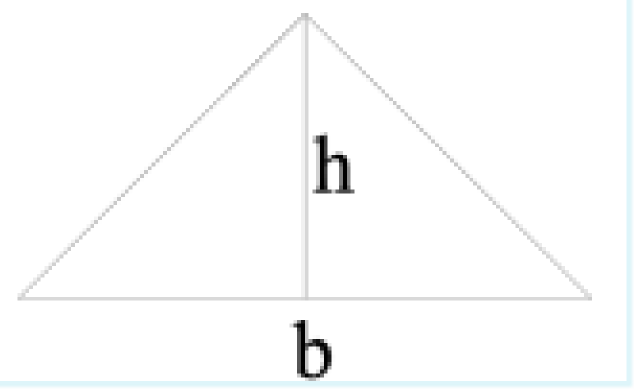
- 1 km2 = 1,000,000 m2
- 1 km2 = 100 Hectare
- 1km2 = 400 Dunam
- 1 km2 = 10,000 Olk (Acre)
- 1 Dunam = 2500 m2 = 25 Olk
- 1 Hectare = 4 Dunam = 100 Olk = 10,000 m2
- 1 Olk = 100 m2

A-Uniform Figures Areas

- Triangles Areas

1) $A = 0.5 * H * B$

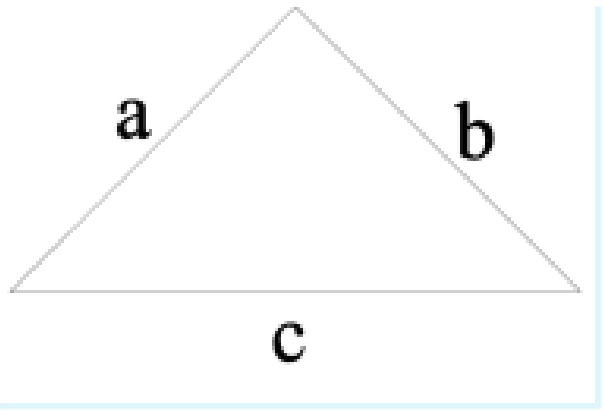
2) *If you know the three sides*



$$A = \sqrt{s (s-a) (s-b) (s-c)}$$

$$S = \frac{a+b+c}{2}$$

S iscircumference of a tringle

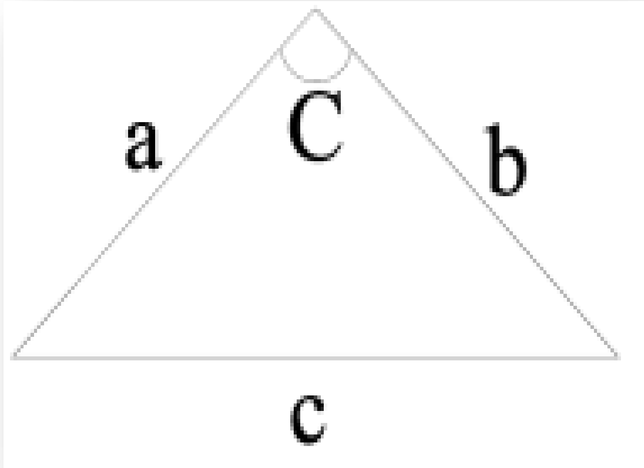


OR

$$A = \frac{b}{2}\sqrt{\left[a^2 - \left(\frac{a^2 + b^2 - c^2}{2 \times b}\right)^2\right]}$$

a , b , c = sides of triangles

4) If two adjacent sides and an included angle are known,



$$A = 0.5 * a * b * \sin <C$$

a , b = sides of triangles
= angle between
tow sides

5) If you know a side and two angles

$$A = \frac{b^2}{2}\left[\frac{\tan A \times \tan C}{\tan A + \tan C}\right]$$

sin rule

$$\frac{a}{\sin < A} = \frac{b}{\sin < B} = \frac{c}{\sin < C}$$

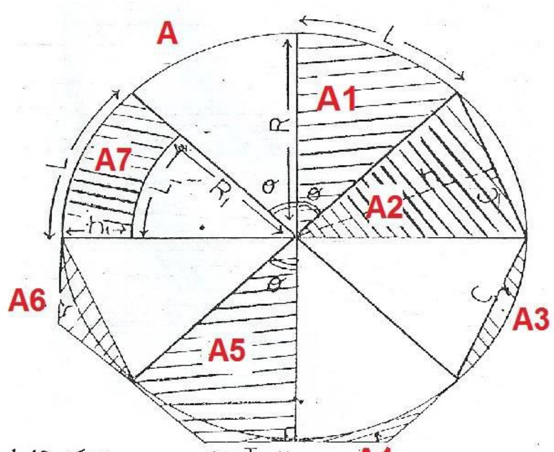
cos rule

$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos < C$$

2) Circle Areas

- $A = \pi \times R^2$

area of a circle sector



الشكل 1.12

$$A_{sector} = \frac{\theta^r}{2} \times R^2 \ where\ \theta^r angle\ in\ radiand$$

Or

$$A_{sector} = \frac{2\pi}{360} \times \frac{\theta^{\circ}}{2} R^2 \ where\ \theta^{\circ} angle\ in\ deggre$$

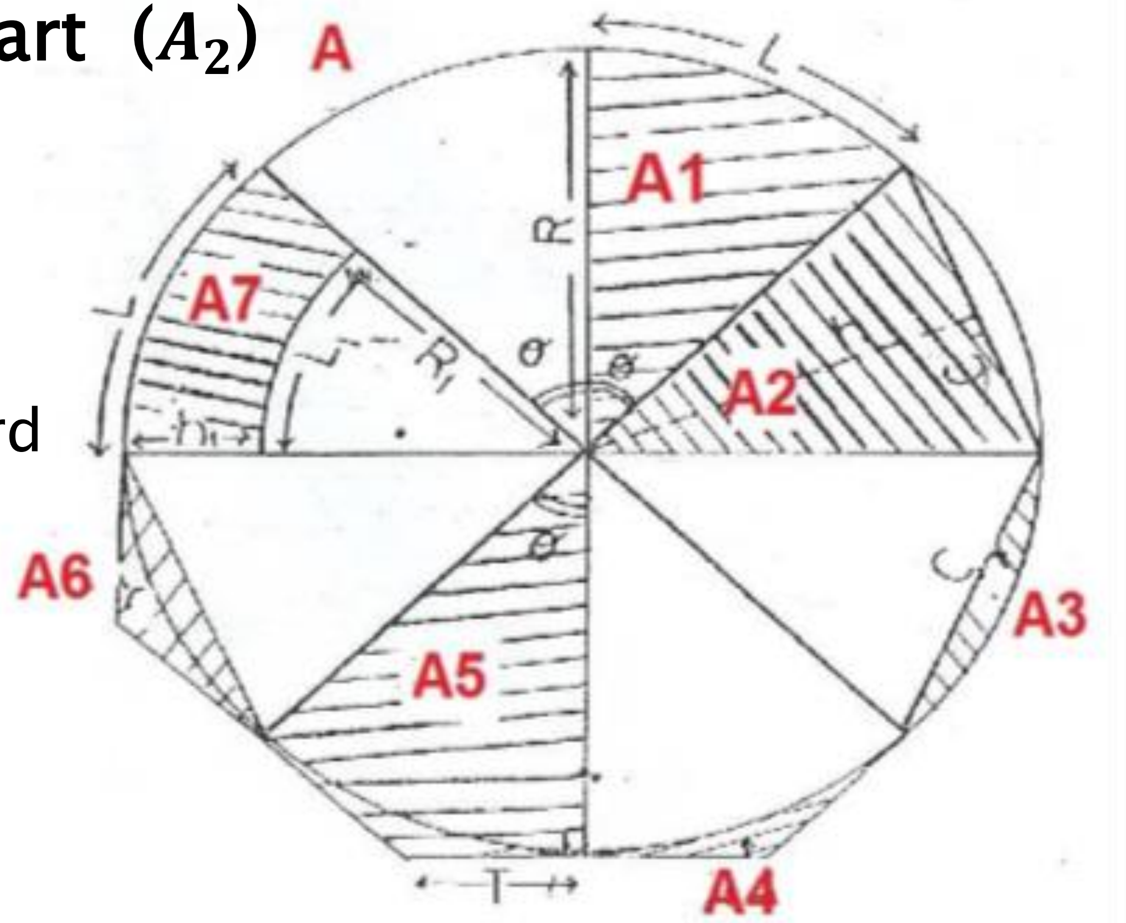
circumference of a circle

$D = 2 \times \pi \times R$

Circle sector length

$L_{sector} = \theta^r \times R^2$ OR $L_{sector} = \frac{2\pi}{360} \times \theta^{\circ} \times R^2$

1- The area of the triangle part (A_2)



$$A_2 = \frac{1}{2} \times h \times C \text{ OR}$$
$$= 0.5 \times R^2 \times \text{SIN}\,\theta$$

where C is the length of the chord

The area of a circular segment or segment (A_3), which is (the part between the hypotenuse and the arc)

$$A_3 = \frac{1}{2} R \, . \, L - \frac{1}{2} h \, . \, C$$
$$= R^2 \left(0{\cdot}008727\,\theta^o - \frac{1}{2} \sin \theta^o \right)$$
$$= \frac{R^2}{2} (\theta^r - \sin \theta^o)$$

- The area of the outer segment (A_4), which is (the part between the arc and the tangents)

$$A_4 = R . T - \frac{1}{2} R . L$$

$$= R^2 \left(\tan \frac{\theta^o}{2} - \frac{\theta^r}{2} \right)$$

Area of sector and outer segment (A_5)

$$A_5 = R^2 \times \tan \frac{\theta^\circ}{2} \text{ or } = \mathbf{R} \times T$$

Area of secant and exterior segment (A_6) where (T) is the length of the tangent

$$A_6 = R . T - \frac{1}{2} h . C \quad = \tan \frac{\theta^o}{2} \left(R . \sin \frac{\theta^o}{2} \right)^2$$

- Area for part of the ring (A_7)

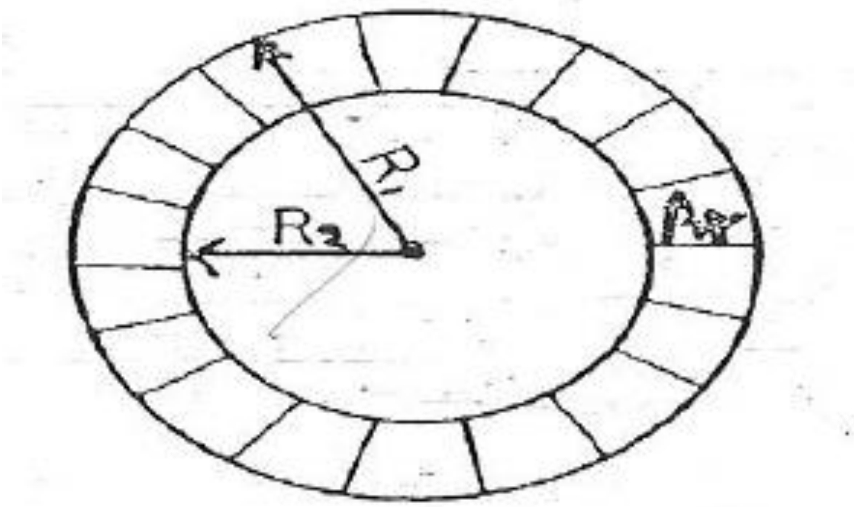
$$A_7 = \frac{1}{2} R \,.\, L - \frac{1}{2} R_1 \,.\, L$$

$$= \frac{1}{2} \theta^r (R^2 - R_1^2)$$

$$= \left(\frac{L + L_1}{4} \right) h_1$$

$$= \frac{\theta^r}{2} (R + \quad) h_1$$

$$= 0.008727\, \theta^\circ (R^2 - R_1^2)$$

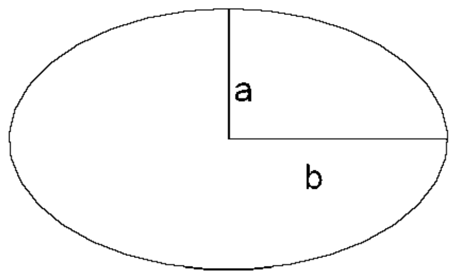


The area of the circular ring (A_r)

$$A_r = \pi \times (R_1)^2 \times (R_2)^2$$

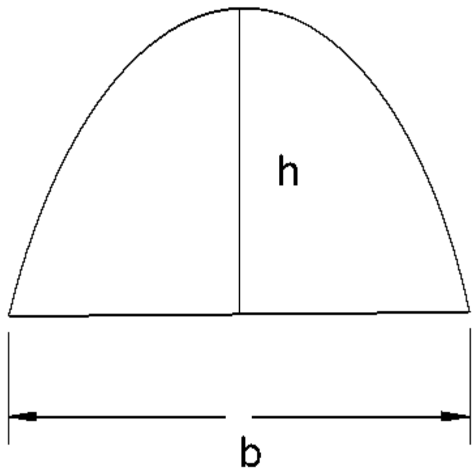
Area of Ellipse

$$A = \pi * a * b$$



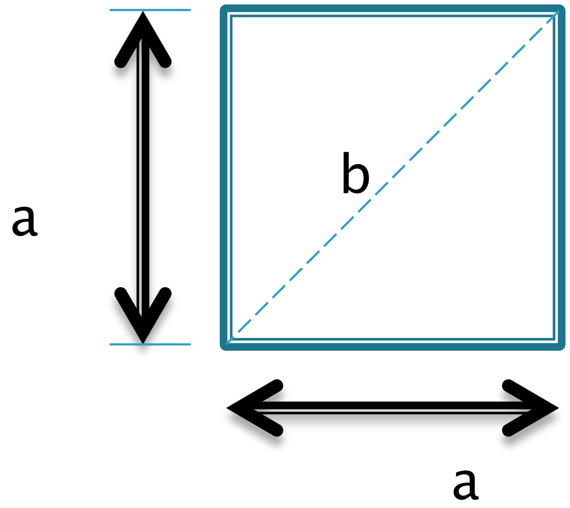
Area of Parabola

$$A = 2/3 * h * b$$



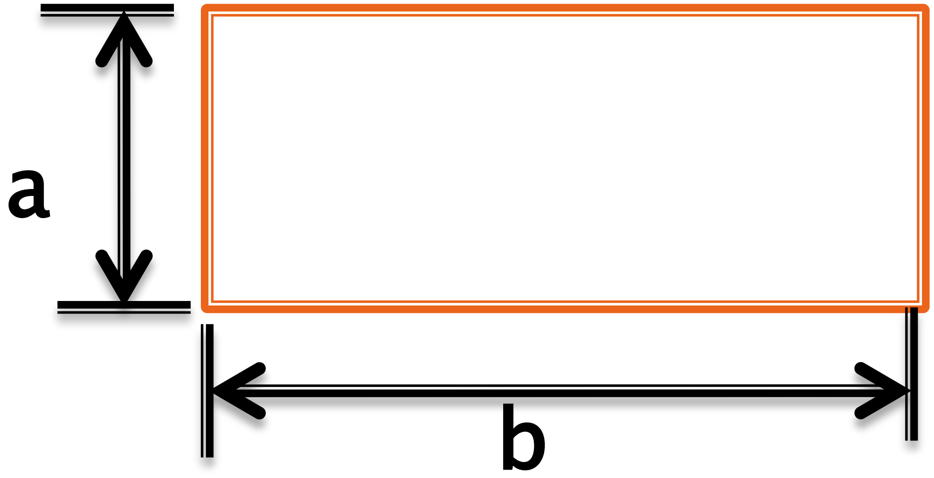
square area

$$A = a^2 \quad or \quad A = \frac{1}{2} \times b^2$$



Rectangle area

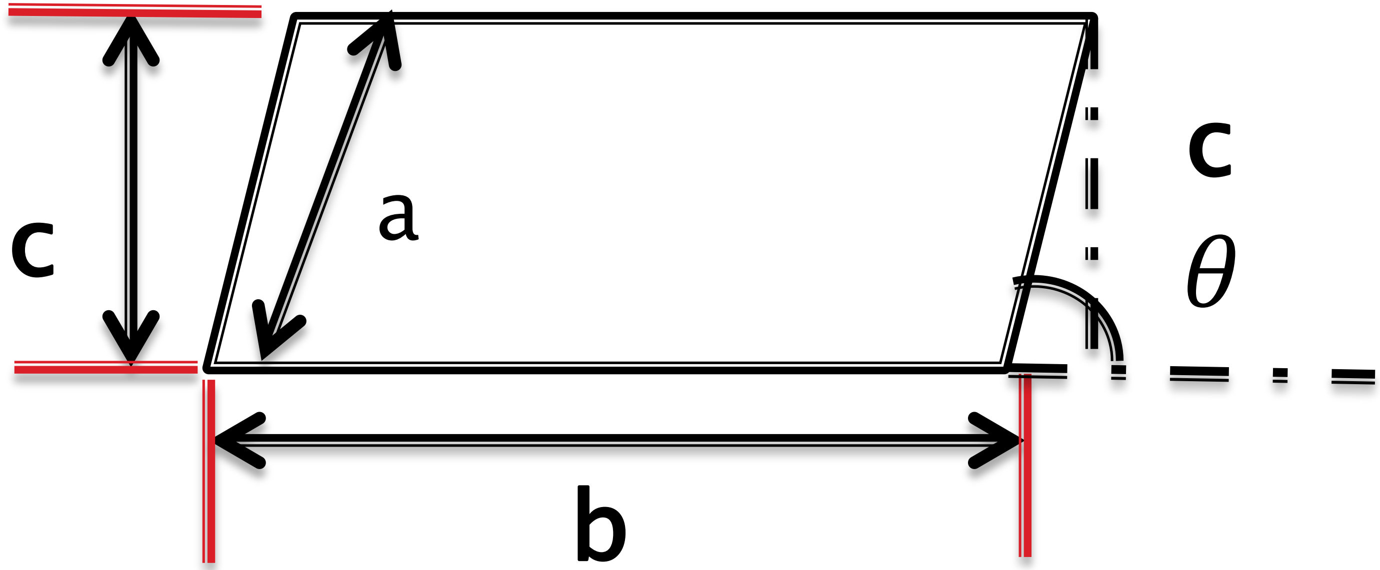
$$A = a \times b$$



The area of a parallelogram:

That is, each two opposite sides are parallel and equal in measure

$$A = b \times c$$
$$or \quad = a \times b \times \text{SIN}\,\vartheta$$

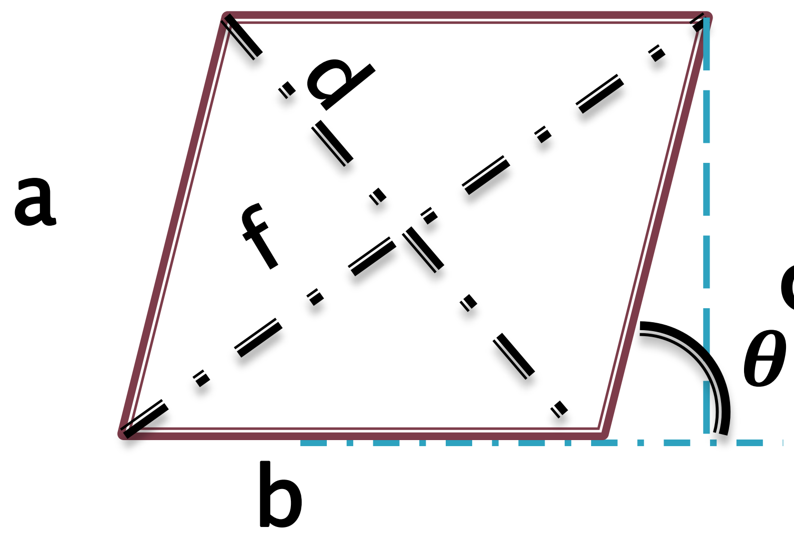


Rhombus space:

each two opposite sides are parallel, and all sides are equal in measure, and the diagonals are perpendicular

$$A = b \times c$$

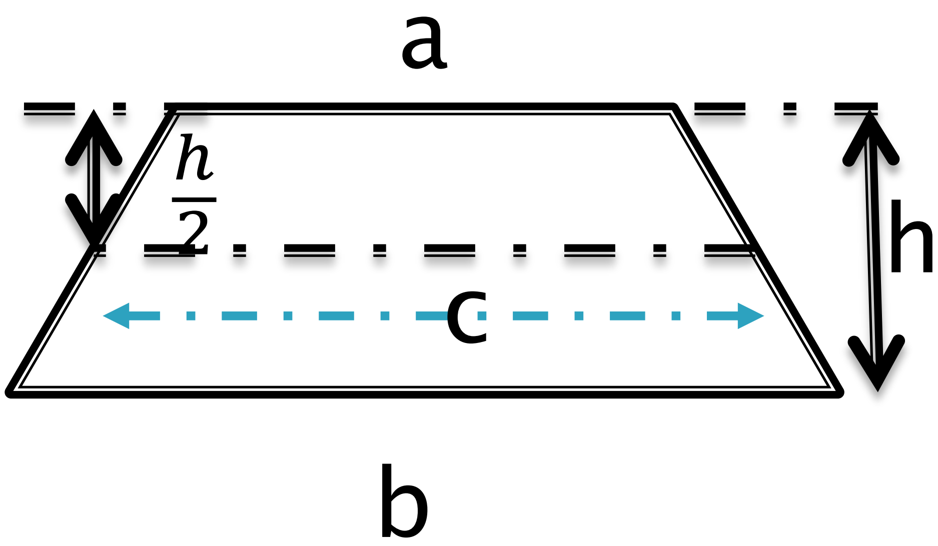
$$or = \frac{1}{2} \times d \times f = b^2 \times \mathrm{SIN}\,\theta$$



Trapezoid area

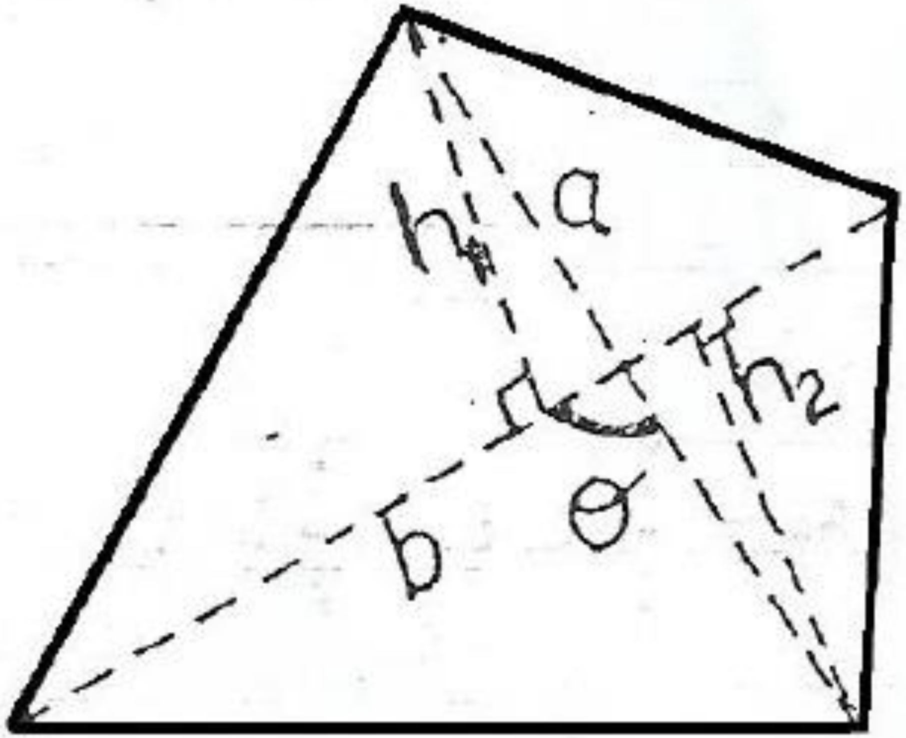
Only two sides are parallel

$$A = h \times \frac{a+b}{2} \quad or \quad h \times c$$



The area of the quadrilaterals

$$A = 0.5 \times a \times b \times sin\theta \quad or \quad = \frac{b}{2} \times (h_1 + h_2)$$



Method of measurements and computation of areas

1. *Field Measurement*
2. *Map Measurements*

- ***Field Measurement***
- ***Subdividing to Triangles***
- Dividing the piece into triangles This method is used when the ground is flat and free of beams, where the piece is divided into triangles so that the lengths of its sides can be measured or some of the lengths and angles confined using the tape only. As in figure 1-1a
- Or use the flat board with the tape (where the angles confined by the protractor are measured from the diagram drawn on the plate in Figure 1-1b

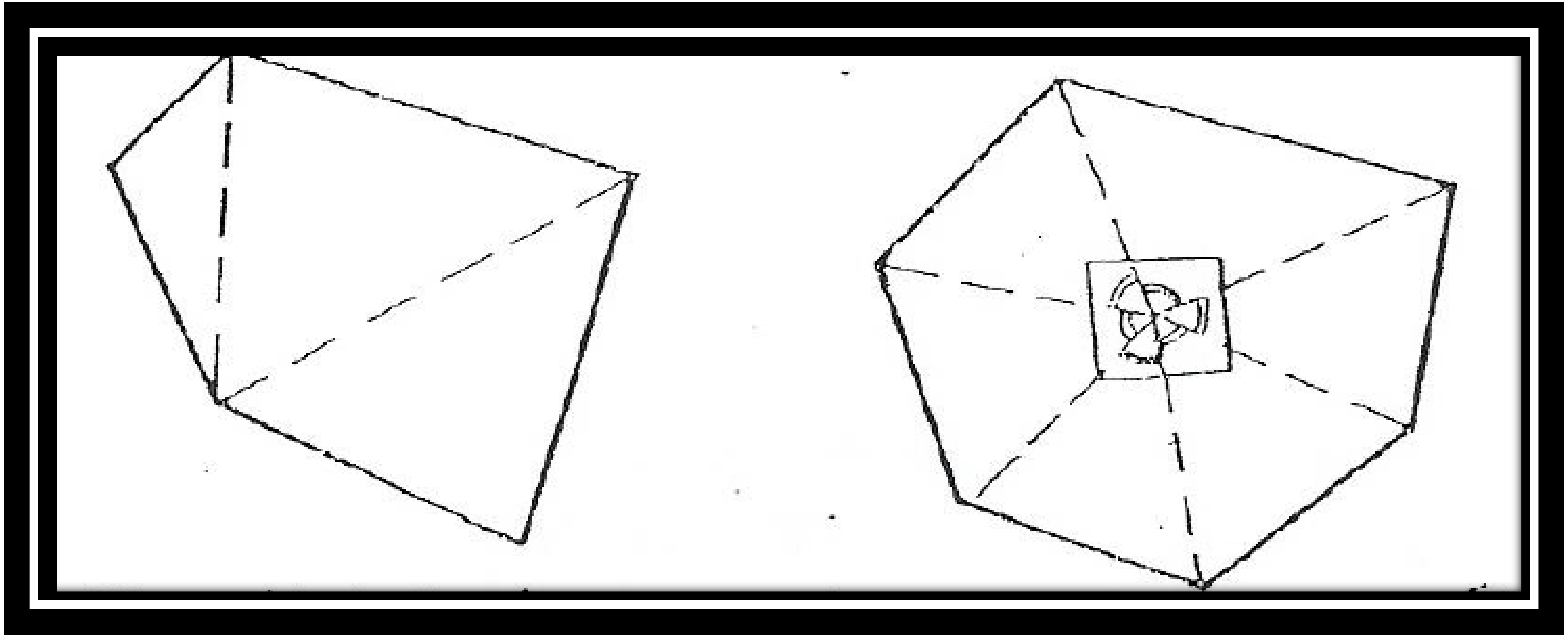


Figure 1-1 a

Figure 1-1 b

If the segment's borders are irregular, the adjacent parts can be divided into small triangles (as in Figure 1-1 c) or by following the method of addition and deletion (as in Figure 1-1 d) or erecting columns from the ribbing line or scanning near the zigzag borders to those borders as shown in the immediate method (where all lengths are measured using the tape).

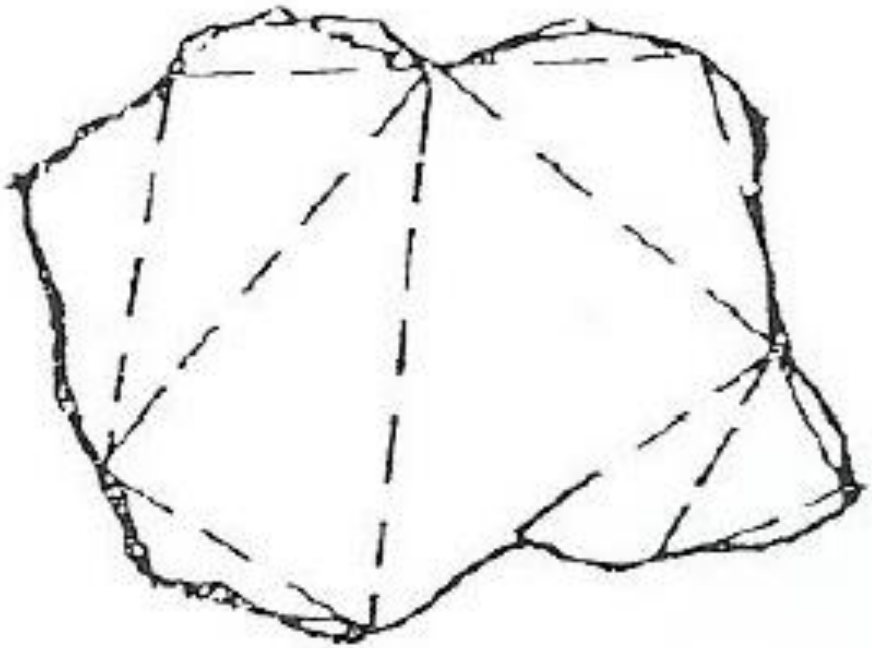


Figure 1-1 c

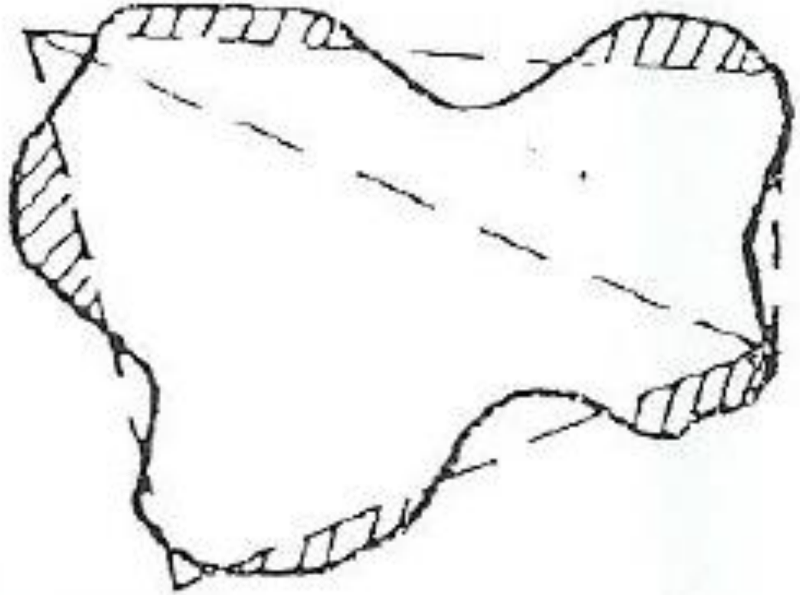


Figure 1-1 d

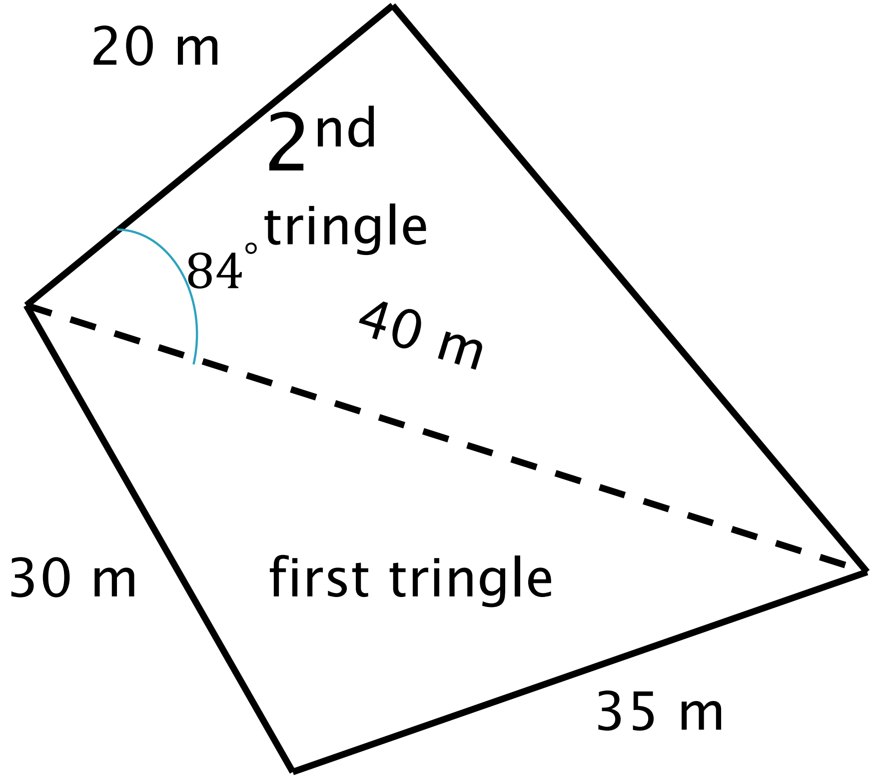
The area of a castle land was evaluated by dividing it into two triangles, where the lengths of the three sides of the first triangle and the lengths of two illustrated ministerial sides of the second triangle were measured, as shown in the figure below. It is required to calculate the area of the plot of land in meters.

Solution /
For the first tringle

.

$$S=\frac{a+b+c}{2}$$

$$S=\frac{30+35+40}{2}=52.5\text{ m}$$



$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

$$A_1 = \sqrt{[52.5 \times (52.5 - 30) \times (52.5 - 35) \times (52.5 - 40)]}$$

$$A_1 = 508.33\ m^2$$

- For the 2nd tringle

$$A = 0.5 * a * b * \sin <C$$

$$A = 0.5 * 20 * 40 * \sin 84$$

$$= 397.81\ m^2$$

$$A_{total} = A_1 + A_2 = 508.33 + 397.81$$

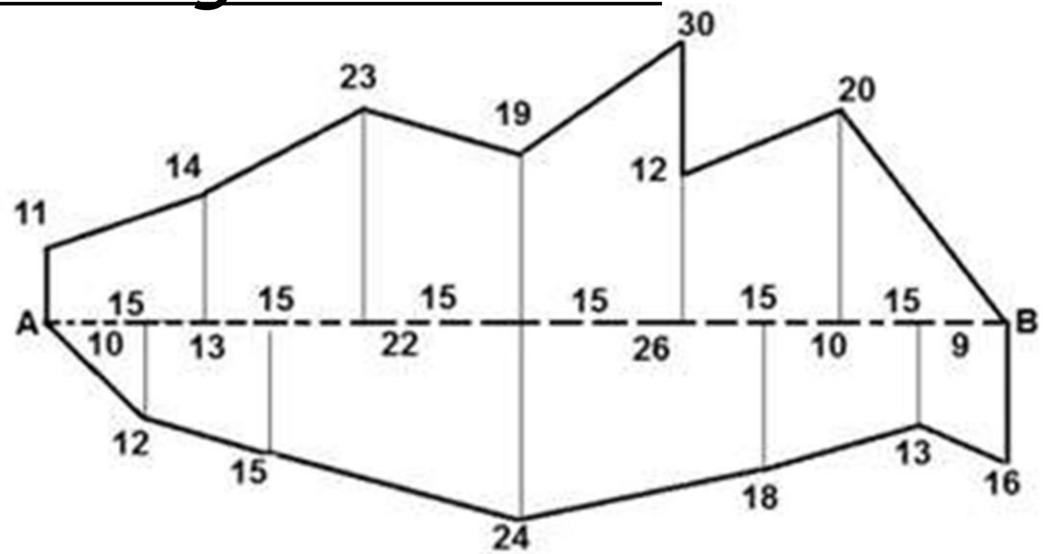
$$= 906.14\ m^2$$

- The area of land parcels or regions is often needed as part of a volume calculation, for
- instance to determine the amount of fertilizer to be applied to a paddock or to determine runoff for stream flow analysis. The legal title description of a land allotment shows the area as well

- as the dimensions, for example:
- Land parcels are not always contained by regular straight line or circular arc boundaries, especially when they front water courses or ridge lines. Methods for surveying these
- boundaries and computing the enclosed areas are as follows
- 2- *Setting_Out_Offsets*
- *3-_Setting_Out_Offsets at Regular Intervals*

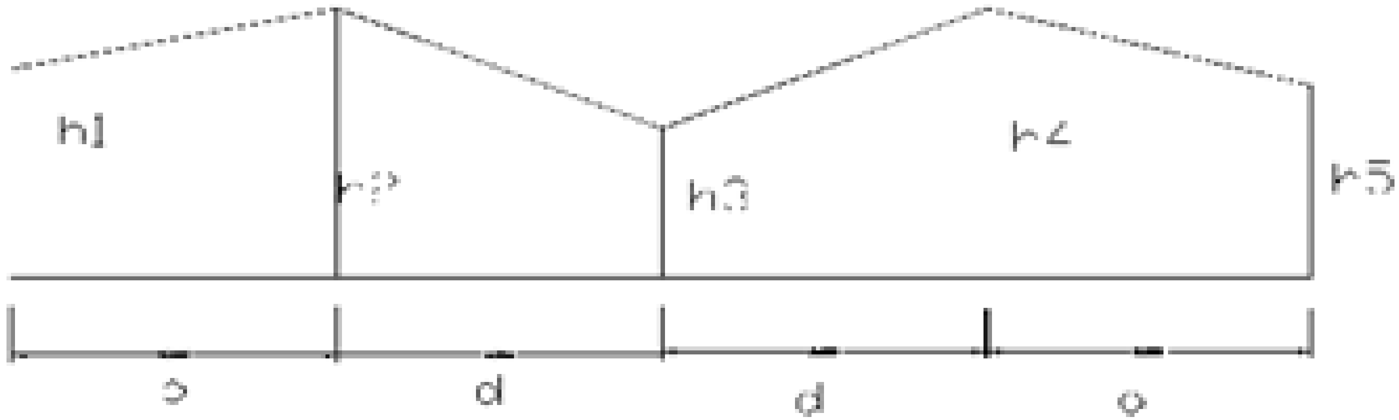
a) Average Offset Formula -

$$A=\left(\frac{h_1+h_2+h_3+\ldots\ldots\ldots\ldots+h_n}{n}\right).L$$



b) Trapezoidal Rule

- This law is used when the segment boundaries are straight or broken, or when the equal interval between the columns is short so that the curved boundaries approach to the straight lines between the columns.



$$a_1 = \left(\frac{h_1 + h_2}{2}\right) * d \qquad a_2 = \left(\frac{h_2 + h_3}{2}\right) * d$$

$$a_3 = \left(\frac{h_3 + h_4}{2}\right) * d \qquad a_4 = \left(\frac{h_4 + h_5}{2}\right) * d$$

$$A_{total} = \left(\frac{h_1 + h_5}{2} + h_2 + h_3 + h_4\right) * d$$

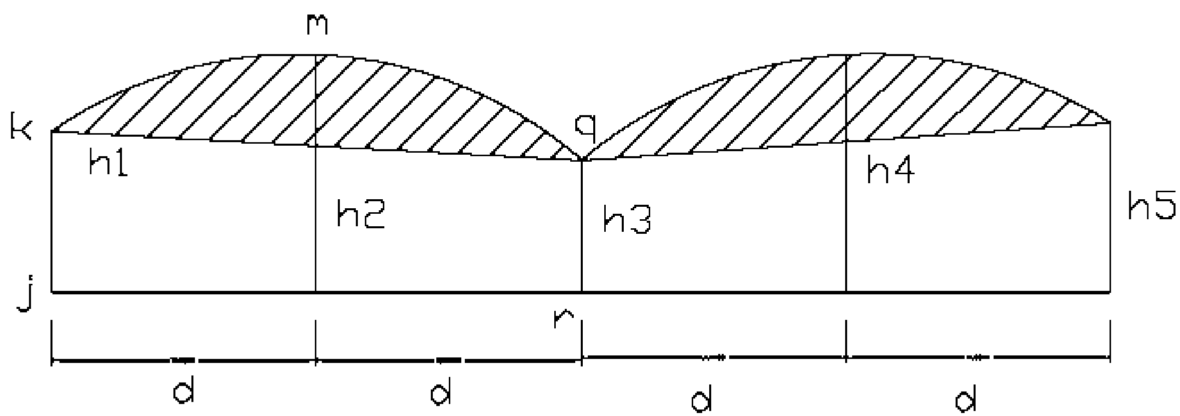
$$A = \left(\frac{h_1 + h_n}{2} + h_2 + h_3 + \ldots\ldots + h_{n-1} \right) * d$$

c) Simpson's Rule

This law is used in the form of arcs or curves, which give more accurate results than the trapezoid law.

$$A_{1\rightarrow 3} = A_{(K\,M\,Q)} + A_{(J\,K\,Q\,R)}$$

$$= \frac{2}{3}\left[2d\left(h_2 - \frac{h_1 + h_3}{2} \right) \right] + 2d\left(\frac{h_1 + h_3}{2} * \frac{3}{3} \right)$$



$$= \frac{d}{3}\left[4h_2 - 2h_1 - 2h_3 + 3h_1 + 3h_3\right]$$

$$A_{1\to 3} = \frac{d}{3}\left[h_1 + 4h_2 + h_3\right]$$

$$A_{3\to 5} = \frac{d}{3}\left[h_3 + 4h_4 + h_5\right]$$

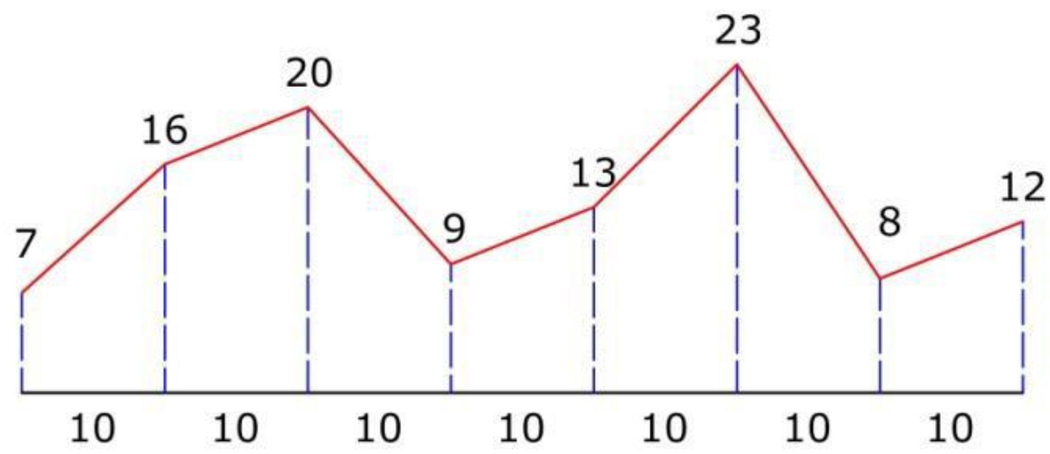
$$A_{total} = \frac{d}{3}\left[h_1 + h_5 + 4(h_2 + h_4) + 2(h_3)\right]$$

The Simpson's Rule become

$$A = \frac{d}{3}\left[h_1 + h_n + 4(h_2 + h_4 + \ldots.. + h_{n-1}) + 2(h_3 + h_5 + \ldots. + h_{n-2})\right]$$

n = odd number

Ex:- Find the area of the fig. Below by using (Average & Trapezoidal) methods .



SOL/

1- Method : Average

$$A = \left(\frac{h_1 + h_2 + h_3 + \dots\dots\dots\dots + h_n}{n} \right) . L$$

$$A = \left(\frac{7+16+20+9+13+23+8+12}{8} \right) * 70$$

$$A = 945\ m^2$$

2- *Trapezoidal Method :*

$$A = d\left(\frac{h1 + hn}{2} + h2 + h3 + \dots\dots\dots\dots\dots + hn\text{-}1\right)$$

$$A = 10\left(\frac{7+12}{2} + 16 + 20 + 9 + 13 + 23 + 8\right)$$

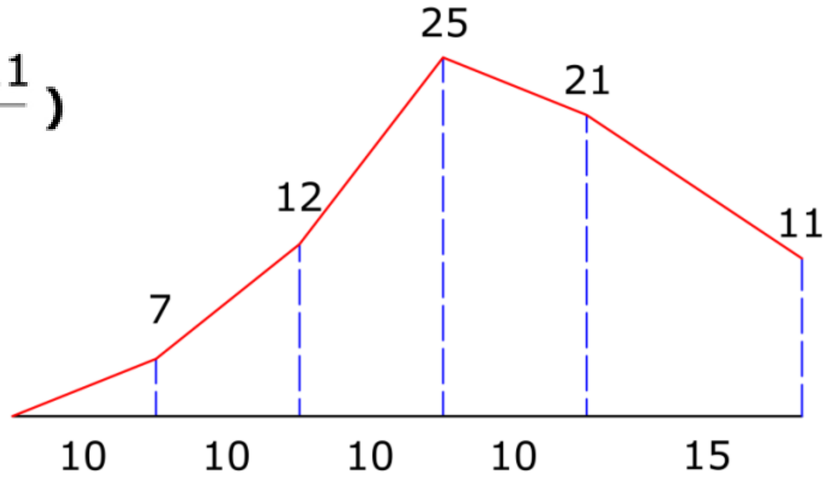
A= 985 m^2

Ex Find the area of the fig. Below by using Appropriate Method .

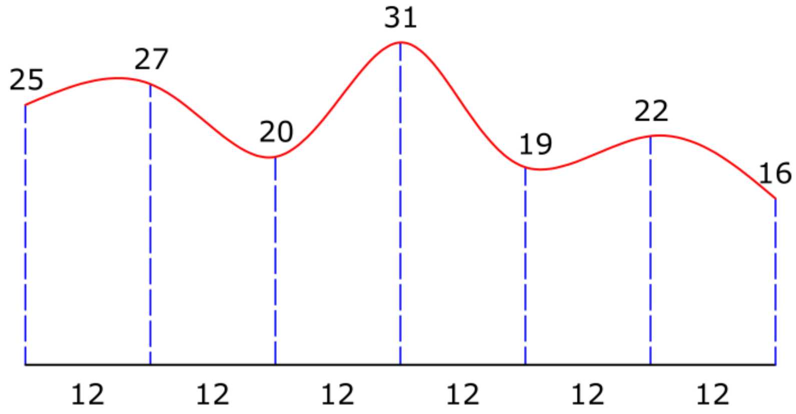
Sol :

$$A = 10\left(\frac{0+21}{2} + 7 + 12 + 25\right) + 15\left(\frac{21+11}{2}\right)$$

A= 755 m^2



Ex : Find the area of the fig. Below by using (Simpsons Rule) .



Sol/

$$A = \frac{d}{3}\left[(h1 + hn) + 4(h2 + h4 + ... + hn\text{-}1) + 2(h3 + h5 + ... + h\text{-}2)\right]$$

$$A = \frac{12}{3}(25+16)+4\,(27+31+22)+2(20+19)$$

A= 1736 m^2

2- Setting out Offset at Irregular Intervals

This method is used in the event that it is not possible to erect columns at equal intervals due to the nature of the land or the presence of obstacles. Its law can be formulated as follows:

$$2A = d1\ (h1-h3) + (d1+d2)(h2-h4) + (d1+d2+d3)(h3-h5) +$$

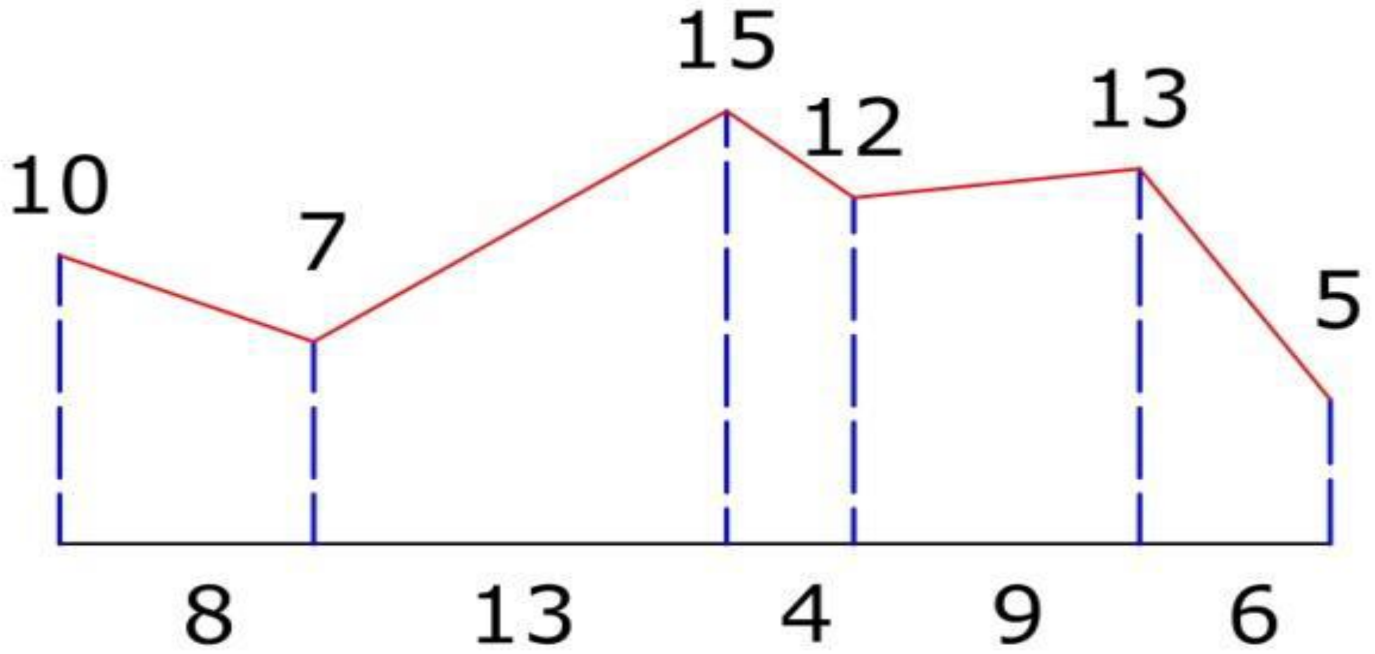
$$(d1+d2+d3+d4)(h4-h6) + (d1+d2+d3+d4+d5)((h5-h7) +$$

$$(d1+d2+d3+d4+d5+d6)(h6+h7) \rightarrow \left|\frac{\pm X}{2}\right| = \mathbf{A}$$

.

Ex : Find the area of the fig. Below by using Appropriate Method .

Sol.



2 A= d1 (h1−h3) + (d1+d2)(h2−h4) + (d1+d2+d3)(h3−h5) + (d1+d2+d3+d4)(h4−h6) + (d1+d2+d3+d4+d5)(h5+h6)

2A= 8(10-15) + (8+13)(7-12) + (8+13+4)(15-13) + (8+13+4+9)(12-5) + (8+13+4+9+6)(13+5)

2A= 863 → A=431.5 m^2

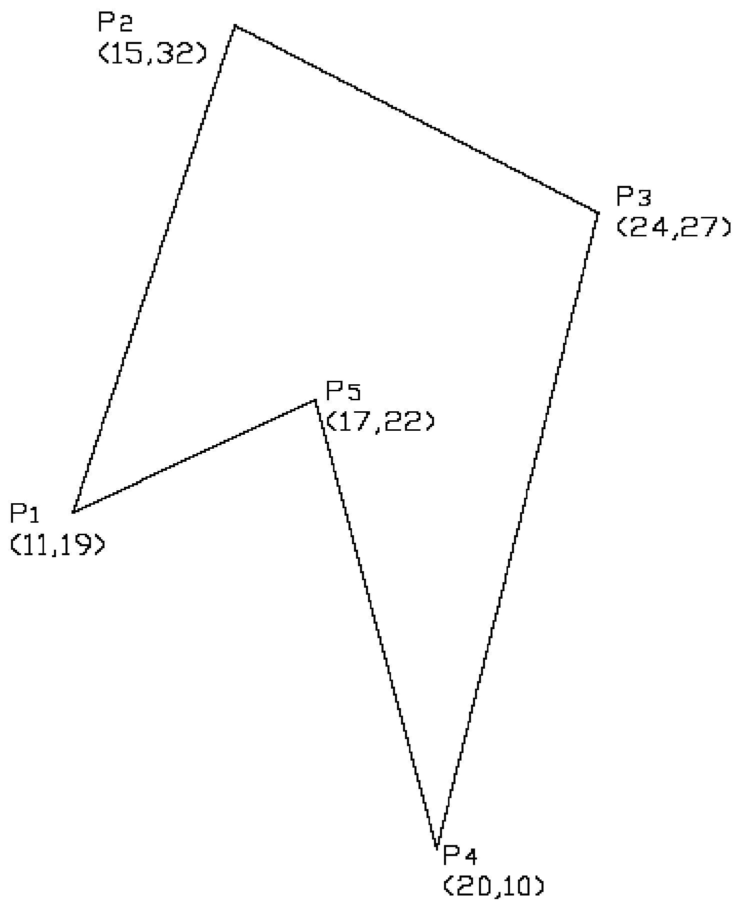
3- Using Coordinates

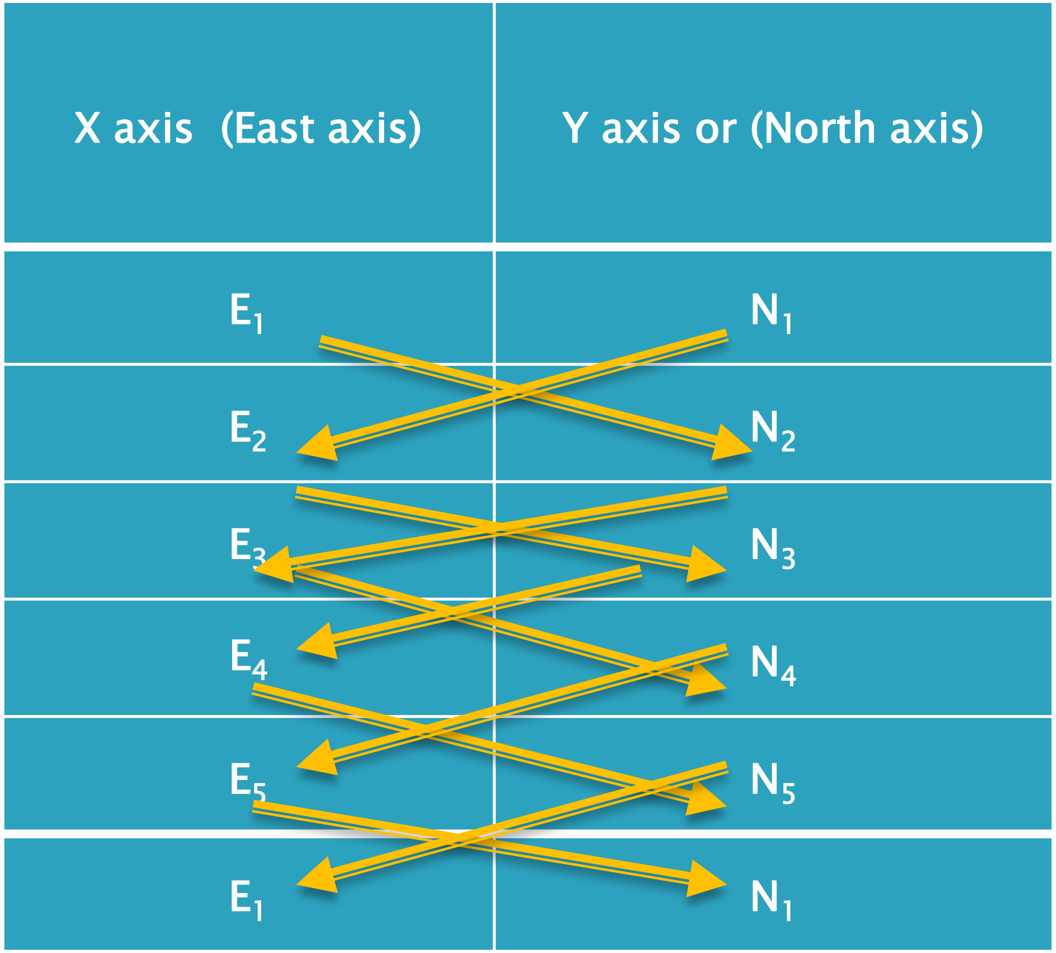
If the corners of the boundaries of a closed plot of land are known, then it is possible to calculate twice the area of that plot by using the coordinate law. Twice the area = the algebraic sum of the product of the x-coordinate (dispersal coordinate) for each point * the difference of the y-coordinate (inclusion coordinate) for the points before and after that point.
Note:- It starts from a certain point and in a certain direction until returning to the same point (the starting point).

Example:-

from corrected coordinates for a closed traverse is shown in figure bellow.

Calculate the area of land by using coordinates rule.





$$\frac{E_1}{N_1} \Leftrightarrow \frac{E_2}{N_2} \Leftrightarrow \frac{E_3}{N_3} \Leftrightarrow \frac{E_4}{N_4} \Leftrightarrow \frac{E_5}{N_5} \Leftrightarrow \frac{E_1}{N_1}$$

2A = [(19*15)+(32*24)+(27*20)+(10*17)+(22*11)] – [(11*32)+(15*27)+(24*10)+(20*22)+(17*19)] =

$$\left|2005-1760\right|$$

$$A=\left|\frac{245}{2}\right|=122.5m^2$$

4-Using Double Meridian Distance Method (DMD)

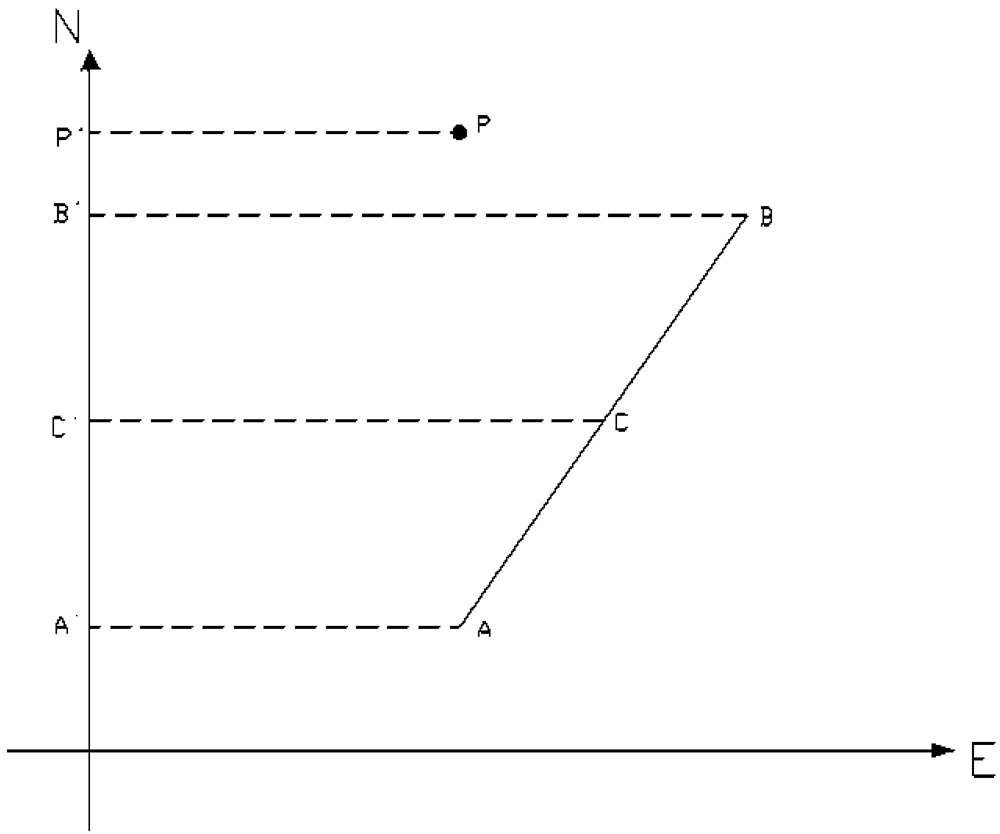
This method is used when the horizontal and vertical components of the sides of the polygon are known and it may be called the composite method.

- Meridian distance for point = the distance of the point from the north line.
- Meridian distance for line = the distance of the midpoint of the line from the north axis.
- Double the distance of the longitude of the (Double-meridian-Distance for line) line = is the sum of the distances of the starting and ending points of that line from the north axis.

- Point longitude distance P = P´ P
- Straight meridian distance AB = C´ C
- Double the longitude distance of the straight line AB = A´A+B´B
- Twice the area = twice the longitude * height
-

  2A = (A´A + B´B) * A´ B´

- The formula for calculating twice the area of the general longitude is:



$$D.M.D_{\text{الضلع}} = Dep_{\text{الضلع السابق}} + D.M.D_{\text{الضلع السابق}} + Dep_{\text{الضلع نفسه}}$$

So the law for the first side becomes: -

D.M.D = Dep of the first side

The last side is:-

Last leg D.M.D = - Dep

Rules for calculating area by methods (DMD .)

1) Find twice the longitude distance of the first side (DMD = the horizontal component of the side.
2) Find twice the longitude distance of any other side (DMD) = twice the longitude distance of the previous side +

The horizontal component of the previous side + the horizontal component of the present side.

3) Twice the longitude distance of the last side = - (the horizontal component of the other side).

Notes to verify work:

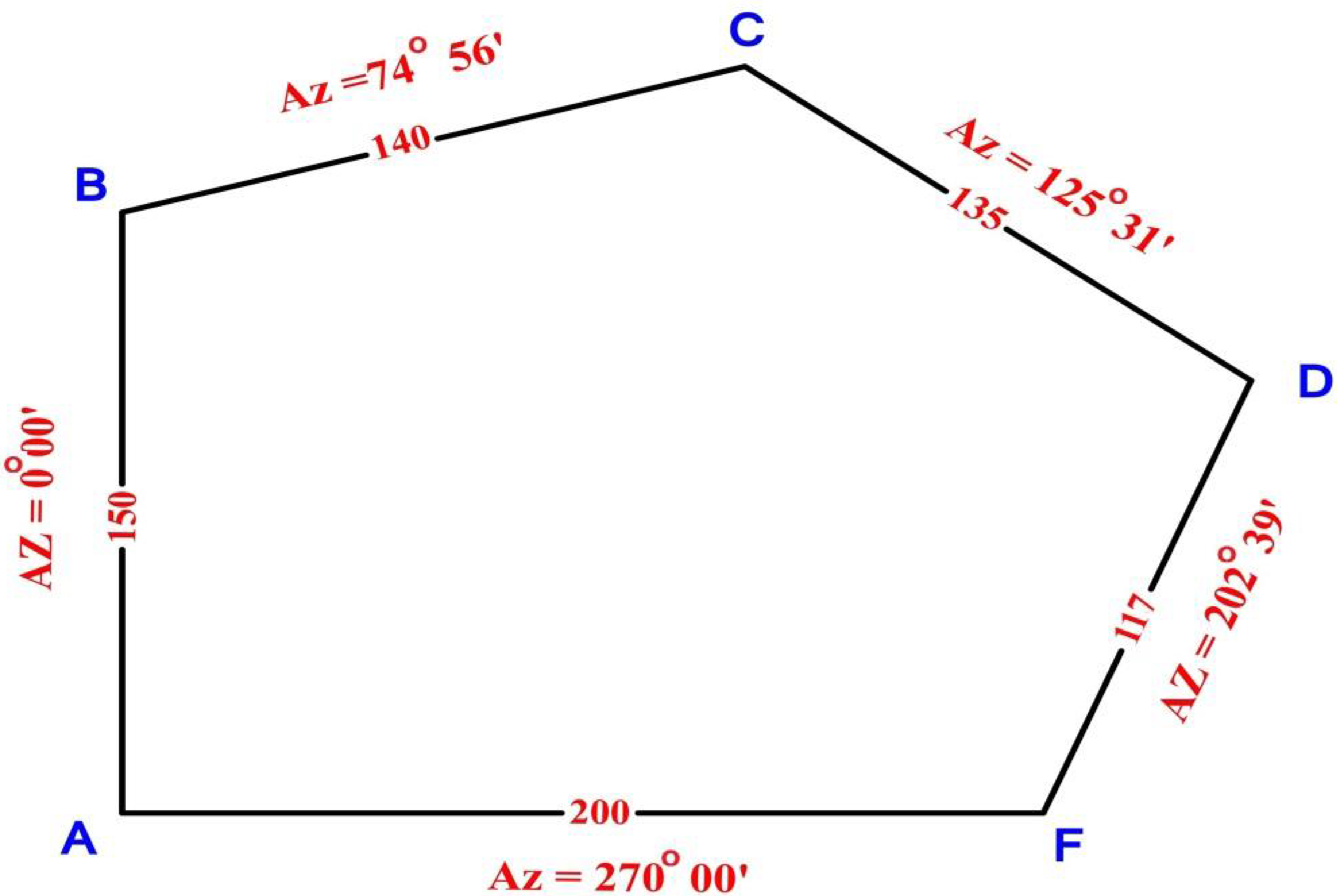
1 - The sum of the vertical components of the sides of the polygon = zero.

2 - The sum of the horizontal components of the sides of the polygon = zero.

3 - Twice the longitude distance of the last side = - (the horizontal component of the last side).

Ex

Find the area of the Fig. below by using DMD



given data			Required	
Side	Length (m)	AZ	Dep.	Lat.
AB	150	$0^\circ 00'$	0	150
BC	140	74° 56'	135.19	36.39
CD	135	125° 31'	109.88	-78.43
DE	117	202° 39'	-45.06	-107.98
EA	200	270° 00'	-200	0

Side	Dep.	DMD	Lat.	2A
AB	0	0	150	0
BC	135.19	135.19	36.39	4919.56
CD	109.88	380.26	−78.43	−29823.80
DE	−45.06	445.08	−107.98	-48059.74
EA	−200	200	0	0
$\sum$	0	-----	0	−72962

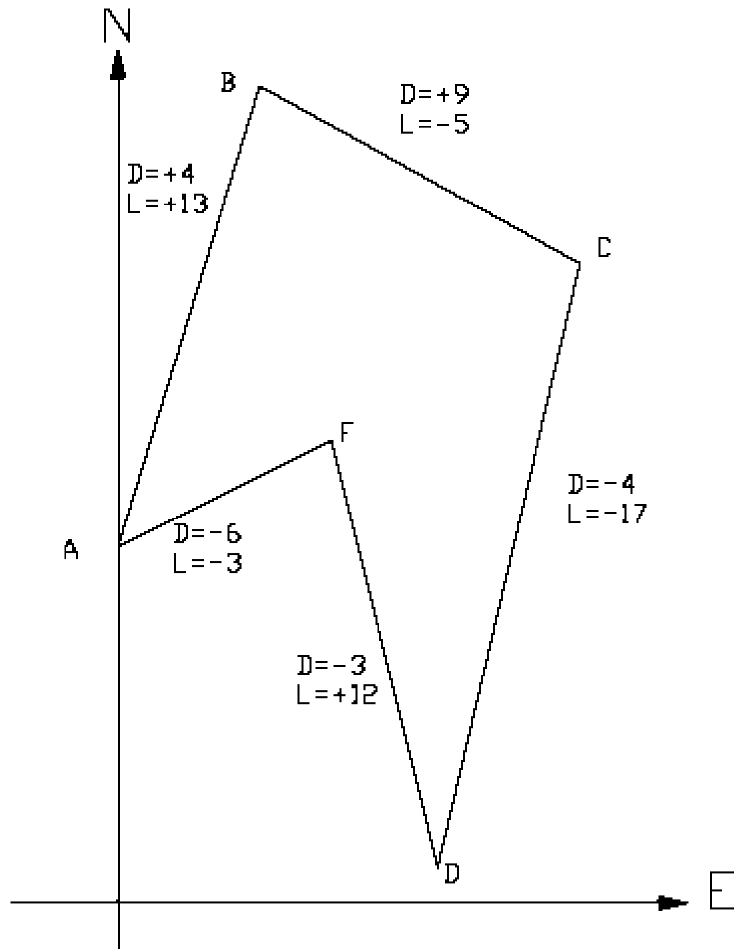
$$A = \left|\frac{-72964}{2}\right| = 36482\ m^2$$

Example:-
From Dep. And Lat. To the closed traverse is shown in figure bellow.

Calculate the area of the closed traverse by using D.M.D.

SoL..

Side	Dep.	D.M.D.	Lat.	2Area
AB	+4	+4	+13	+52
BC	+9	+4 +4 +9 = +17	-5	-85
CD	-4	+9 +17 -4 = +22	-17	-374
DF	-3	-4 +22 -3 = +15	+12	+180
FA	-6	-3 +15 -6 = +6	-3	-18
Sum				-245

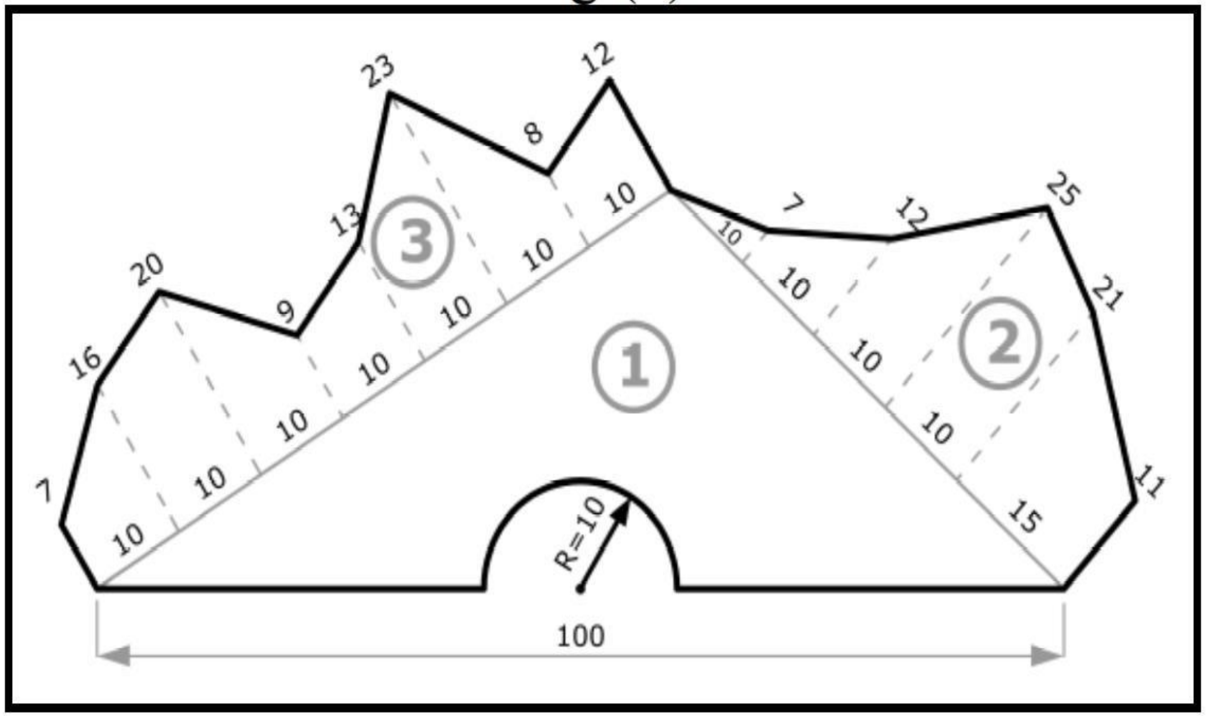


$A = |-245/2| = 122.5\ m^2$

Test

Q1/ Find the area that surrounded by a dark line of the fig (1) Below in (m^2) . using appropriate method .

Fig (1)



Q4 : Find the area of the land that have the following coordinates , by using DMD method : A (50,50) , B (54,63) , C (63,58) , D (59,41) , E (56,53) .

Area of Cross Section

- The longitudinal section of a land surface along the central line obtaining by using the longitudinal leveling process, where the levels calculated at intervals of every 100 m, and they are usually called stations or chains or distances. Then the longitudinal section of the land line is drawn and called (Ground Line). Then draws in the profile the longitudinal section of the construction line called (Grade Line).
- After that, we take the cross sections which are perpendicular to the longitudinal section using the Cross-section-levelling process. Cross-section- levelling Calculates the levels of points on either side of the center line for short distances, especially in the changes in the nature of the land, it is calculated for each full station per 100 m. If necessary, and when the terrain changes, the settlement between the full stations is used, and it is called partial stations, then the cross-sections of the full stations and the partial stations are drawn.

EX :Find the Missing (Elevation & Horizontal Distance) for the fig. Below , if the side slope (1:4).

Sol

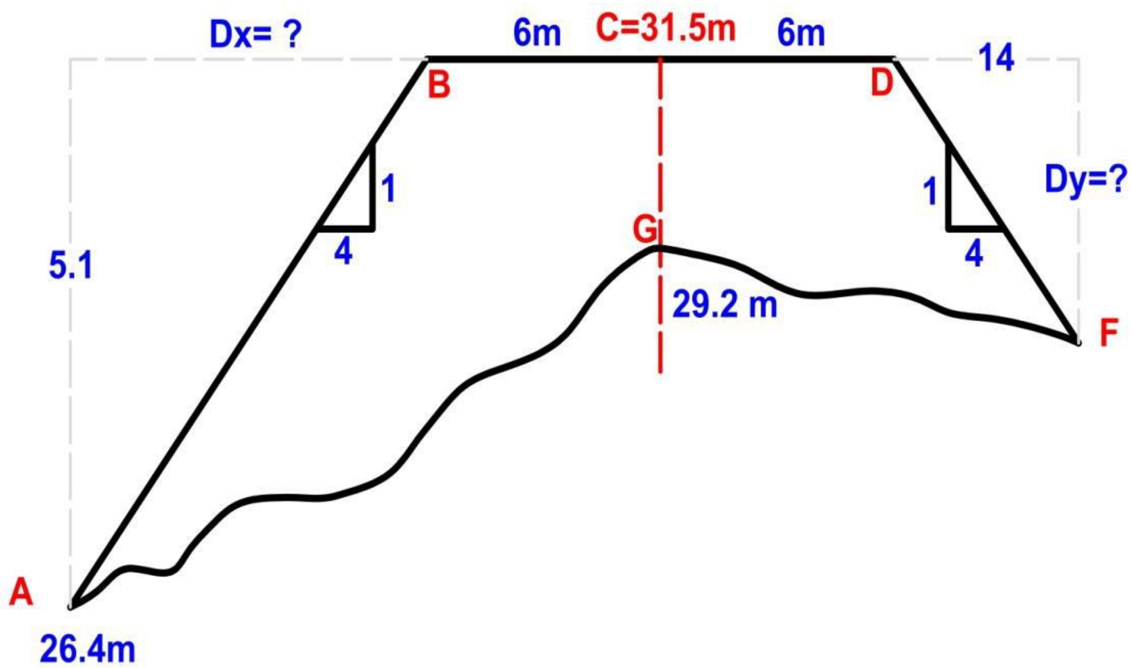
1- Elev. of Point (F).

$$\frac{1}{4} = \frac{Dy}{14} \longrightarrow \text{Dy=3.5 m}$$

ELEV. OF (F) = 31.5-3.5 = 28 M

2- Dist. (Dx) from Point (A).

$$\frac{1}{4} = \frac{5.1}{Dx} \longrightarrow \text{Dx=20.4 m}$$



Dist. From (A) to Centre line = 6+20.4 =26.4 m

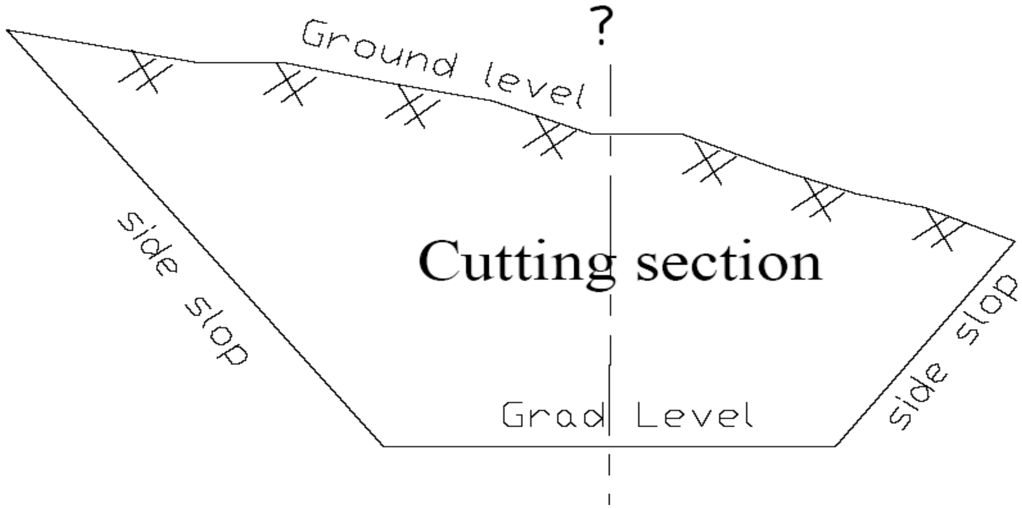
The Coordinates :

A(0 , 26.4) , B(20.4 , 31.5) , C(26.4 , 31.5) , D(32.4 , 31.5) , F(46.4 , 28) ,G(26.4 , 29.2)

The cross-sections are of three main forms:

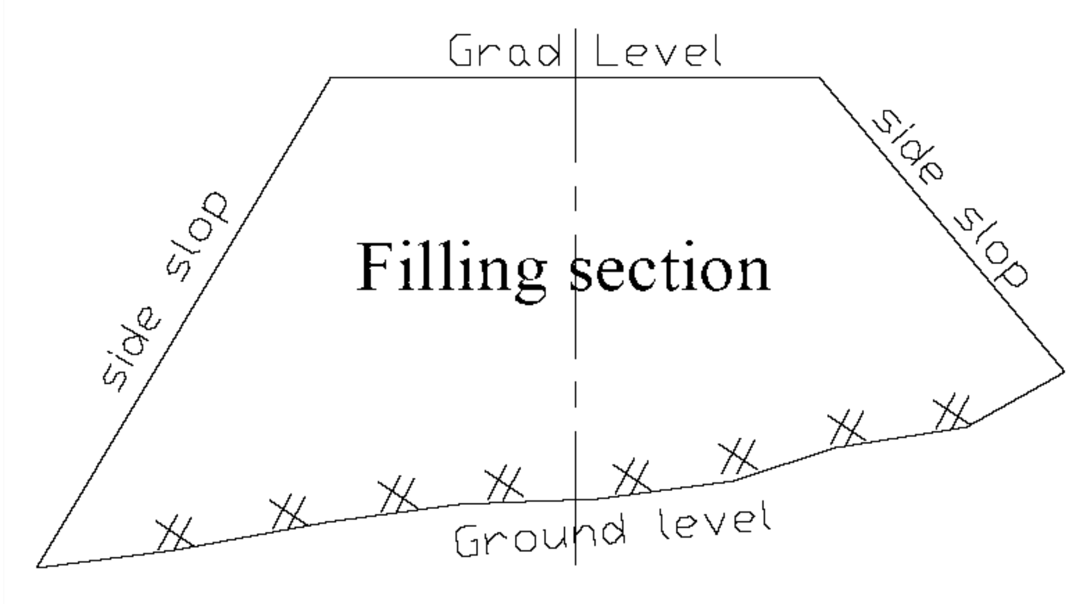
A. Cutting section

Where the ground level is higher than the grad level.



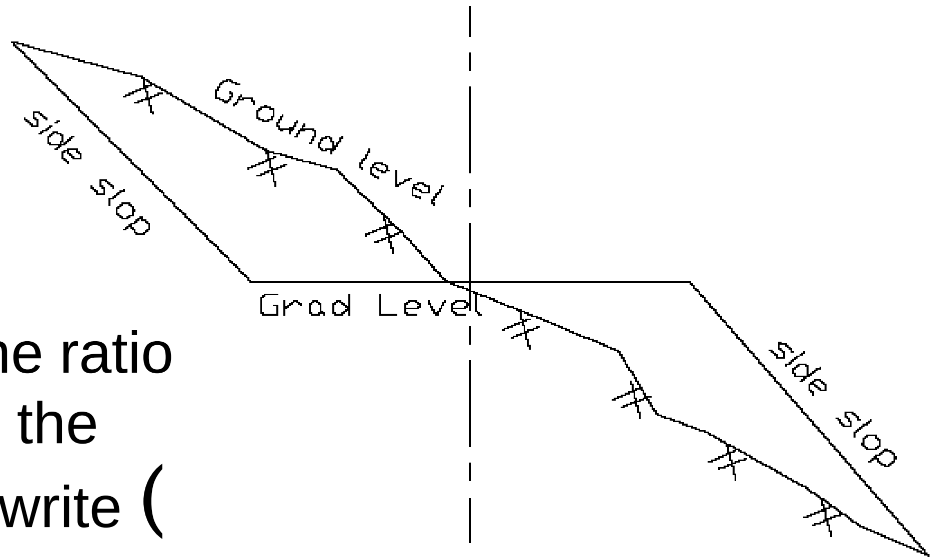
B. Filling section

Where the ground level is lower than the grad level



C. Side–Hill section

Where the ground level is higher than the grad level on one side and lower than it on the other.



Side slop is known for any cross section the ratio between the unit of the vertical distance to the unit of the horizontal distance and usually write ($\frac{1}{S}$) or $1 : S$

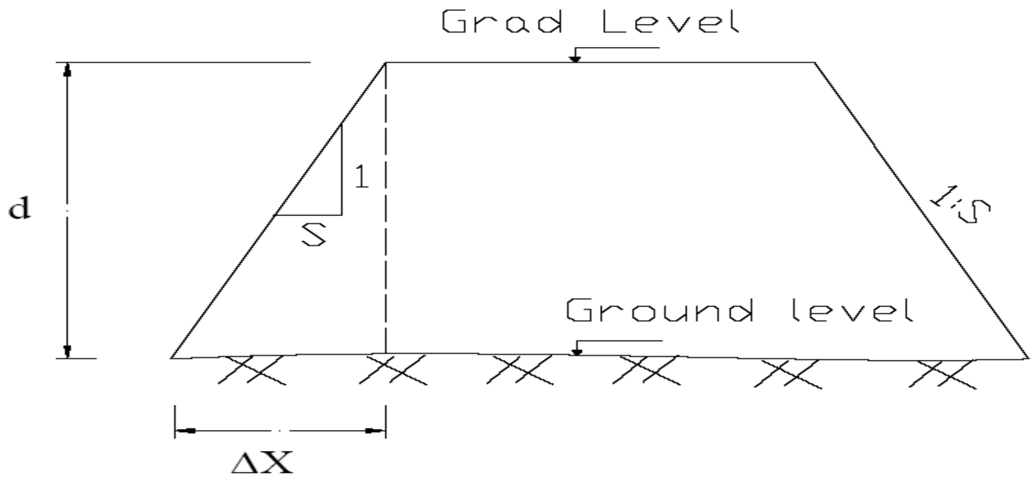
1) FOR A LEVEL SECTION COMPUTATION

d = (Ground Level – Grad Level)

d = + then depth of cut

d = – then depth of fill

$\frac{1}{S}$ = side slop



1;S= $\frac{1}{S} = \frac{d}{\Delta x}$ then $\Delta x = S.d$

$$A = \left[\frac{b + (b + 2S.d)}{2}\right]d$$

$\Rightarrow$ $A = d\,(b + S.d)$

2. FOR A THREE-LEVEL SECTION COMPUTATION

b/2= half of grad line.

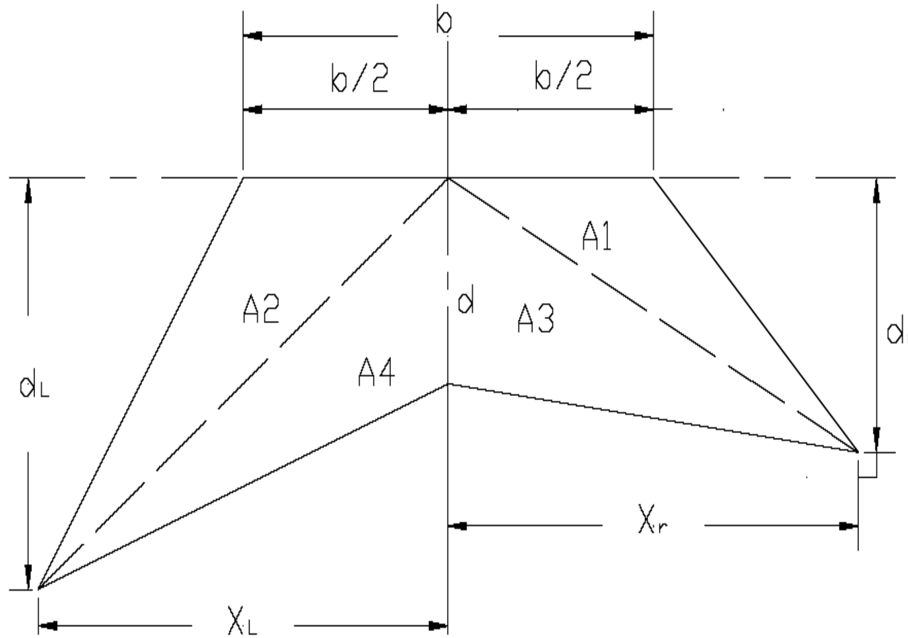
d = depth of cut or fill.

d_r =depth of cut or fill to the right side.

d_l = depth of cut or fill to the left side.

x_r = horizontal distance from ℄ to the point of right side.

x_l horizontal distance from ℄ to the point of left side



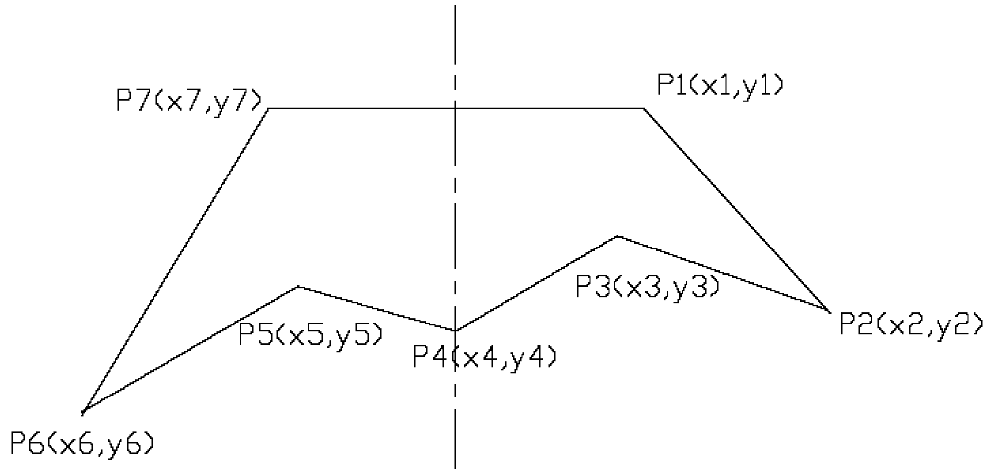
$$A_1 = \frac{1}{2} * d_r * \frac{b}{2} \qquad A_2 = \frac{1}{2} * d_l * \frac{b}{2}$$

$$A_3 = \frac{1}{2} * x_r * d \qquad A_4 = \frac{1}{2} * x_l * d$$

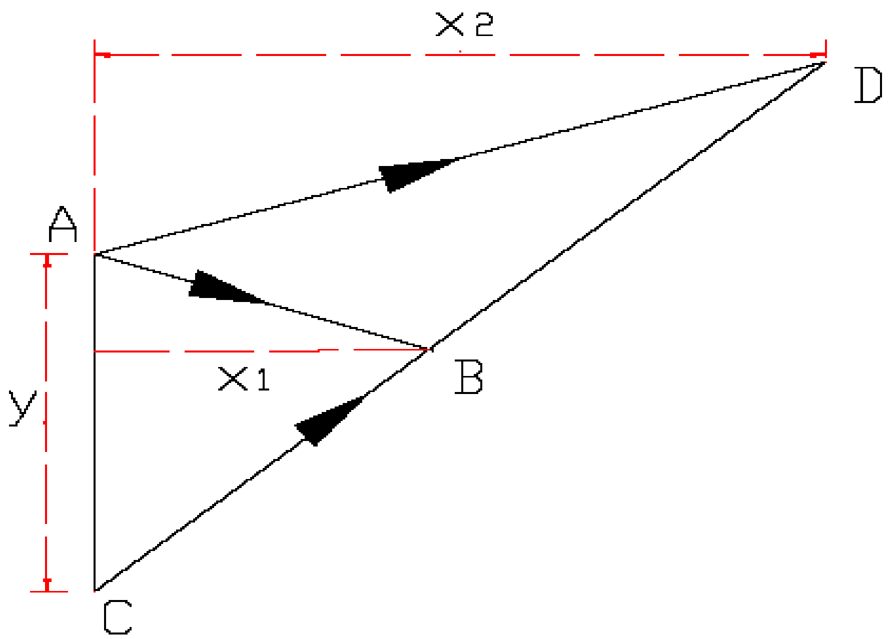
$$A_{total} = \frac{1}{2}\left[\frac{b}{2}(d_r + d_l) + d(x_r + x_l)\right]$$

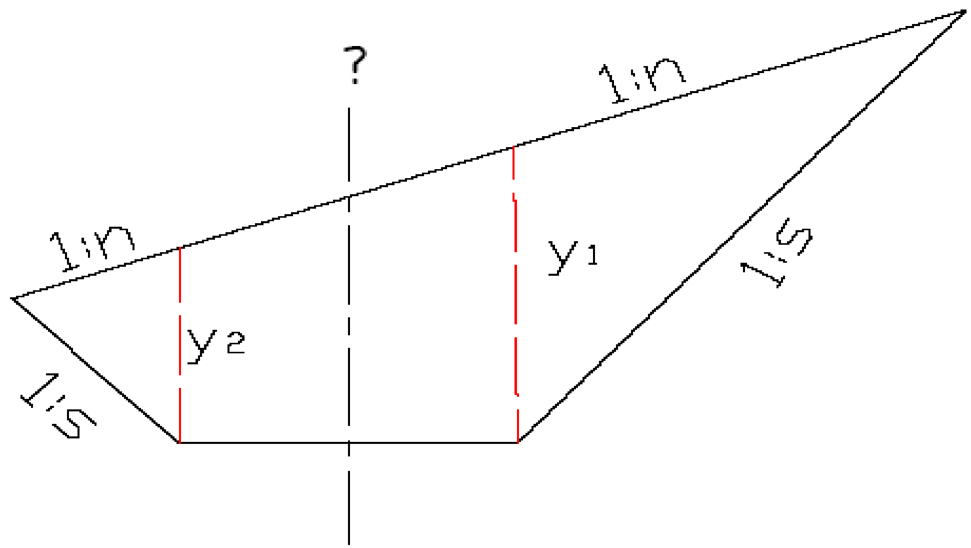
3. FOR A MULTI-LEVEL SECTION COMPUTATION

$$2A = \frac{x_1}{y_1} \times \frac{x_2}{y_2} \times \frac{x_3}{y_3} \times \frac{x_4}{y_4} \times \frac{x_5}{y_5} \times \frac{x_6}{y_6} \times \frac{x_7}{y_7} \times \frac{x_1}{y_1}$$



4. COMPUTATION A CROSS SECTION IF KNOWN A SIDE SLOPE FOR THE GROUND





Calculated by the rate of approach

If we get the height (y) and the two miles as in the triangle ABC, the distance x1 can be calculated, which is calculated by summing the two miles, then flipping the result and multiplying it by the value of y.

When the two slopes are in opposite directions, as in the triangle ABC, we add the two slopes as in the following equation

$$\left(\frac{1}{s}+\frac{1}{n}\right)^{-1} * y = x_1$$

When the two slopes are in the same direction as in the triangle ACD, we subtract the two slopes as in the following equation

$$\left(\frac{1}{s}-\frac{1}{n}\right)^{-1} * y = x_2$$

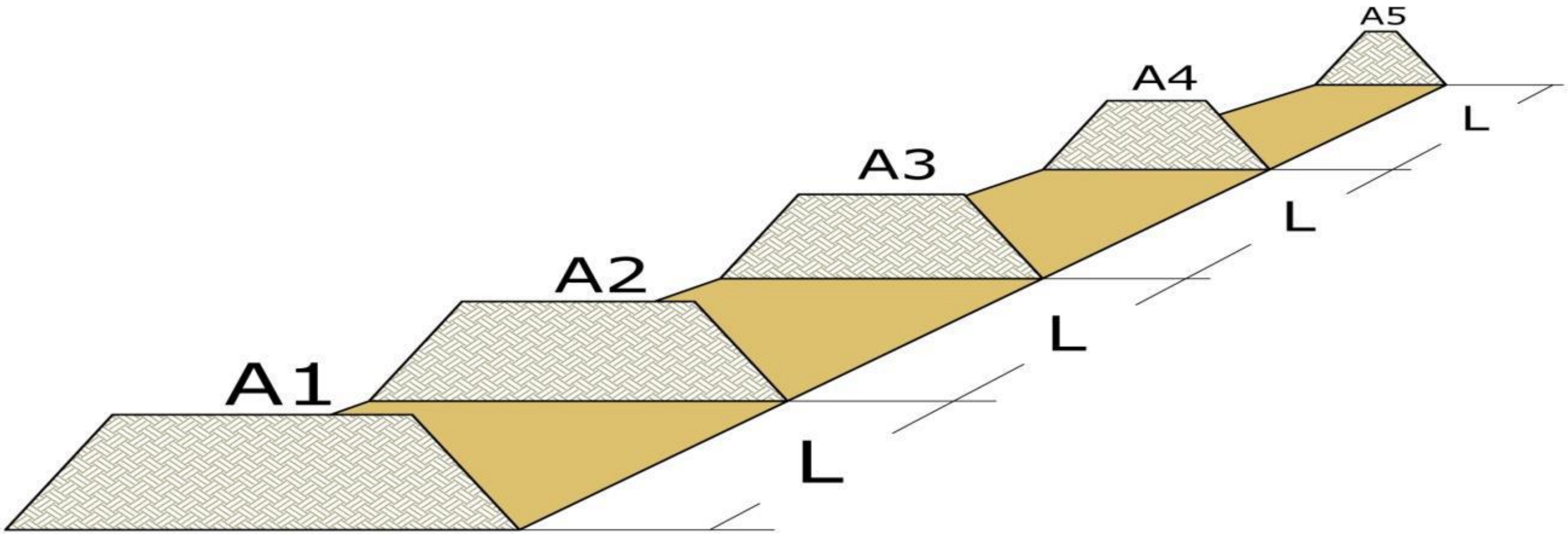
Then we calculate the area of the triangles and the area of the trapezoid and add them to get the area of the cross section

VOLUMES

Just as there are various methods for measuring areas, the processes of finding earth volumes vary according to the previous measurement methods, as well as the nature of the land for which earth volumes are required to be calculated. The process of calculating earth volumes for roads is different from those volumes that are calculated for depressions and heights that can be represented by contour lines.

1) AREA FORMULA –AVERAGE END

As shown in the figure below. The size of the road section is calculated using the following relationship:



$$T_{vol.} = L\left(\frac{A1+An}{2} + A2 + A3 + \ldots\ldots + A(n-1)\right)$$

This method is used if the spaces between the stations change, where the segment whose period changes are taken and calculated individually as in the following relationship:

$$T_{vol.} = L\left(\frac{A1+A2}{2}\right)$$

2) PRISMOIDAL FORMULA

It is used in the event that the number of syllables is odd (3, 5, 7 ......) and its mathematical relationship is similar to (Simpson's rule) to be used in calculating areas.

$$V = \frac{L}{3}\left[(A1 + An) + 4(A2 + A4 + ... + An\text{-}1) + 2(A3 + A5 + ... + An\text{-}2)\right]$$

Ex : For the data below , find the total volume of (Cut & Fill) by using average & Prismoidal methods .

St.	Area (m^2)		Vol. (m^3)	
	Cut	Fill	Cut	Fill
5+00	12			
6+00	16			
7+00	22			
8+00	9			
9+00	6		6/3*100 = 200	15/3 *100 = 500
10+00		15		
11+00		19		
12+00		21		
13+00		24		

Sol. : (For Cut)

1) BY AVG. –END METHOD

Vol. of cut = 100 [(12+6)/2 + 16 + 22 + 9)] = 5600 m^3

T vol. of cut = 5600 +200 =5800 m^3

1) BY PRISMOIDAL FORMULA

$$V=\frac{L}{3}[(A1+An)+4(A2+A4+......+An\text{-}1)+2(A3+A5+.....+An\text{-}2)]$$

Vol. of cut = 100/3[(12+6) + 4(16+9)+2(22)] = 5400 m^3

T vol. of cut = 5400 +200 =<u>5600</u> m^3

B. (For Fill)

1. by Avg.– End = T vol. of fill=

$=100[(15+24)/2+19+21]= 5950+500 = 6450\ m^3$

2– by Prismoidal: Vol. of fill (6 → 8) =

$\frac{100}{3} \times [15+21]+4(19)]=3730\ m^3$

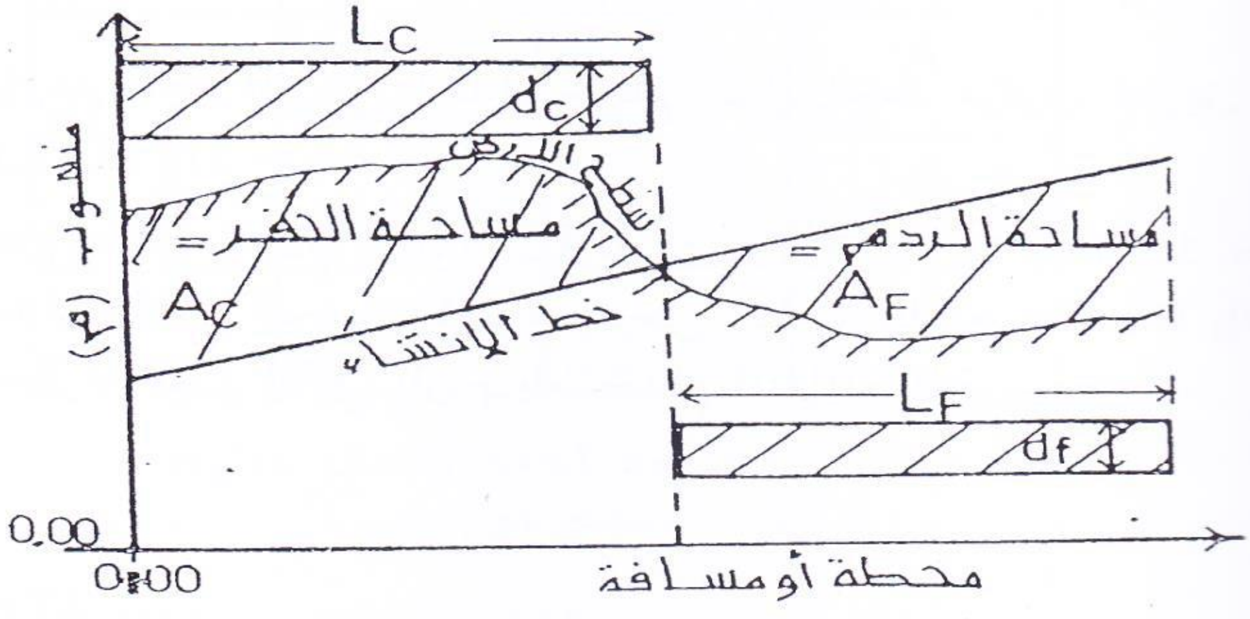
Vol. of fill (8 → 9)$=100\times[(21+24)/2]= 2250\ m^3$

T vol. of fill $= 3730 + 2250 + 500 = 6480\ m^3$

3) APPROXIMATE METHOD FOR COMPUTING VOLUME FROM PROFILE:

This method is used to calculate volumes when the ground level on the side of the road line is equal to the level of the land on that line.

The size of the cutting and filling was also found without finding the areas of the cross-sections of the stations. As for the method of finding the volumes, it is done by following the following steps, and as in the figure:



1- Finding the area of the closed segment (Ac) by one of the familiar methods for calculating the areas of shapes (palmetres, AutoCAD, coordinates method).

2- Finding the drilling area distance (Lc), which is the calculated distance from the sum of the drilling stations.

3- Calculate the average drilling depth (dc) by dividing the drilling area (Ac) by the drilling distance (Lc).

$$dc = \frac{Ac}{Lc} \qquad df = \frac{Af}{Lf}$$

4- Calculate the area of an ideal drilling cross-section (ac) using the following relationship:

$$ac = dc\,(bc + sc \times dc)$$

5-The total volume of drilling is calculated by multiplying the drilling distance (Lc) by the area of the drilling cross section (ac .).

T VOL. OF CUT = AC * LC

Ex : Compute the (Vf & Vc) by using approximate method if :

Ac = 472 m^2, Lc = 324 m , bc = 12 m , 1/sc = 1:2
Af = 318 m^2 , Lf = 356 m , bf = 10 m , 1/Sf = 1:3

Sol : (1) Cut

$dc = \frac{Ac}{Lc}$ ⟶ 472/324 = 1.46 m

ac = dc (bc + Sc ×dc) ⟶ 1.46 (12 + 2 × 1.46) = 21.78m^2

$$Vc = ac \times Lc \longrightarrow 21.78 \times 324 = 7058\ m^3$$

(2) Fill

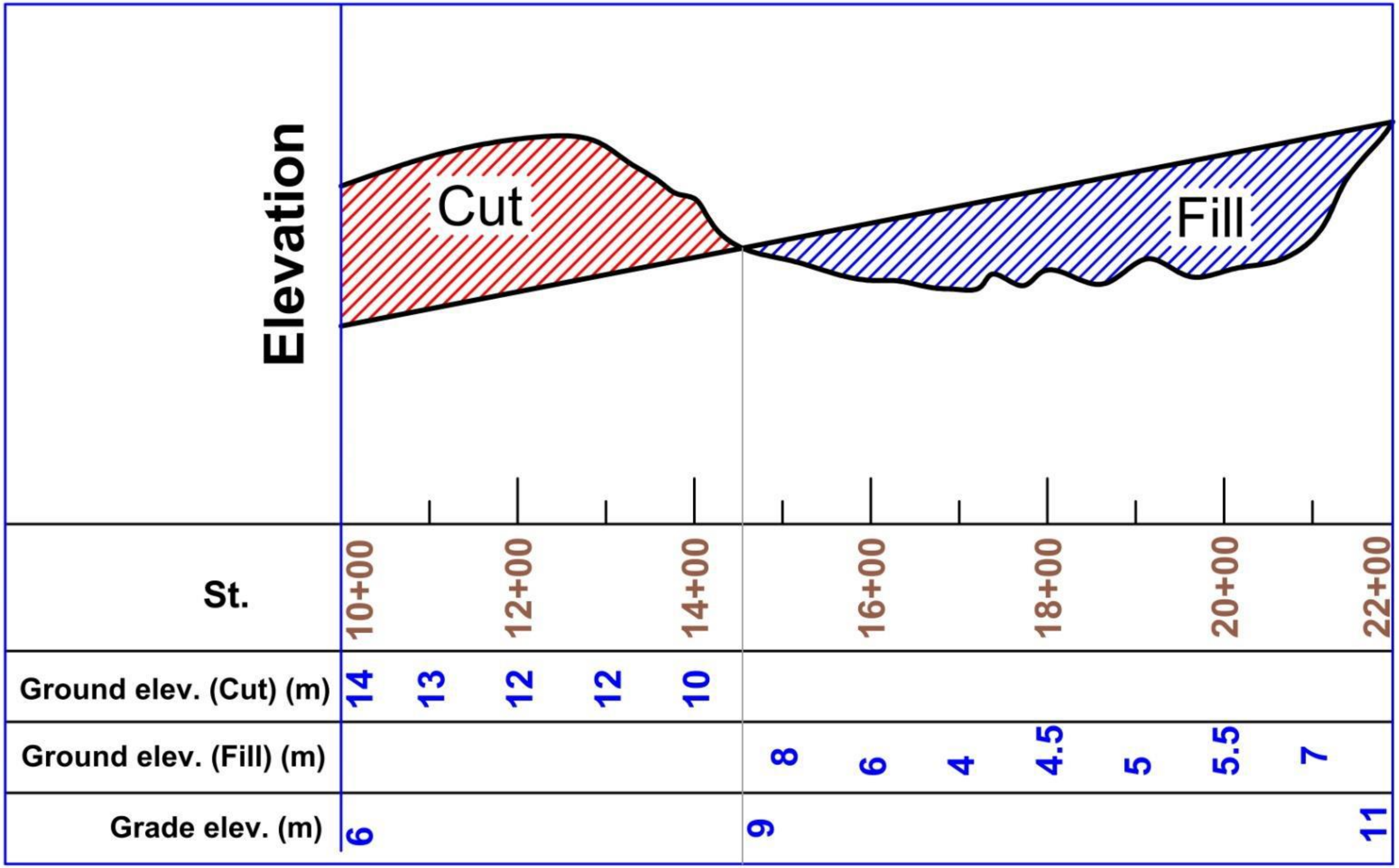
$$df = \frac{Af}{Lf} \longrightarrow 318/356 = 0.89\ m$$

$$af = df\,(\,bf + Sf \times df) \longrightarrow 0.89\,(10 + 3 \times 0.89)$$
$$= 11.28\ m^2$$

$$Vf = af \times Lf \longrightarrow 11.28 \times 356 = 4014 m^3$$

Note: The question may be in the form of a longitudinal section equipped with levels and stations, through which the coordinates of the points of the section can be calculated.. Then the area is calculated using the coordinates method, as in the following example:

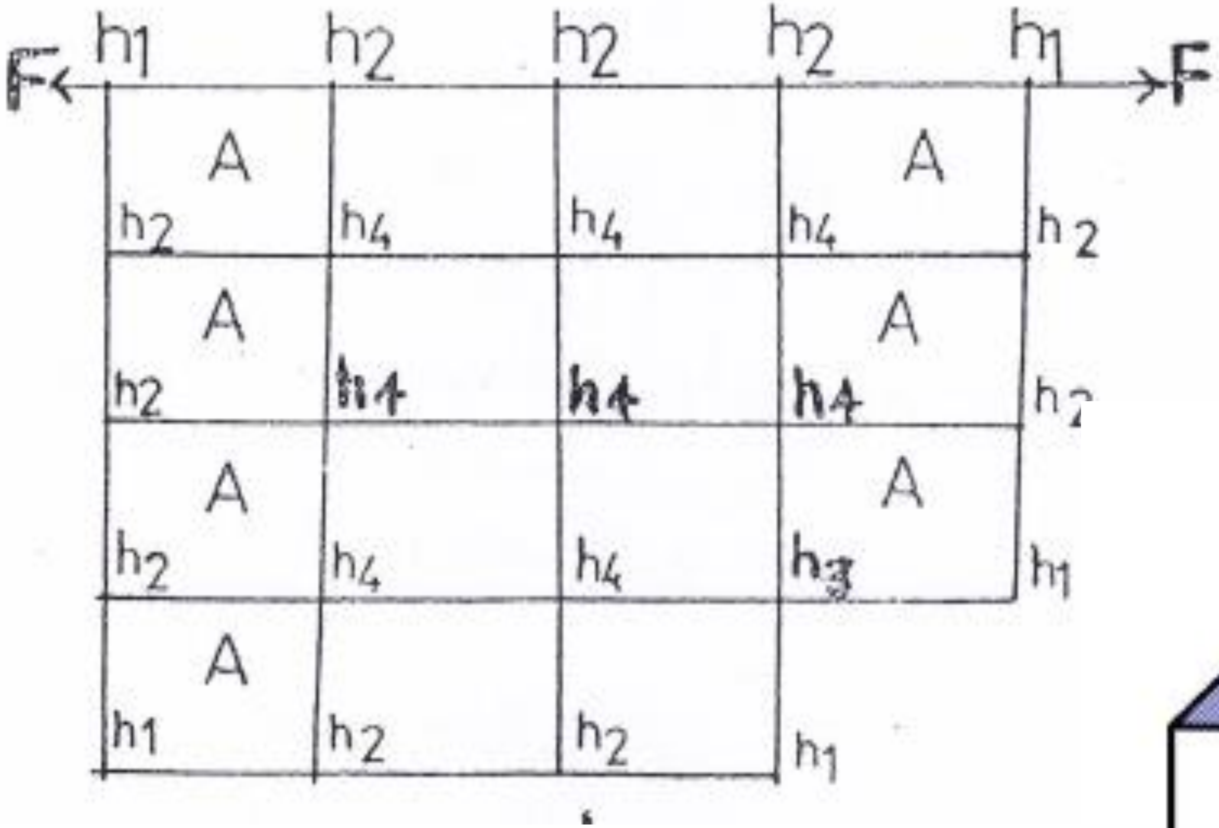
Ex : Compute the volume of Cut & Fill by using approximate & Average method for the fig. below , if you know Road width = 10 m , Side slope = 1:2 .

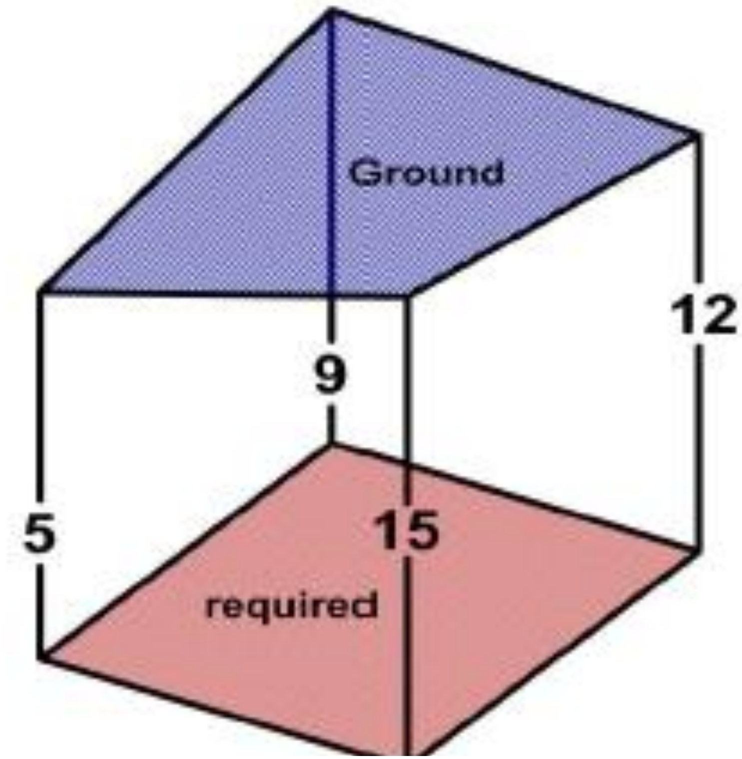


4- Volume of borrow – pit or Vol. Of spot height :

* This method is used if there is a network of squares in which corners level (elevation) are available before and after cutting (that is, the depth of cut known at the corners)

* The concept of the method is to find the average depth (average digging depth) and given the areas of the squares. The depth can be multiplied by the known area to find the volumes of the cubes, as follows:



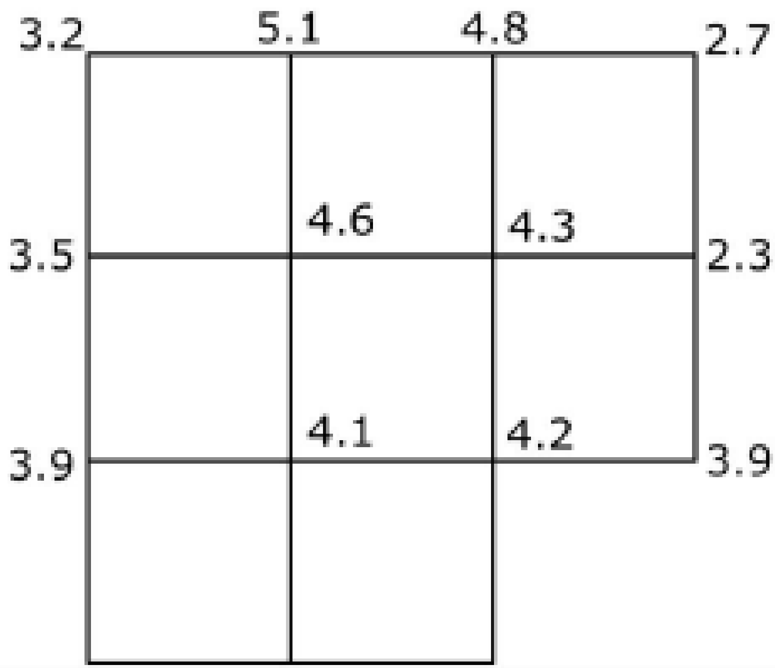


$$T_{vol.} = A * \left(\frac{\sum h1 + 2\sum h2 + 3\sum h3 + 4\sum h4}{4} \right)$$

Ex : For the fig. below , find the vol. Of cut if you know the side length of the square = 10m

Sol :

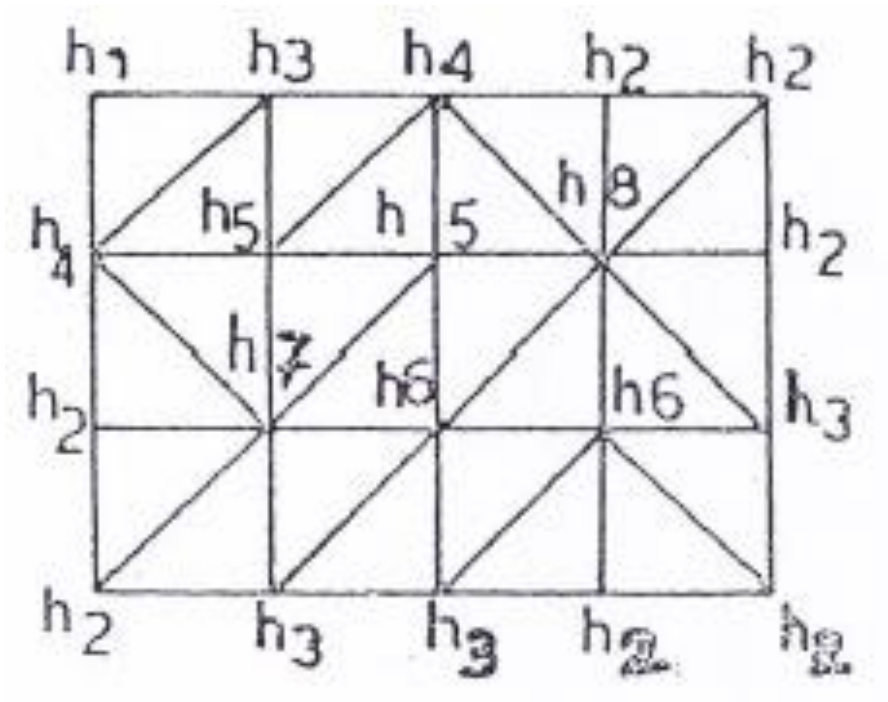
$$T_{vol.} = A * \left(\frac{\sum h1 + 2\sum h2 + 3\sum h3 + 4\sum h4}{4} \right)$$

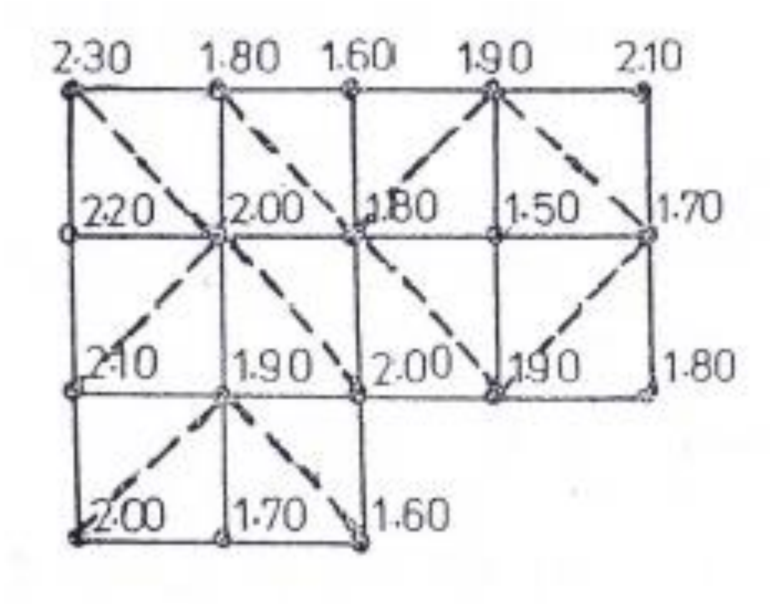


$$T_{\{VOL.} = 100 \times \left[\frac{(3.2+4.5+2.7+3.9+3.8)+2\times(5.1+4.8+2.3+3.7+3.9+3.5)+3\times(4.2)+4\times(4.3+4.6+4.1)}{4}\right]$$

$$\text{T vol. Cut} = 3232.5\ m^3$$

Note: Squares or rectangles can be divided into triangles by adding diagonals. The division is in the direction of the least cutting slope (the hypotenuse connects the two cutting depths with the least difference within one square) as in the following figure:





The mathematical relationship is as follows:

T vol. Of cut=

$$\frac{A_t}{3} \times \left(\sum h_1 + 2 \times \sum h_2 + 3 \times \sum h_3 + 4 \times \sum h_4 + 5 \times \sum h_4 + 6 \times \sum h_6 + 7 \times \sum h_7 + 8 \times \sum h_8 \right)$$

Ex : Compute the total volume of borrow – pit by using triangle method

Sol. : $\sum h1 = [1.5+1.5] = 3$ 2

$\sum h2 = 2[1.2+1.8+1.5+1.2+1.3] = 14$

$3 \sum h3 = 3[1.9+1.1] = 9$

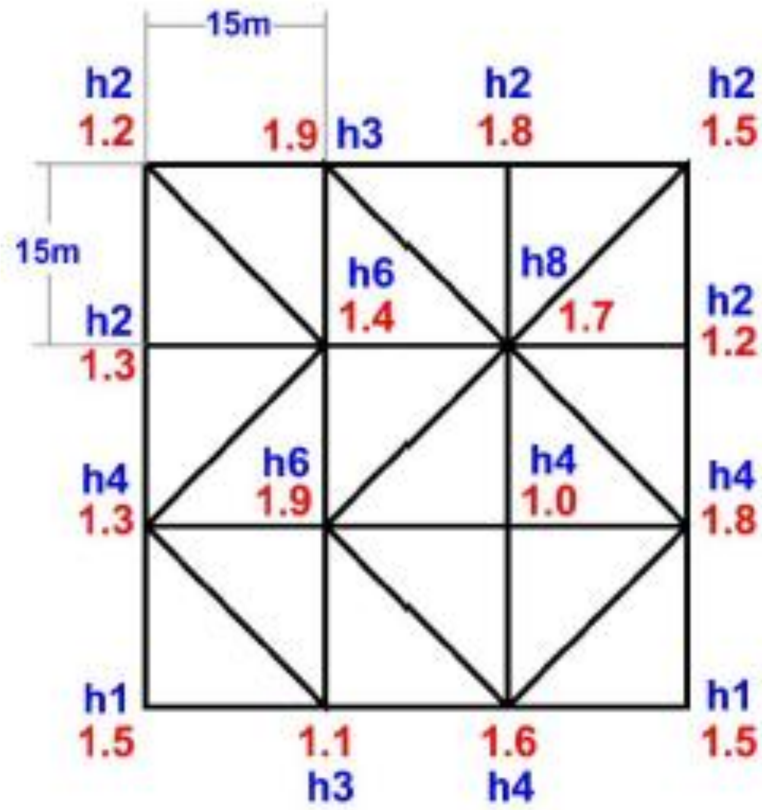
$4 \sum h4 = 4[1.3+1.0+1.8+1.6] = 22.8$

$5 \sum h5 = 5[0] = 0$

$6 \sum h6 = 6[1.4+1.9] = 19.8$

$7 \sum h7 = 7[0] = 0$

$8 \sum h8 = 8[1.7] = 13.6$



$$T_{vol.} = \frac{1/2 \times 15 \times 15}{3} [3+14+9+22.8+0+19.8+0+13.6]$$

$$T_{VOL.} == 3082\ m^3$$

5- Volume from Contour lines :

Be calculating the areas of earthen quantities through the area defined by the contour line and the period Contour and as in the mathematical relationship similar to the (Trapezoidal) method for measuring areas, and the (Average) method for calculating volumes, as follows:

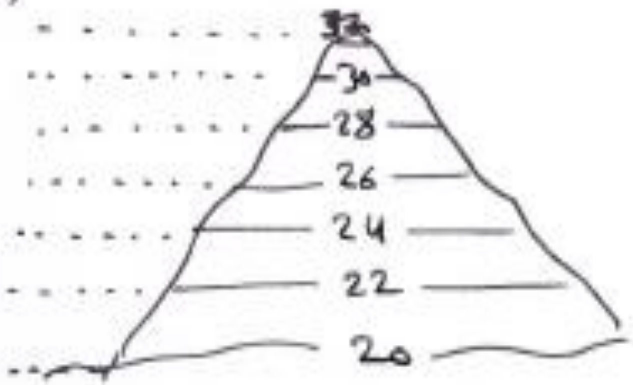
Note: the increase in the level of the contour line inward means (hills), the increment outward (low).

$$T_{vol.} = C.I\left(\frac{A1+An}{2} + A2 + A3 + \ldots\ldots + A(n-1)\right)$$

Ex1: find the volum of the Contour map if you know the C.I = 2 m & the area surrounded by the contour was

Sol.

Area (m²)
10 m²
15
22
30
42
55
67



$$T\ Vol. = 2*\left[\frac{67+10}{2}+55+42+30+22+15\right]$$

$$T_{Vol.} = 405\ m^3$$

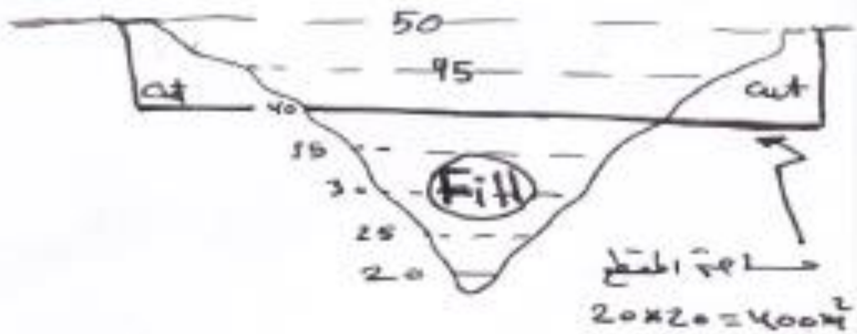
Ex2: Compute the total vol. of cut & fill, if the areas bounded by contour was as follows :-

Contour elev.	20	25	30	35	40	45	50
Area bounded m^2	60	130	200	210	240	260	320

that was to build a factories at elev. (40 m) of foundation with dimension (20 * 20) m.

$$T_{vol.\ of\ fill} = 5\left(\frac{60+240}{2} + 130+200+210\right)$$

$$T_{vol.\ fill} = 3450\ m^3$$



$T_{vol.\ cut}$:

$400 - 240 = 160\ m^2$

$400 - 260 = 140\ m^2$

$400 - 320 = 80\ m^2$

$$T_{vol.\ cut} = 5\left(\frac{160+80}{2} + 140\right) = 1300\ m^3$$

Note: In general...
The change is direct between the level of the contour line and the area means (low). The change is inverse between the level of the contour line and the area means (high).

(Q: 2) Use **Table (1) & Figure (2)** and to calculate the **total volume of cut and fill** when the ground (250mx250m) is to be leveled at **33m**. The contours areas are measured on a map scaled **1:2500** as follows.

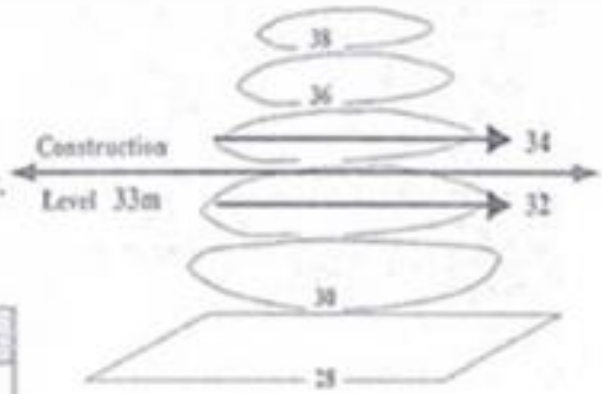


Figure (2)

Table (1)

Contour line	28m	30m	32m	33m	34m	36m	38m
Area in $(cm)^2$	100	80	70	?	50	35	25

* نحول المساحات من (cm^2) الى الأمتار المربعة

مقياس الرسم 1/2500 $\times$ $1cm = 25m$ $\therefore 1cm^2 = 625\ m^2$

$A_{28} = 62500\ m^2$ * $A_{30} = 50000\ m^2$ * $A_{32} = 43750\ m^2$

$A_{33} = 37500\ m^2$ * $A_{34} = 31250\ m^2$ * $A_{36} = 21875$

$A_{38} = 15625\ m^2$

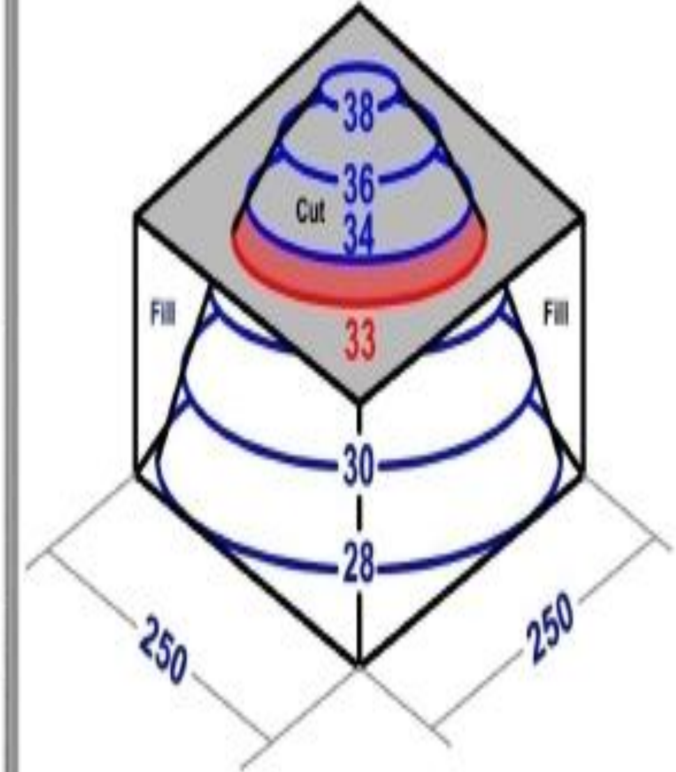
$$V_c = 2\left[\frac{a_{34} + a_{38}}{2} + a_{36}\right] + \left[\frac{a_{33} + a_{34}}{2} \times 1\right]$$

$$V_c = 125000\ m^3$$

$$V_F = a_{28}(33-28) - \left[\frac{a_{28} + a_{32}}{2} + a_{30}\right] \times 2 - \left[\frac{a_{32} + a_{33}}{2}\right] \times 1 =$$

(250x250)x(33-28)

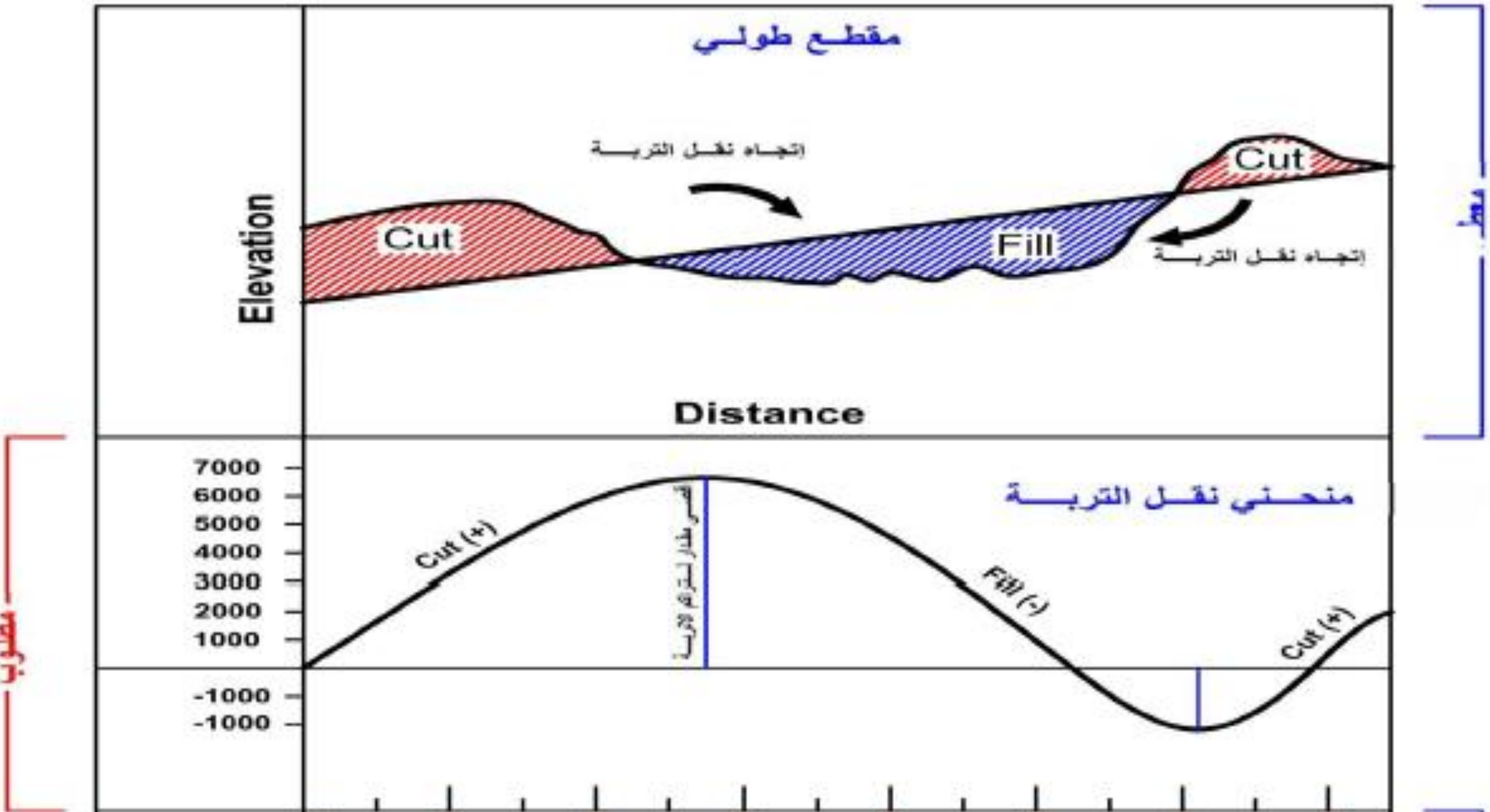
$$= \boxed{312500} - 206250 - 40625 = 65625\ m^3$$



Mass – Haul Diagram

The soil transfer scheme is a method for studying the quantities of earth resulting from excavations of the longitudinal sections of roads. It gives a general idea of the volumes of the available soil quantities and the areas of excavation and backfilling, as well as the method of balancing the excavation and backfilling quantities, and thus it is possible to calculate the costs of transporting soil and the requirements of this work.

Ex : Compute an area of cross-section by leveling work , then compute the total vol. Of cut & fill by using Avg. method , then draw the Mass – Haul Diagram after adding (10%) to the fill volume, use the data showing in the fig. Below .



St.	10+00		12+00		14+00		16+00		18+00		20+00		22+00		24+00	
Area of Cut (m2)	14	20	18	12	10								8	12	13	16
Area of fill (m2)						6	11	15	14	12	16	9				
Vol. of Cut (m3)		1700	1900	1500	1100	330							270	1000	1250	1450
Vol. of fill (m3)						200	850	1300	1450	1300	1400	1250	300			
Vol. of fill + 10%						220	935	1430	1595	1430	1540	1375	330			
Camulative Vol. (m3)	0.00	1700	3600	5100	6200	6310	5375	3945	2350	520	-620	-1995	-2055	-1055	195	1645

معط

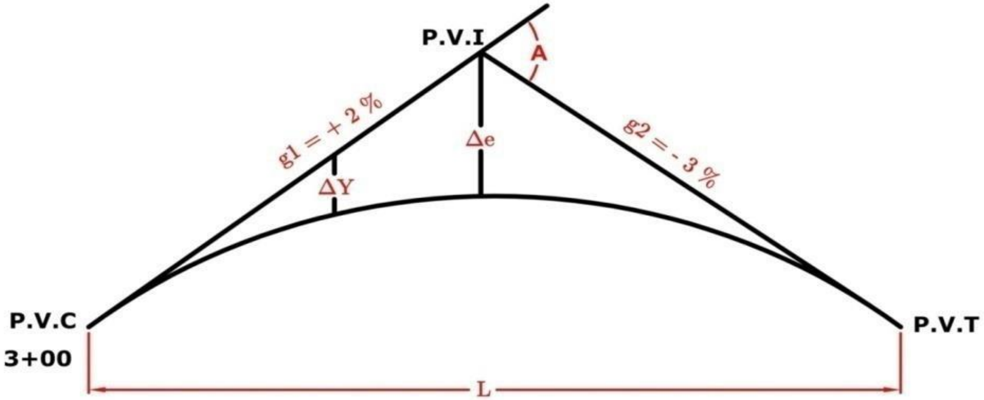
مطلوب

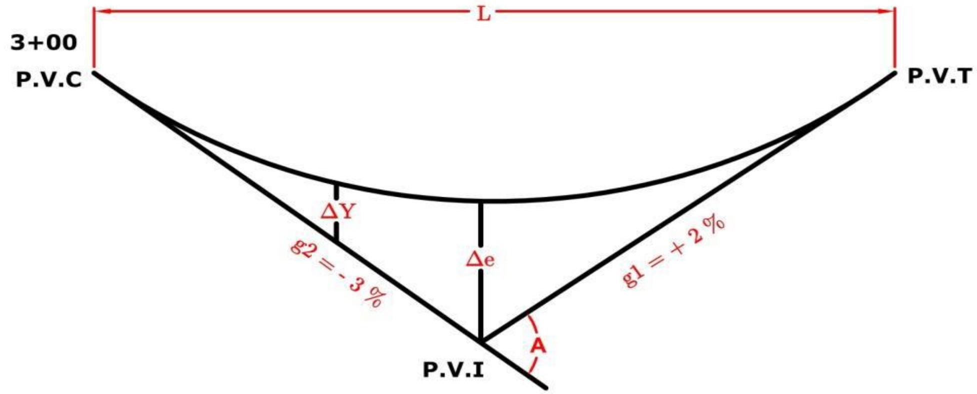
Camulative Vol. (m3)	Vol. of fill + 10%	Vol. of fill (m3)	Vol. of Cut (m3)	Area of fill (m2)	Area of Cut (m2)	St.
0.00					14	10+00
			1700			
1700					20	
			1900			
3600					18	12+00
			1500			
5100					12	
			1100			
6200					10	14+00
	220	200	330			
6310				6		
	935	850				
5375				11		16+00
	1430	1300				
3945				15		
	1595	1450				
2350				14		18+00
	1430	1300				
920				12		
	1540	1400				
- 620				16		20+00
	1375	1250				
-1995				9		
	330	300	270			
-2055					8	22+00
			1000			
-1055					12	
			1250			
195					13	24+00
			1450			
1645					16	

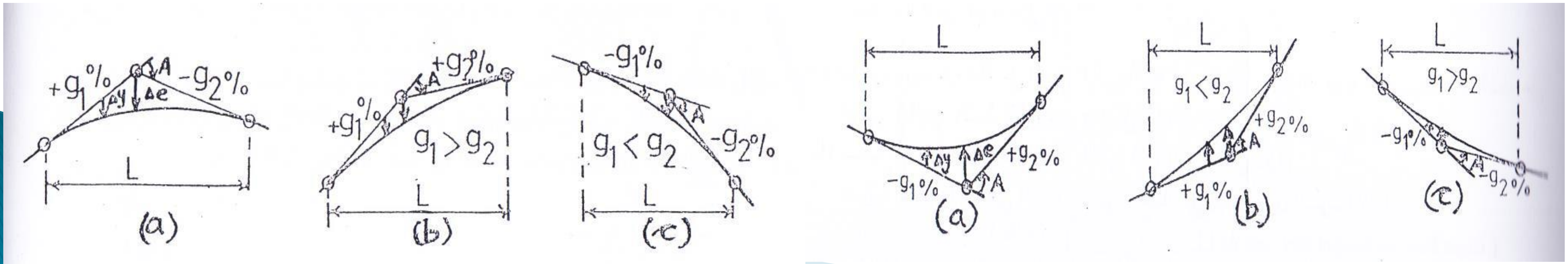
LECTURE 7 : *VERTICAL CURVES*

Steps of rout surveying

- Reconnaissance
- Preliminary survey .
- Location survey .
- Construction survey .



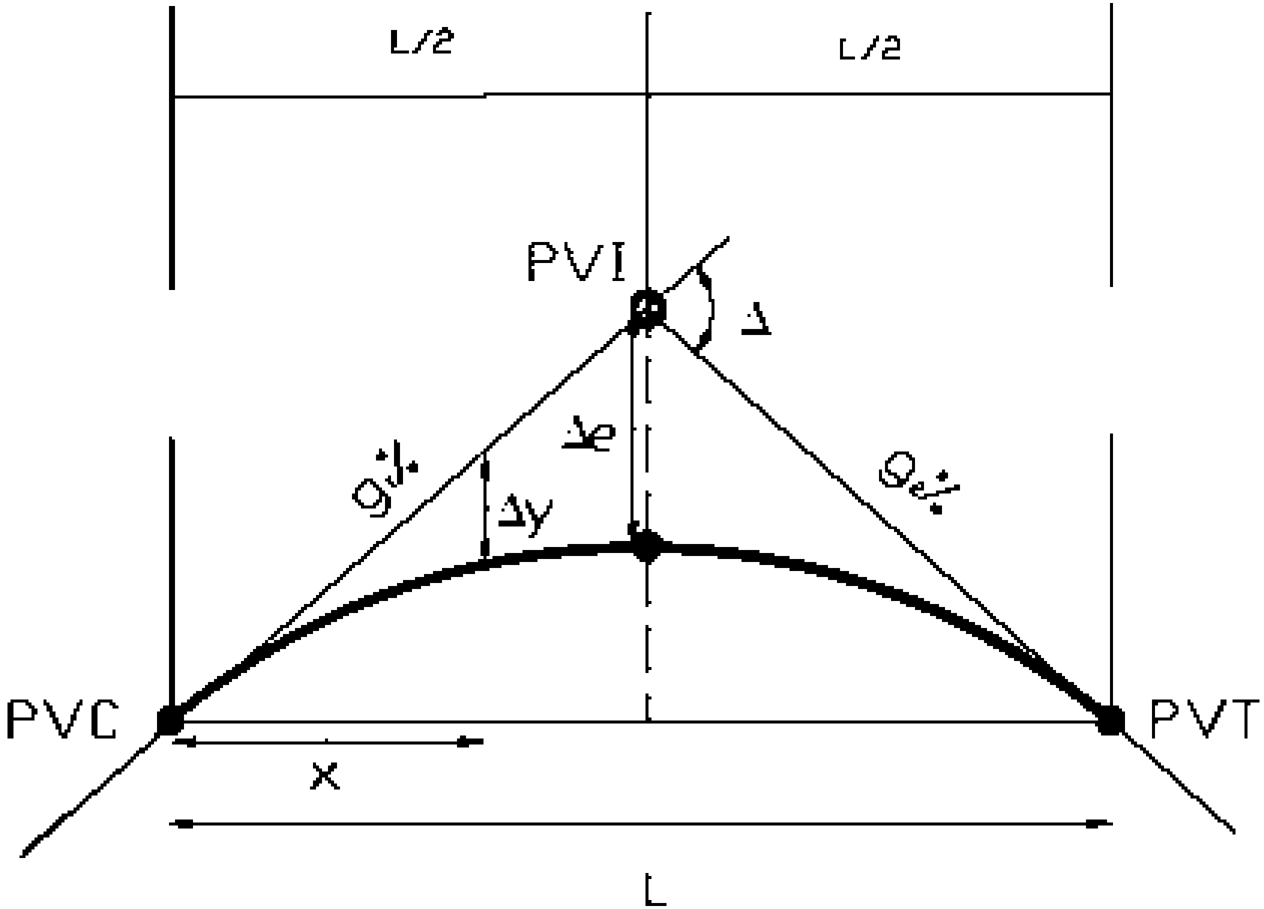




Types of vertical Curves

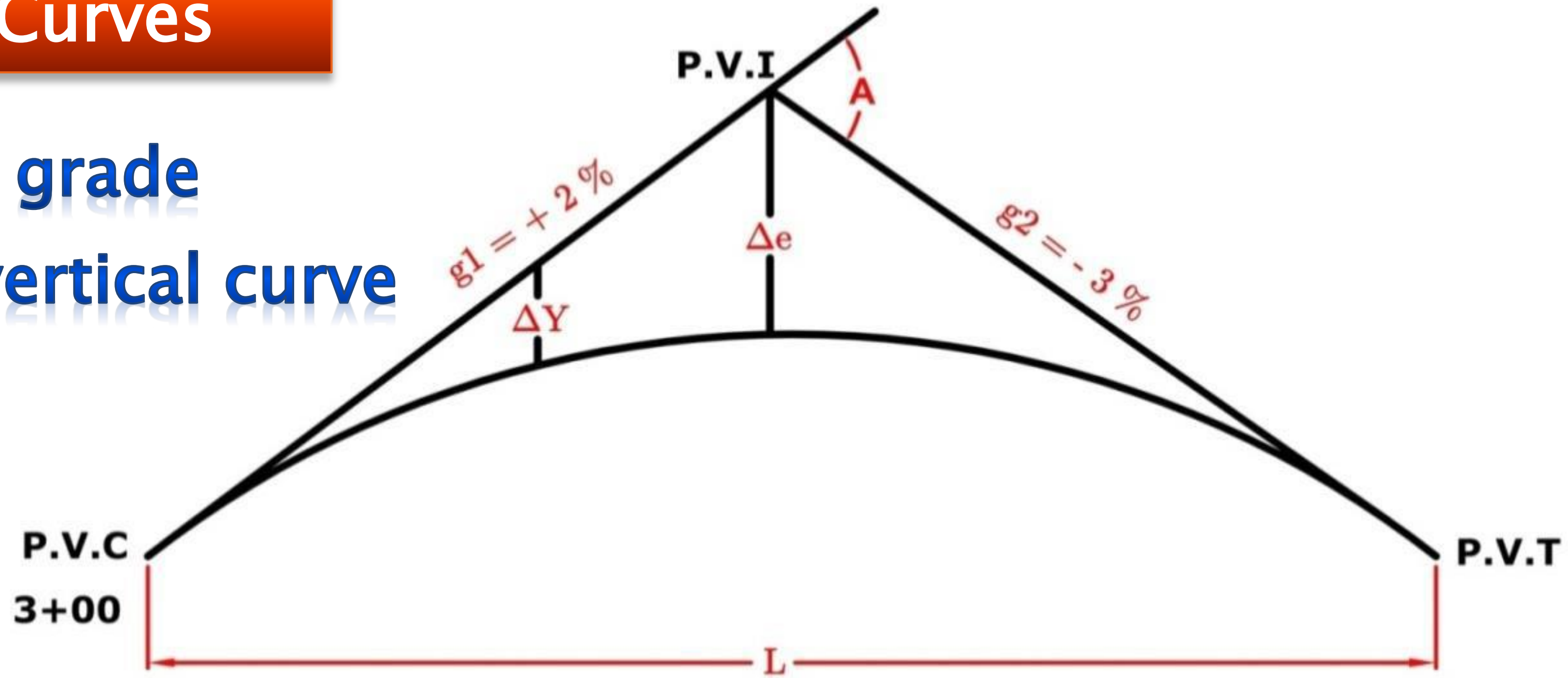
Vertical curve:-

Using to connected two lane have changeable gradient in the vertical level, that to using gradual change in vertical movement.



Types of vertical Curves

- ❑ g1,g2 : percent grade
- ❑ . L : Length of vertical curve



- ❑ r : Rate of change of grade per station.
- ❑ Δy: Difference in elevation between tangent & curve .
- ❑ Δe : Difference in elevation at P.V.I
- ❑ A : Algebraic difference in grade .
- ❑ P.V.C : Point of Vertical Curvature .
- ❑ P.V.I : Point of Vertical Intersection .
- ❑ P.V.T : Point of Vertical Tangency .

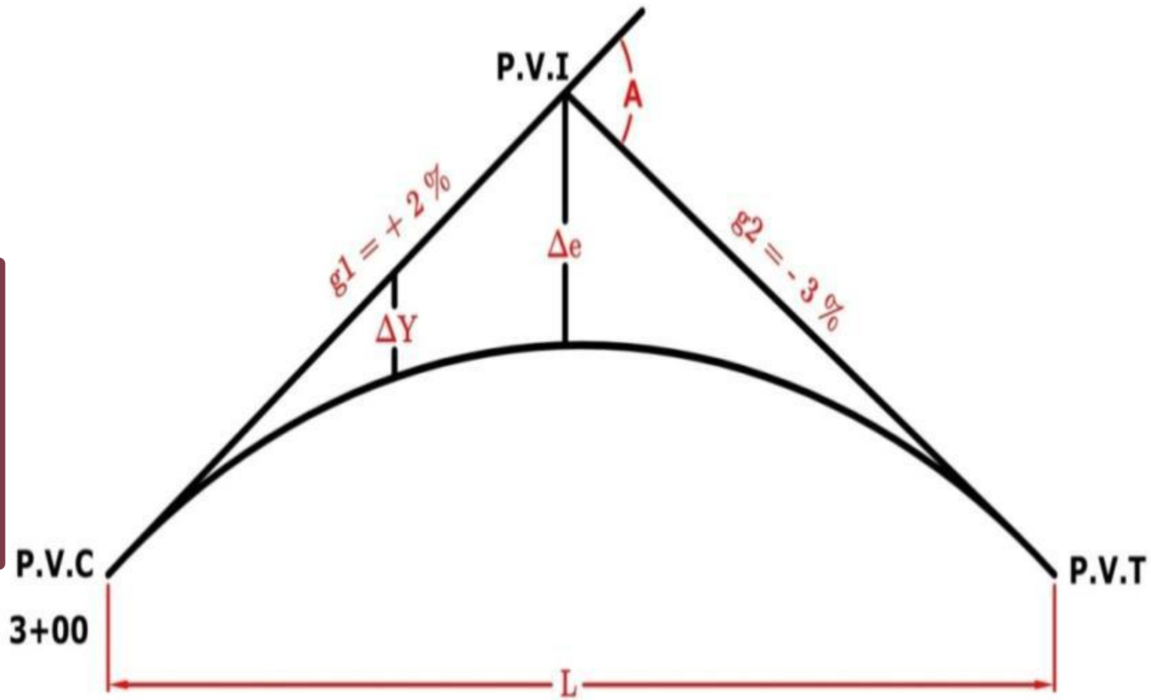
A-Geometric Method

1- A = g2 - g1 .... (1)

2- $r = \frac{A}{L}$, *where L length of curveby stations* … … . (2)

3- $\Delta y = \frac{r}{2} \times x^2$, *where x is the distance of station from start or end* … … … (3)

4- $\Delta e = \frac{A \times L}{8}$, *where L length of curveby stations* … . . .(4)



5- elev. $P.V.I = elev.P.V.C \pm \frac{g_1}{100} \times \frac{L}{2}$......(5) , where L is in meter.........(5)

6- $\mathbf{elev.}\boldsymbol{P.V.T} = \boldsymbol{elev.P.V.I} \pm \frac{\boldsymbol{g_2}}{\mathbf{100}} \times \frac{\boldsymbol{L}}{\mathbf{2}}$......(6) , where L is in meter.........(5)

7- st.$\boldsymbol{P.V.I} = \boldsymbol{st.P.V.C} + \frac{\boldsymbol{L}}{\mathbf{2}}$......(6) , where L is in meter.........(7)

8- **st**.$\boldsymbol{P.V.T} = \boldsymbol{st.P.V.I} + \frac{\boldsymbol{L}}{2}$......(6) , where L is in meter.........(8) *OR*

$$\boldsymbol{st.P.V.T = st.P.V.C + L}$$

Parabola equation (vertical curve equation)

$$y = ax^2 + bx + c$$

@ x = 0 ----- y = Elev.PVC

c = Elev.PVC

$$\frac{dy}{dx} = 2ax + b$$

$b = g_1$

@ **x = 0** **-----** $\frac{dy}{dx} = g_1$

$$\frac{d^2y}{dx^2} = 2a$$

@ x = 0 ----- $\frac{d^2y}{dx^2} = r$ a = (r/2)

$$y = \frac{r}{2}x^2 + g_1 x + Elev.PVC$$

$$\frac{dy}{dx} = rx_0 + g_1 = 0$$

$$x_0 = -\frac{g_1}{r}$$

$$y_0 = \frac{r}{2}x_0^2 + g_1 x_0 + Elev.PVC$$

Where (X_0) Represents the horizontal distance measured in stations from the beginning to the lowest or highest point and by substituting it with the following equation we get the level of the lowest or highest point on the curve

Type of vertical curve:-

1.convex curve

$A = g_2 - g_1 = (-)$

2. concave curve

$A = g_2 - g_1 = (+)$

Computation Methods of Vertical Curve:-Engineering Methods:-

Using difference elevation dependent on (r) value:-

1. Calculate the PVT, PVI, PVC terminals with their levels
2. Calculate the value of Δe, r/2,r, A,
3. Plan a table in which stations and levels are placed, as shown in the figure
4. The stations are divided according to what is required
5. Calculate the levels of the points on the first tangent using the g1 plane, PVC and the distances between the starting point and other points in the direction of PVI
6. The levels of the points on the second tangent are calculated using the g2 ratio, PVT and the distances are calculated from PVT towards PVI

station	Tangent Elev.	Difference Elev. $\Delta y = (r/2)*x^2$	Curve Elev.
PVC	Elev.PVC	0	Elev.PVC
:	:	:	:
PVI=13+70	:	Δe OK.	:
:	:	:	:
PVT=16+20	Elev.PVT	0.00	Elev.PVT

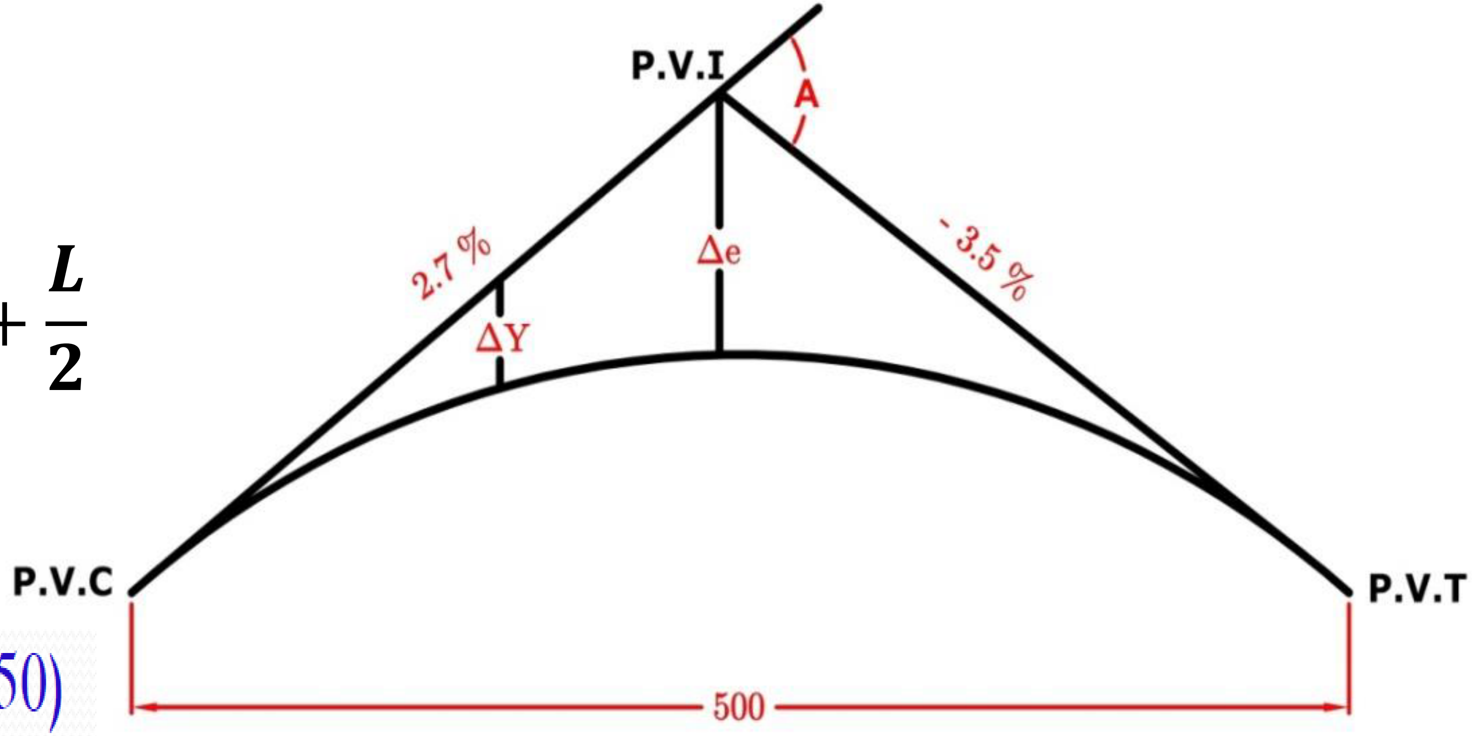
<u>Analytical method</u>

Using parabola equation (or vertical curve equation)

In this method, the levels of points are calculated directly on the curve using equation No. 7 and using the table shown below.

Station	x	$y = \frac{r}{2}x^2 + g_1 x + Elev.PVC$
PVC	0	Elev.PVC
:	:	:
PVI	$x=(L_{sta}/2)$	:
:	:	:
PVT	$x=L_{sta.}$	Elve.PVT OK.

Ex : compute the elevation of main & full stations on vertical curve , if the station P.V.I = 34+20 , L= 500 m, Elevation
P.V.I = 31.70 m.



Sol: (1)

$$St.P.V.I = St.P.V.C + \frac{L}{2}$$

$$St.P.V.C = St.P.V.I - \frac{L}{2}$$

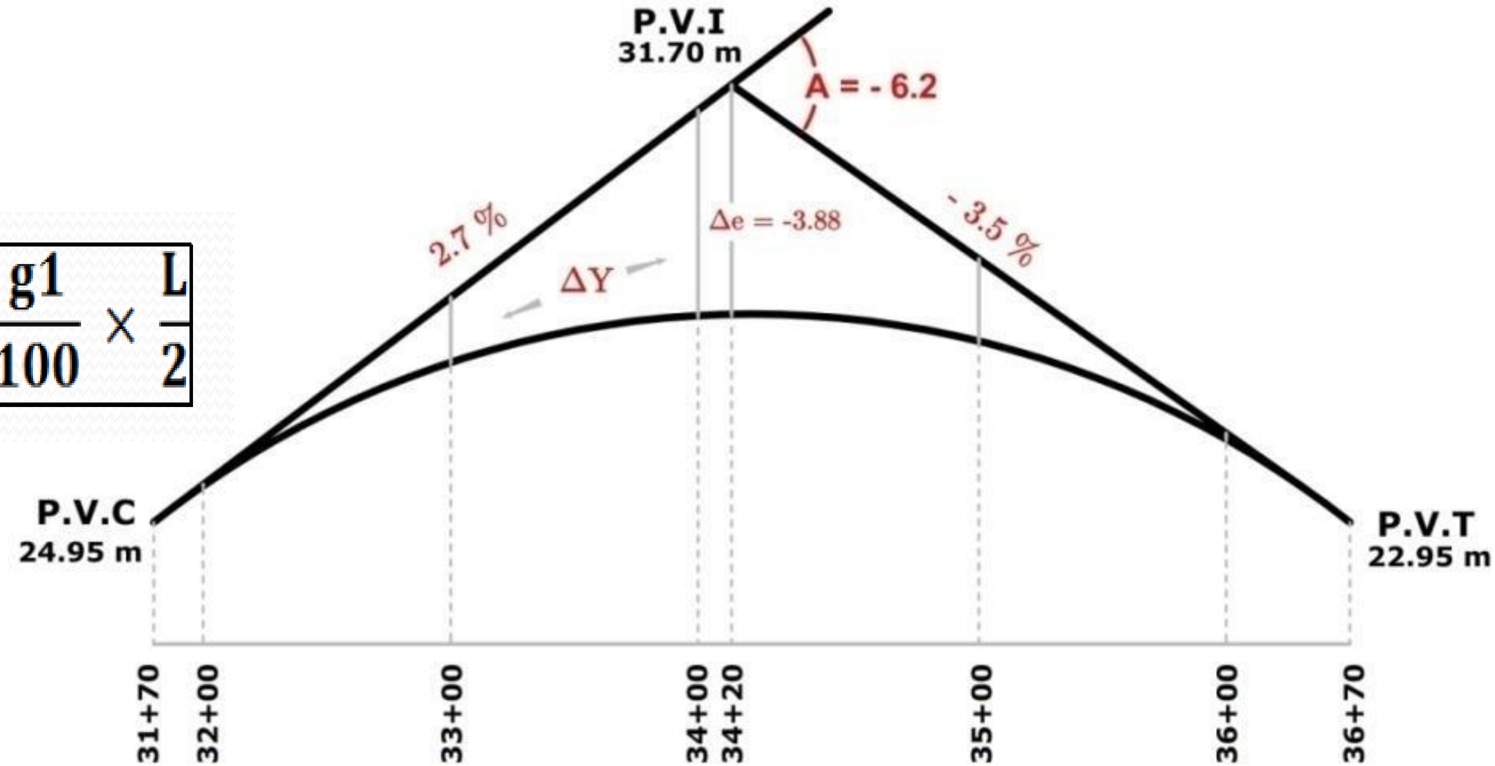
→ St. P.V.C = (34+20) – (2+50)

St. P.V.C = (31+70) .

(b) St. P.V.T = St. P.V.I + (L/2) → St. P.V.T = (34+20) +(2+50)

(2) a :

$$\text{elev. P.V.I} = \text{elev. P.V.C} \pm \frac{g1}{100} \times \frac{L}{2}$$



elev. P.V.C = elev. P.V.I – (g1/100)x(L/2)

elev. P.V.C = 31.7– (2.7/100)x(250) elev.

P.V.C = 24.95 m

b : $\text{elev. P.V.}T = \text{elev. P.V.I} \pm \frac{g2}{100} \times \frac{L}{2}$

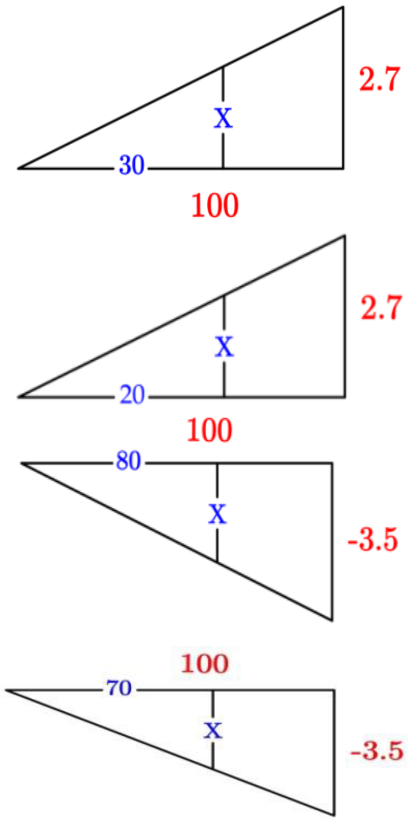
elev. P.V.T = 31.70–(3.5/100)x(250)

elev. P.V.T = 22.95m

(3) A = g2−g1 = (−3.5) – (+2.7) = −6.2

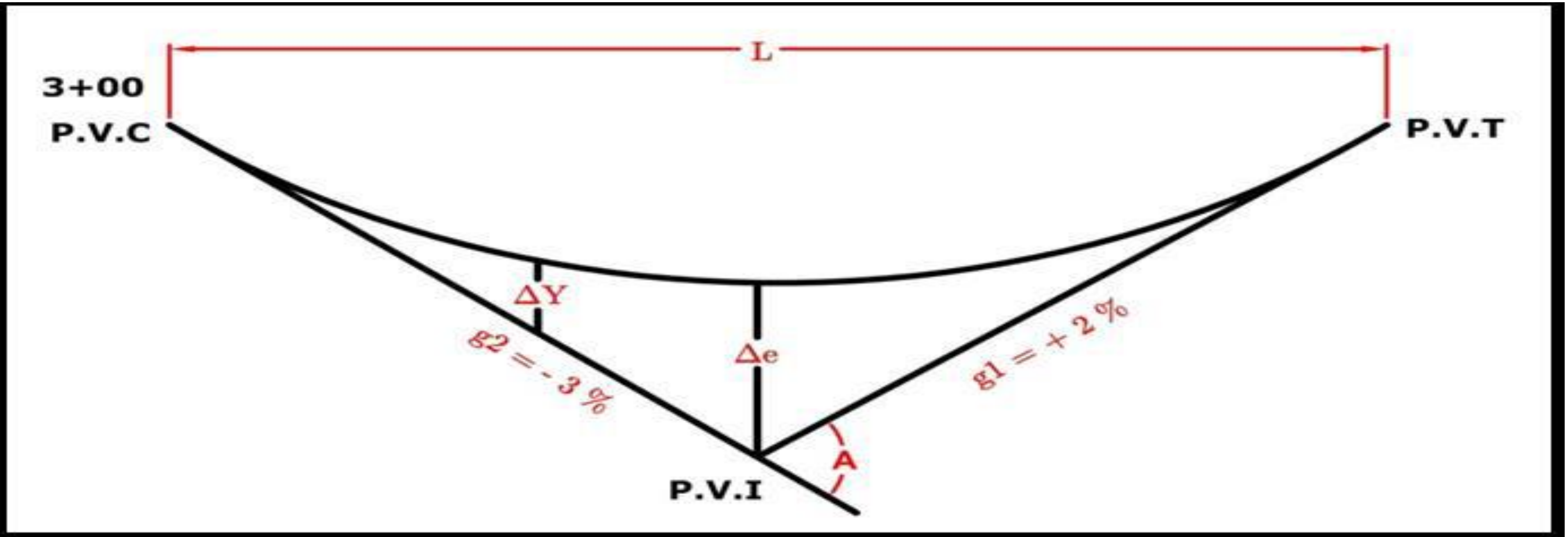
(4) r=A/L = −6.2/5 = −1.24

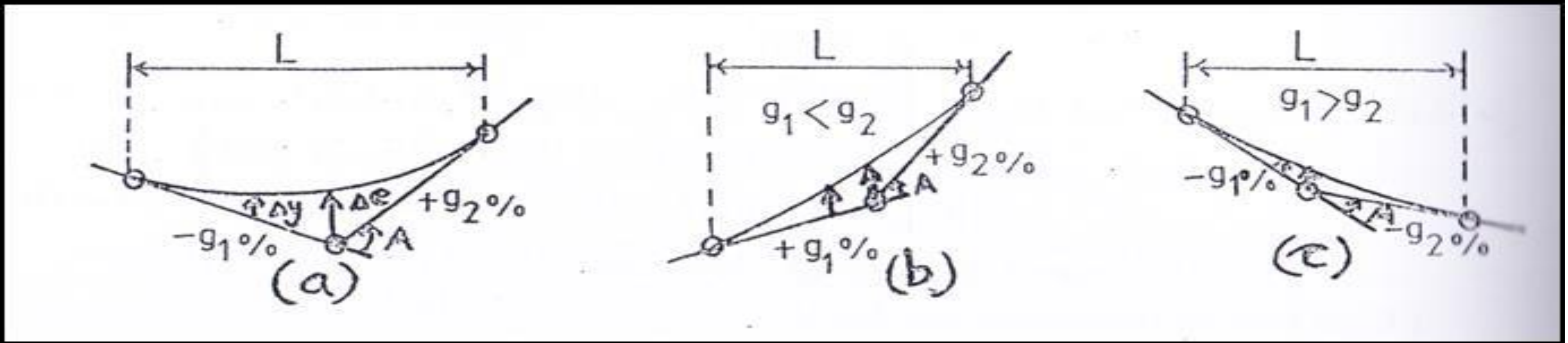
(5) $\Delta e = A \times L/8 = -6.2 \times 5/8 = -3.88$m



Station	Tangent Elev. (m)	$\Delta y = (r/2) \times X^2$	Curve Elev.(m)
P.V.C. 31+70 محسوب	24.95 محسوب	0.00	24.95
	(+2.7/100) ×30 = +0.81	-1.24 / 2	
32+00	25.76 +	$\Delta y = -0.62 \times (0.3)^2 = -0.06$ =	25.70
	(+2.7/100)×100 = +2.7		
33+00	28.46	$\Delta y = -0.62 \times (1.3)^2 = -1.05$	27.41
	(+2.7/100)×100 = +2.7		
34+00	31.16	$\Delta y = -0.62 \times (2.3)^2 = -3.28$	27.88
	(+2.7/100)×20= +0.54	$\Delta y = -0.62 \times (2.5)^2 = -3.88$	
P.V.I. 34+20 معطى	31.70 معطى	- 3.88 (To check)	27.82
		$\Delta y = -0.62 \times (2.5)^2 = -3.88$	
	(-3.5/100)×80 = -2.8		
35+00	28.90	$\Delta y = -0.62 \times (1.7)^2 = -1.79$	27.11
	(-3.5/100)×100 = -3.5		
36+00	25.40 +	$\Delta y = -0.62 \times (0.7)^2 = -0.30$ =	25.10
	(-3.5/100)×70 = -2.45		
P.V.T. 36+70 محسوب	22.95 محسوب	0.00	22.95

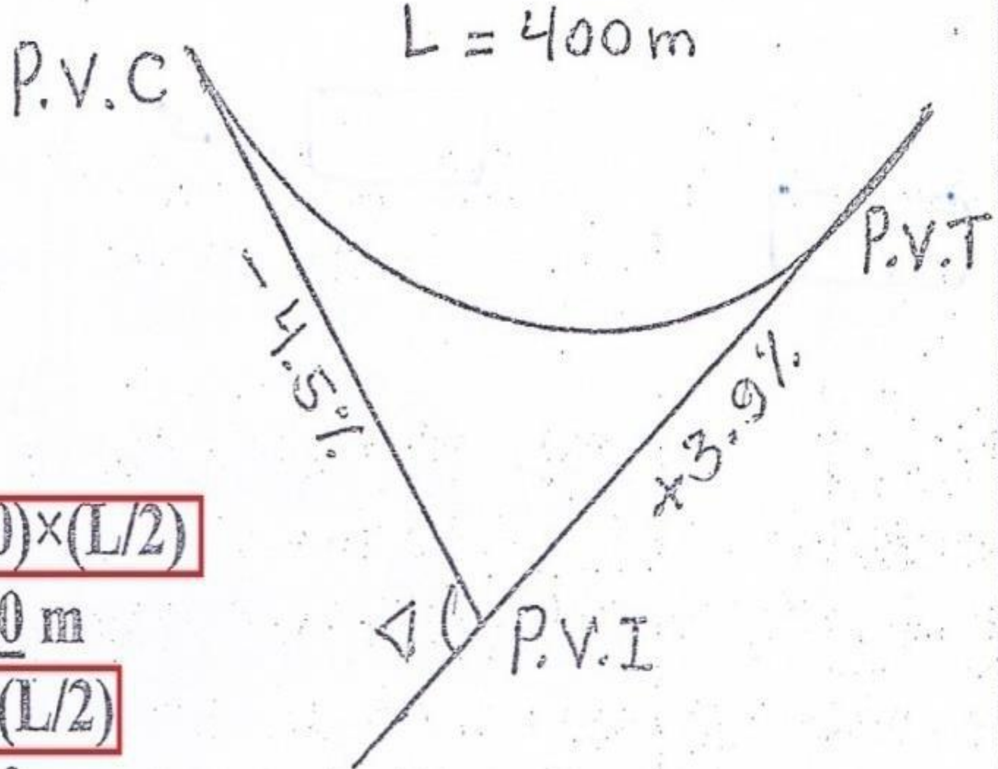
2-CONCAVE


L
3+00
P.V.C
P.V.T
ΔY
Δe
g2 = - 3 %
g1 = + 2 %
A
P.V.I


L
Δy
Δe
A
-g1%
+g2%
(a)
g1<g2
+g2%
A
+g1%
(b)
g1>g2
-g1%
-g2%
(c)

Compute the elevations of main and full stations of vertical curve if , L = 400 m
Station P.V.I. = 20+10 , Elevation P.V.I. = 40.50 , g_1 = -4.5% , g_2 = 3.9%

Solution*



Station P.V.C. = Sta. P.V.I. – L/2

Sta. P.V.C. = (20+10) – (2+00) = 18+10

Station P.V.T. = Sta. P.V.I. + L/2

Sta. P.V.T. = (20+10) + (2+00) = 22+10

Elevation P.V.C. = Elevation P.V.I. +(g_1/100)×(L/2)

Eleva. P.V.C. = 40.50 +(4.5/100)×200 = 49.50 m

Elevation P.V.T. = Eleva. P.V.I. + (g_2/100)×(L/2)

Eleva. P.V.T. = 40.50 + (3.9/100)×200 = 48.30 m

A = $g_2 - g_1$ = 3.9 – (-4.5) = +8.4 , *r = A/L = +4.8/4 = + 2.1 m , *r/2 = 1.05 m

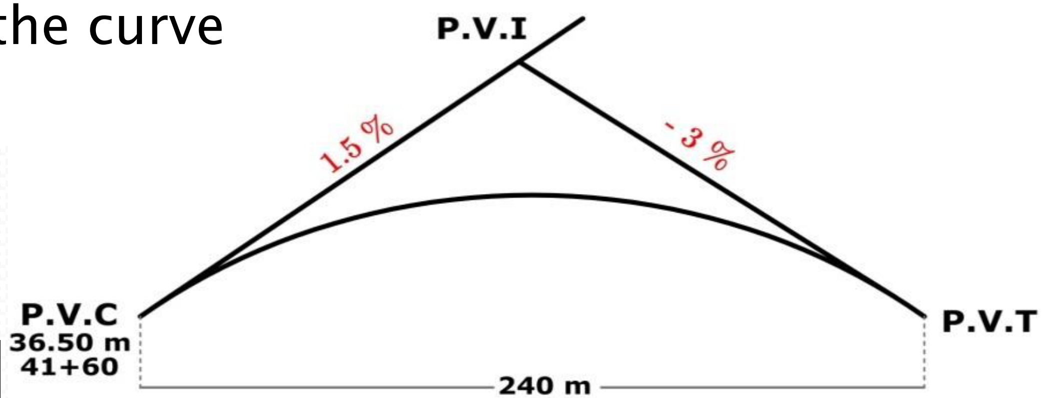
Δe = A×L/8 = +8.4×4/8 = +4.2

Station	Tangent Elevation (m)	$\Delta y = (r/2)\times X^2$ (m)	Curve Eleva.
P.V.C. 18+10	49.50	00 .00	49.50
	-4.05		
19+00	45.45	$\Delta y = 1.05\times(0.9)^2 = +0.85$	46.30
	-4.5		
20+00	40.95	$\Delta y = 1.05\times(1.9)^2 = +3.79$	44.74
	-0.45		
P.V.I. 20+10	40.50	+4.20	44.70
	+3.5		
21+00	44.01	$\Delta y = 1.05\times(1.1)^2 = +1.27$	45.28
	+3.9		
22+00	47.91	$\Delta y = 1.05\times(0.1)^2 = +0.01$	47.92
	+0.39		
P.V.T. 22+10	48.30	0.00	48.30

ANALYTICAL METHOD OR PARABOLA METHOD

In this method, the levels on the curve are calculated directly through the parabola equation.

Ex: use the following fig. to calculate the curve elevation of full stations .

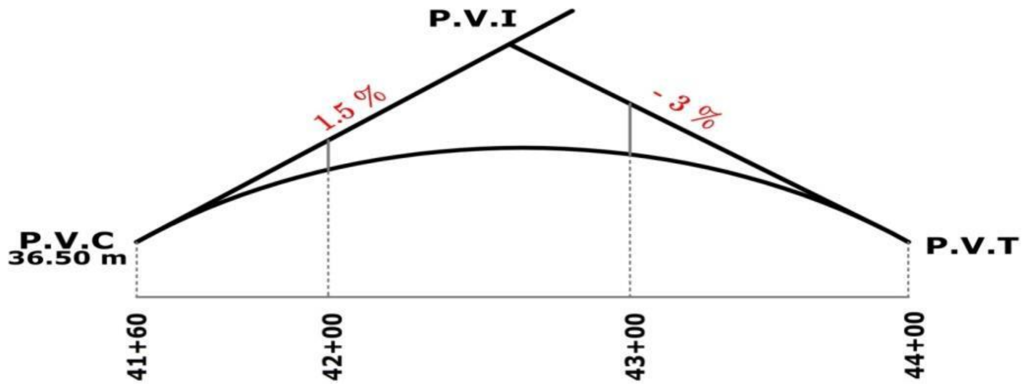


Sol : $r = \frac{A}{L} = \frac{g2 - g1}{L} = \frac{-3 - 1.5}{2.4} = -1.875$

$$y_{(42+00)} = \frac{-1.875}{2} \times (0.4)^2 + 1.5(0.4) + 36.5 = 36.952\ m$$

$$y_{(43+00)} = \frac{-1.875}{2} \times (1.4)^2 + 1.5(1.4) + 36.5 = 36.787\ m$$

$$y_{(44+00)} = \frac{-1.875}{2} \times (2.4)^2 + 1.5(2.4) + 36.5 = 34.772\ m$$



Finding the highest point (in a convex curve) and the lowest point (in a concave curve) in the analytical way.

$$X_o = \frac{-g1}{r}$$

$$y_o = \frac{r}{2} X_o^2 + g_1 X_o + elev.P.V.C$$

$X_0 =$ is the distance in stations from the start to the highest or lowest point on the curve.

y_0 =(point level) .

For the previous example, the plane of the lowest point on the curve would be:

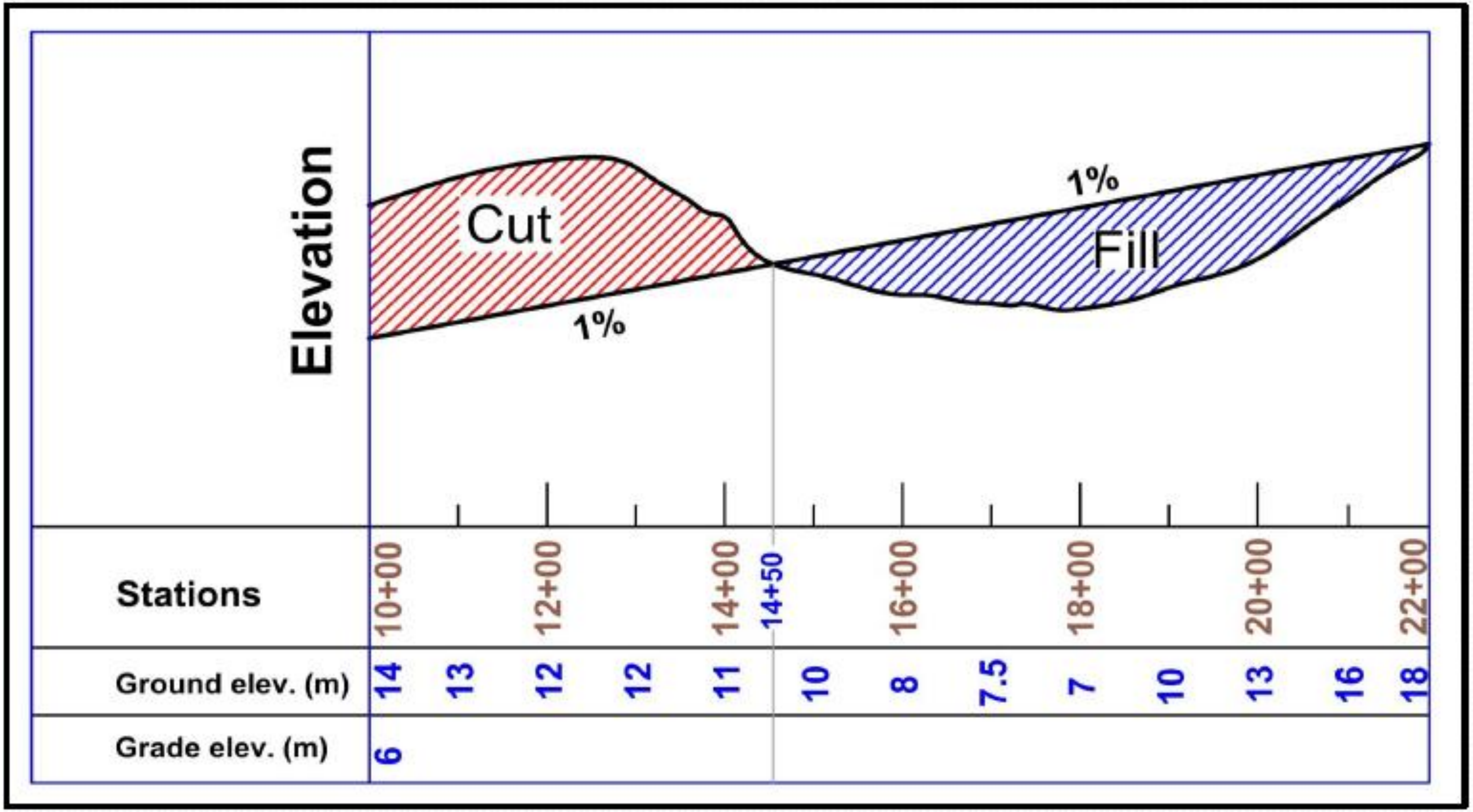
Sol : $X_o = \frac{-g1}{r}$ = -1.5 / -1.875 → (0.8) Station

St. Of highest point = (41+60) + (0+80) = (42+40)

$$y_o = \frac{r}{2} X_o^2 + g_1 X_o + elev.P.V.C$$

$$y_o = \frac{-1.875}{2} \times (0.8)^2 + 1.5(0.8) + 36.5 = 37.10\, m$$

Q : Compute the volume of **Cut** & **Fill** by using **Average** method for the fig. below **, if you know : Road width = 12 m , Side slope = 1: 3 .**

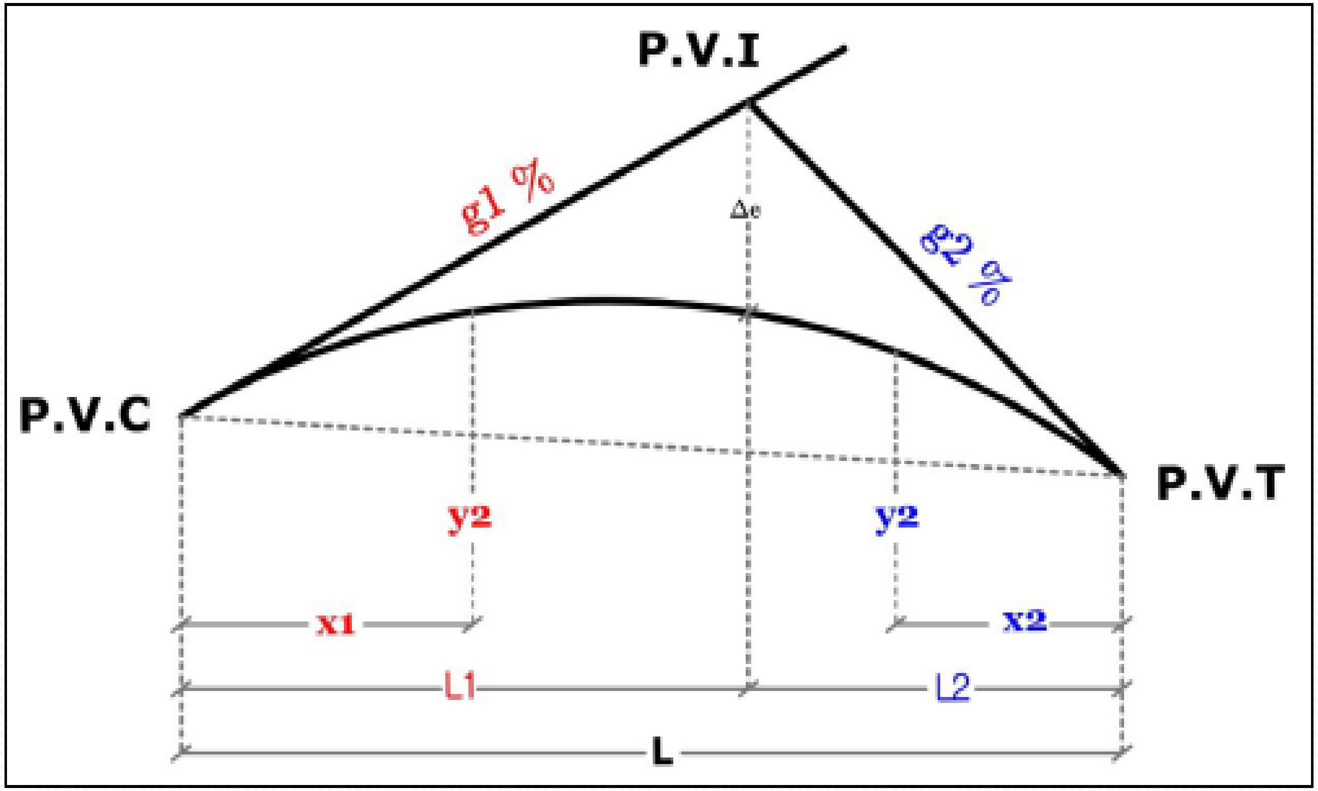


UNSYMMETRICAL CURVE VERTICAL :

$$\Delta e = \frac{g_2 - g_1}{2L} \times l_1 \times l_2$$

$$r_1 = \frac{g_2 - g_1}{L} \times \frac{l_2}{l_1}$$

$$r_2 = \frac{g_2 - g_1}{L} \times \frac{l_1}{l_2}$$



$$Elev.P.V.C = Elev.\text{P.V.I} \pm \frac{g_1}{100}(l_1)$$

$$Elev.P.V.T = Elev.\text{P.V.I} \pm \frac{g_2}{100}(l_2)$$

$$y_1 = Elev.\text{P.V.C} + g_1(x_1) + \frac{r_1}{2}(x_1)^2$$

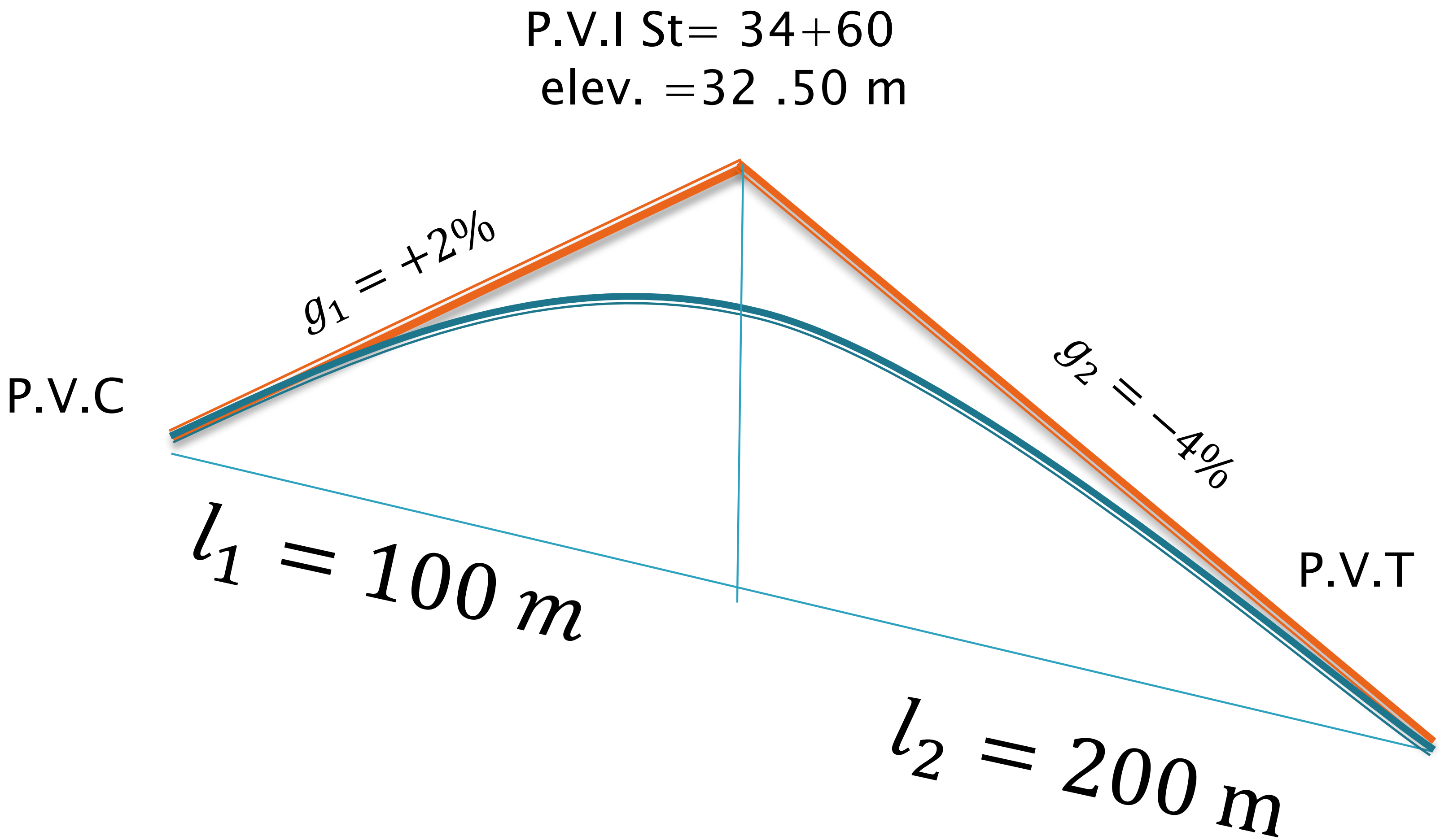
$$y_2 = Elev.\text{P.V.T} - g_2(x_2) + \frac{r_2}{2}(x_2)^2$$

NOTE: The sign surrounded by a circle above was placed in the negative direction because the calculations are in the direction of the left.. That is, the point whose level is required to be found has a negative slope $(-g_2)$
In order to obtain an increase from the point (P.V.T), a negative sign must be placed for an object attributed to (y), so it increases with respect to (P.V.T). In other words, this The reference depends on the desired affiliate site

$$\text{St. P.V.}C = \text{St. P.V.I} - l_1$$

$$\text{St. P.V.T} = \text{St. P.V.I} + l_2$$

EX:_ calculate the elevations of the half stations for the asymmetric vertical curve shown bellow using the analytical method



Sol:-

$St.pvc = St.PVI - L_1 = 12 + 60 - 1 + 00$

$= 11+60$

$St.pvt = St.PVI + L_2 = 12 + 60 + 2 + 00 = 14 + 60$

$$elev.pvc = elev.PVI - \frac{g_1}{100} \times L_1 = 34.5 - \frac{2}{100} \times 100$$

$$= 32.5\, m$$

$$elev.pvt = elev.PVI - \frac{g_2}{100} \times L_2 = 34.5 - \frac{4}{100} \times 200$$

$$= 26.5\, m$$

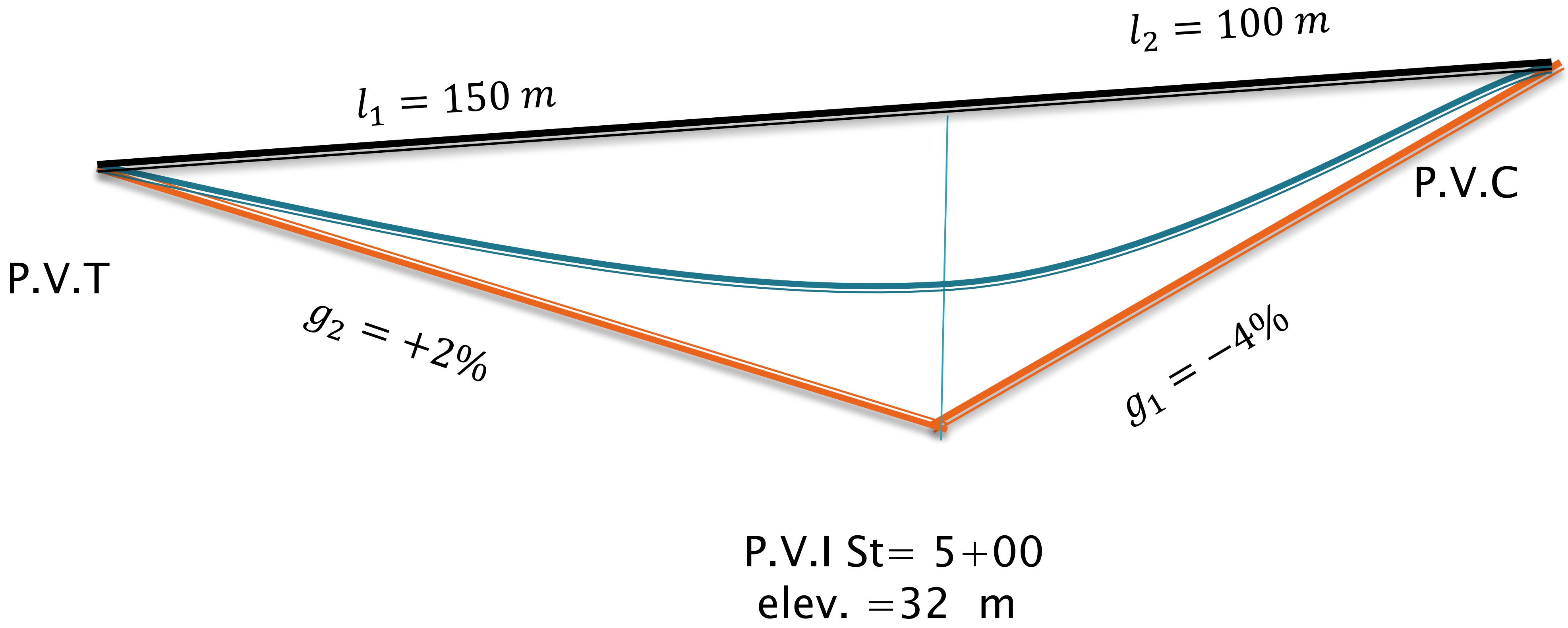
$A = g_2 - g_1 = -4 - (+2) = -6$

$r_1 = \frac{A}{L} \times \frac{l_2}{l_1} = \frac{-6}{3} \times \frac{2}{1} = -4$, $r_2 = \frac{A}{L} \times \frac{l_1}{l_2} = \frac{-6}{3} \times \frac{1}{2} = -1$

- $for\ any\ point\ in\ first\ part\ Elev = \frac{r_1}{2}(x)^2 + g_1(x) + elev.pvc$
- $for\ any\ point\ in\ second\ \ part\ Elev = \frac{r_2}{2}(\bar{x})^2 + g_1(\bar{x}) + elev.pvt$

Station	X	Elev.	Notes
P.V.C 11+60	0	32.5	Equation of the first part l_1 $= = \frac{-4}{2}(0.4)^2 + 2(0.4) + 32.5 = 32.98$
12+00	0.4	32.98	
`12+50	0.9	32.68	
P,V,I 12+60	1.00 2.00	32.5	
13+00	1.6	31.62	Equation of the second part l_2 Putted to rise form pvt $= \frac{-1}{2}(0.1)^2 - (-4)(0.1) + 26.5 = 26.895$
13+50	1.1	30.295	
14+00	0.6	28.720	
14+50	0.1	26.895	
14+60 P.V.T	0	26.5	

EX:_ calculate the elevations of the half stations for the non-symmetric vertical curve shown bellow using the geometric method , then find the station and elevation of lowest point of it



$Sol.$

$$r = \frac{A}{L} = \frac{2-(-4)}{2.5} = 2.4 \qquad r_1 = r \times \frac{l_2}{l_1} = 2.4 \times \frac{1.5}{1} = 3.6$$

$$r_2 = r \times \frac{l_1}{l_2} = 2.4 \times \frac{1}{1.5} = 1.6$$

If we work in the direction from left to right and the Axis don't rotate

$$St.PVT = St.PVI - l_1 = 5+00-1+50 = 3+50$$

$$St.PVC = St.PVI + l_1 = 5+00+1+00 = 6+00$$

$$\text{Elve.}\,PVC = St.PVI \pm \frac{g_1}{100} \times l_1 = 32 + \frac{4}{100} \times 100 = 36\,m$$

$$\text{Elve.}\,PVT = St.PVI \pm \frac{g_2}{100} \times l_2 = 32 + \frac{2}{100} \times 150 = 35\,m$$

If the solution track the direction from left to right with Axis rotation the solution be as bellow

$$r = \frac{A}{L} = \frac{2-(-4)}{2.5} = 2.4 \qquad r_1 = r \times \frac{l_2}{l_1} = 2.4 \times \frac{1.5}{1} = 3.6$$

$$r_2 = r \times \frac{l_1}{l_2} = 2.4 \times \frac{1}{1.5} = 1.6$$

$$St.PVC = St.PVI - l_1 = 5+00 - 1+00 = 4+00$$

$$St.PVT = St.PVI + l_2 = 5+00 + 1+50 = 6+50$$

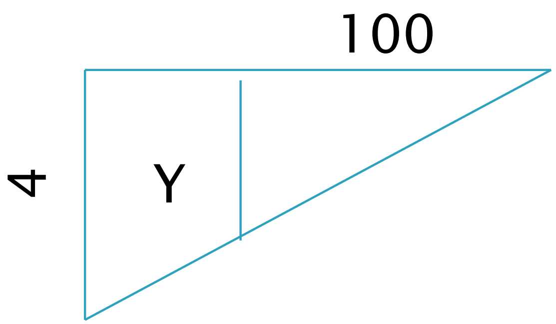
$$\text{Elve.}\, PVC = St.PVI \pm \frac{g_1}{100} \times l_1 = 32 + \frac{4}{100} \times 100 = 36\, m$$

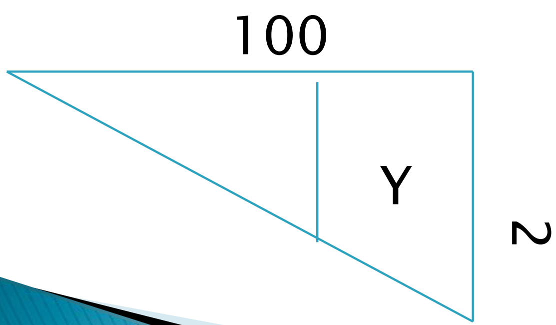
$$\text{Elve.}\, PVT = St.PVI \pm \frac{g_2}{100} \times l_2 = 32 + \frac{2}{100} \times 150 = 35\, m$$

$$\Delta e = \frac{g_2 - g_1}{2 \times L} \times l_1 \times l_2$$

$$\Delta e = \frac{2 - (-4)}{2 \times 2.5} \times 1 \times 1.5 = 1.8\ m$$

$$\Delta y_1 = \frac{r_1}{2} \times {x_1}^2$$





Station	Tangent elev.	Δy	Curve elev.
P.V.C 4+00	36	0	36
4+50	36 −2=34	$\frac{3.6}{2} \times (0.5)^2 = 0.45$	34.45
P.V.I 5+00	34−2=32.0	1.8	32+1.8=33.8
5+50	33 −1	$\frac{1.6}{2} \times (1)^2 = 0.4$	33+0.8=33.8
6+00	34.0 −1	$\frac{1.6}{2} \times (0.5)^2 = 0.2$	34+0.2=34.2
P.V.T 6+50	35.0	0	35.0

- The lowest point can found by checking from two sides
- $from\ the\ P.V.C\ station\ x_0 = \frac{-g_1}{r_1} = \frac{-(-4)}{3.6} = 1.111$ $> 1\ that\ is\ not\ ok$
- $from\ the\ P.V.T\ station\ \ x_0 = \frac{-g_2}{r_2} = \frac{-(-2)}{3.6} = 1.25$ $< 1.5\ that\ is\ ok$
- $Y_0 = \frac{r_2}{2} \times (x_0)^2 - g_2 \times x_0 + elev\ P.V.T$
- $Y_0 = \frac{1.6}{2} \times (1.25)^2 - (+2) \times 1.25 + 35 = 33.75\ M$
- Lowest point station= 36+50–(1+25)=35+25

Compute the Cut & Fill to the Vertical Curve

Working in this way required adding columns to the previous tables representing (excavation depth, area section width at each station, the total size required between each two stations), and this means that the demands

The addition will be:

1 - Depth of excavation: an appropriate calculation is required between the natural ground stations (by subtracting the natural ground level from

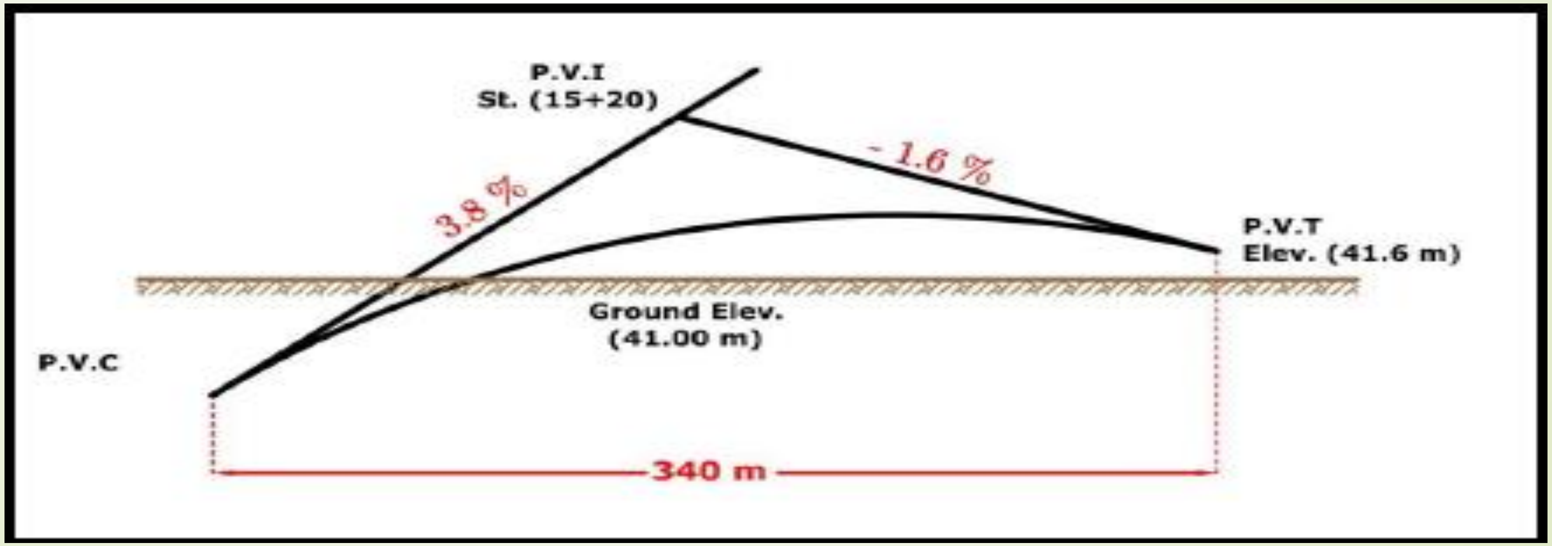
The plane of the curve, we will get the depth of excavation.

2- Width section area: it requires knowledge of (S & b) represented by the side inclination of the excavation and the width of the roads, Through the mathematical relationship A=d (b+sd), the cross-sectional areas of the stations can be obtained.

3- Volumes: It can be calculated using the average method, using the areas of two consecutive sections and the distance between them.

Sta.	Tang. Elev.	$\Delta y=(r/2)X^2$	Curve Elev.	Ground Elev.	Depth(m)		Area(m^2)		Volume(m^3)	
					fill	Cut	fill	cut	fill	Cut

EX:-Compute the total vol. of cut & fill , if the ground Exelevation is constant that equal to (41.00m) , and road width (13m) , side slope (1:4) and use the fig. below for other data .



Solution*

Sta. P.V.C. = Sta. P.V.I. – L/2
Sta. P.V.C. = (15+20) – (1+70) = (13+50)
Sta. P.V.T. = Sta. P.V.I. + L/2
Sta. P.V.T. = (15+20) + (1+70) = (16+90)

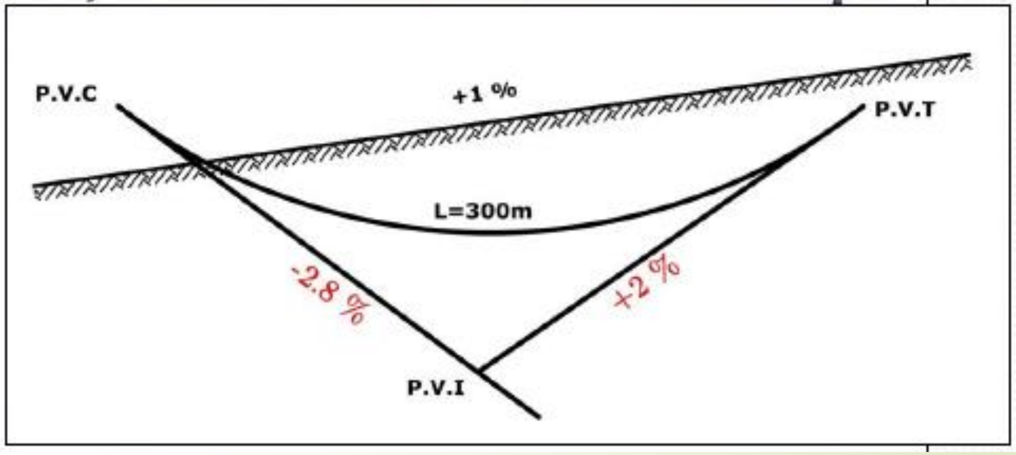
Elev. P.V.I. = Elev. P.V.T. + $(g_2/100)\times(L/2)$
Elev. P.V.I. = 41.60 + $(1.6/100)\times170$
Elev. P.V.I. = 44.32m

Elev. P.V.C. = Elev. P.V.I. – $(g_1/100)\times(L/2)$
Elev. P.V.C. = 44.32 – $(3.8/100)\times170$ = 37.86m

$A = g_2 - g_1 = -1.6 - (+3.8) = -5.4$m
$r = A/L = -5.4/3.4 = -1.59$m → $r/2 = -1.59/2 = -0.8$m
$\Delta e = A\times L/8 = -5.4\times3.4/8 = 2.3$m

Station	Tang. Elev.(m)	$\Delta y = (r/2)X^2$	Curve Elev. (m)	Ground Elev.(m)	Depth (m)		$A=d(b+S\times d)(m^2)$		Volume(m^3)	
					Fill	Cut	Fill	Cut	Fill	Cut
P.V.C. 13+50	37.86	0.00	37.86	41.00		3.14		80.26		2681.75
14+00	+(3.8/100)×50= +1.9 39.76	$\Delta y = -0.8\times(0.5)^2 = -0.20$	39.54	41.00		1.44		27.01	4 06.3	900.3
15+00	+(3.8/100)×100= +3.8 43.56	$\Delta y = -0.8\times(1.5)^2 = -1.8$	41.76	41.00	0.76		12.19		296.10	
P.V.I. 15+20	(3.8/100)×20= +0.76 44.32	= -2.3	42.02	41.00	1.02		17.42		1738.40	
16+00	(1.6/100)×80= -1.28 43.04	$\Delta y = -0.8\times(0.9)^2 = -0.65$	42.40	41.00	1.40		26.04		1587.66	
P.V.T. 16+90	-(1.6/100)×90= -1.44 41.60	0.00	41.60	41.00	0.60		9.24			

*Compute total volume of cut and fill if ground elevation at station P.V.C. = 32.50m and slope of ground elevation +1% , width of road 9m and side slope 1:2 and vertical curve on road having :



*Sta. P.V.C. = 15+60 , L = 300m

*Elev. P.V.C. = 36+40 ,

*$g_1 = -2.8\%$, $g_2 = +2.0\%$

Solution

*$A = g_2 - g_1 = +2.8 - (-2.8) = +4.8m$

*$r = A/L = +4.8/3 = +1.6 \longrightarrow r/2 = +1.6/2 = +0.8m$

*$\Delta e = A \times L/8 = +4.8 \times 3/8 = +1.8m$

*Station P.V.I. = Sta. P.V.C. + L/2

Station P.V.I. = (15+60) + (1+50) = 17+10

*Station P.V.T. = Sta. P.V.I. + L/2

Station P.V.T. = (17+10) + (1+50) = 18+60

*Elevation P.V.I. = Elev. P.V.C. – $(g_1/100) \times (L/2)$

Elevation P.V.I. = 36.40 – (2.8/100)×150 = 32.20m

*Elevation P.V.T. = Elev. P.V.I. + $(g_2/100) \times (L/2)$

Elevation P.V.T. = 32.20 +(2/100)×150 = 35.20m

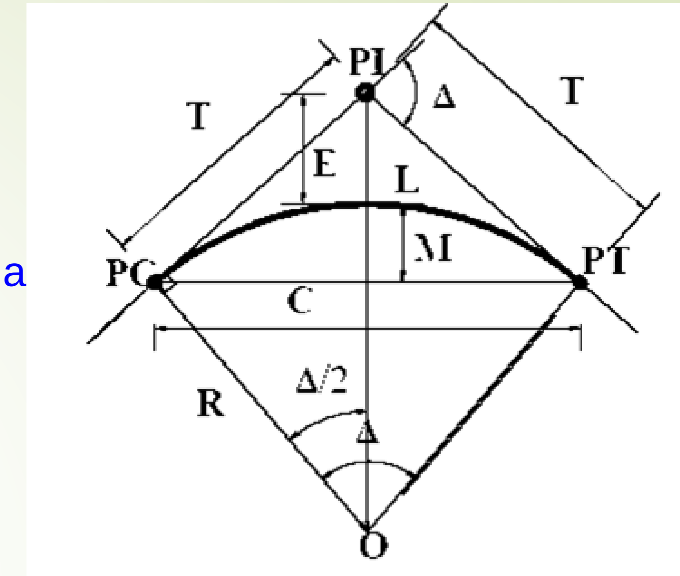
Station	Tang. Elev.(m)	$\Delta y = (r/2)X^2$	Curve Elev. (m)	Ground Elev.(m)	Depth (m)		$A = d(b+S\times d)(m^2)$		Volume(m^3)	
					Fill	Cut	Fill	Cut	Fill	Cut
P.V.C. 15+60	36.40	0.00	36.40	32.50	3.90		=3.9(9+2×3.9)=65.52		2014	
16+00	- (2.8/100)×40= -1.12 35.28	$\Delta y = 0.8\times(0.40)^2 = +0.13$	35.41	(1/100)×40 =0.40 32.90	2.51		=2.51(9+2 ×2.51) =35.19		1830.0	
17+00	= -2.80 32.48	$\Delta y = 0.8\times(1.40)^2 = +1.57$	34.05	(1/100)×100 =+100 33.90	0.15		=1.40		7.0	
P.V.I. 17+10	= -0.28 32.20	= +1.8	34.00	(1/100)×10 =+0.10 34.00	0.00		0.00	0.00		280
18+00	(2 /100)×90= +1.8 34.00	$\Delta y = 0.8\times(0.6)^2 = 0.29$	34.29	(1/100)×90 =0.90 34.90		0.61		=6.23		273
P.V.T. 18+60	+(2.0/100)×60= +1.20 35.20 Check	0.00	35.20	(1/100)×60 =0.60 35.50		0.30		2.88	∑fill= 3851	∑Cut= 553

<u>HORIZONTAL CURVE</u>

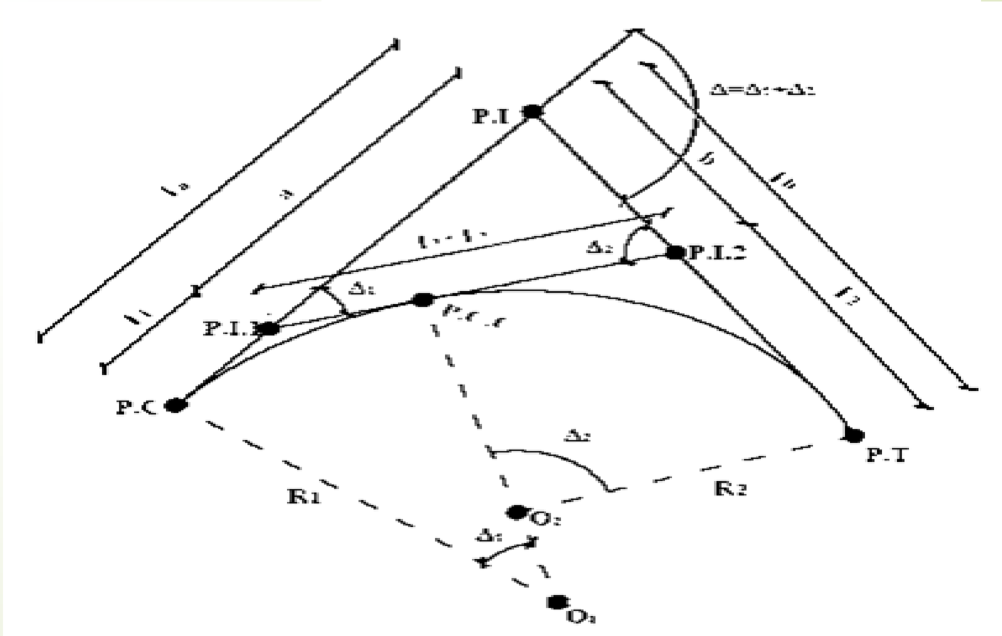
1- CIRCULAR CURVE

Horizontal curves are used to connect two paths associated with a changing Turn right or left at the horizon level gradually.

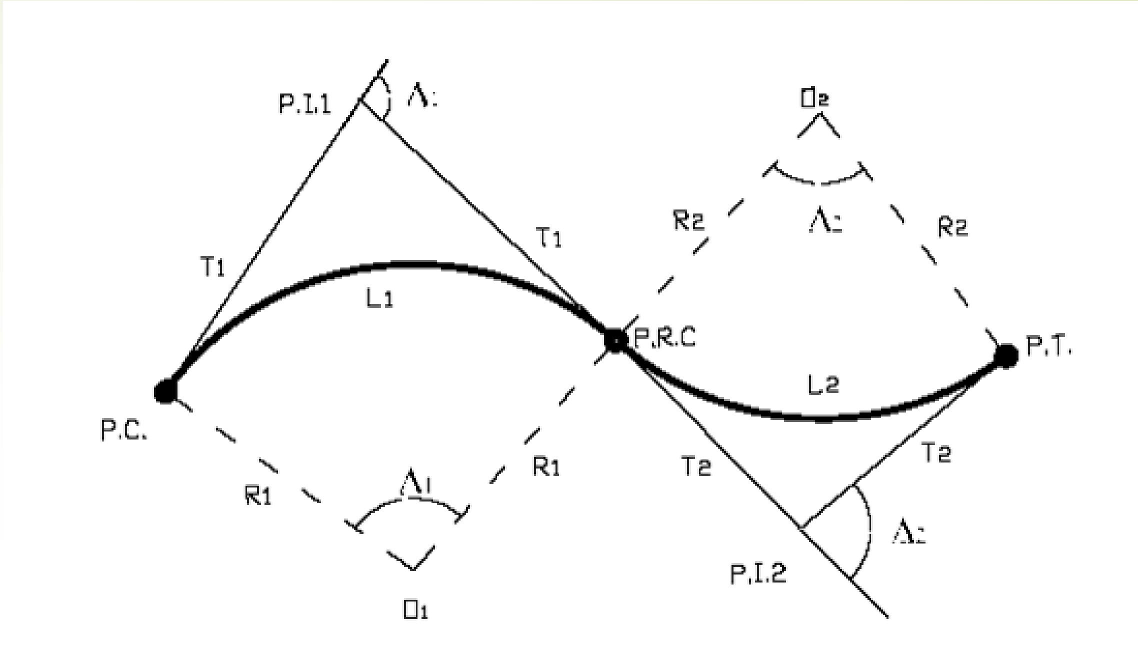
a) Simple circular curve



b) Compound Circular Curve



c) Reverse Circular Curve



2) Transition or Spiral Curve

Simple Circular Curve:-

Symbols and Characters:-

P.I. = Point of Intersection {V. (Vertex)}

P.C. = Point of Curvature {B.C. (Beginning of Curve)}

P.T. = Point of Tangency {E.C. (End of Curve)}

T. = Tangent Length

Δ. = Deflection Angle {Intersection Angle}

R. = Radius of Curve

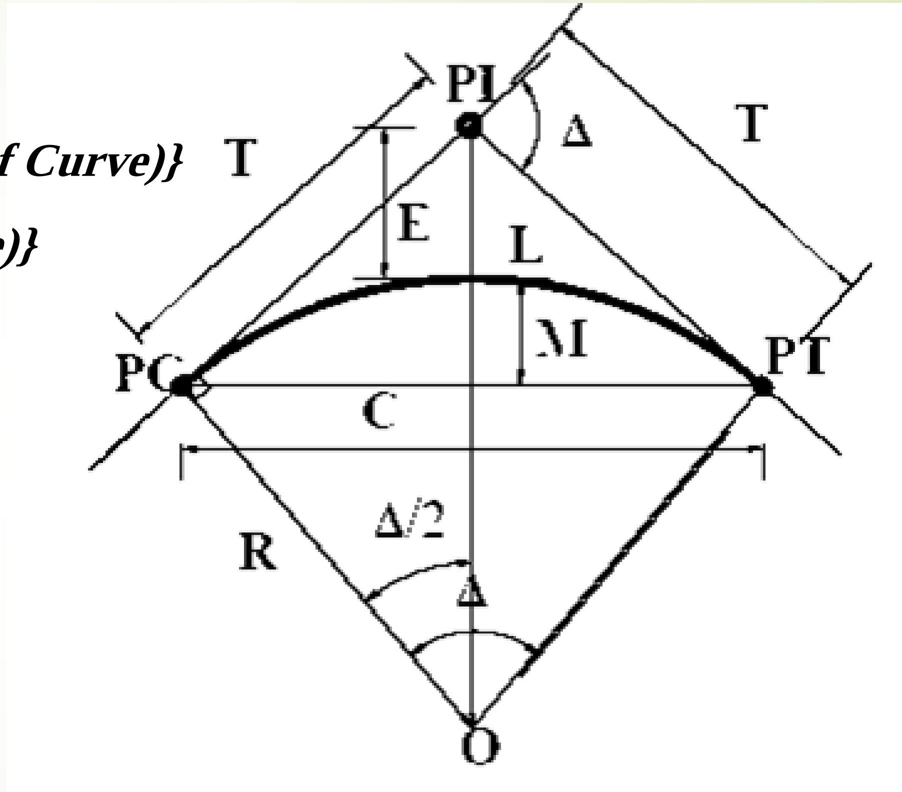
C. = Chord Length

L. = Length of Curvature

E. = External Distance

M. =Middle Distance

D = Degree of Curvature (Degree of Curve)



The degree of curvature is the angle subtended by an arc or (chord) of length 10 m

$$E = R\left(\frac{1}{\cos\frac{\Delta}{2}} - 1\right)$$

$$D = \frac{573}{R_{(m)}}$$

$$L = \frac{\pi . R . \Delta}{180} \text{ or } \frac{10\Delta}{D}$$

$$T = R \tan \frac{\Delta}{2}$$

$$M = R\left(1 - \cos\frac{\Delta}{2}\right)$$

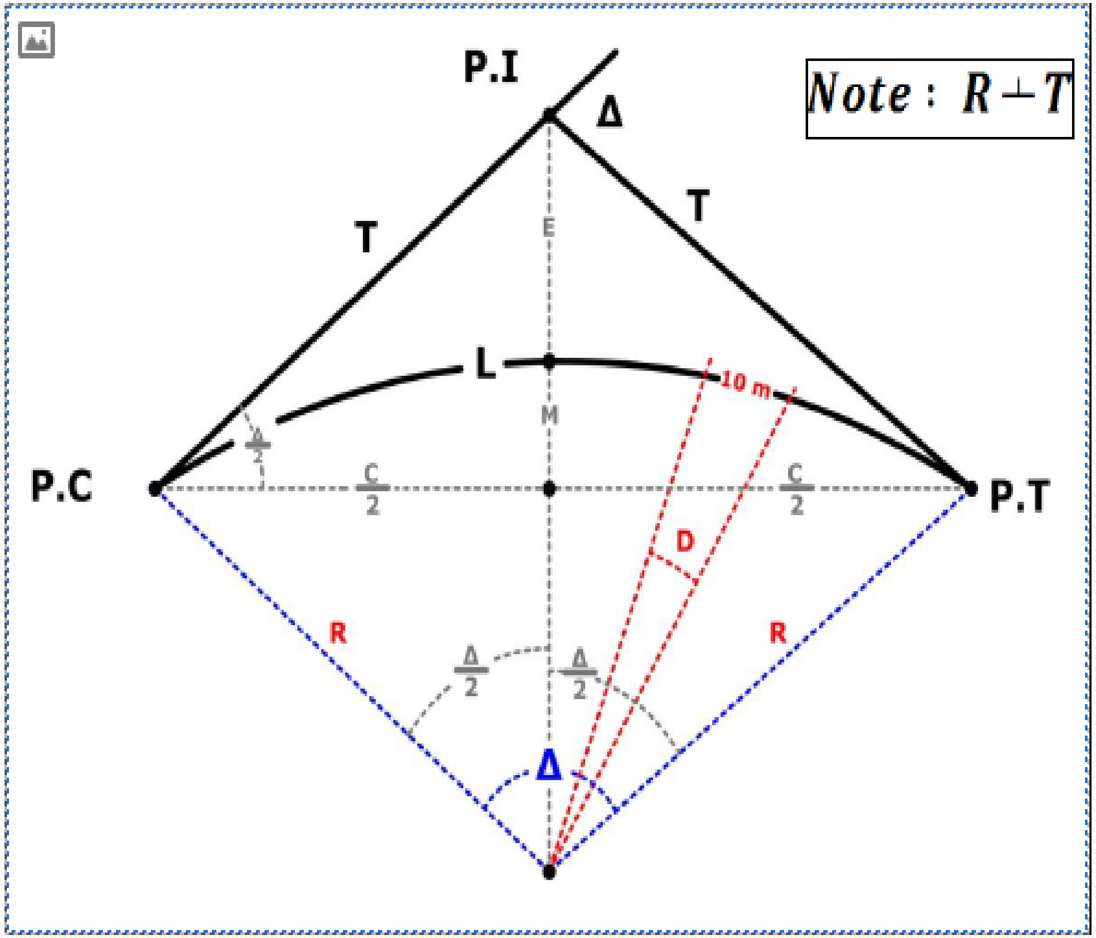
$$C = 2R \sin \frac{\Delta}{2}$$

ملاحظة : E دائما أكبر من M

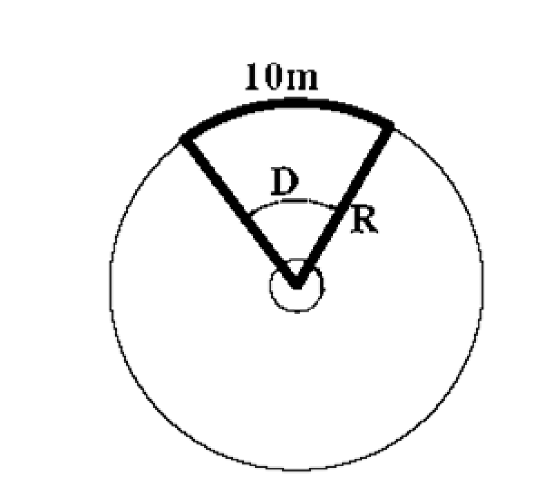
$$Sta.P.T = Sta.P.C + L$$

$$Sta.P.C = Sta.P.I - T$$

$$Sta.P.T = Sta.P.I - T + L$$



$$\frac{10}{D} = \frac{2\pi R}{360} = \frac{L}{\Delta}$$ ---------- (قياس الزوايا بالدرجات)



L=Δ.R (قياس الزاوية بالرديان) ------1a

L=(10* Δ /D) (قياس الزوايا بالدرجات) -----1c

L= Δ.R *2π/360 (قياس الزاوية بالدرجات) -----1b

$$R = \frac{10*360}{D*2\pi} = \frac{573}{D}$$ (قياس الزاوية بالدرجات)-----2a

$$D = \frac{573}{R}$$ (قياس الزاوية بالدرجات) -----2b

From triangle (PC. PI. O.)

$$\tan\frac{\Delta}{2} = \frac{T}{R}$$

T=R*tan(Δ/2) -------------- 3

From triangle (PC. P. O.)

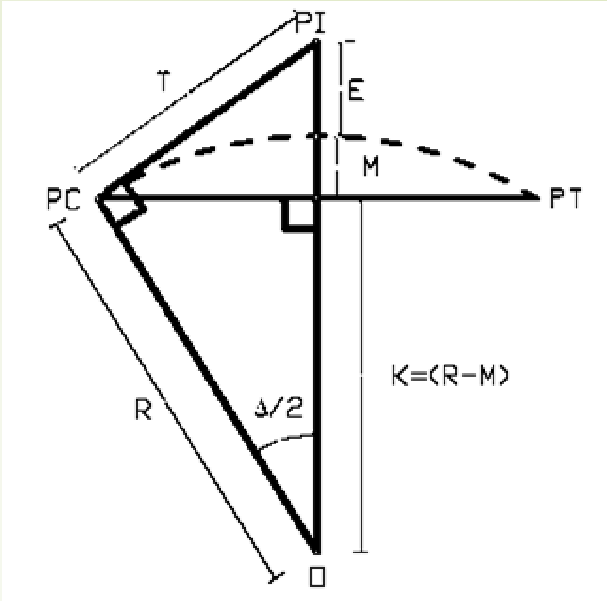
$$\sin\frac{\Delta}{2} = \frac{C/2}{R}$$

C =2R*sin (Δ/2) -------------- **4**

K = (R-M)

$$\cos\frac{\Delta}{2} = \frac{K}{R}$$

K =R*cos(Δ/2)
R–M= R*cos(Δ/2)
M = R[1– cos(Δ/2)] ---------- 5



From triangle (PC. PI. O.)

$$\sec\frac{\Delta}{2} = \frac{(R+E)}{R}$$

R+E =R*sec(Δ/2)

E = R [sec(Δ/2)–1] -------------- **6**

Note: - It is not permissible to calculate PT from (PI + T) because the length of the two tangents is not equal to the length of the curve.

$$2T > L > C$$

$$sta.PC = sta.\ PI - T$$

$$sta.PT = sta.\ PC + L$$

Example 1:-

A simple circular curve is to connect two tangent that intersection at angle Δ of 52º 36′ at station (14+80), radius of curve equal 250m.

Compute values of (C, L, T, M, E, D, PC and PT).

Solution:

$$\frac{10}{D} = \frac{2\pi R}{360} = \frac{L}{\Delta} \qquad L = \frac{\pi R}{180} 52^o 36'$$

$= 229.51m$

$D=1800/\pi R=573/250=2^o\ 17'\ 31''$

$C\ =2R*\sin\ (\Delta/2)$

$=2*250*\sin 26^o 18'=221.54m$

$T=R*\tan(D/2)$

$=250*\tan 26^o 18'=123.56m$

$E\ =R\ [\sec(\Delta/2)-1]$

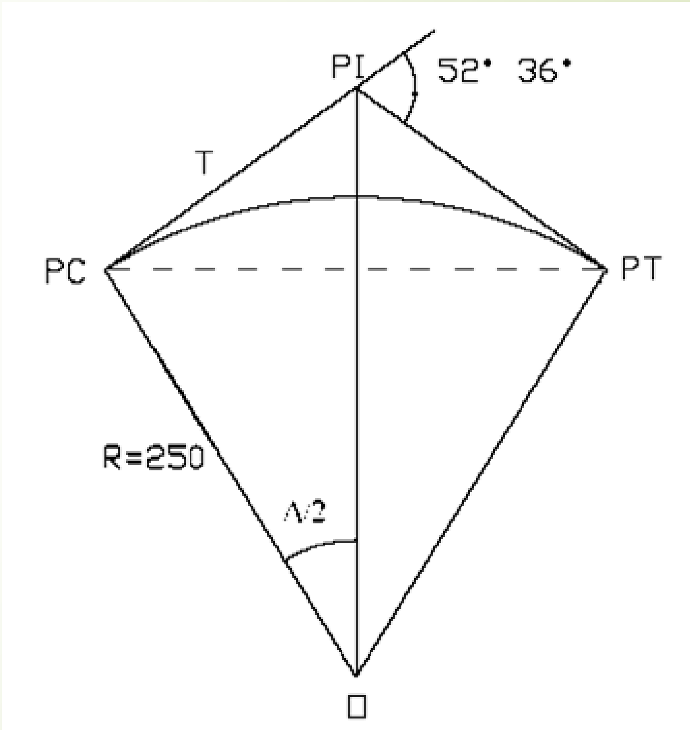
$=250(\sec 26.3-1)\ =28.87m$

$M\ =R\ [1-\ \cos(\Delta/2)]$

$=250(1-\cos 26.3)\ =25.88m$

$PC=PI-T\ =1480-123.56=13+56.44$

$PT=PC+L=1356.44+229.51=15+85.95$



Example 1:-

At a simple circular curve:

R=388m, C=287m, sta.PT=52+50.

Calculate (D, Δ, L, T, E, M, sta.PC, and sta.PT).

Solution:

$$\frac{10}{D}=\frac{2\pi R}{360}=\frac{L}{\Delta}$$

D=1800/ (π*388) = 1.48° = 1° 29'

C=2R.sin(Δ/2)

sin(Δ/2) =C/2R Δ =2sin⁻¹(C/2R)

$\Delta = 2\sin^{-1}$ [287/ (2*388)] =43.4° = 43° 24'

(Δ/2) =21.7° = 21° 42'

(L/Δ)=πR/180

L= (43.4*π*388)/180 =293.9m

T= R*tan(Δ/2)

=388*tan21.7° =154.4m

E=R [sec(Δ/2)−1]

=388 [sec21.7° −1] =29.59m

M= R [1−cos(Δ/2)]

=388*[1−cos21.7°] =27.5m

sta.PC=PT−L

=5250−293.9=49+56.1

Sta.PI=PC+T

=4956.1+14.4=51+10.5

HORIZONTAL CURVES/SIMPLE CIRCULAR CURVE

1- *Tangential angles Method or Deflections Angles Method:-*

The projection of any point on the ground requires a distance and direction from a known point, so if the point P.C is the required point, then the projection of the points of the curve requires the calculation of the distance of the chord C from point P.C in addition to the direction Ø to that point calculated from the contact line

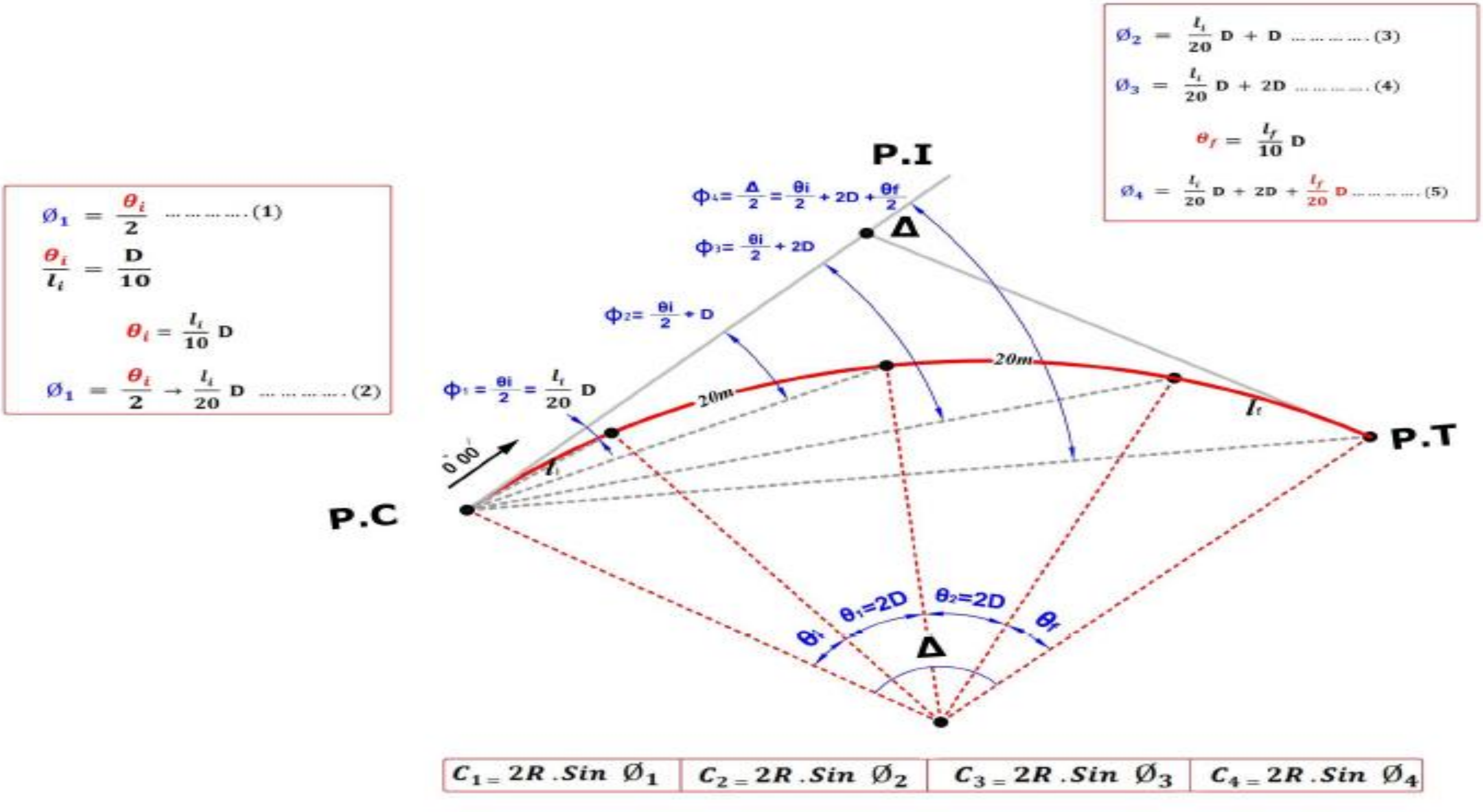
STEPS TO WORK IN THIS WAY:

1- Determine the length of the first arc: it represents the distance that must be added to station P.C in order for the first station after station P.C to become a multiple of 20.

Ex : *St. (P.C) = (15+25) → First St. = (15+40)*

And then assign the following stations for each (20m) until reaching the last station, which represents the remaining distance.

Calculating the angles of deviation (deflection) from the contact line, as shown in the following figure:



Furthermore, the values of · can be calculated from the above law. Thus, by the theodolite device, the points can be projected by horizontal angles, after pointing the device to the PI point, then the device is zeroed, after that we rotate the device according to the calculated

α values.

$$d = l * \frac{D}{10} = l * \frac{360^{o}}{2\pi R} = l * \frac{573}{R}$$

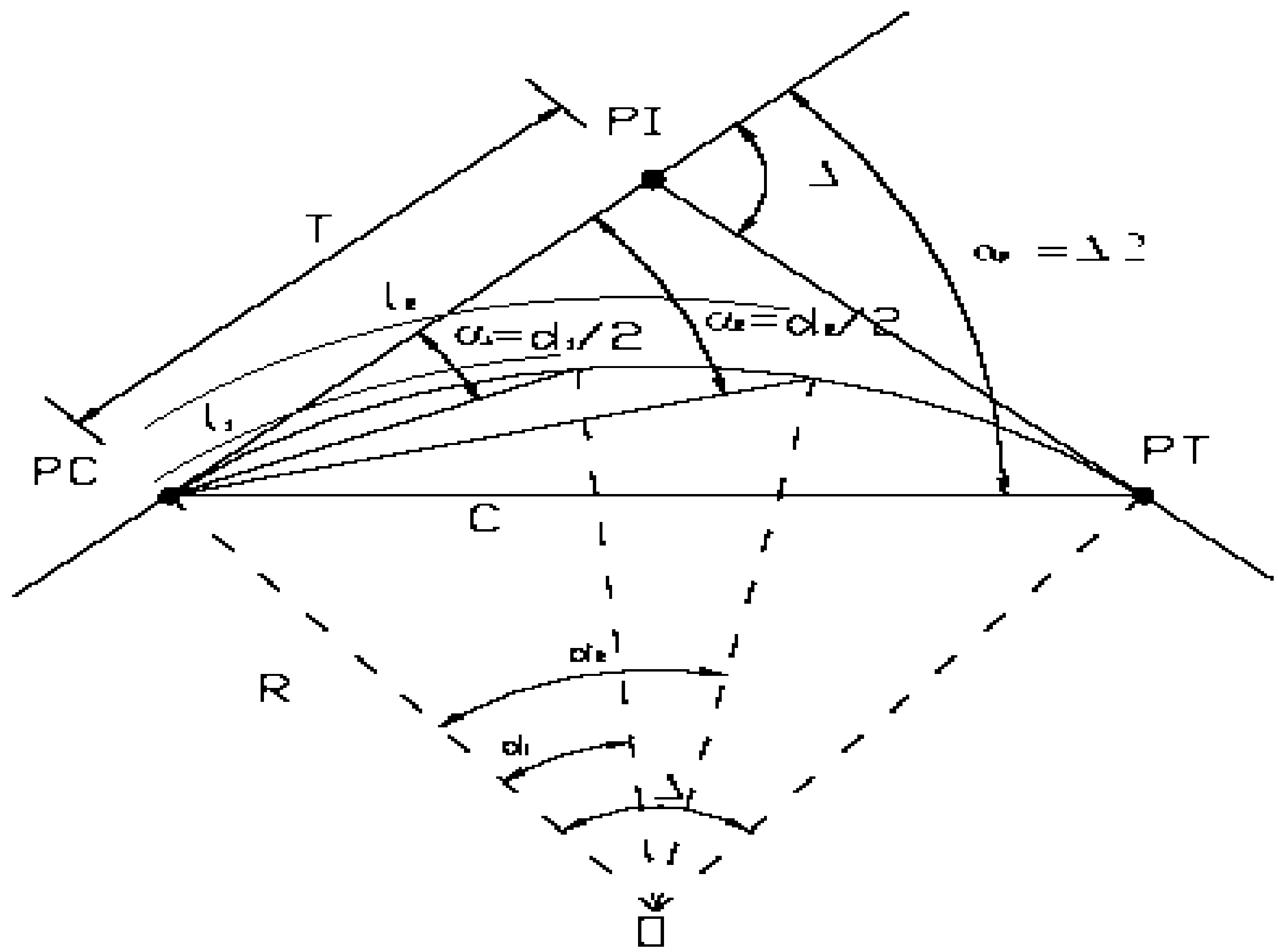
Central angle subtended by an arc of length ( )

$$\alpha = \frac{1}{2} d$$

The angle of deviation = ( ) the central angle corresponding to that arc

$$\alpha = l * \frac{D}{20}$$

The values of · can be calculated from the above law. Thus, by the theodolite device, the points can be projected by horizontal angles, after pointing the device to the PI point, then the device is zeroed, after that we rotate the device according to the calculated · values.



the accounts:-

1. Compute the elements of the L, T, M, E, and C curves
2. Calculation of PC, PT, MC curve stations
3. Calculation of substations based on the division period
4. Calculate the distance of each station for the PC starting point
5. Calculate the tangent angle for each station, and the tangent angle for the last station should be Δ /2
6. Makes a chart of accounts
7. Projection at the site according to the angles fixed in the table. Note: The period of the main stations must be mentioned, whatever the period of division

8. The theodolite device is installed on the PC point and directs the line of sight towards PI and the horizontal angle zeroes

NOTE: – If there is an obstacle towards PI, we start from PT and confirm the reading of the device (Δ/2), then we return to PI until we obtain an angle of (o 00' 00"0), then we proceed to work, and to ensure the correctness of the work, we measure the distance between the PC And PT must be equal to the calculated value of T

NOTE: Distances are measured from the previous stations and are equal to the lengths of the arcsThe following table shows the method of projecting a simple horizontal curve by tangent angles

Station	away from the previous point C	distance from the starting point l	$\alpha = l * \frac{D}{20}$	Notes
Sta. PC.	0	0	0	
:	:	:	:	
Sta. MC.		L/2		
:	:	:		
Sta. PT.		L	$\alpha = \frac{\Delta}{2}$	

Ex: Setting out a simple circular curve by using a tangential angles method and length of (Total & single) chord , if you know :

D= 4° 48' , Δ = 45° 18' , St. P.I = 27+25

$$D = \frac{573}{R} \rightarrow R = \frac{573}{D} \rightarrow \frac{573}{4^o\ \ 48'} = 119.38\ m$$

$$T = R\tan\frac{\Delta}{2} = 119.38\ tan\ 22^o\ \ 39' = 49.82\ m$$

$$L = \frac{\pi R \Delta}{180} = \frac{\pi \times 119.38 \times 45^o\ \ 18'}{180} = 94.39\ m$$

$$C = 2R.\,Sin\frac{\Delta}{2} = 2 \times 119.38 \times Sin\ 22^o\ \ 39' = 91.95\ m$$

$$St.P.C = St.P.I - T = (27+25) - (0+49.82) = (26+75.18)$$

$$St.P.T = St.P.C + L = (26+75.18) + (0+94.39) = (27+69.57)$$

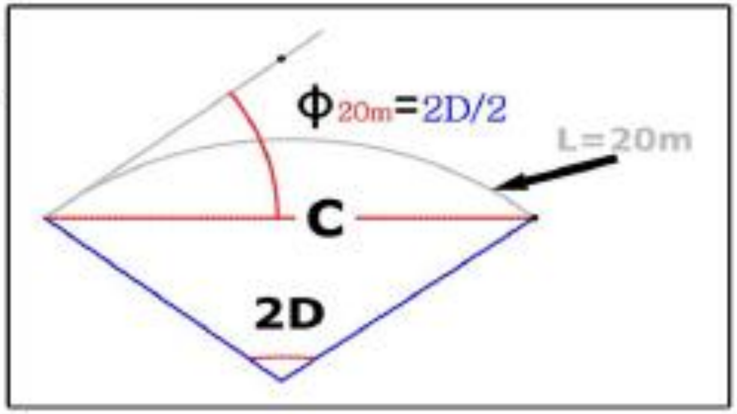
Station	Def. Angle	Total Chord (m) $C = 2R \sin$ angle	Single Chord $C = 2R \sin$ angle
P.C 26+75.18	0 00	0.00	0.00
26+80	$\varnothing_1 = \frac{\theta_i}{2} \rightarrow \frac{l_i}{20} D$ $= \frac{4.82}{20} \times 4^\circ 48' = 1^\circ 9.4'$	$2R \sin 1^\circ 9.4'$ $= 4.82$ m	$2R \sin 1^\circ 9.4'$ $= 4.82$ m
27+00	$\frac{\Theta i}{2} + D = 1^\circ 9.4' + 4^\circ 48'$ $= 5^\circ 57.4'$	$2R \sin 5^\circ 57.4'$ $= 24.78$	$2R \sin 4^\circ 48'$ $= 19.98$
27+20	$\frac{\Theta i}{2} + 2D = 1^\circ 9.4' + 2 \times 4^\circ 48' = 10^\circ 45.4'$	$2R \sin 10^\circ 45.4'$ $= 44.56$	$2R \sin 4^\circ 48'$ $= 19.98$
27+40	$\frac{\Theta i}{2} + 3D$ $= 15^\circ 33.4'$	$= 64.03$	$= 19.98$
27+60	$\frac{\Theta i}{2} + 4D = 20^\circ 21.4'$	$= 83.06$	$= 19.98$
P.T 27+69.57	$\varnothing_6 = \frac{\Theta i}{2} + 4D + \frac{\Theta f}{2} =$ $\varnothing_6 = \frac{\Theta i}{2} + 4D + \frac{l_f}{20} D$ $20^\circ 21.4' + \frac{9.57}{20} \times 4^\circ 48'$ $= 22^\circ 39.2'$ $* = \frac{\Delta}{2}$ Check *	$= 91.95$ m * Check * $C = 91.95$ m	$= 2R \sin 2^\circ 17.8'$ $= 9.57$

* Deflection Angle $= \Delta / 2 = 22^\circ 39'$ * Total Chord $= C = 91.95$ m

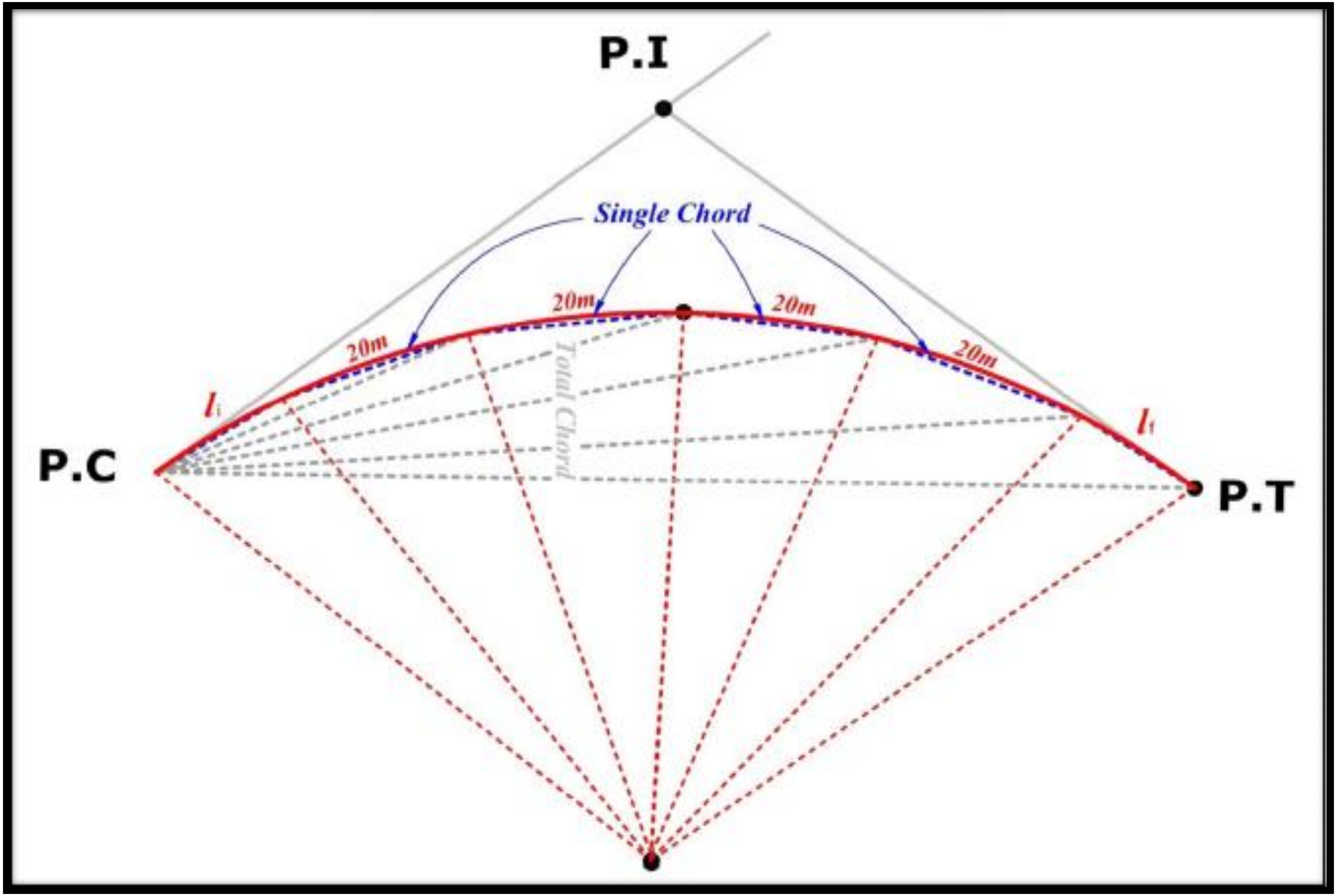
$$\varnothing_i = \frac{l_i}{20} D$$

$$C = 2R \times \sin \varnothing_{20m}$$

$$\varnothing_{20m} = \frac{2D}{2}$$



$$\varnothing_f = \frac{l_f}{20} D$$



P.I
Single Chord
20m
20m
20m
20m
Total Chord
P.C
P.T

The process of shedding by the method of total strings is done through: zeroing on (P.I), then directing on each point through the angles, then measuring all the strings from point (P.C). . As for the method of single strings, it is done as before with regard to angles, but the strings are counted from point (P.C) to the first point, then from the first to the second, then from the second to the third... and so on.

2- TANGENT OFFSET METHOD

This method is called the coordinate method, as it depends on the basis of measuring horizontal and vertical distances from a point of origin. The columns are installed as follows:

* Determine the x-axis, and it is in the direction of (P.C) → (P.I), and it represents the x-axis distance from point P.C.

* The y axis is identical to the radius, and the point (P.C) represents the first point of origin.

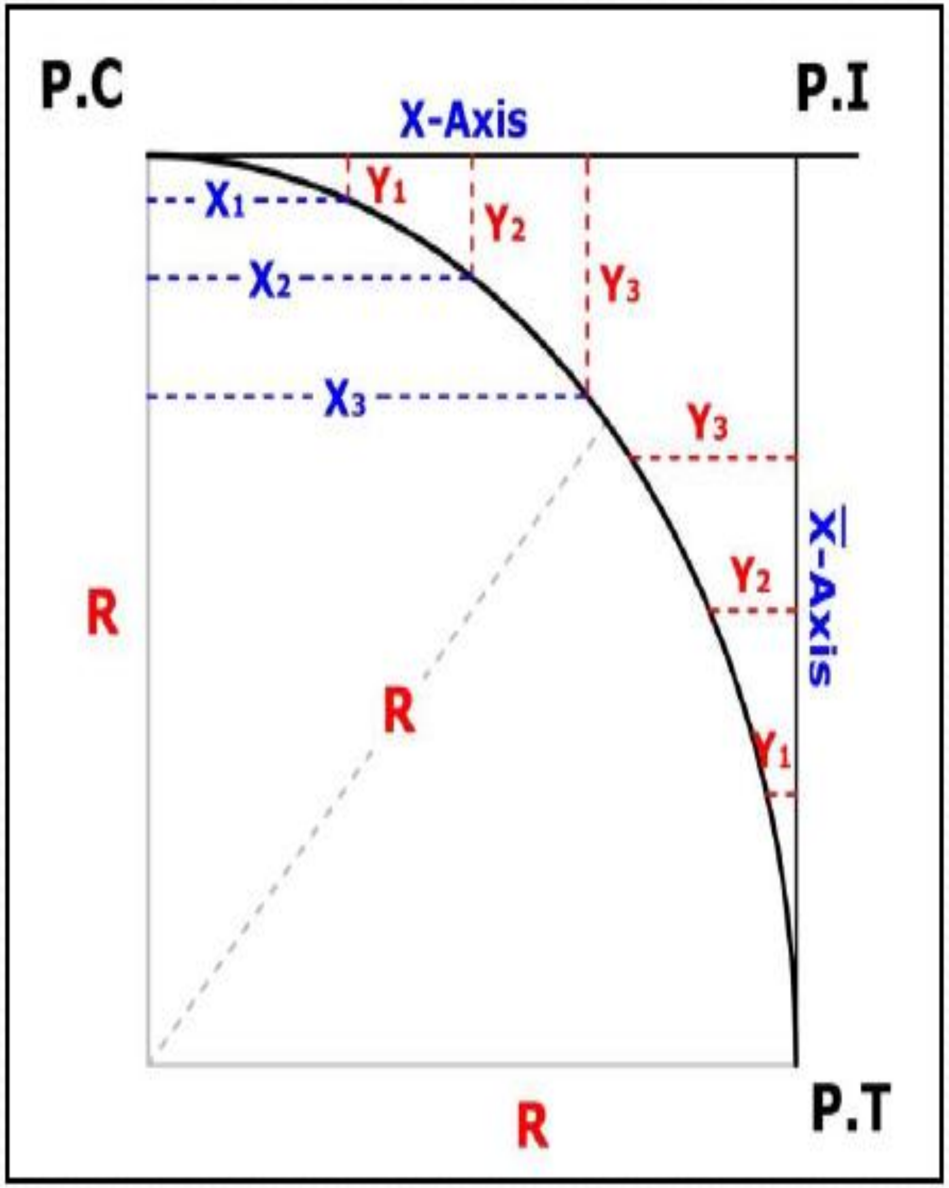
* The same process is applied from point (P.T), which represents the second point of origin.

* The y distance is measured perpendicular to the tangent and at every sigmoidal distance.

The y distances are calculated according to the following relationship:

$$y = R[1 - \sqrt{1 - (\frac{x}{R})^2}$$

Point	x	$y = R\left[1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right]$
(1)	$x_1 =$	$y_1 =$
(2)	$x_2 =$	$y_2 =$
(3)	$x_3 =$	$y_3 =$
(4)	$x_4 =$	$y_4 =$
⋮	⋮	⋮



OR this method is used most of the time if it is not possible to determine the center of the curve because of an obstruction.

1. Determine the starting and ending points of the curve.

2. We divide the tangent into a group of equal distances, and these distances represent the x-axis.

3. We establish columns of division points on the x-axis.

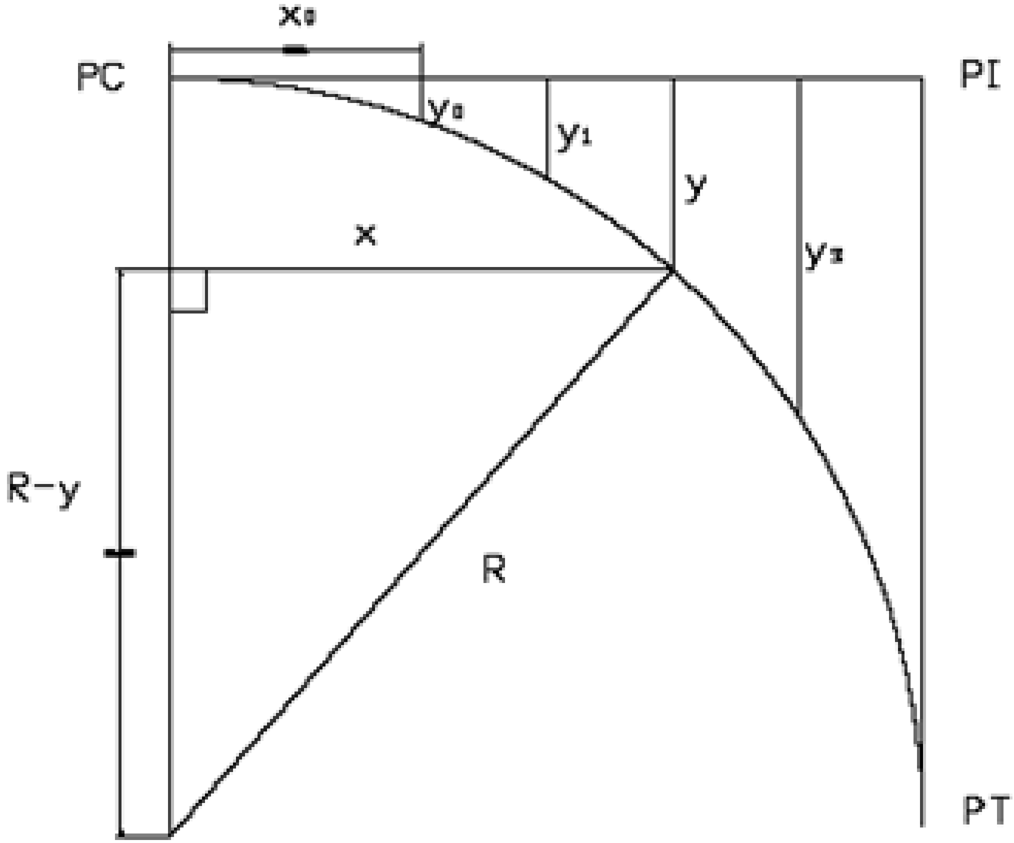
4. We cut the columns built on the x-axis by the value of y calculated in the table shown below and from the law shown below as well, and prove it with arrows.

5. To ensure accuracy in projecting the curve, we establish columns on the second tangent, thus the points are doubled and the curve is more accurate.

$$(R-y)^2 = R^2 - x^2$$

$$R - y = \sqrt{R^2 - x^2}$$

$$y = R - \sqrt{R^2 - x^2} \quad \text{------- 1}$$

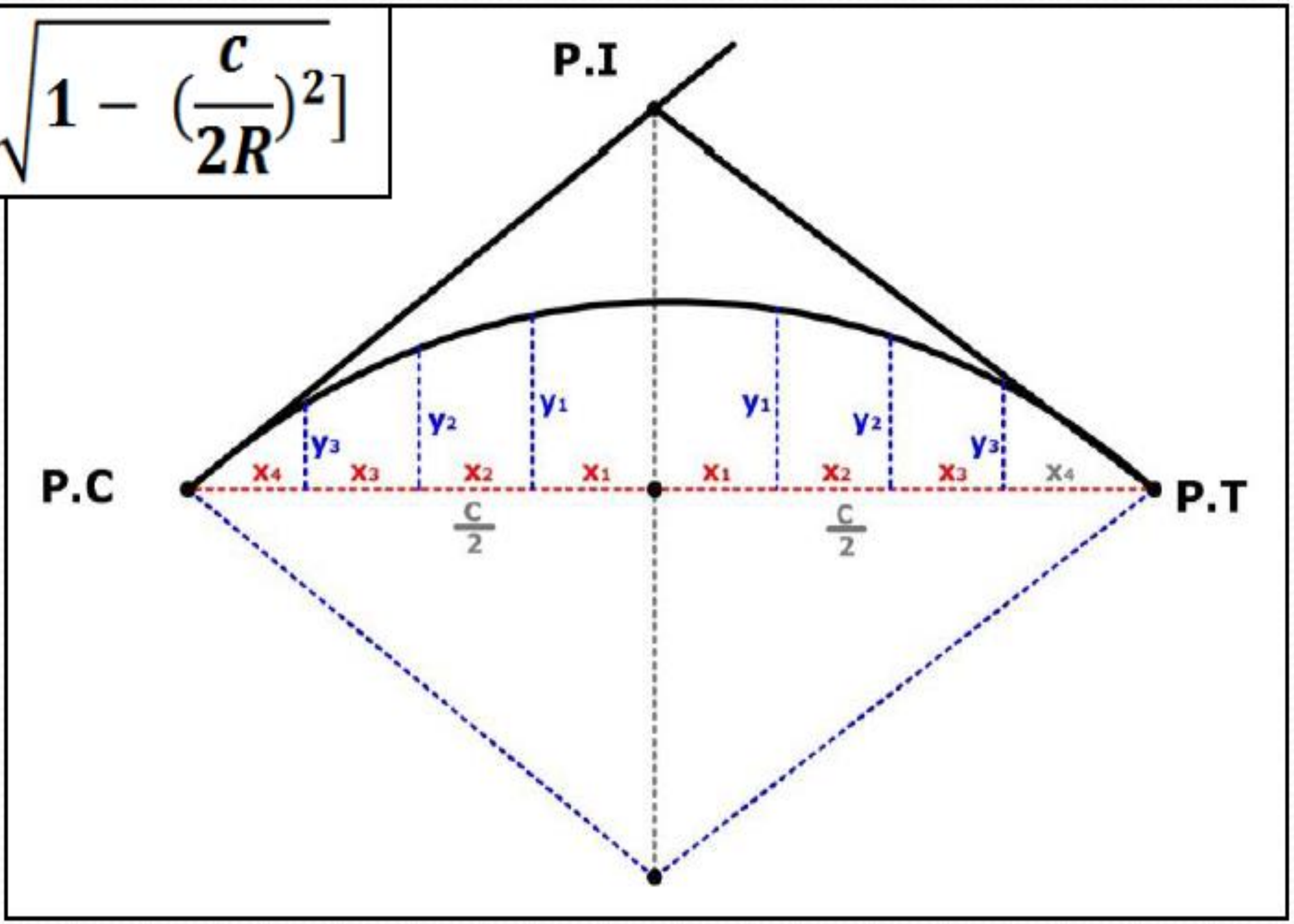


3- CHORD OFFSET METHOD

1. - In this method, the chord between (P.I & P.T) is first determined, then it is divided into a number of equal parts.

2. Fixed default distances are fixed from the middle of the string towards the right and left (representing the x-coordinates).
3. The columns that represent the y-axis are calculated from the points established in the previous step and using the relationship the following mathematical, as shown in the figure.

$$y = R\left[\sqrt{1 - \left(\frac{x}{R}\right)^2} - \sqrt{1 - \left(\frac{c}{2R}\right)^2}\right]$$



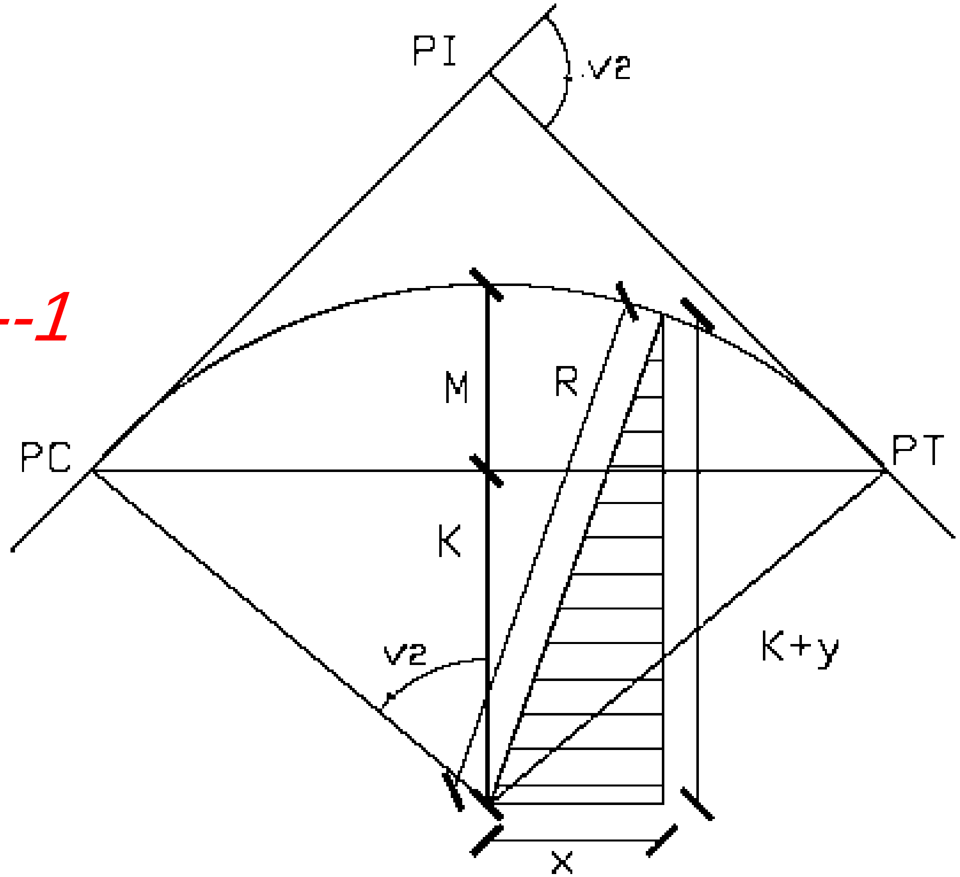
OR

1. The chord is drawn between the starting and ending points of the curve (PC, PT) and determines its straightness by means of signs.
2. Divide half of the string into an appropriate number of parts, and with the same number we divide the other half of the string.
3. We prove the locations of the points by measuring the lengths of the segments from the midpoint of the hypotenuse (which represents the x-coordinate).
4. We measure the lengths of the columns built on those parts that represent the y-coordinates and calculated from the table shown below by the law shown below as well.
5. Columns shall be erected by tape and by the triangle method, or by one of the previously studied methods.

$$(K+y)^2 = R^2 - x^2$$

$$K + y = \sqrt{R^2 - x^2}$$

$$y = \sqrt{R^2 - x^2} - K \quad \text{---------1}$$



$$K = R - M \quad \text{------2a}$$

$$K = R * \cos(\Delta/2) \text{-----2b}$$

$$K = \sqrt{R^2 - (C/2)^2} \quad \text{-----2c}$$

DETERMINED CENTER OF CURVE METHOD

A. If PC, PT and R are known and the radius value is less than 30 m.1. We identify PC and PT on nature, then open the bar from PC as far as the value of R, and then draw an arc on the ground.2. We repeat the operation from PT and from the intersection of the brackets we determine the center (O).3. From point (O), we open the bar by the value of R, and then define the simple circular arc as in figure (a).

B. If the value of R is known and the directions of the two tangents.1. Determine the direction of the two tangents and let the lines AB and AC be on the ground.2. Move a certain distance on the line AB, and let it be greater than the value of R.3. A column of length R is erected from the point fixed on line AB4. From another point, with a distance greater than the first, on line AB, another column of length R is erected.5. Connect the ends of the two columns, so we get the line DE.6. Repeat the process from 1 to 5 on line AC to get line FG as shown in Figure (b).7. From the intersection of straight lines DE and FG, we get a point (O), which represents the center of the curve and by means of the bar, and from point (O) at the opening of the bar with a value of R, we get the curve.

C. If the value of T is known and the directions of the two tangents.1. I measure a distance T from PI on lines AB and AC to get PC and PT.2. From these points PC and PT, establish perpendiculars to lines AB and AC.3. From the point of intersection we get the center (O).4. We calculate the value of R from the length of the shaft between O and PC or PT.5. By opening the tape from the center with a length of R, we get the required curve as in Figure C.

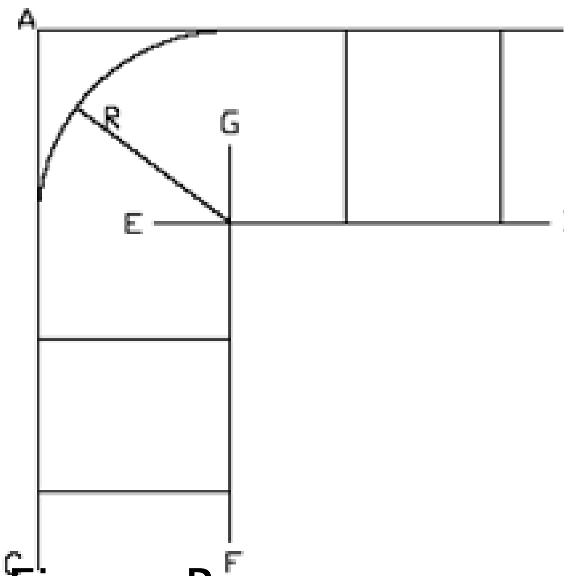


Figure B

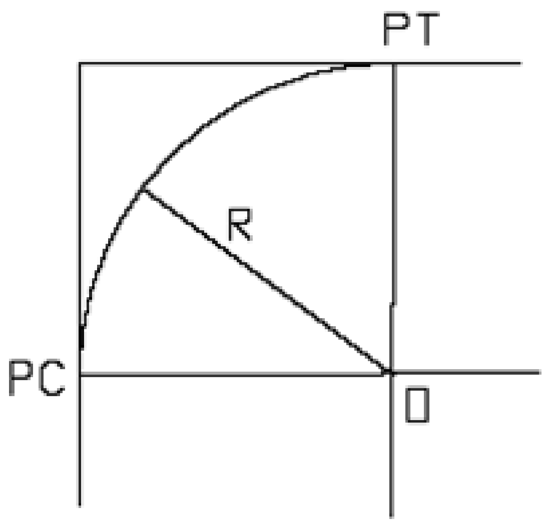


Figure C

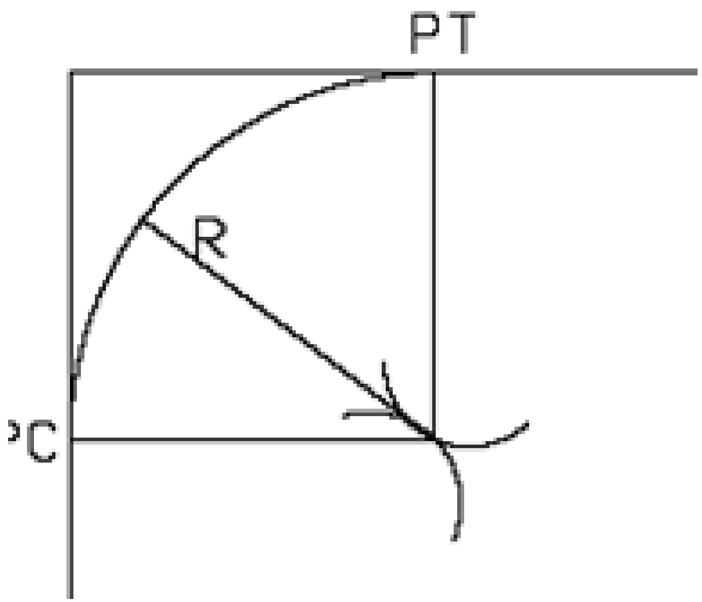
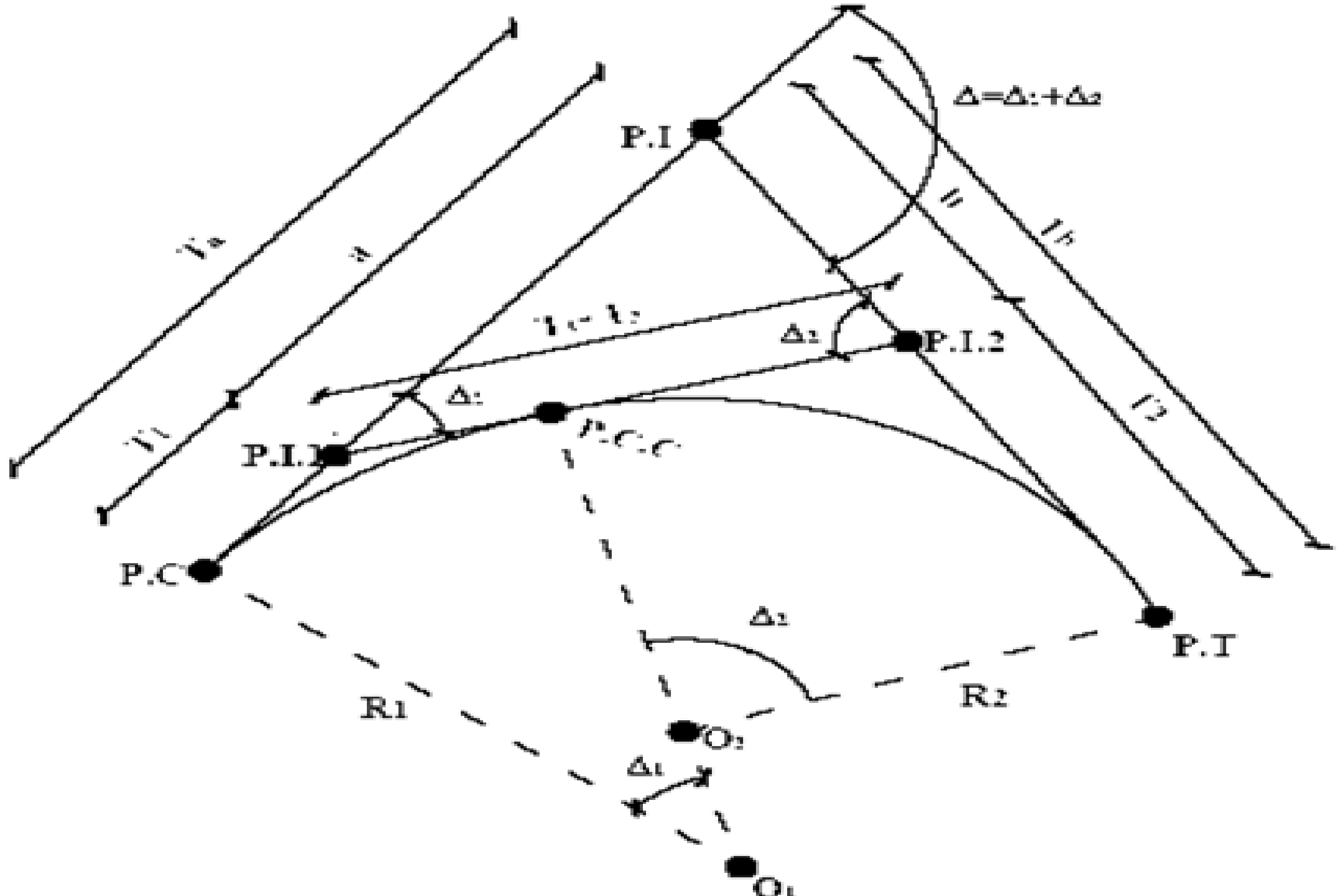


Figure A

Compound circular curve:-

A compound circular curve consists of two or more simple circular curves. The centers of the curves being on the same side of the curve. Such is shown in fig. below.



OR: compound circular curve is that curve that consists of two segments of two or more circles, each with a radius and a center, and the direction of rotation of the two curves is unified. The calculations of each curve are identical to the calculations of the simple circular curve, given that each of them represents a simple curve, except that there are a number of common elements between the two curves, as shown in the figure and the following relationships:

$$T_a = T_1 + a$$

$$T_b = T_2 + b$$

$$\Delta = \Delta_1 + \Delta_2$$

$$C.T = T_1 + T_2$$

$$\frac{T_1 + T_2}{\sin(180 - \Delta)} = \frac{a}{\sin \Delta_2} = \frac{b}{\sin \Delta_1}$$

$$a = \frac{T_1 + T_2}{\sin(180 - \Delta)} \times \sin \Delta_2$$

$$b = \frac{T_1 + T_2}{\sin(180 - \Delta)} \times \sin \Delta_1$$

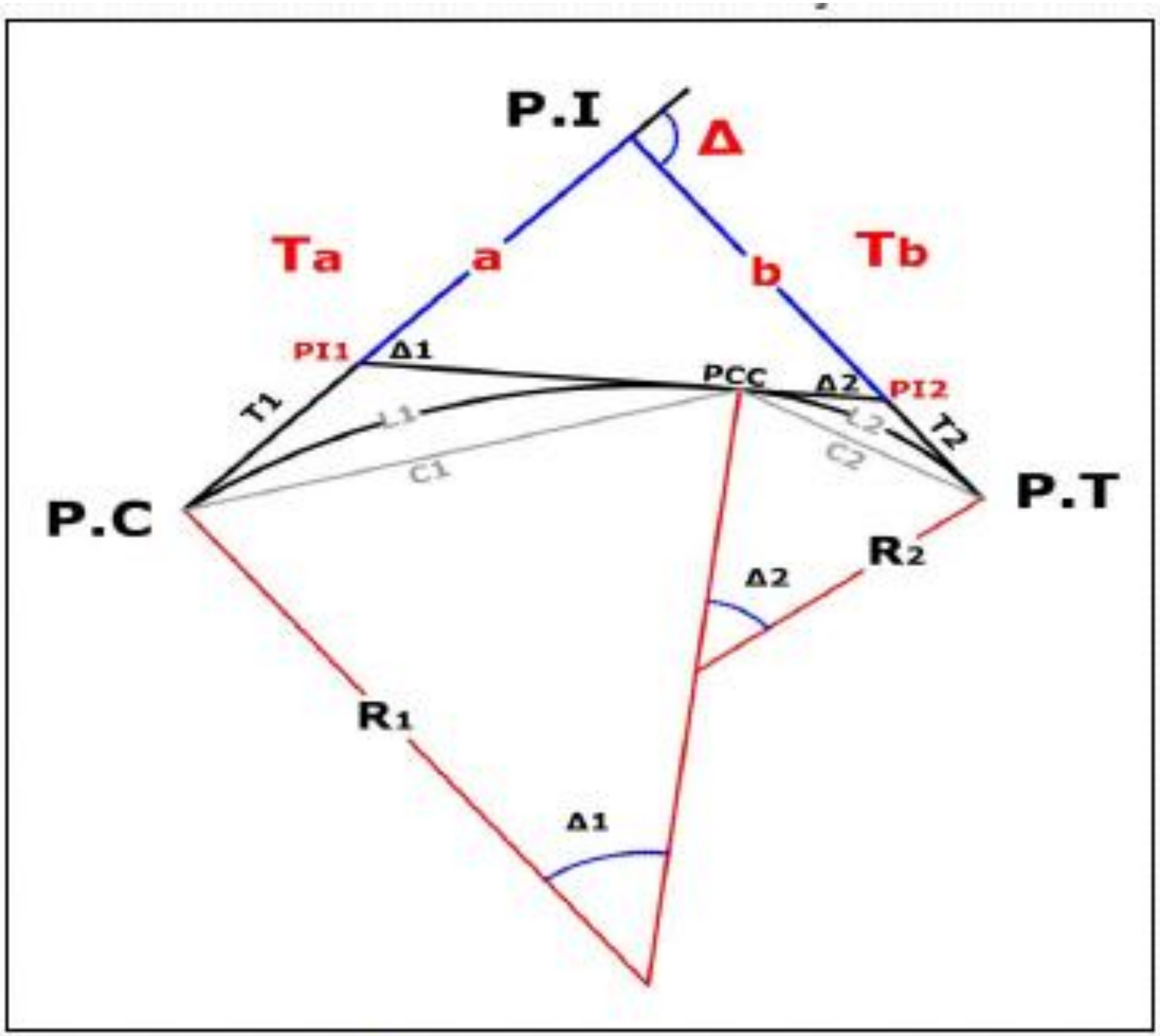
$$St.P.I_1 = St.P.C + T_1$$

$$St.P.C.C = St.P.C + L_1$$

$$St.P.T = St.P.C.C + L_2$$

$$St.P.C = St.P.I - T_a$$

$$St.P.I_2 = St.P.C.C + T_2$$



* The tangents (Ta, Tb) are used for the purpose of fixing the two stations (P.T, P.C) from the station (P.I).
* The station (P.C.C) is the Point of Compound Curvature.
* C.T: represents the length of the common tangent (T1+T2), and is collinear

TO UNDER STAND THIS RELATIONSHIPS WE CAN SAY

For computation compound circular curve:-

a) *Compute elements for all curves at alone, that using* Δ_1, R_1 *for a first curve and* Δ_2, R_2 *for a second curve.*

b) *To compute length of a first major tangent* (T_a) *and length of a second major tangent* (T_b) *that is following:-*

1. *compute length of common tangent*

$$C.T. = T_1 + T_2$$

a)Compute total deflection angle Δ for compound curve, which equal $(\Delta = \Delta_1 + \Delta_2)$

b)Compute distance (a) between sta.PI to sta. PI_1 and distance (b) between sta.PI to sta. PI_2 by using sin rules.

For triangle (PI_1 PI PI_2)

$$\frac{(T_1 + T_2)}{\sin(180 - \Delta)} = \frac{a}{\sin \Delta_2} = \frac{b}{\sin \Delta_1}$$

$$a = \frac{(T_1 + T_2)}{\sin(180 - \Delta)} * \sin \Delta_2 \quad \text{------------- 1}$$

$$b = \frac{(T_1 + T_2)}{\sin(180 - \Delta)} * \sin \Delta_1 \quad \text{------ 2}$$

1. compute length of a first major tangent (T_a)

$$T_a = T_2 + a \qquad \text{-------------3}$$

1. compute length of a second major tangent (T_b)

$$T_b = T_1 + a \qquad \text{-------------4}$$

Using T_b & T_a for set-out sta.PC & sta.PT from sta.PI when set-out compound curve.

a)Compute major stations (PC, PCC, PT) after known sta.PI.

$$sta.PC = sta.PT + T_a \qquad \text{--------------5}$$

$$sta.PCC = sta.PC + L_1 \qquad \text{--------------6}$$

$$sta.PT = sta.PCC + L_2 \qquad \text{--------------7}$$

$$sta.PI_1 = sta.PC + T_1 \qquad \text{--------------8}$$

$$sta.PI_2 = sta.PC + T_2 \qquad \text{--------------9}$$

Example :

Compute stations for compound circular curve. If known:-

Sta.PI=28+50 R_1=500m D_1=38° 20´ R_2=750m

D_2=41° 30´

SOLUTION:

T_1 = 500*TAN(19° 10') = 173.79M

L_1 = (Π*500*38.333)/ 180 = 334.52M

T_2 = 750*TAN(20° 45') = 284.15M

L_2 = (Π*750*41.5)/ 180 = 543.23M

$$\frac{(T_1 + T_2)}{\sin(180 - \Delta)} = \frac{a}{\sin \Delta_2} = \frac{b}{\sin \Delta_1}$$

$$\frac{(173.79 + 284.15)}{\sin(180 - 79^o 50')} = \frac{a}{\sin 41^0 30'} = \frac{b}{\sin 38^o 20'}$$

$A = (457.94 * SIN(41^0\ 30')/ SIN(79^0\ 50') = 308.28M$

$T_A = 173.79 + 308.28 = 482.07M$

$B = (457.94 * SIN(38^0\ 20')/ SIN(79^0$

$50') = 288.56M$

$T_B = 288.56 + 284.15 = 572.71$

$STA.PC = PI - T_A$

$= (28+50)-(4+82.07) = 23+67.93$

$STA.PI_1 = PC - T_1$

$= (23+67.93)-(1+73.79) = 25+41.72$

$STA.PCC = PC - L_1$

$= (23+67.93)-(3+34.52) = 27+02.45$

$STA.PI_2 = PCC - T_2$

$= (27+02.45)-(2+84.15) = 29+86.60$

$STA.PT = PCC - L_2$

$= (27+02.45)-(5+43.23) = 32+45.68$

Ex 2:- compute station and coordinates of main points for a compound circular curve if :-

Δ_1=49°30′, Δ_2=28°24′,C_1=383.50 m, C_2=383.50 m, E_2=7.5 m ,Sta. P.C.C=(31+47),P.C.C(1500,800),

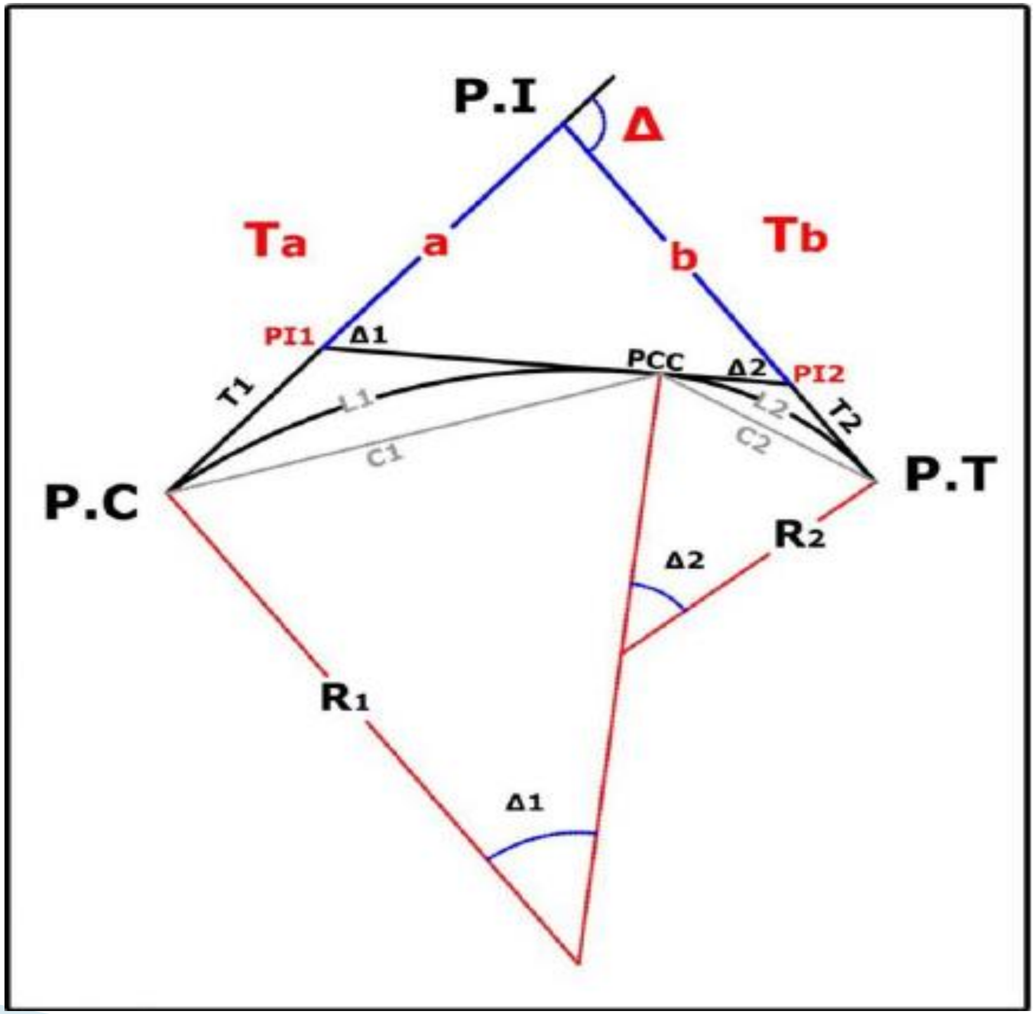
Berg P.C ⟶ P.I=N30°45′

Solution :-

$$C = 2R \times \sin(\frac{\Delta}{2})$$

$$R_1 = \frac{C_1}{2\sin(\frac{\Delta_1}{2})} = \frac{383.5}{2\sin(\frac{49°30'}{2})}$$

= 458.01m



$$T = R \times \tan(\frac{\Delta}{2})$$

$$L = \frac{\pi \, \Delta \, R}{180} \quad or \quad \frac{\mathbf{10}\,\Delta}{\boldsymbol{D}}$$

$$T_1 = \frac{R_1}{\tan(\frac{\Delta_1}{2})} = \frac{\mathbf{458.01}}{\tan(\frac{49°30'}{2})}$$

=211.15m

$$L_1 = \frac{\pi \times \Delta_1 \times R_1}{180}$$

$$= \frac{22 \times 49°30' \times 458.01}{180 \times 7}$$

=395.69m

$$E = R \times \left(\frac{1}{\cos\frac{\Delta}{2}} - 1\right)$$

$$R_2 = \frac{E_2}{\times\left(\frac{1}{\cos\frac{\Delta_2}{2}} - 1\right)} = \frac{\mathbf{7.5}}{\times\left(\frac{1}{\cos\frac{28°24'}{2}} - 1\right)}$$

=237.96m

$$T = R \tan \frac{\Delta}{2}$$

$$*T_2 = R_2 \tan \frac{\Delta_2}{2} = 237.96 \tan 14^\circ 12' = 60.21 \text{ m}$$

$$C = 2R \sin \frac{\Delta}{2}$$

$$*C_2 = 2R_2 \sin \frac{\Delta_2}{2} = 2 \times 237.96 \sin 14^\circ 12' = 116.75 \text{ m}$$

$$L = \frac{\pi . R . \Delta}{180} \text{ or } \frac{10\Delta}{D}$$

$$*L_2 = \frac{\Pi R_2 \Delta_2}{180^\circ} = \frac{3.14159 \times 237.96 \times 28^\circ 24'}{180^\circ} = 117.95 \text{ m}$$

$$C.T = T_1 + T_2$$

$$*C.T = T_1 + T_2 = 211.15 + 60.21 = 271.36 \text{ m}$$

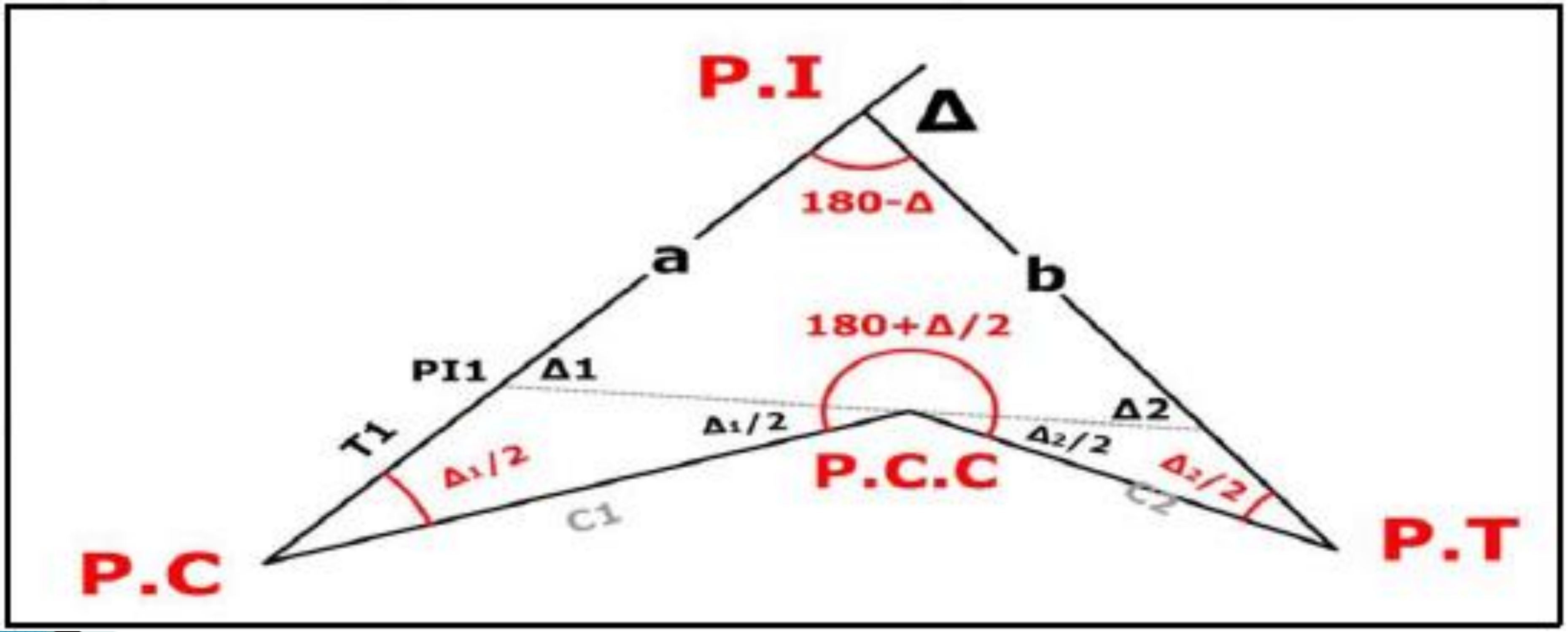
$$\frac{T_1 + T_2}{\sin(180 - \Delta)} = \frac{a}{\sin \Delta_2} = \frac{b}{\sin \Delta_1}$$

$$\frac{C.T}{\sin(180^\circ - \Delta)} = \frac{a}{\sin \Delta_2} = \frac{b}{\sin \Delta_1}$$

$$a = \frac{C.t \times \sin \Delta_2}{\sin(180^\circ - \Delta)} = \frac{271.36 \sin 28^\circ 24'}{\sin(180^\circ - 77^\circ 54')} = 132 \text{ m}$$

$$b = \frac{C.t \times \sin \Delta_1}{\sin(180^\circ - \Delta)} = \frac{271.36 \sin 49^\circ 30'}{\sin(180^\circ - 77^\circ 54')} = 211.03 \text{ m}$$

Ta = $T_1 + a = 211.15 + 132 = 343.15$ m
Tb = $T_2 + b = 60.21 + 211.03 = 271.24$ m
Sta. P.C. = Sta. P.C.C – L
Sta. P.C. = (31+47) – (3+95.69) = (27+51.31)
Sta. P.I. = Sta. P.C. + Ta
Sta. P.I. = (27+51.31) + (3+43.15) = (30+94.46)
Sta. P.T. = Sta. P.C.C. + L_2
Sta. P.T. = (31+47) + (1+17.95) = (32+64.95)

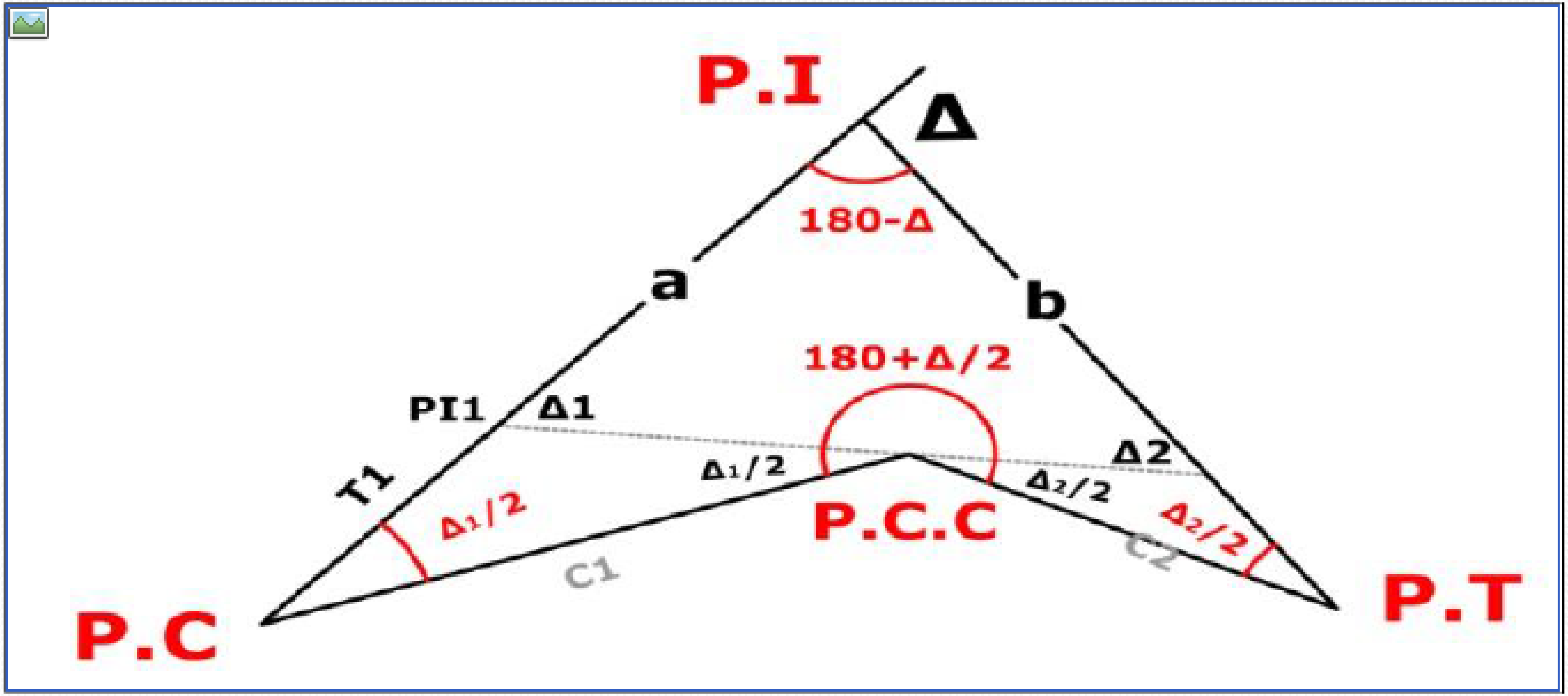


Sta.	Side	Length (m)	AZ	Dep.	Lat	X	Y
				$L \times Sin\ AZ$	$L \times Cos\ AZ$		
P.C.C						1500	800
	P.C.C→P.C	(C_1) 383.50	235° 30′	- 316.05	- 217.22		
P.C						1183.95	582.78
	P.C→P.I	(Ta) 343.15	30° 45′	175.45	294.90		
P.I						1359.40	877.68
	P.I→P.T	(Tb) 271.24	108° 39′	257	- 86.74		
P.T						1616.40	790.94
	P.T→P.C.C	(C_2) 116.75	274° 27′	- 116.40	9.06		
P.C.C						1500	800
				$\sum$=0.0	$\sum$=0.0		

$$Az_{P.C.C\to P.C} = Az_{P.C\to P.I} + \frac{\Delta_1}{2} + 180^o \to 30^o\ 45' + \frac{49^o\ 30'}{2} + 180^o$$

$$Az_{P.I\to P.T} = Az_{P.C\to P.I} + \Delta = 30^0\ 45' + 77^0\ 54'$$

$$Az_{P.T\to P.C.C} = Back\ Az_{P.I\to P.T} - \frac{\Delta_2}{2} = (180^o + 108^o\ 39') - \frac{28^o\ 14'}{2}$$



$$\text{Angle at } (P.I) = (180^0 - \Delta) = 102^0\ 06'$$

$$\text{Angle at } (P.T) = \frac{\Delta_2}{2} = 14^o\ 12'$$

$$\text{Angle at } (P.C) = \frac{\Delta_1}{2} = 24^o\ 45'$$

$$\text{Angle at } (P.C.C) = 180^0 + \frac{\Delta}{2} = 218^0\ 57'$$

$$\sum = 360^o\ 00'$$

*Draw and compute stations and coordinates of main points for a compound circular curve if ; $\Delta = 56^\circ 48'$, $\Delta_1 = 24^\circ 24'$, $L_1 = 165$ m , $D_2 = 1^\circ 30'$, Sta. P.C. = (47+26) , P.C. = (1750 , 900) , Berg P.C. → P.I = N $49^\circ 50'$ E

Solution

$$R_1 = \frac{180^\circ \times L_1}{\Pi\ \Delta_1} = \frac{180^\circ \times 165}{3.14 \times 24^\circ 24'}$$

$$R_1 = 387.45 \text{ m}$$

$$T_1 = R_1 \tan\frac{\Delta_1}{2} = 387.45 \tan\frac{24^\circ 24'}{2}$$

$$T_1 = 83.77 \text{ m}$$

$$C_1 = 2R_1 \sin\frac{\Delta_1}{2} = 2 \times 387.45 \sin 12^\circ 12'$$

$$C_1 = 163.76 \text{ m}$$

$$\Delta_2 = \Delta - \Delta_1 = 56^\circ 48' - 24^\circ 24'$$

$$R_2 = \frac{573}{D_2} = \frac{573}{1^\circ 30'} = 382 \text{ m}$$

$$T_2 = R_2 \tan\frac{\Delta_2}{2} = 382 \times \tan 16^\circ 12' = 110.98 \text{ m}$$

$$C_2 = 2R_2 \sin\frac{\Delta_2}{2} = 2 \times 382 \sin 16^\circ 12' = 213.15 \text{ m}$$

$$L_2 = \frac{\Pi\ R_2\ \Delta_2}{180^\circ} = \frac{3.14159 \times 382 \times 32^\circ 24'}{180^\circ} = 216.02 \text{ m}$$

$$C.T = T_1 + T_2 = 83.77 + 110.98 = 194.75 \text{ m}$$

$$\frac{C.T}{\sin(180^\circ - \Delta)} = \frac{a}{\sin \Delta_2} = \frac{b}{\sin \Delta_1}$$

$$a = \frac{C.T \times \sin\Delta_2}{\sin(180^\circ - \Delta)} = \frac{194.75 \times \sin 32^\circ 24'}{\sin(180^\circ - 56^\circ 48')} = 124.71 \text{ m}$$

$$b = \frac{C_t \times \sin \Delta_1}{\sin(180^\circ - \Delta)} = \frac{194.75 \times \sin 24^\circ 24'}{\sin(180^\circ - 56^\circ 48')} = 96.15 \text{ m}$$

Ta = $T_1 + a = 83.77 + 124.71 = 208.48$ m
Tb = $T_2 + b = 110.98 + 96.15 = 207.13$ m
Sta. P.I = Sta. P.C + Ta = (47+26) + (2+08.48) = (49+34.48)
Sta. P.C.C = Sta. P.C + L_1 = (47+26) + (1+65) = (48+91)
Sta. P.T = Sta P.C.C. + L_2 = (48+91) + (2+16.02) = (51+07.02)

Point	Side	Length	AZ	Dep.	Lat.	X	Y
P.C						1750	900
	P.C – P.I	208.48	49° 50'	+159.32	+134.48		
P.I						1909.32	1034.48
	P.I – P.T	207.13	106° 38'	+198.46	-59.29		
P.T						2107.78	975.19
	P.T – P.C.C	213.15	270° 26'	-213.14	+1.61		
P.C.C						1894.64	976.80
	P.C.C – P.C	163.76	242° 02'	-144.64	-76.80		
P.C						1750	900
				∑=0.00	∑=0.00