Southern Technical University
Technical Institute/ Basrah
Department of Electrical Techniques

Power Electronics
Second Year

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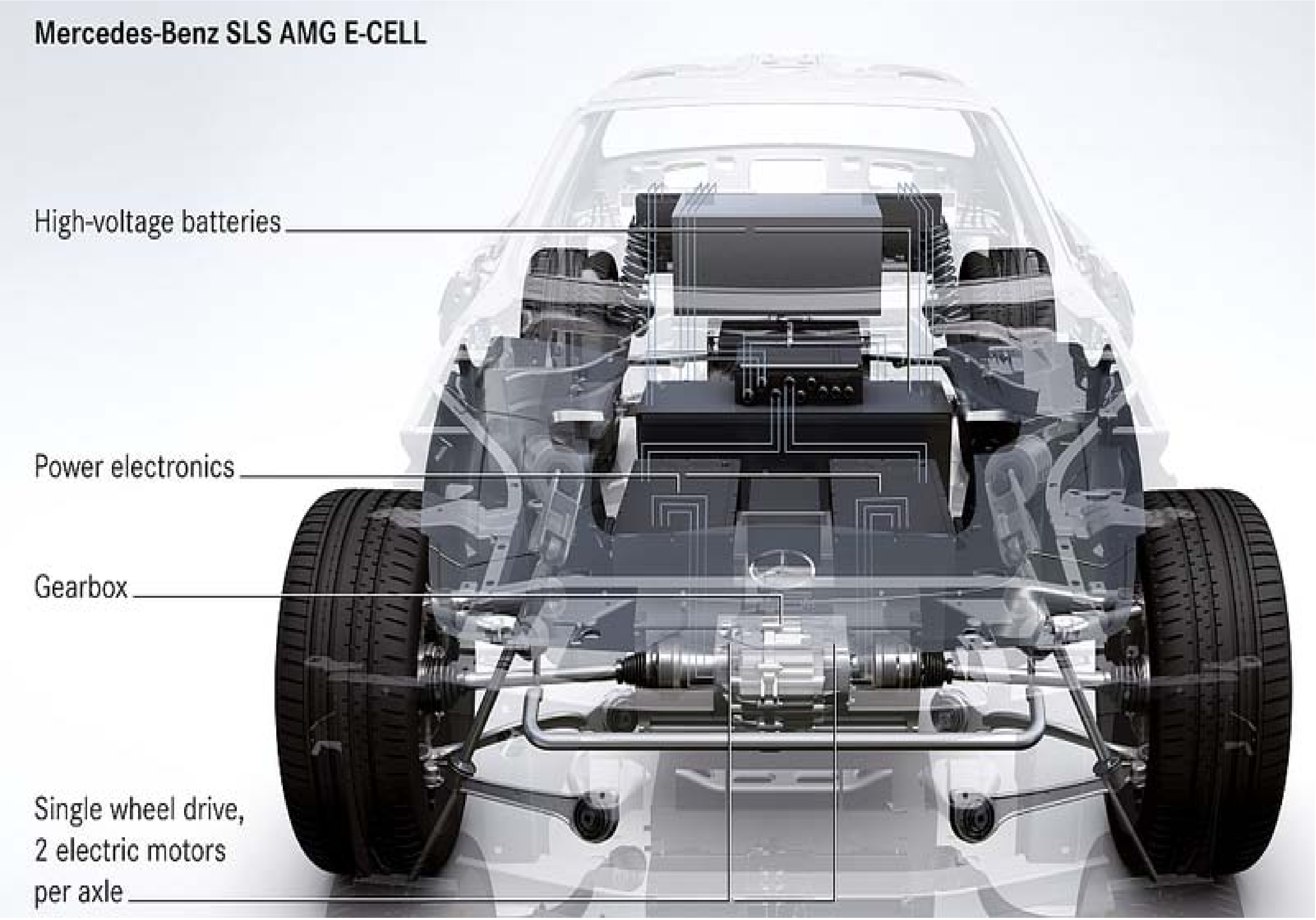

Mercedes-Benz SLS AMG E-CELL
High-voltage batteries
Power electronics
Gearbox
Single wheel drive,
2 electric motors
per axle

Subject Outline

Chapter 1: Power Electronic Devices

Chapter 2: The Transistors

Chapter 3: The Uni-Junction Transistor (UJT)

Chapter 4: The Operational Amplifier (Op-Amp)

Chapter 5: Silicon Controlled Rectifiers (SCRs)

Chapter 6: AC-DC Converters

Chapter 7: DC-DC Converters (DC Choppers)

Chapter 8: DC-AC Inverters

Chapter 9: AC-AC Converters

References

[1] W. Shepherd, and L. Hulley, **Power Electronics and Motor Control**, 2nd edition, Cambridge University Press, 1996.

[2] S.B. Dewan, **Power Semiconductor Circuits**, New York: Wiley, 1975.

[3] M.S. Berde, **Thyristor Engineering: An Introductory Book on Converters, Inverters, Motor Drives and Other Applications of Thyristors in Electrical Control of Power**, 8th edition, Khanna, 1995.

[4] M.H. Rashid, **Power Electronics Handbook: Devices, Circuits and Applications**, 2nd ed. Academic Press: Elsevier, 2007.

[5] T.L. Floyd, **Electronic Devices**, 9th edition, Merrill Publishing Company, 1988.

Other assistive Arabic books on Power Electronics are available.

Subject Aims and Objectives

Aims:

To study and investigate different power electronic devices and circuits used in power control and conversion.

Objectives:

After completion this course, the students will be able to:

- Differentiate between the general electronic circuits and power electronic devices.
- Understand the main functions of power electronics: Power control and Power conversion.
- Use the transistors for switching control.
- Understand the various applications of UJT and op-amp circuits.
- Learn the SCR structure, functions, circuits, and types.
- Understand and differentiate between the different kinds of power conversion: AC-DC (rectifiers), DC-DC (choppers), DC-AC (invertors), AC-AC (convertors).
- Connect the most important power electronic circuits in the Lab. and measure their parameters and characteristics.

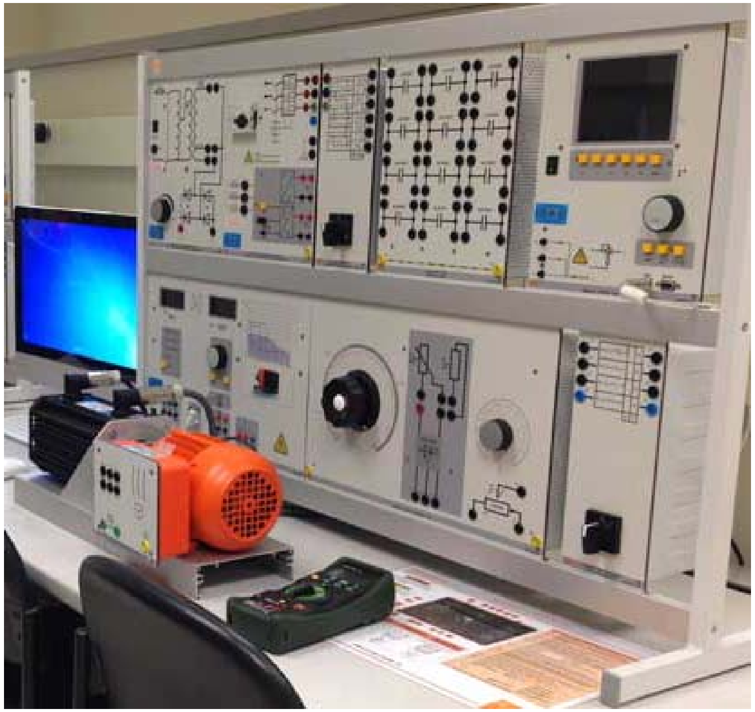
Chapter 1

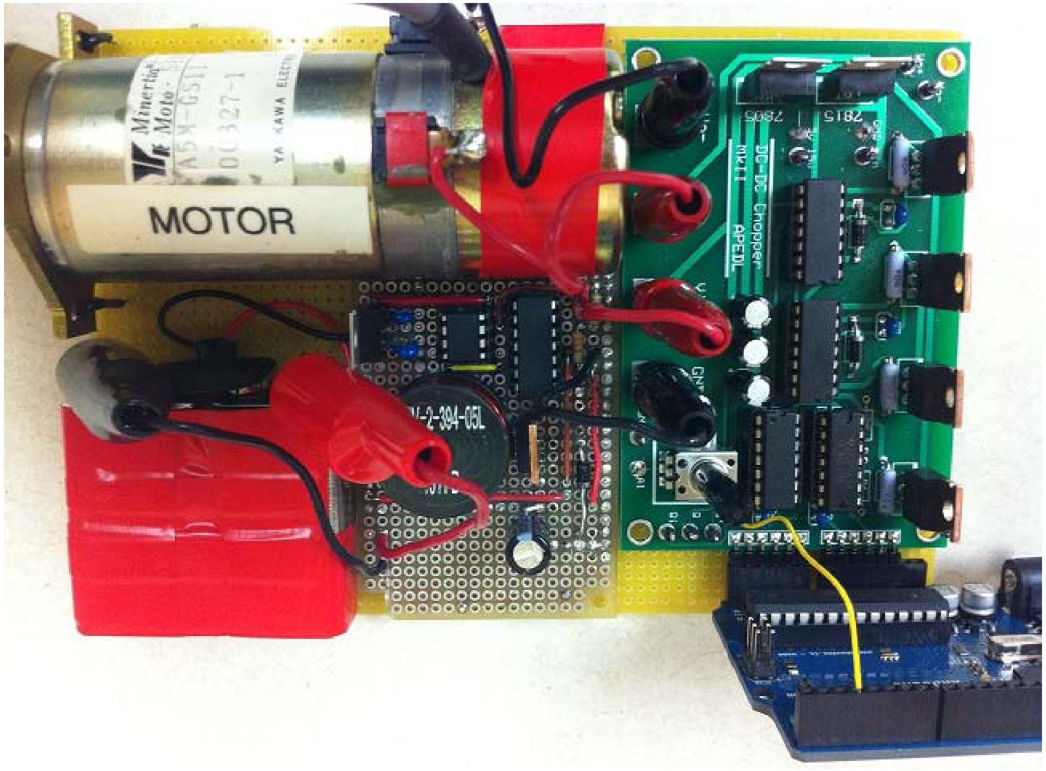
Power Electronic Devices

Introduction

Power Electronic Devices:

Any device that can be used in the power processing circuits to convert or control the electric power.





Classification of Power Electronic Devices

Uncontrolled device: diode
(Uncontrollable device)

has only two terminals and can not be controlled by control signal. The on and off states of the device are determined by the power circuit.

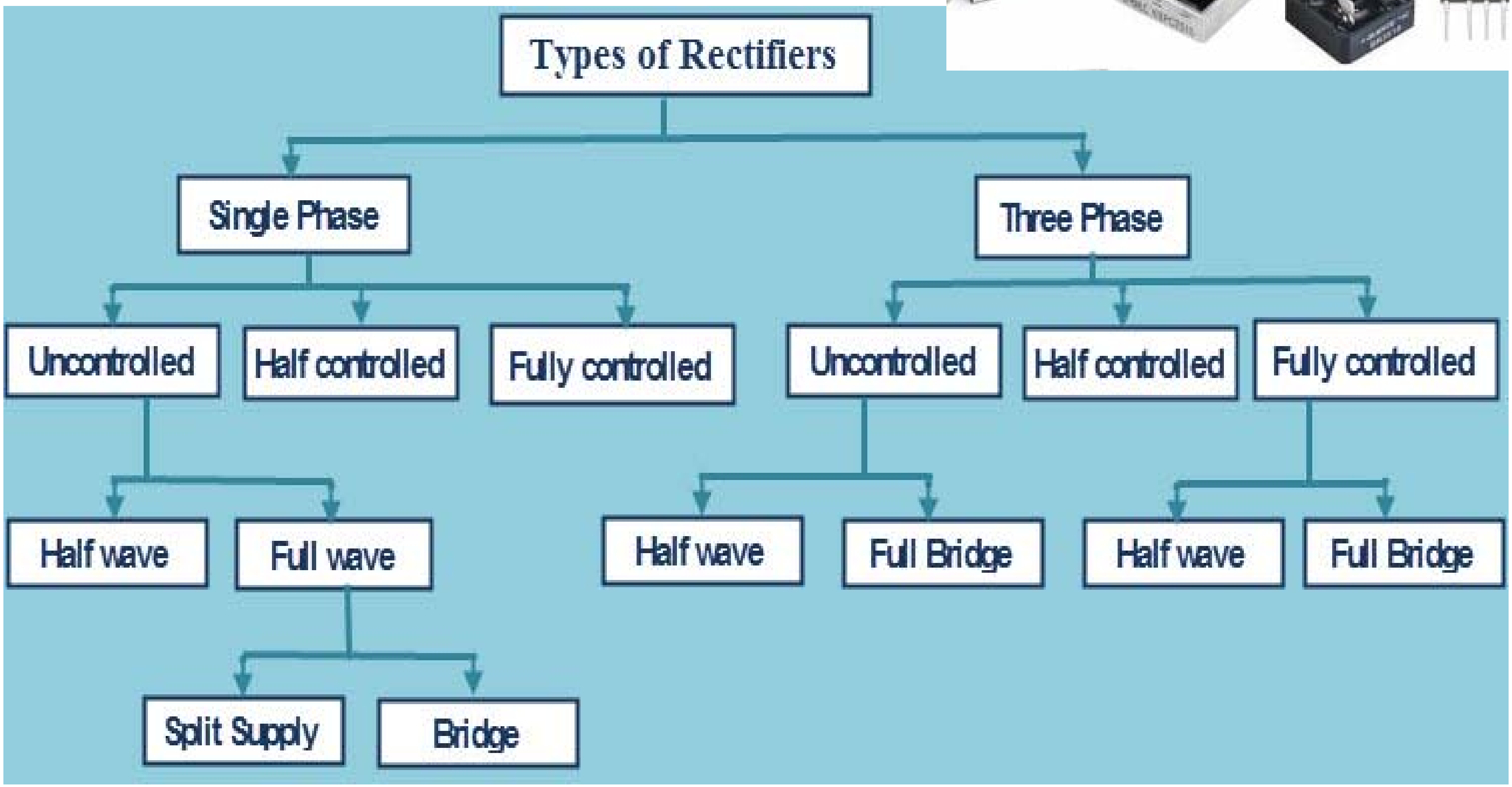
Half-controlled device: thyristor
(Half-controllable device)

is turned-on by a control signal and turned-off by the power circuit

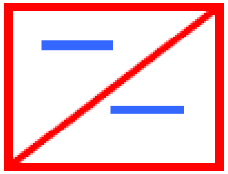
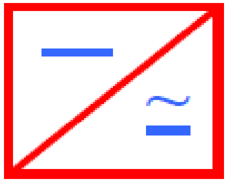
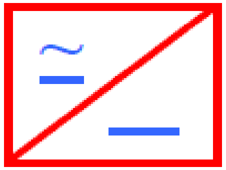
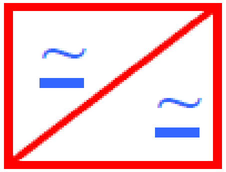
Fully-controlled device: Power MOSFET, IGBT,GTO, IGCT
(Fully-controllable device)

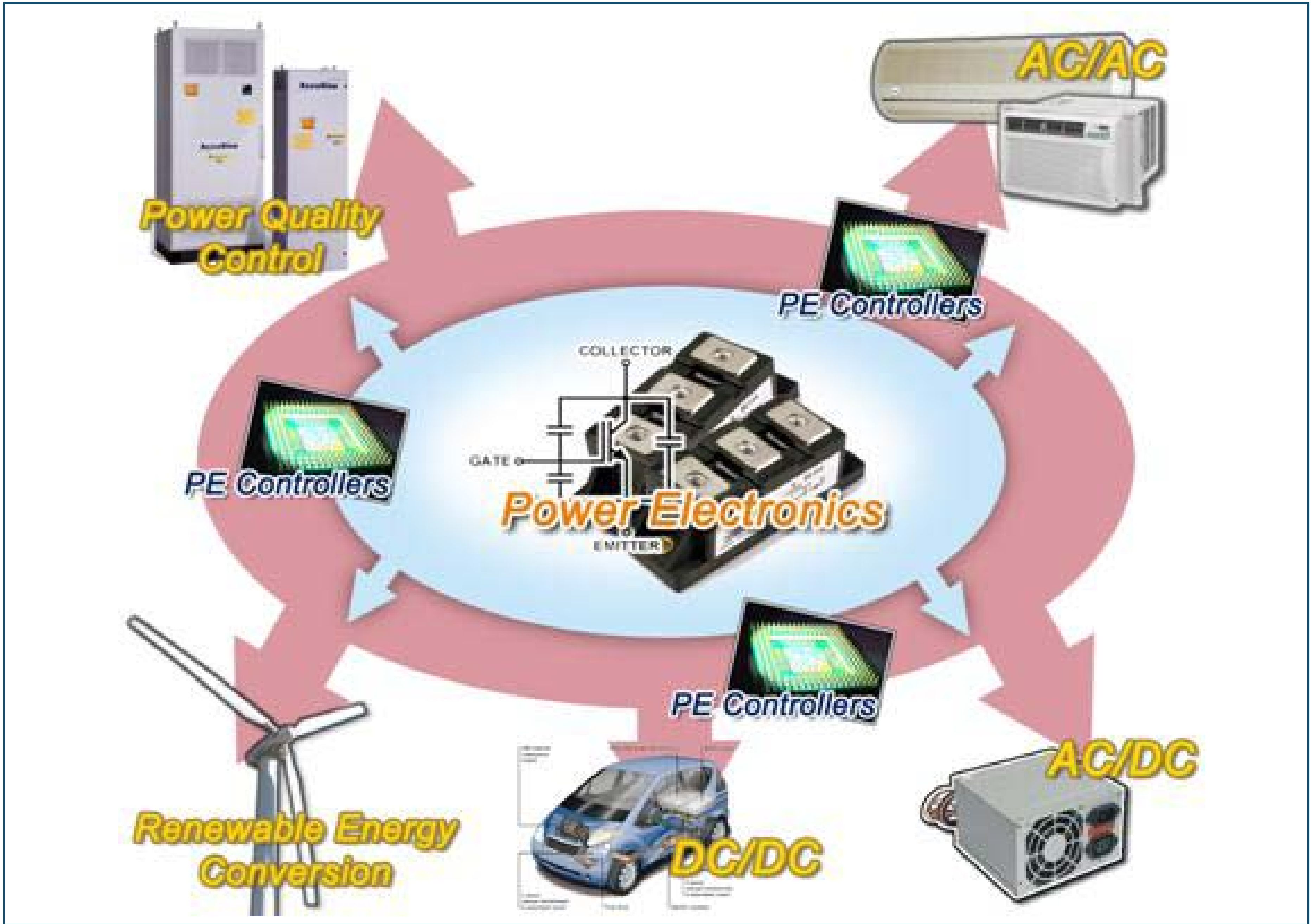
The on and off states of the device are controlled by control signals.

Types of Rectifiers



Power Conversion

CONVERSION FROM/TO	NAME	FUNCTION	SYMBOL
DC to DC	Chopper	Constant to variable DC or variable to constant DC	
DC to AC	Inverter	DC to AC of desired voltage and frequency	
AC to DC	Rectifier	AC to unipolar (DC) current	
AC to AC	AC controllers or AC Regulators	AC of desired frequency and/or magnitude from generally line AC	


Power Quality Control
AC/AC
PE Controllers
COLLECTOR
GATE
EMITTER
Power Electronics
PE Controllers
PE Controllers
Renewable Energy Conversion
DC/DC
AC/DC

Diode Circuits (Uncontrolled Rectifiers)

Rectification: The process of converting the alternating voltage to direct voltage.

Performance Parameters

Rectification Efficiency:

$$\eta = P_{dc} / P_{ac}$$

Form Factor:

$$FF = V_{rms} / V_{dc}$$

Ripple Factor:

$$RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{dc}} = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1}$$

$$RF = \sqrt{FF^2 - 1}$$

Three Phase Half-Wave Uncontrolled Rectifier ($f_o = 3f_i$)

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{2\pi} = 0.827V_m$$

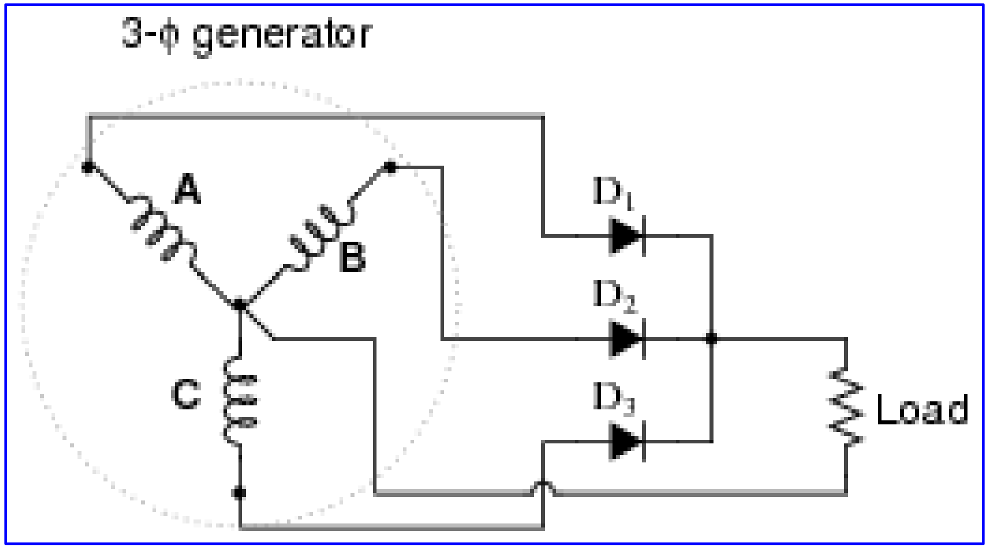
$$I_{dc} = \frac{3\sqrt{3}\,V_m}{2\pi R} = \frac{0.827V_m}{R}$$

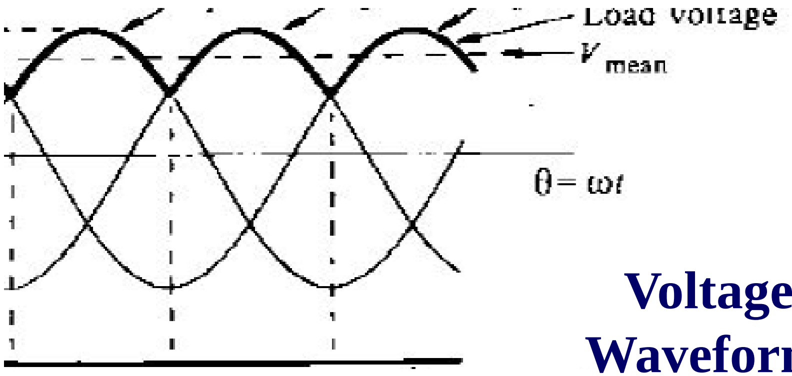
$$V_{rms} = \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{8\pi}}\,V_m = 0.8407\,V_m$$

$$I_{rms} = \frac{0.8407\ V_m}{R}$$

$$I_r = I_S = \frac{08407\ V_m}{R\sqrt{3}} = 0.4854\ \frac{V_m}{R}$$

$$Diode\ \text{PIV} = \sqrt{2}V_{LL} = \sqrt{3}\,V_m$$





Voltage Waveforms

Example 1 The rectifier shown above is operated from 460V/50 Hz supply at the secondary side and the load resistance is R = 20Ω. If the source inductance is negligible, determine: (a) Rectification efficiency, (b) Form factor (c) Ripple factor (d) Peak inverse voltage (PIV) of each diode.

Solution:

$$V_S = \frac{460}{\sqrt{3}} = 265.58 \ \text{Volt} \quad ; \quad V_m = 265.58 \times \sqrt{2} = 375.59 \ \text{Volt}$$

$$V_{dc} = \frac{3\sqrt{3}\ V_m}{2\pi} = 0.827\, V_m \quad ; \quad I_{dc} = \frac{3\sqrt{3}\ V_m}{2\pi\ R} = \frac{0827 V_m}{R}$$

$$V_{rms} = 0.8407\, V_m \quad ; \quad I_{rms} = \frac{0.8407\ V_m}{R}$$

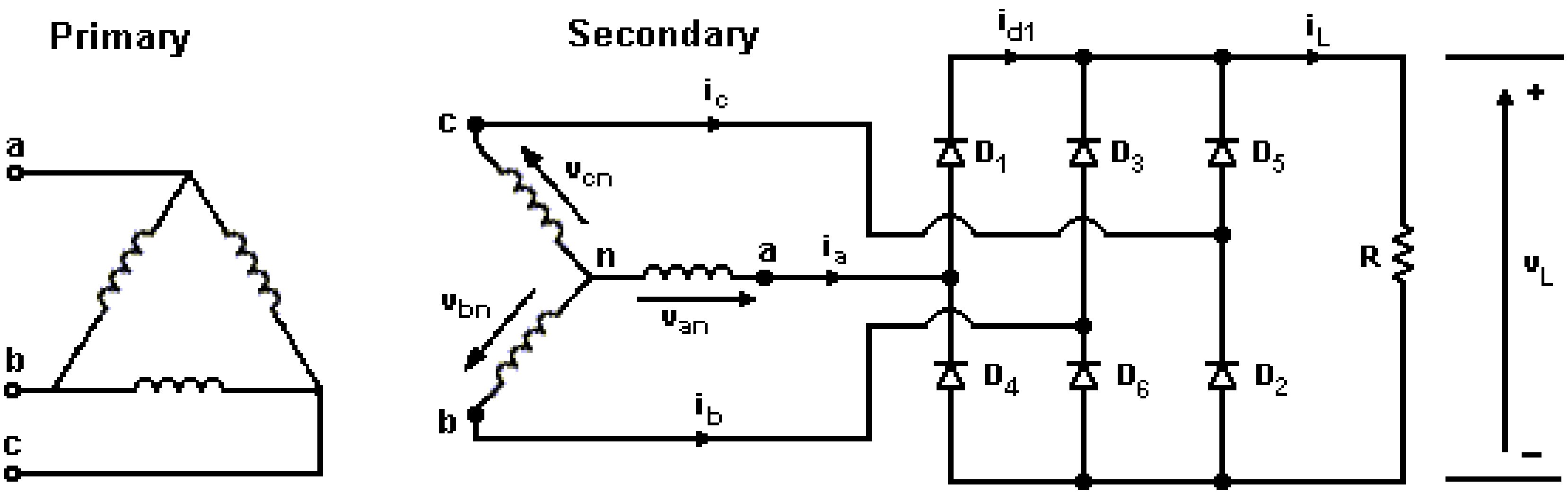
$$\Rightarrow \eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{dc}\ I_{dc}}{V_{rms}\ I_{rms}} = 96.767\ \%$$

$$\Rightarrow FF = \frac{V_{rms}}{V_{dc}} = 101.657\,\%$$

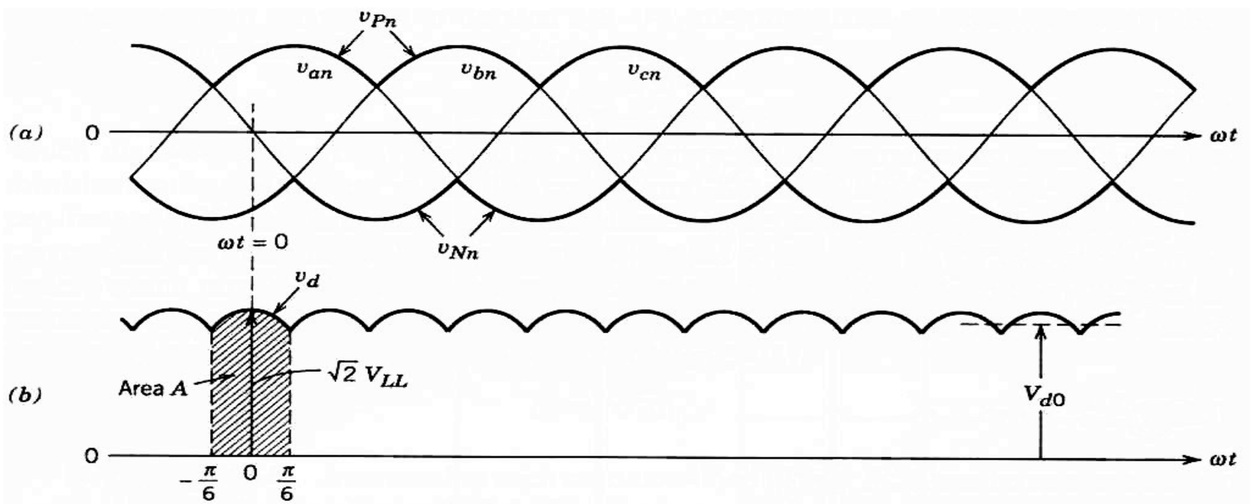
$$\Rightarrow RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{dc}} = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1} = \sqrt{FF^2 - 1} = 18.28\ \%$$

$$\Rightarrow PIV = \sqrt{3}\ V_m = 650.54\ \text{Volt}$$

Three Phase Uncontrolled Bridge Rectifier ($f_o = 6f_i$)



Voltage Waveforms



Three Phase Uncontrolled Bridge Rectifier ($f_o = 6f_i$)

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi} = \frac{3\sqrt{2}\,V_{LL}}{\pi} = 1.654V_m = 1.3505V_{LL}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{1.654V_m}{R} = \frac{1.3505V_{LL}}{R}$$

$$V_{rms} = \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}}\,V_m = 1.6554\,V_m = 1.3516V_{LL}$$

$$I_{rms} = \frac{1.6554\,V_m}{R}$$

$$I_r = \frac{1.6554\,V_m}{R\sqrt{3}} = 0.9667\frac{V_m}{R}$$

$$I_s = 0.9667\sqrt{2}\frac{V_m}{R}$$

$$Diode\ \text{PIV} = \sqrt{2}V_{LL} = \sqrt{3}\,V_m$$

Where:
$V_s = 0.707\,V_m$ is the rms voltage of transformer secondary;
$I_s = 0.7804\,I_m$ is the rms current of one line transformer secondary;
I_m = peak secondary line current.

Example 2 3ϕ Bridge rectifier is operated from 460V/50 Hz supply and the load resistance is $R = 20\Omega$. If the source inductance is negligible, determine (a) The efficiency, (b) Form factor (c) Ripple factor (d) Peak inverse voltage (PIV) of each diode .

Solution:

$$V_S = \frac{460}{\sqrt{3}} = 265.58 \text{ Volt} \quad ; \quad V_m = 265.58 \times \sqrt{2} = 375.5 \text{ Volt}$$

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi} = 1.654 V_m = 621.2 \text{ Volt} \quad ; \quad I_{dc} = \frac{1.654 V_m}{R} = 31.0613 \text{ A}$$

$$V_{rms} = 1.6554\,V_m = 621.75 \text{ Volt} \quad ; \quad I_{rms} = \frac{1.6554\,V_m}{R} = 31.08 \text{ A}$$

$$\Rightarrow \eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{dc}\,I_{dc}}{V_{rms}\,I_{rms}} = 99.83\ \%$$

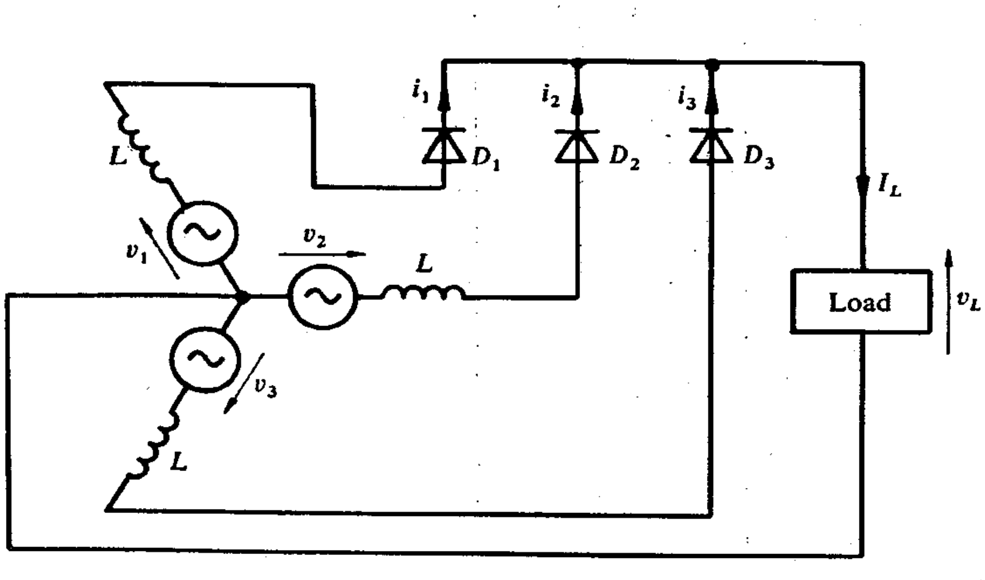
$$\Rightarrow FF = \frac{V_{rms}}{V_{dc}} = 100.08\%$$

$$\Rightarrow RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{dc}} = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1} = \sqrt{FF^2 - 1} = 4\ \%$$

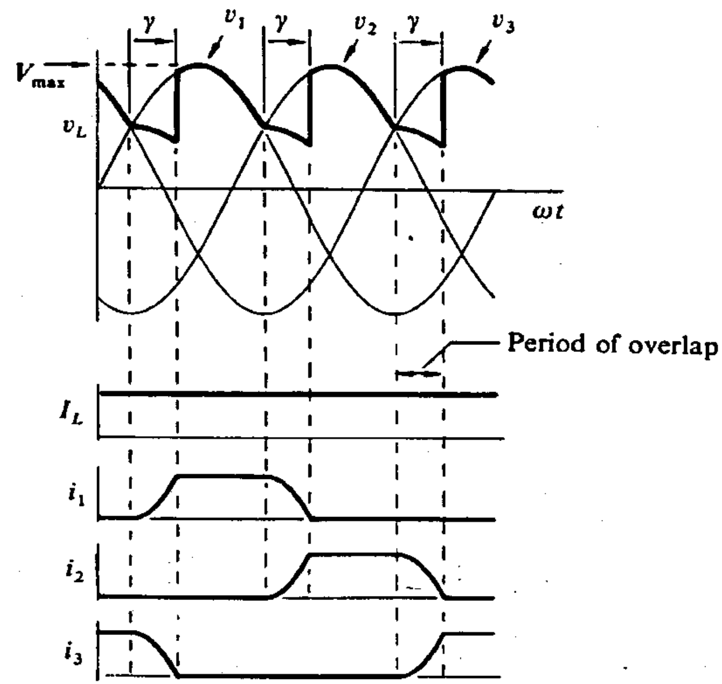
$$\Rightarrow PIV = \sqrt{3}\ V_m = 650.54 \text{ Volt}$$

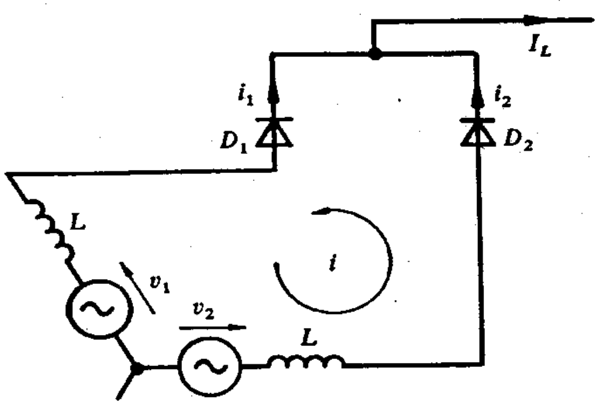
Overlap Phenomenon

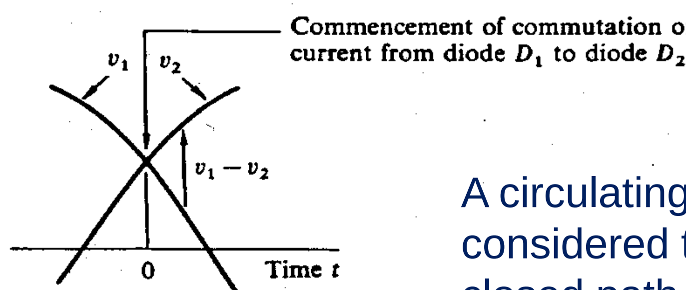
- Overlap is the phenomenon due to the effect of source inductance on the AC side.
- Commutation current is delayed due to the source inductance which is normally the leakage reactance of a transformer (as X >> R for a transformer, the source resistance is usually neglected).
- The waveforms with the commutation period, denoted by [γ] during which both the outgoing diode and incoming diode are conducting. This period is also known as “overlap” period.
- During the overlap period, the load current is the addition of the two diode currents, the assumption being made that the load is inductive enough to give a constant load current.
- The load voltage is the mean of the two conducting phases during overlap period.
- The effect of overlap is to reduce the mean output voltage.



(a)







A circulating current i can be considered to flow in the closed path formed by the 2 conducting diodes D1 and D2.

The average output voltage due to overlap:

$$V_{dc} = \frac{1}{2\pi/3}\left[\int_{\frac{\pi}{6}+\gamma}^{\frac{5\pi}{6}} V_m \sin\theta \, d\theta + \int_0^{\gamma} V_m \sin\frac{\pi}{6}\cos\phi \, d\phi\right] = \frac{3\sqrt{3}}{4\pi} V_m (1+\cos\gamma)$$

$$\boxed{V_{dc} = \frac{3\sqrt{3}}{4\pi} V_m (1+\cos\gamma)}$$

OR

V_{dc} = V_{dc} without overlap - reduction of V_{dc} due to overlap.

$$V_{dc} \text{ without overlap} = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m \sin\omega t \, d\omega t = \frac{3\sqrt{3}\, V_m}{2\pi}$$

Example 3 A three-phase half-wave rectifier circuit is supplied from a [120/208] V, 3phase, 60 Hz source. Determine: 1) the average DC output voltage of the rectifier, 2) the conduction angle θ_C, 3) output frequency f_o.

Solution:

$$V_{ph}=\frac{208}{\sqrt{3}}=120.2 \text{ Volt} \quad ; \quad V_{peak}=V_m=\sqrt{2}V_{ph}=170 \text{ Volt}$$

$$1)\ V_{dc}=3\sqrt{3}\times\frac{V_m}{2\pi}=140.3 \text{ Volt} \quad ; \quad 2)\ \theta_C=\omega t_C=2\pi f_i\times\left(\frac{1}{3f_i}\right)=\frac{2\pi}{3}=120^o$$

$$3)\ f_o=3f_i=3\times 60=180\text{Hz}$$

Example 4 3ϕ – Half-Wave rectifier with Δ-Y transformer is connected to 100Ω resistive load. For 220V secondary line to line voltage and 50Hz supply frequency. Calculate: 1) The dc load power, 2) The conduction time of each diode, 3) If the input transformer has leakage reactance which causes an overlap angle of 20° , calculate V_{dc} .

Solution:

$$1)\ V_{dc}=0.827V_m=0.827\times\sqrt{2}V_{ph}=0.827\times\sqrt{2}\frac{220}{\sqrt{3}}=149\text{ Volt}$$

$$\Rightarrow P_{dc}=\frac{V_{dc}^2}{R}=\frac{(149)^2}{100}=222\text{W}$$

$$2)\ \theta_C=\omega t_C \quad \Rightarrow t_C=\frac{\theta_C}{\omega}=\frac{2\pi/3}{2\pi f_i}=6.67m\sec \text{ OR } t_C=\frac{1}{f_o}=\frac{1}{3f_i}$$

$$3)\ \text{Due to overlap:}V_{dc}=3\sqrt{3}\frac{V_m}{4\pi}(1+\cos 20)=144.1\text{Volt}$$

Problems and Solutions

Question 1 A three-phase half-wave rectifier circuit is supplied from a [220/380] V, 3phase, 60 Hz source. Determine : 1) The average DC output voltage of the rectifier, 2) The conduction time of each diode, 3) If there is an overlap phenomenon of 30° , find V_{dc} and the reduction of V_{dc} due to overlap.

Solution:

$$V_m = 380 \times \sqrt{2} / \sqrt{3} = 311 \text{ Volt}$$

1) $V_{dc} = 0.827 V_m = 0.827 \times 311 = 212 \text{ Volt}$

2) $\theta_C = \omega t_C \Rightarrow t_C = \dfrac{\theta_C}{\omega} = \dfrac{1}{f_o} = \dfrac{1}{3f_i} = \dfrac{1}{3 \times 60} = 5.56\text{msec}$

3) For an overlap angle of $30^o \Rightarrow V_{dc} = 3\sqrt{3}\dfrac{V_m}{4\pi}(1 + \cos\gamma)$

$$\therefore V_{dc} = 3\sqrt{3}\frac{311}{4\pi}\left(1 + \cos 30^o\right) = 198.2\text{Volt}$$

$\therefore$ the reduction in $V_{dc} = V_{dc}$ without overlap $- V_{dc}$ with overlap

$$V_{dc} = 212 - 198.2 = 13.8 \text{ Volt}$$

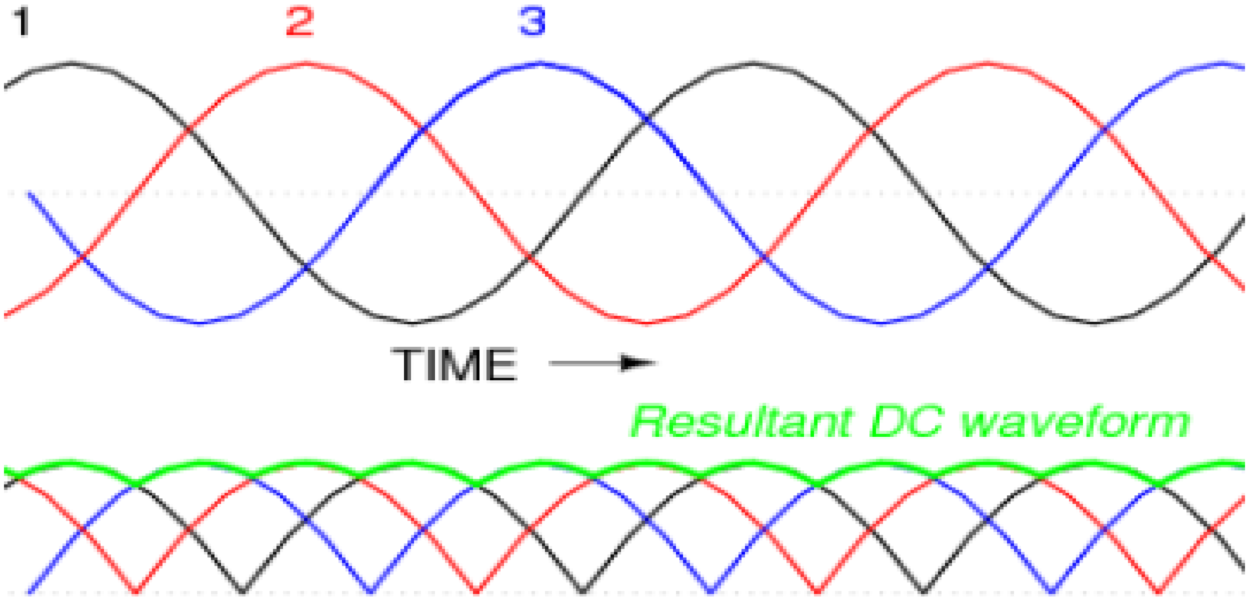
Question 2 A three-phase Bridge rectifier is supplied from a [220/120] V, 3phase, 60 Hz source. 1) Determine the average DC output voltage of the rectifier, 2) Sketch the load voltage and supply current waveforms.

Solution:

$$V_m = 120 \times \sqrt{2} / \sqrt{3} = 97.97 \text{ Volt}$$

1) $V_{dc} = 1.654 V_m = 1.654 \times 97.97 = 162 \text{ Volt}$

2) Load voltage and supply waveforms as below:



Question 3

3ϕ-Bridge rectifier with Δ-Y transformer and [1200w, 3Ω] resistive load. Find : 1) The secondary line voltage, 2) The Form Factor, and 3) The efficiency of rectification.

Solution:

$$1)\ P_{dc} = \frac{V_{dc}^2}{R} \Rightarrow V_{dc} = 3\times\sqrt{1200} = 60\ \text{Volt}$$

$$V_{dc} = 1.654 V_m \Rightarrow V_m = \frac{60}{1.654} = 36.27\ \text{Volt}$$

$$V_{line} = \frac{\sqrt{3}\,V_m}{\sqrt{2}} = 44.26\ \text{Volt}$$

$$2)\ V_{rms} = 1.6554 V_m = 60.041\ \text{Volt}$$

$$\text{FF} = \frac{V_{rms}}{V_{dc}} = \frac{1.6554 V_m}{1.654 V_m} = 100.8\%$$

$$3)\ I_{dc} = \frac{V_{dc}}{R} = \frac{60}{3} = 20\ \text{A}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{60.041}{3} = 20.014\ \text{A}$$

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{dc}\,I_{dc}}{V_{rms}\,I_{rms}} = \frac{1200}{60.041\times 20.014} = 99.86\ \%$$

Question 4 3ϕ – Half-Wave rectifier with Δ-Y transformer and [250w, 5A] resistive load. Find the value of the secondary line voltage.

Solution:

$$P_{dc} = V_{dc} \times I_{dc} = 250\text{W} \Rightarrow V_{dc} = 250/5 = 50\text{ V} = 0.827 V_m$$

$$\Rightarrow V_m = 60.5\text{V} ;\; V_{ph} = V_m/\sqrt{2} = 42.79\text{V};\; V_{line} = \sqrt{3} V_{ph} = 74\text{V}$$

Question 5 3ϕ – Bridge rectifier with Δ-Y transformer is connected to 100Ω resistive load. For 220V secondary line to line voltage and 50Hz supply frequency, calculate: 1) The dc load power, 2) The conduction time.

Solution:

$$V_{dc} = 1.654 V_m = 1.654 \times \sqrt{2} \times 220/\sqrt{3} = 297\text{ V} \Rightarrow P_{dc} = V_{dc}^2/R = (297)^2/100 = 883\text{W}$$

$$t_C = \theta_C/\omega = 1/f_o = 1/6f_i = 1/(6 \times 50) = 3.33\text{msec}$$

Question 6 3ϕ – Bridge rectifier with Δ-Y transformer and [250w, 5A] resistive load. Find the value of secondary line voltage V_{ab}.

Solution:

$$P_{dc} = V_{dc} \times I_{dc} = 250\text{W} \Rightarrow V_{dc} = 250/5 = 50\text{ V} = 1.654 V_m$$

$$\Rightarrow V_m = 52.3\text{V} ;\; V_{ph} = V_m/\sqrt{2} = 36.98\text{V};\; V_{line} = \sqrt{3} V_{ph} = 64\text{V}$$

Question 7 Choose the correct or best alternative in the following:

1) Power electronics is the application of……………..for the control and conversion of electric power.

   A. Conductors B. Transformer C. Inductive load **D.** solid-state electronics

2) Most power semiconductor devices are only used in ……………

   A.commutation mode B. Rectification mode C. Amplification mode D. Non of the above

3) The three phase half-wave rectifier with resistive has an average value = …………..

   A. 0.87 V_{ph} **B.** 1.17 V_{ph} C. 1.41 V_{ph} D. 1.99 V_{ph}

4) When voltage applied to a diode is more than PIV, it is likely to result in…………

   A. More distortion on output side B. Poor regulation C. Conduction in both direction **D.** Breakdown at the junction.

5) The ripple factor of a full-wave rectifier compared to half wave rectifier is …………

   A. half that of half -wave rectifier **B.** less than half that of half-wave rectifier
   C. equal to that of half -wave rectifier D. none of the above.

6) Switched mode power supplies are preferred over the continuous types, because they are……………and……………………..

   A. suitable for use in both ac and dc **B.** more efficient **C.** suitable for low-power circuits.
   D. suitable for high-power circuits.

12) Electric current is the flow of …………..

   A. Neutrons B. Photons **C.** Electrons D. none of the above

Question 8 **Fill in the blanks with the appropriate words**

1) In a rectifier, electrical power flows from the……………..side to the………………side.

2) Uncontrolled rectifiers employ………………whereas controlled rectifiers employ……………….

3) For any waveform “Form factor” is always……………..than or equal to unity.

4) Three phase half-wave uncontrolled rectifier uses……………….. diodes .

5) Three phase half-wave uncontrolled rectifier requires………… phase……… wire power supply .

Answers:

1) ac, dc

2) diodes, thyristors

3) greater

4) three

5) three , four

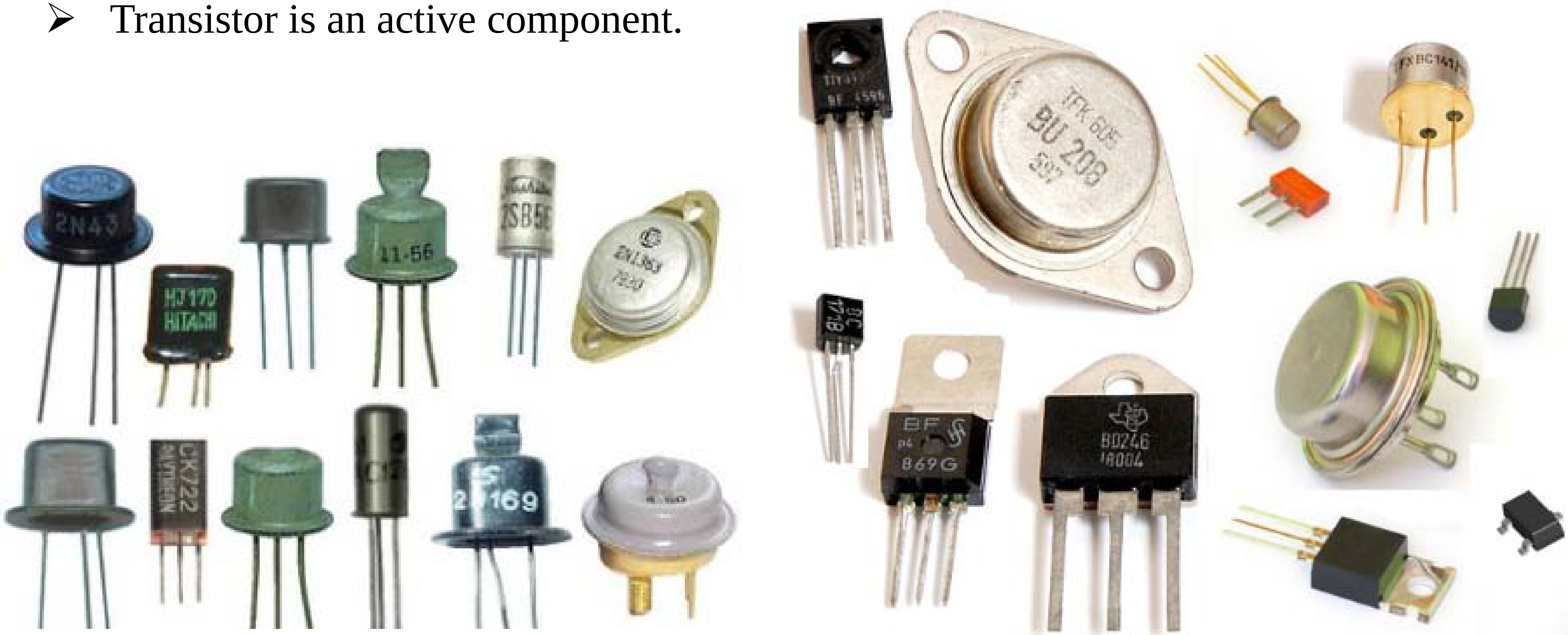
Chapter 2

The Transistors

The Transistor

Transistor: It is a three-terminal, solid-state semiconductor device used to control the electric current or voltage between two of the terminals by applying an electric current or voltage to the third terminal.

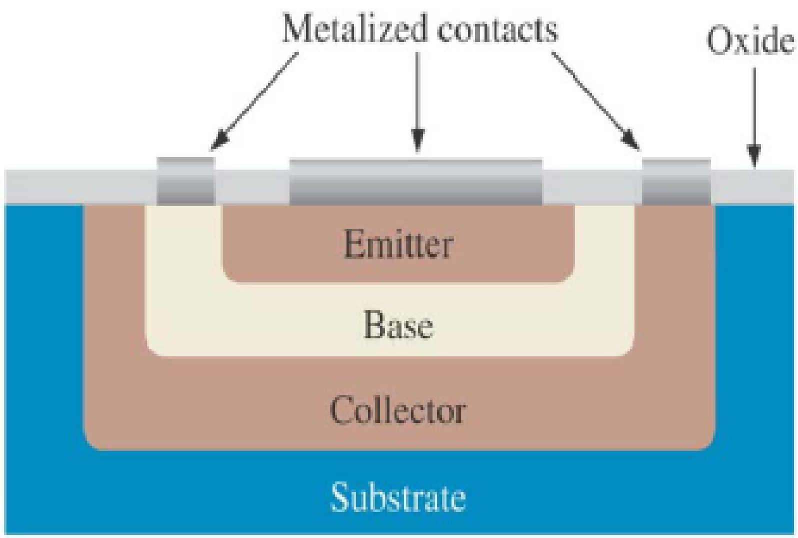
- Transistor is an active component.



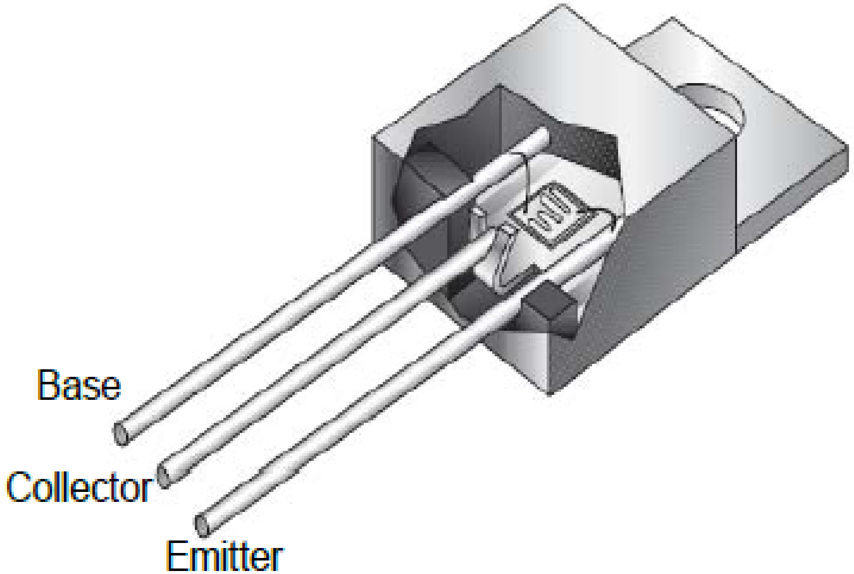
Examples of Different Transistors Packages

Bipolar Junction Transistor (BJT)

- It is constructed with *three* doped semiconductor regions separated by *two* PN junctions.
- The three regions are called **emitter**, **base**, and **collector**.
- The base is lightly doped and very narrow compared with the heavily doped emitter and moderately doped collector.
- It is used as an electrical signal amplifier or an electronic switch.
- Bipolar: refers to the use of both holes and electrons as current carriers.



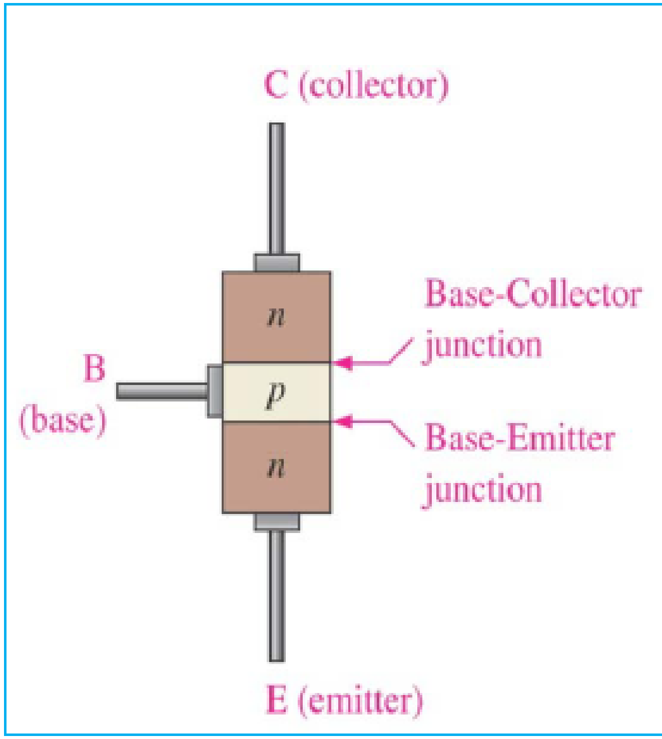
Basic BJT construction

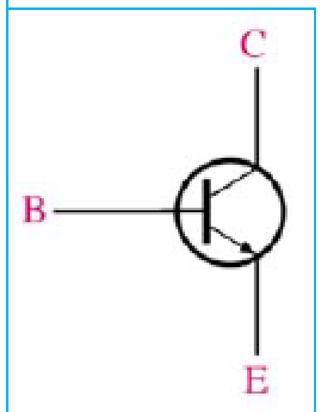


BJT package Example

Types of BJTs

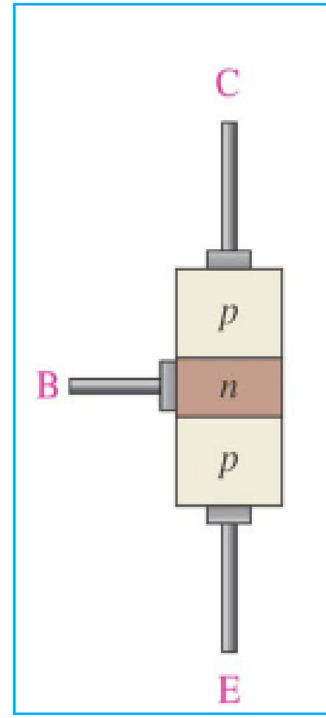
npn

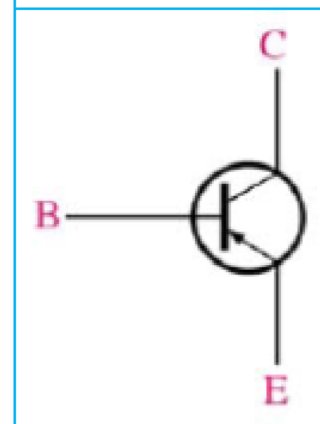




npn Symbol

pnp





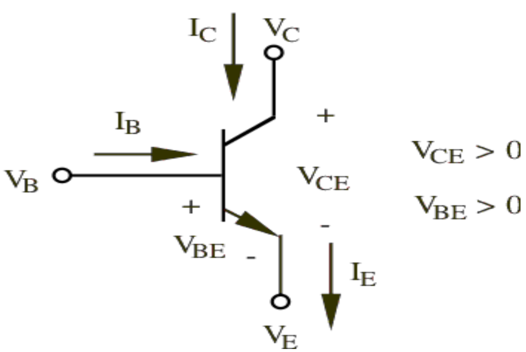
pnp Symbol

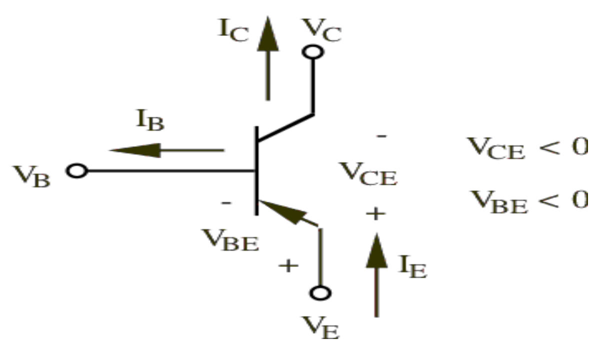
$$V_{BE} \simeq 0.7\ \mathrm{V}$$

$$I_E = I_C + I_B$$

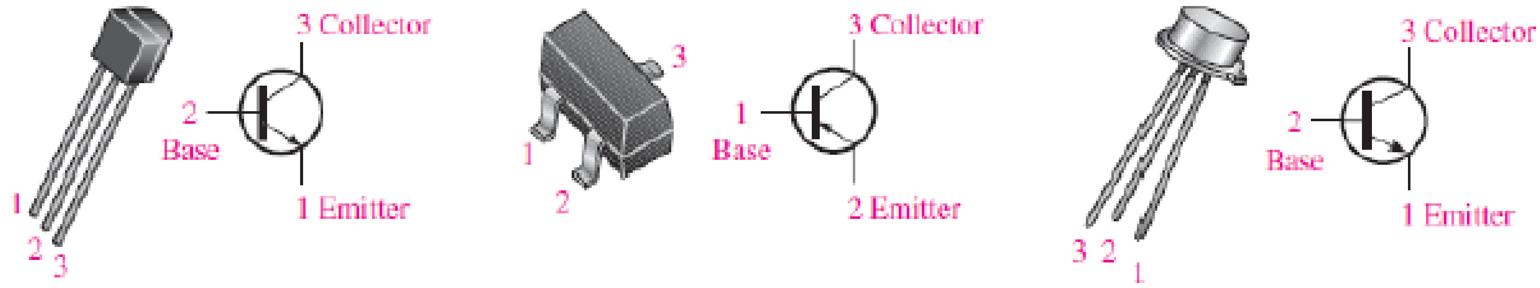
$$V_{BE} = V_B - V_E$$

$$V_{CE} = V_C - V_E$$



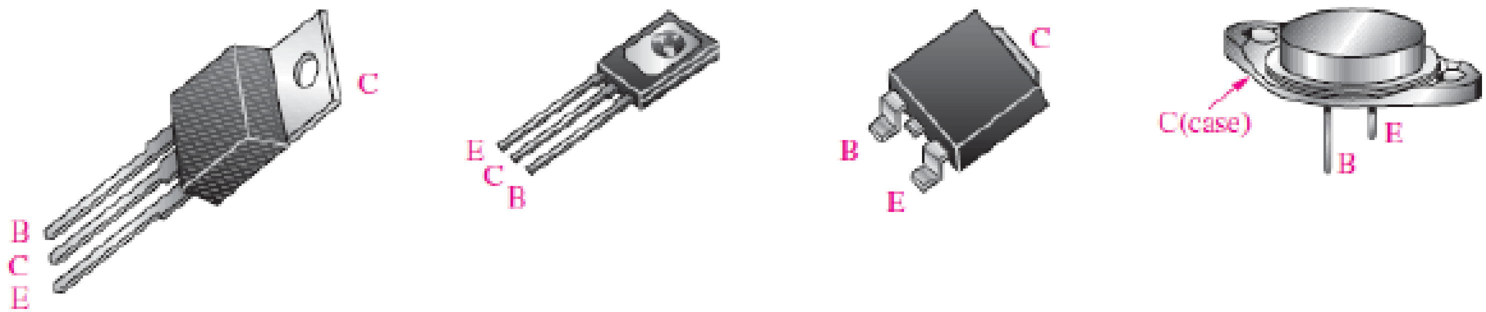


Examples of BJTs



(a) TO-92 (b) SOT-23 (c) TO-18. Emitter is closest to tab.

General-purpose/small-signal transistors



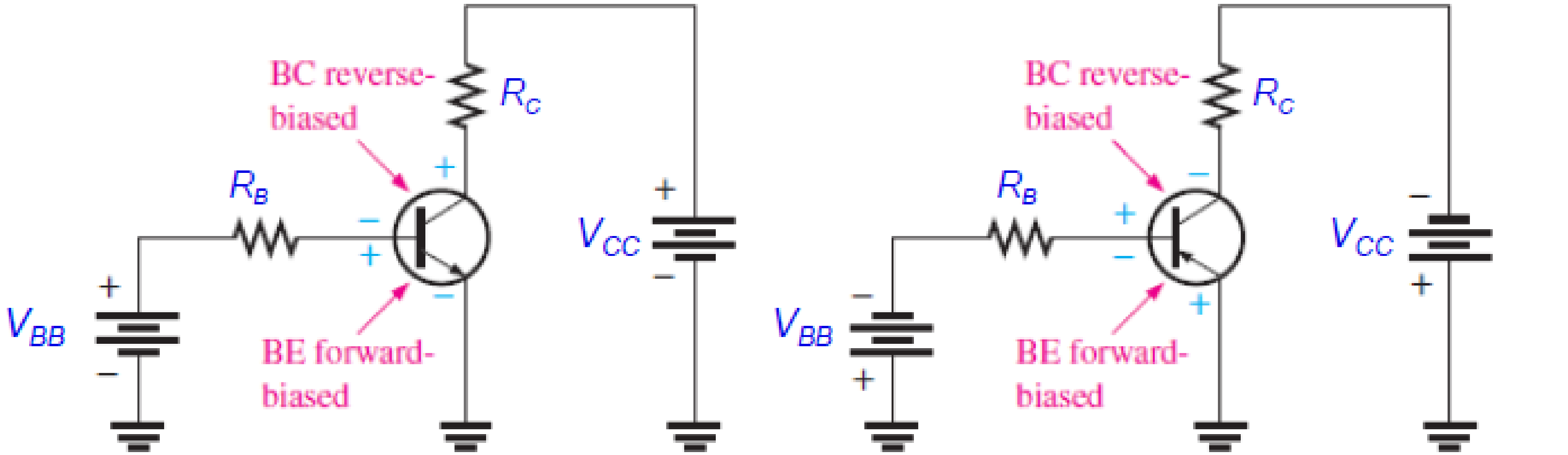
(a) TO-220 (b) TO-225 (c) D-Pack (d) TO-3

Power transistors

Common Emitter BJT Circuit

- In order for a **BJT** to operate properly as an amplifier, the two **pn** junctions must be correctly biased with external **dc** voltages.
- The operation of **pnp** is similar to the **npn** except that the roles of electrons and holes, bias voltage polarities, and current directions are all reversed.

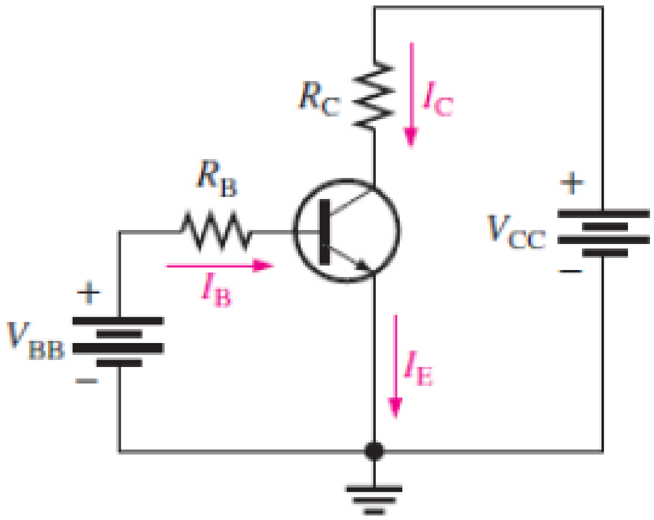
Forward-Reverse Bias for BJT Operation as an amplifier



(a) *npn* (b) *pnp*

- The BE junction in both cases is forward-biased .
- The BC junction in both cases is reversed-biased.

BJT Currents



(a) *npn* (b) *pnp*

For Conventional current direction (holes current) we have:

$$I_E = I_B + I_C$$

I_B is very small due to light doping

$$I_E \simeq I_C$$

Collector current is much greater than base current and hence the current gain

BJT Parameters

DC Beta (β_{DC}) and DC Alpha (α_{DC})

- The **dc current gain** of a transistor (β_{DC}) is the ratio of dc collector current (I_C) to the dc base current (I_B).

$$\beta_{DC} = \frac{I_C}{I_B}$$

Typical values of β_{DC} from less than 20 to 200 or higher.

- In data sheets, the hybrid ***h***-parameters are used for transistors where $h_{FE} = \beta_{DC}$.

- α_{DC} is the ratio of dc collector current (I_C) to the dc emitter current (I_E).

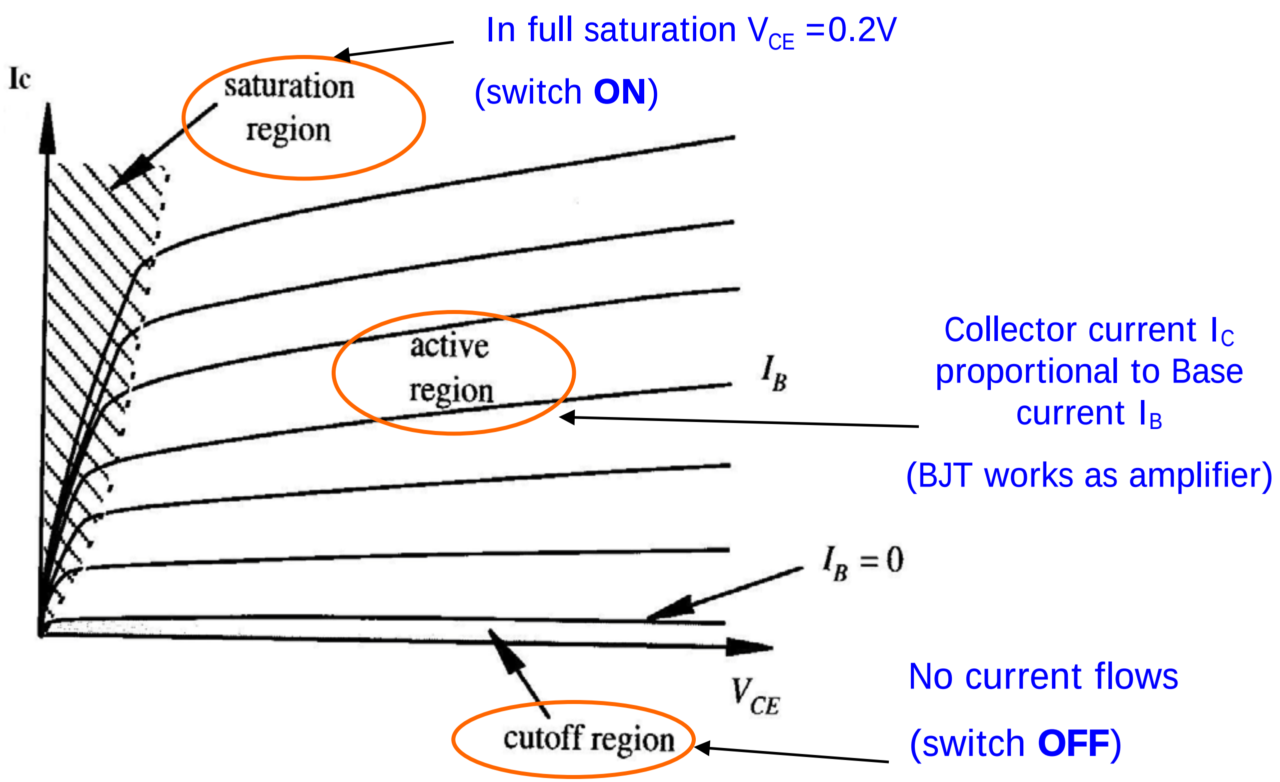
$$\alpha_{DC} = \frac{I_C}{I_E}$$

Typical values of α_{DC} from 0.95 to 0.99 or less than 1.

BJT Operation

- Emitter is grounded and input voltage is applied to Base.
- Base-Emitter starts to conduct when V_{BE} is about 0.7V, I_C flows with $I_C = \beta_{\mathrm{DC}}\, I_B$
- As I_B further increases, V_{BE} slowly increases to 0.7V, IC rises exponentially.
- As I_C rises, voltage drop across R_C increases and V_{CE} drops toward ground (transistor in saturation, no more linear relation between I_C and I_B)

Common Emitter Characteristics



BJT Operation

- Electrons diffuse from **E**mitter to **B**ase (from *n* to *p*) With $V_C > V_B > V_E$
- Depletion layer on the Base-Collector junction → no flow of electron allowed.
- The Base is thin and Emitter region is heavily doped → electrons have enough momentum to cross Base into Collector.
- Small base current I_B controls large current I_C through β_{DC} which functioning as a current gain of the amplifier.
- β_{DC} is temperature and voltage dependent.

Example 1

(a) For a given BJT: I_B = 50 μA and I_C = 3.65 mA . Find the dc current gain (β_{DC}), I_E , and α_{DC} .

(b) In a certain BJT: I_B = 50 μA and β_{DC} = 200. Find I_C , I_E , and α_{DC} . (HOME WORK)

SOLUTION

$$\beta_{DC} = \frac{I_C}{I_B} = \frac{3.65 \text{ mA}}{50 \ \mu\text{A}} = \frac{3.65 \times 10^{-3}}{50 \times 10^{-6}} = 73$$

$$I_E = I_B + I_C = 50 \times 10^{-6} + 3.65 \times 10^{-3} = 3.7 \times 10^{-3} = 3.7 \text{ mA}$$

$$\alpha_{DC} = \frac{I_C}{I_E} = \frac{3.65 \text{ mA}}{3.7 \text{ mA}} = \frac{3.65 \times 10^{-3}}{3.7 \times 10^{-3}} = 0.986$$

DC Analysis of BJT Circuit

$$V_{R_B} = V_{BB} - V_{BE}$$

Also, by Ohm's law,

$$V_{R_B} = I_B R_B$$

Substituting for V_{R_B} yields

$$I_B R_B = V_{BB} - V_{BE}$$

Solving for I_B,

$$I_B = \frac{V_{BB} - V_{BE}}{R_B}$$

The voltage at the collector with respect to the grounded emitter is

$$V_{CE} = V_{CC} - V_{R_C}$$

Since the drop across R_C is

$$V_{R_C} = I_C R_C$$

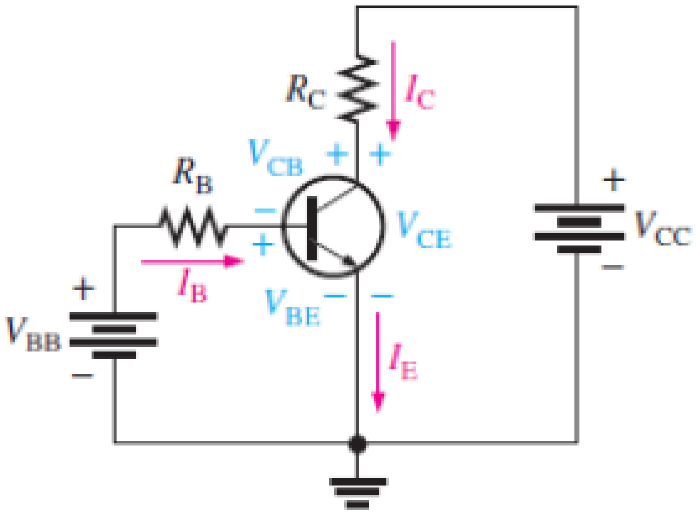
the voltage at the collector with respect to the emitter can be written as

$$V_{CE} = V_{CC} - I_C R_C$$

where $I_C = \beta_{DC} I_B$.

The voltage across the reverse-biased collector-base junction is

$$V_{CB} = V_{CE} - V_{BE}$$

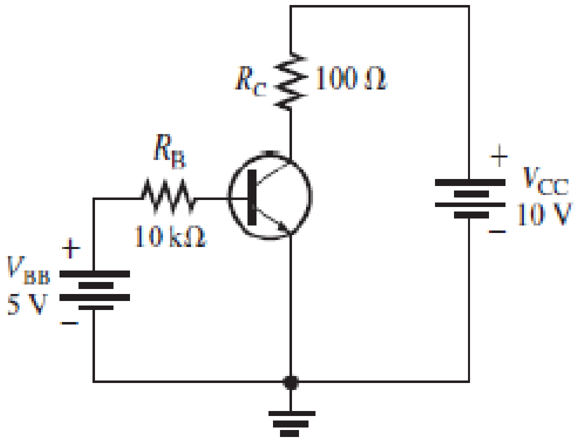


DC Bias Circuit of *npn* Transistors

Example 2

1) For the given npn BJT circuit if $\beta_{DC} = 150$, find: I_B, I_C, I_E, V_{BE}, V_{CE}, and V_{CB}.
2) Repeat the problem if $\beta_{DC} = 90$, $R_B = 22\ k\Omega$, $R_C = 220\ \Omega$, $V_{BB} = 6V$, $V_{CC} = 9V$. (HOME WORK)

SOLUTION



$$V_{BE} \cong \mathbf{0.7\ V}$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{5\ V - 0.7\ V}{10\ k\Omega} = \mathbf{430\ \mu A}$$

$$I_C = \beta_{DC} I_B = (150)(430\ \mu A) = \mathbf{64.5\ mA}$$

$$I_E = I_C + I_B = 64.5\ mA + 430\ \mu A = \mathbf{64.9\ mA}$$

Solve for V_{CE} and V_{CB}.

$$V_{CE} = V_{CC} - I_C R_C = 10\ V - (64.5\ mA)(100\ \Omega) = 10\ V - 6.45\ V = \mathbf{3.55\ V}$$

$$V_{CB} = V_{CE} - V_{BE} = 3.55\ V - 0.7\ V = \mathbf{2.85\ V}$$

Since the collector is at a higher voltage than the base, the collector-base junction is reverse-biased.

BJT As Switch

OFF State

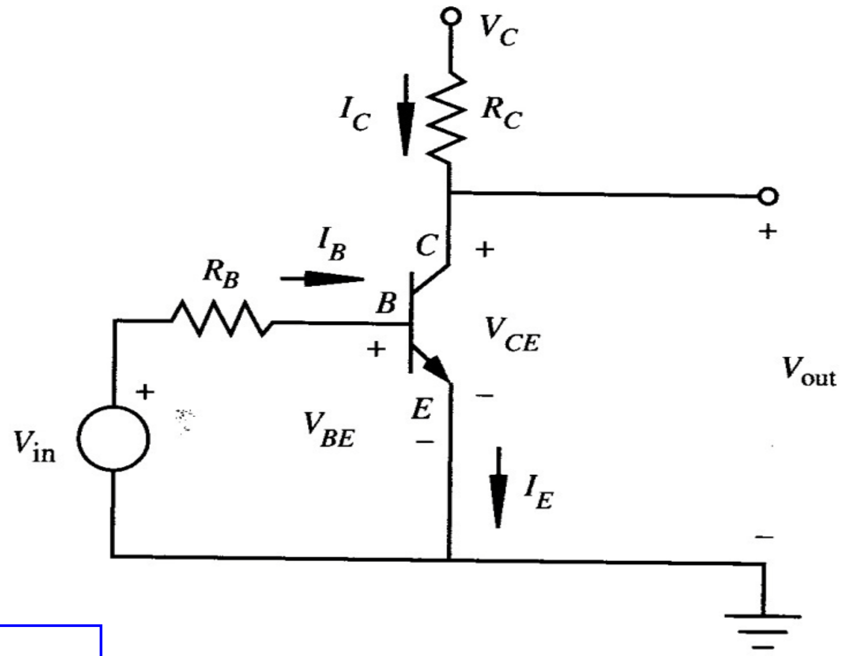
When $V_{in} < 0.7V$

- BE junction not forward biased
- **Cutoff state** of transistor
- $I_C = I_E = 0$
- $V_{out} = V_{CE} = V_C$
- V_{out} = High

ON State

When $V_{in} > 0.7$

- BE junction forward biased ($V_{BE} = 0.7V$)
- $I_B = (V_{in} - V_{BE}) / R_B$
- **Saturation** region
- V_{CE} small (~0.2 V for saturated BJT)
- V_{out} = Low



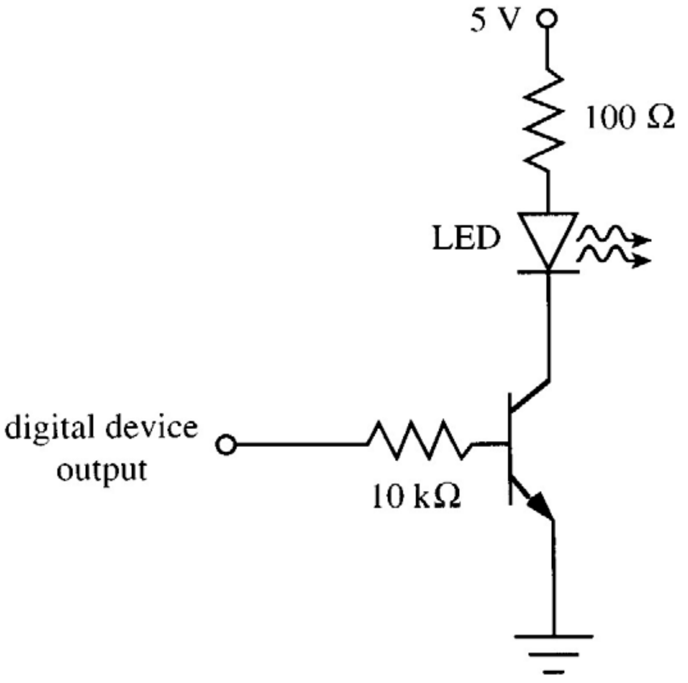
Example 3

Practical LED Switch: For the given 2N3904 npn BJT circuit assuming the LED requires 20-40 mA to provide a bright display and has 2 voltage drop when forwarded biased. Find that the circuit works properly with ON and OFF states.

Solution

- When digital output is 0V, transistor is **OFF**.
- When digital output is 5V, the transistor is in **saturation** (**ON**), with base current:

$$I_B = (5V - 0.7V)/10K\Omega = 0.43\,mA$$



Collector current (LED current) is limited by collector resistor as:

$$I_C = (5V - 2V - 0.2V)/100\Omega = 28\,mA$$

Hence: LED is lighted properly

Phototransistors

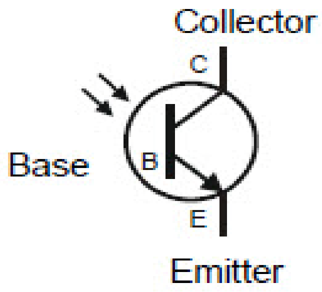
- **Phototransistors** are solid-state light detectors that possess internal gain.
- They are photodiode-amplifier combinations (2 or 3 terminals) integrated within a single silicon chip to overcome the major fault of photodiodes (unity gain).
- They can be used to provide either an analog or digital output signal.

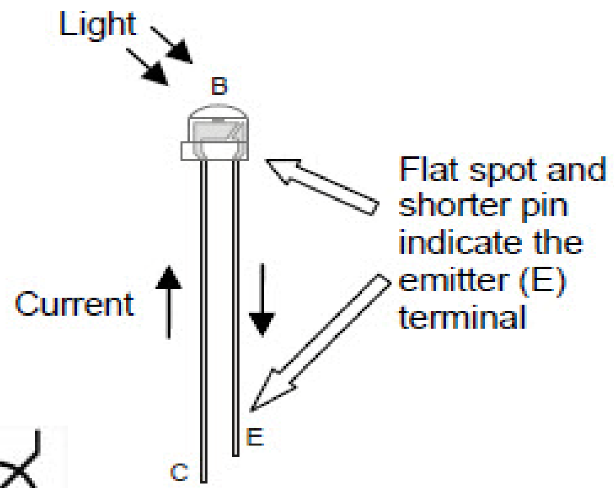
NPN Phototransistors

0.25", small area, high speed

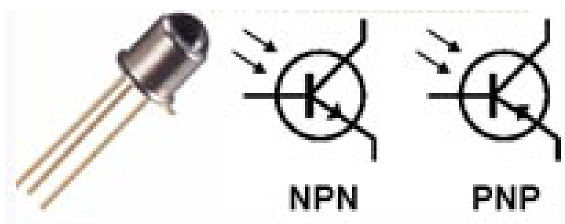
0.04", medium area, high sensitivity

0.05", large area, high sensitivity





Two and Three Terminal Phototransistors



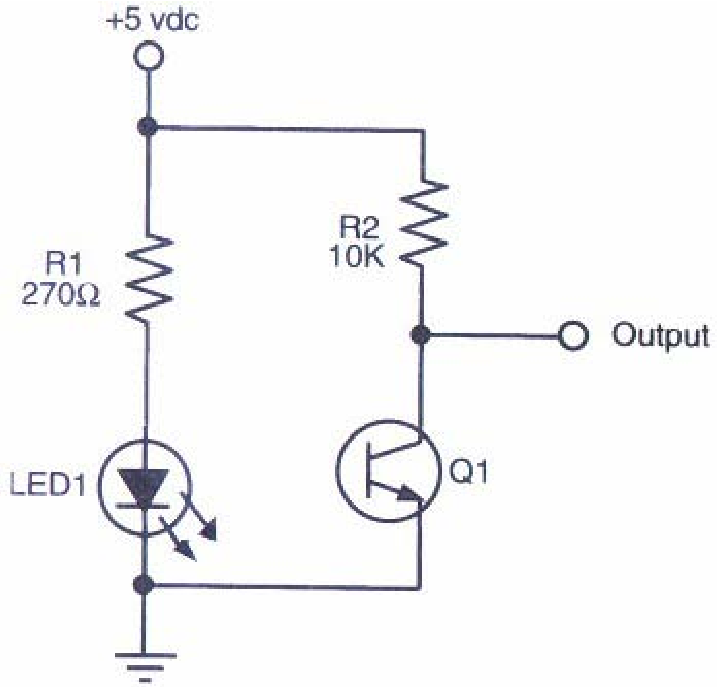
Features and Applications of Phototransistors

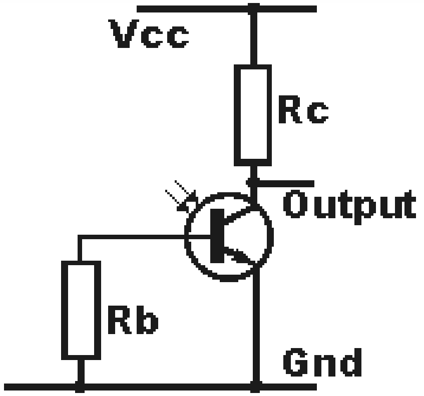
Features

- Low-cost visible and near-IR region.
- Available with gains from 100 to over 1500.
- Moderately fast response times.
- Available in a wide range of packages.
- Same general electrical characteristics as BJTs.

Applications

- Computer/business equipment: write-protect control and margin controls in printers.
- Industrial: security systems and safety shields.
- Consumers: coin counters, lottery card readers, audio/visual equipments, games, and camera shutter control.





Circuit Diagrams for Phototransistor Connections

Problems and Solutions

Question 1 Using ideal BJT switch of $\beta_{DC} = 200$, $V_{CC} = 10V$ and $I_B = 20\mu A$.

1) Find the value of base resistor R_B required to switch the load "ON" when the input terminal voltage exceeds 2.5V and $V_{BE} = 0.7V$.
2) Calculate R_C for point (1).
3) Find the minimum base current I_B required to turn the transistor "Fully-ON" (saturated) for a load that requires 200 mA when the input voltage is increased to 5.0V. Also calculate the new value of R_B.

Solution:

1) $$R_B = \frac{V_{in} - V_{BE}}{I_B} = \frac{2.5 - 0.7}{20 \times 10^{-6}} = 90\ \text{k}\Omega$$

2) For ideal BJT at saturation $V_{CE} = 0$ Volt

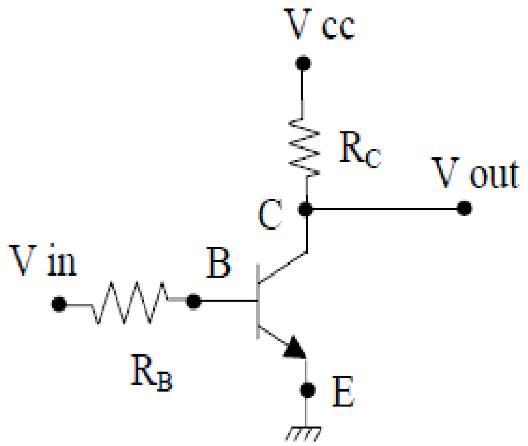
$$I_C = \beta_{DC} I_B = 200 \times 20 \times 10^{-6} = 4\ \text{mA}$$

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{10 - 0}{4 \times 10^{-3}} = 2.5\ \text{k}\Omega$$

3) At ON state, the load current is equal to I_C

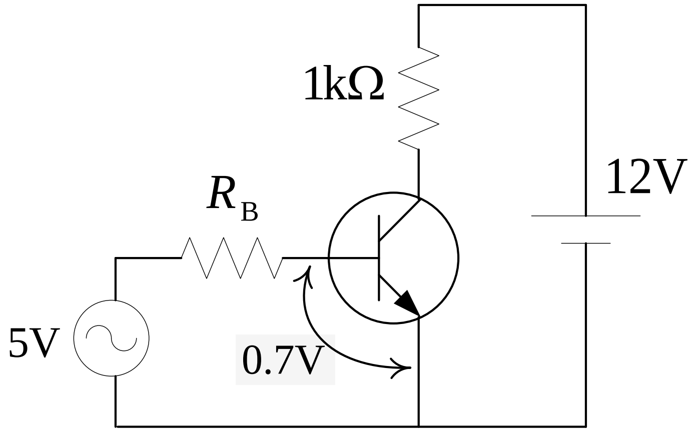
$$I_B = \frac{I_C}{\beta_{DC}} = \frac{200\ \text{mA}}{200} = 1\ \text{mA}$$

$$R_B = \frac{V_{in} - V_{BE}}{I_B} = \frac{5 - 0.7}{1 \times 10^{-3}} = 4.3\ \text{k}\Omega$$



Question 2 The BJT in the given circuit has: $50 \le \beta_{DC} \le 150$, $V_{CC} = 12$V, $V_{BE} = 0.7$V, and $V_{CE} = 0.2$V. Find R_B that saturates the BJT with the so called Over Driven Factor (ODF) of at least 10.

Solution:



$$I_{Csat} = \frac{V_{CC} - V_{CE}}{R_C} = \frac{12 - 0.2}{1 \times 10^3} = 11.8 \text{ mA}$$

$$I_B = \frac{I_{Csat}}{\beta_{DC\,\min}} = \frac{11.8 \times 10^{-3}}{50} = 0.236 \text{ mA}$$

For OFD of at least10, we have:

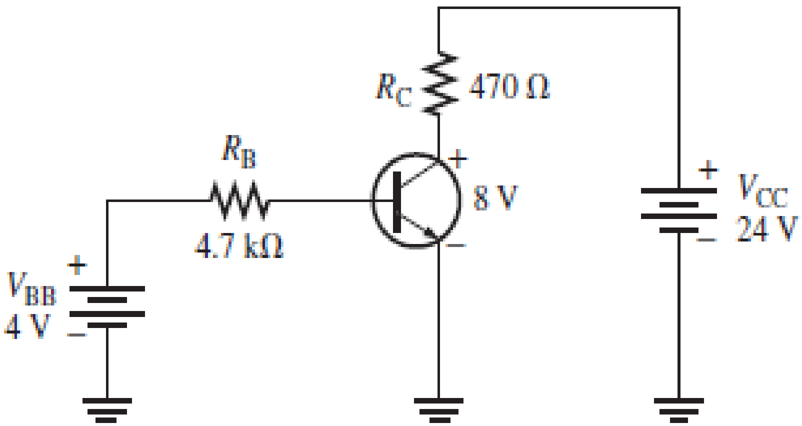
$$I'_B = 10 \times I_B = 10 \times 0.236 \times 10^{-3} = 2.36 \text{ mA}$$

$$R_B = \frac{V_{in} - V_{BE}}{I'_B} = \frac{5 - 0.7}{2.36 \times 10^{-3}} = 1.82 \text{ k}\Omega$$

Question 3 Find for the given npn BJT circuit: I_B , I_C , I_E , V_{CB} , β_{DC} , α_{DC}

Solution:

We have $V_{BE} = 0.7$ V and form the circuit $V_{CE} = 8$ V



$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{4 - 0.7}{4.7 \times 10^3} = 0.7 \text{ mA}$$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{24 - 8}{470} = 34 \text{ mA}$$

$$I_E = I_B + I_C = 0.7 \text{ mA} + 34 \text{ mA} = 34.7 \text{ mA}$$

$$V_{CB} = V_{CE} - V_{BE} = 8 - 0.7 = 7.3 \text{ V}$$

$$\beta_{DC} = \frac{I_C}{I_B} = \frac{34 \text{ mA}}{0.7 \text{ mA}} = 48.57$$

$$\alpha_{DC} = \frac{I_C}{I_E} = \frac{34 \text{ mA}}{34.7 \text{ mA}} = 0.9798$$

Question 4 For the given BJT switch circuit: $\beta_{DC} = 100$, $V_{CC} = 5V$, $V_{BE} = 0.7V$, $V_{CE} = 0V$, $V_{LED} = 2V$ and $I_{LED} = 15mA$. Assuming the minimum average voltage of microcontroller I/O port V_{port} with logical "1" is about 4.2V.

1) Find the value of R_B and R_C.
2) Calculate the power dissipation on R_B and R_C.

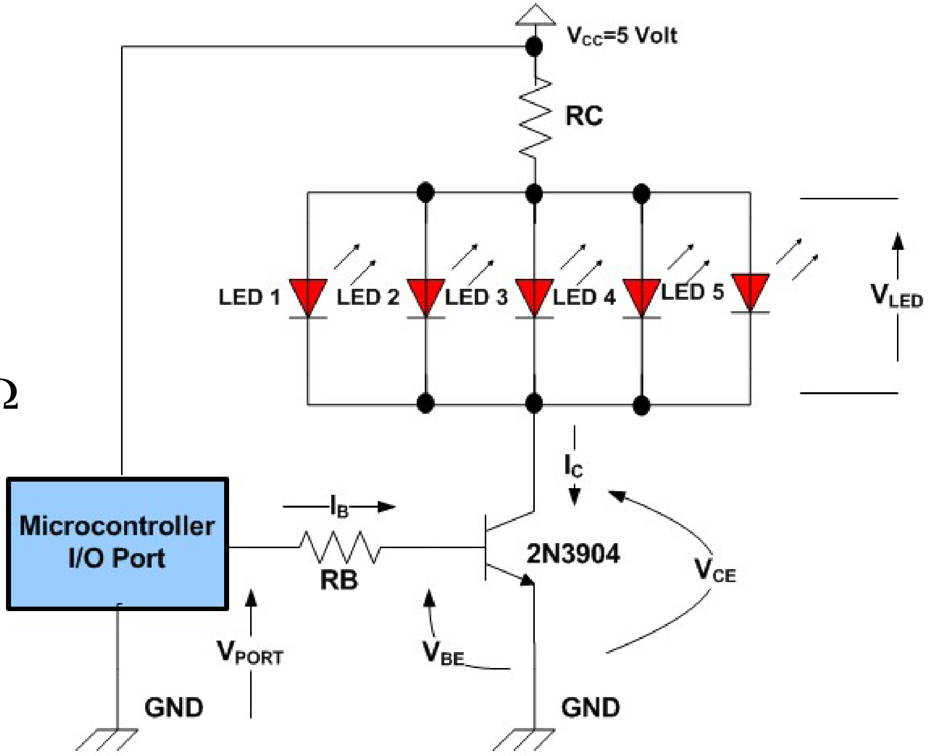
Solution:

1) $$I_C = 5 \times I_{LED} = 5 \times 15 \times 10^{-3} = 75 \text{ mA}$$

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{75 \text{ mA}}{100} = 0.75 \text{ mA}$$

$$R_B = \frac{V_{port} - V_{BE}}{I_B} = \frac{4.2 - 0.7}{0.75 \times 10^{-3}} = 4.6 \text{ k}\Omega$$

$$R_C = \frac{V_{CC} - V_{LED} - V_{CE}}{I_C} = \frac{10 - 2 - 0}{75 \times 10^{-3}} = 40\Omega$$



2) Power dissipation on $R_B = I_B^2 R_B$

$$= \left(0.75 \times 10^{-3}\right)^2 \times 4.6 \times 10^3 = 2.58 \text{ mW}$$

Power dissipation on $R_C = I_C^2 R_C$

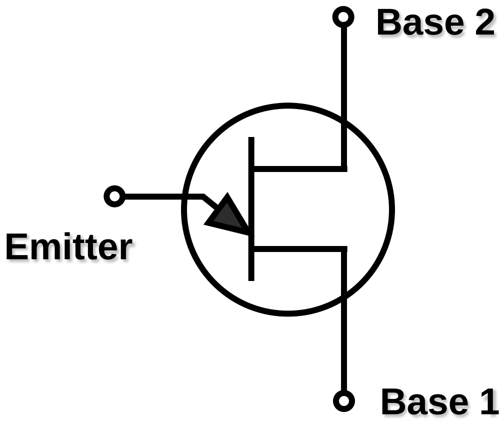
$$= \left(75 \times 10^{-3}\right)^2 \times 40 = 0.225 \text{ W}$$

Chapter 3

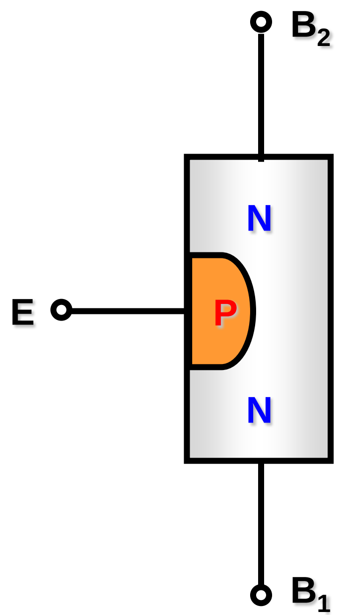
The Uni-Junction Transistor
(UJT)

The Uni-Junction Transistor UJT

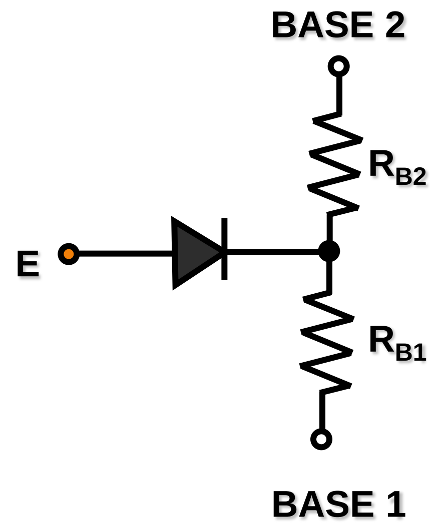
UJT is a semiconductor device having only one PN junction and three terminals, Base1, Base2, and Emitter.



Symbol



Structure

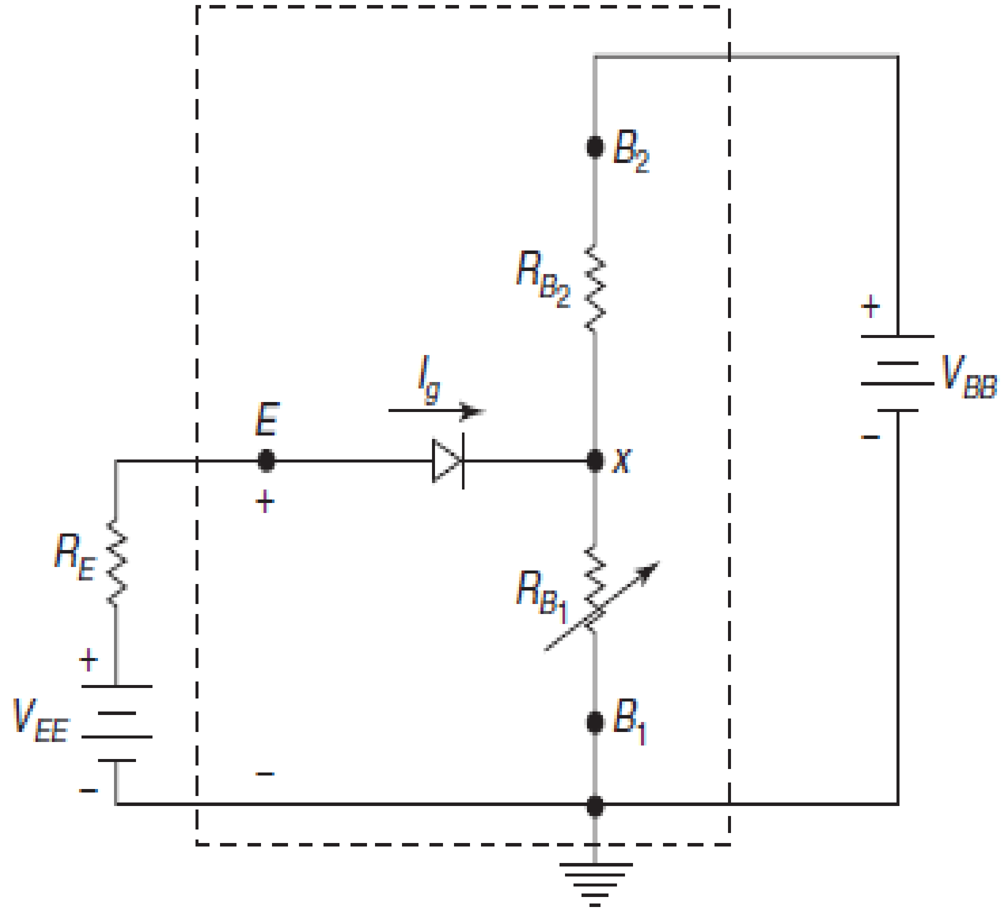


Equivalent Circuit

The Inter-Base Resistor: $R_{BB}=R_{B1}+R_{B2}$

Equivalent Circuit Of UJT

- The V_{BB} source is generally fixed and provides constant voltage from $\mathbf{B_2}$ to $\mathbf{B_1}$.
- The UJT is normally operated with both $\mathbf{B_2}$ and **E** is positive biased relative to $\mathbf{B_1}$.
- $\mathbf{B_1}$ is always the UJT reference terminal and all voltages are measured relative to $\mathbf{B_1}$.
- $\mathbf{V_{EE}}$ is a variable voltage source.



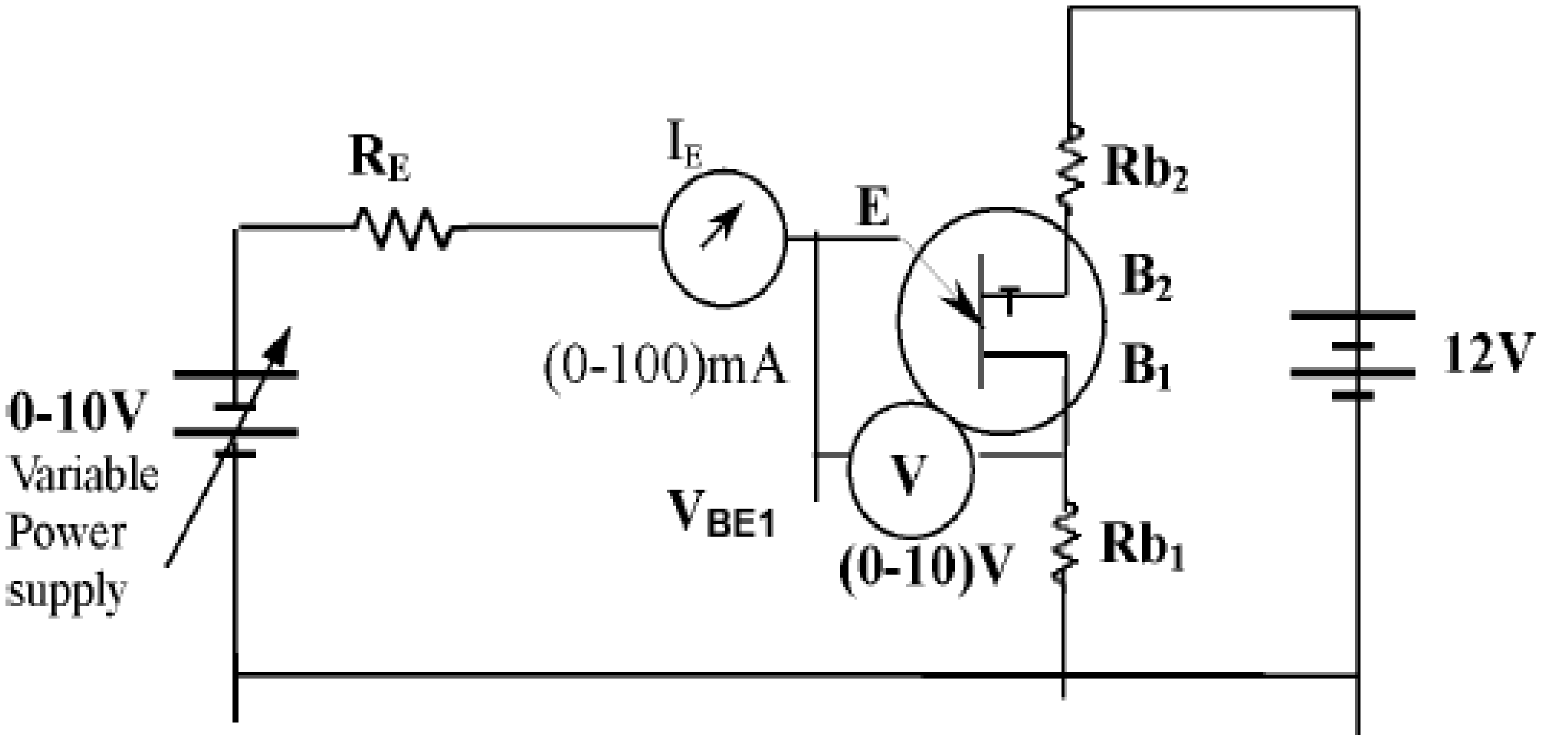
- η is the **Intrinsic standoff ratio**:

$$\eta = \frac{R_{B1}}{R_{B1} + R_{B2}} = \frac{R_{B1}}{R_{BB}}$$

$$R_{BB} = R_{B1} + R_{B2}$$

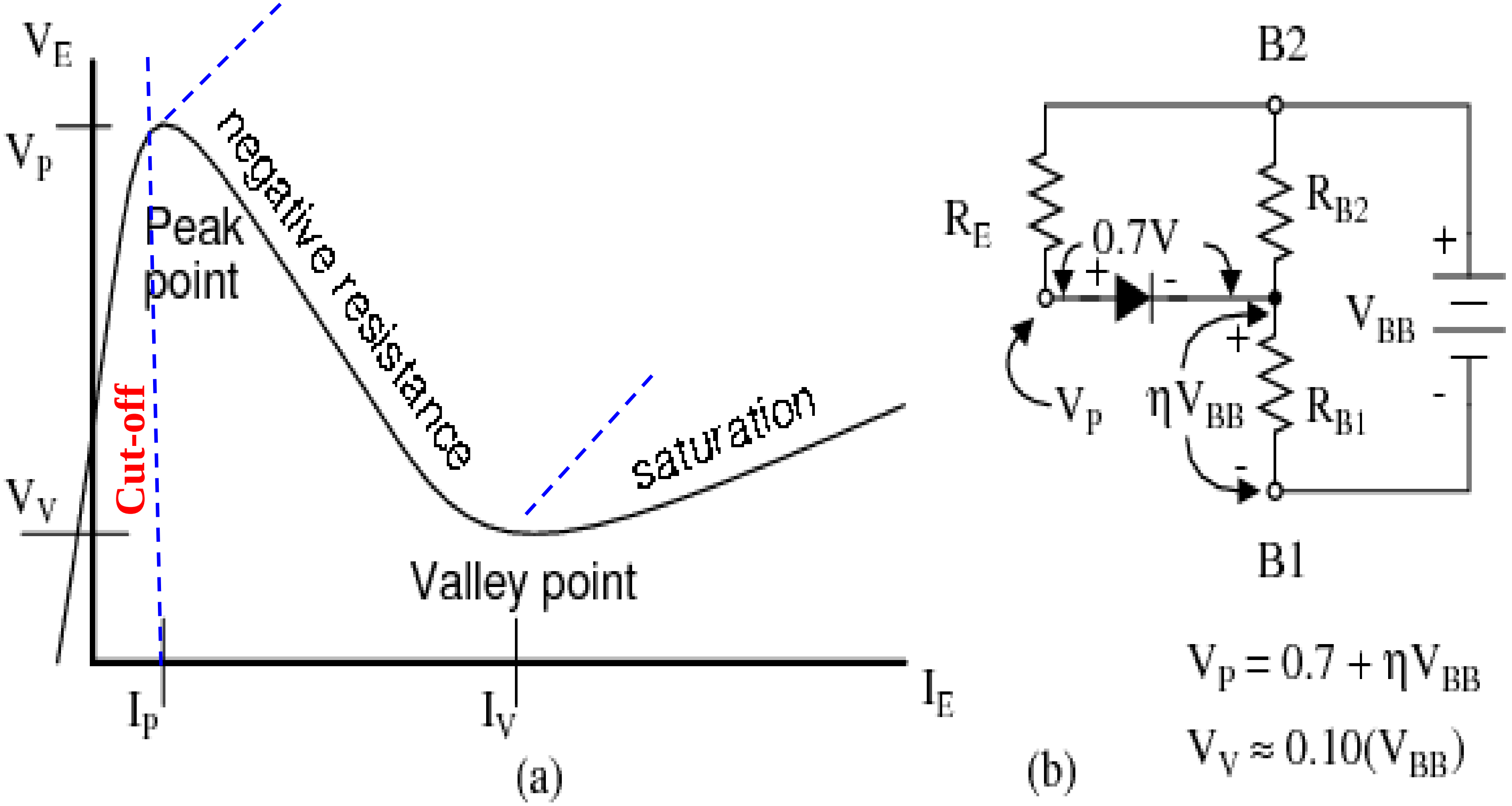
Input Forward Characteristics Of UJT

CIRCUIT DIAGRAM:



Standard values : R_E = 1KΩ, R_{b2}= 2.2KΩ, R_{b1} = 220Ω, T= 2N2646

Characteristic Curve



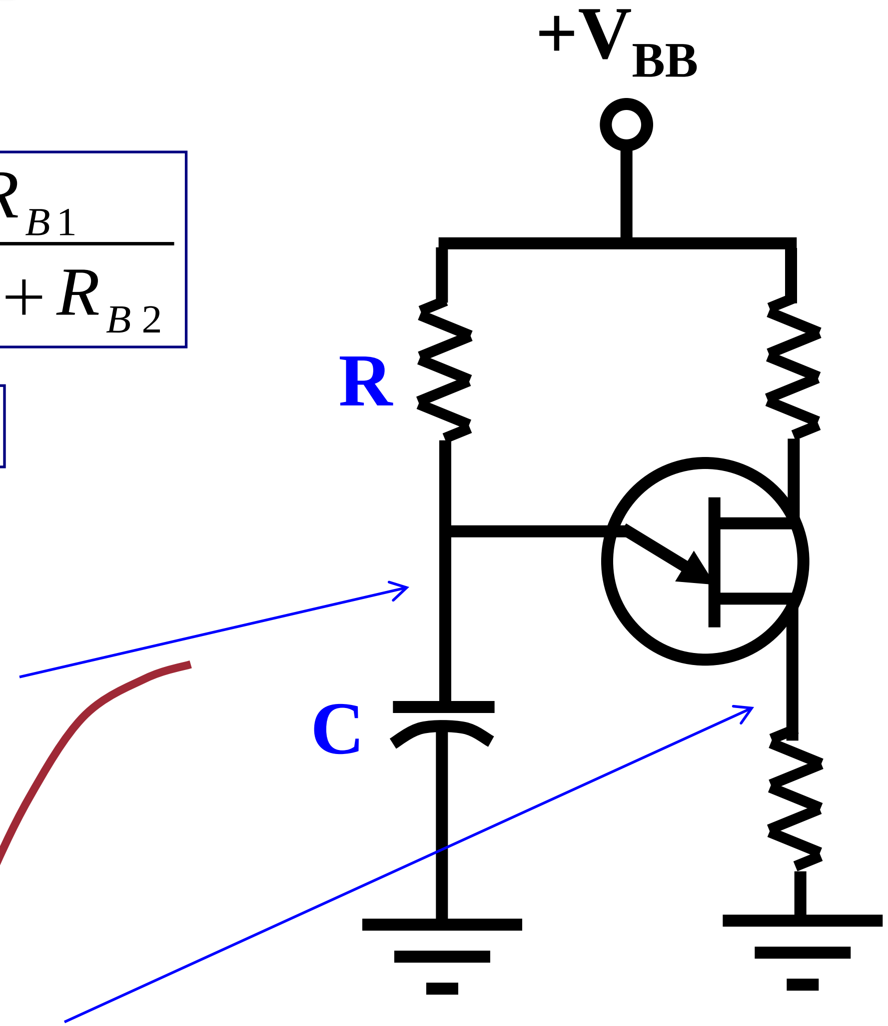
UJT Relaxation Oscillator

It provides two waveforms.

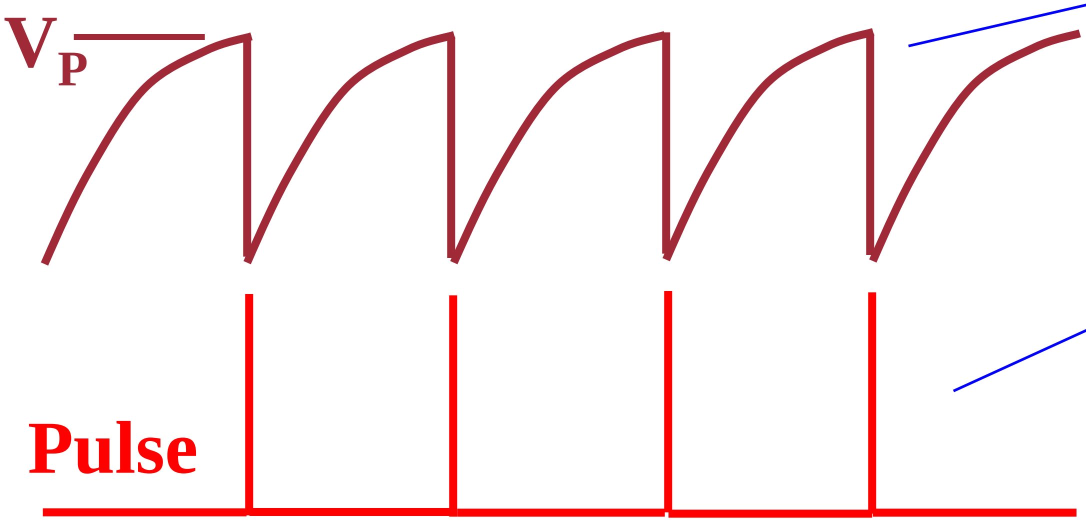
$$f \cong \frac{1}{RC\,\ln\left(\frac{1}{1-\eta}\right)}$$

$$\eta = \frac{R_{B1}}{R_{B1}+R_{B2}}$$

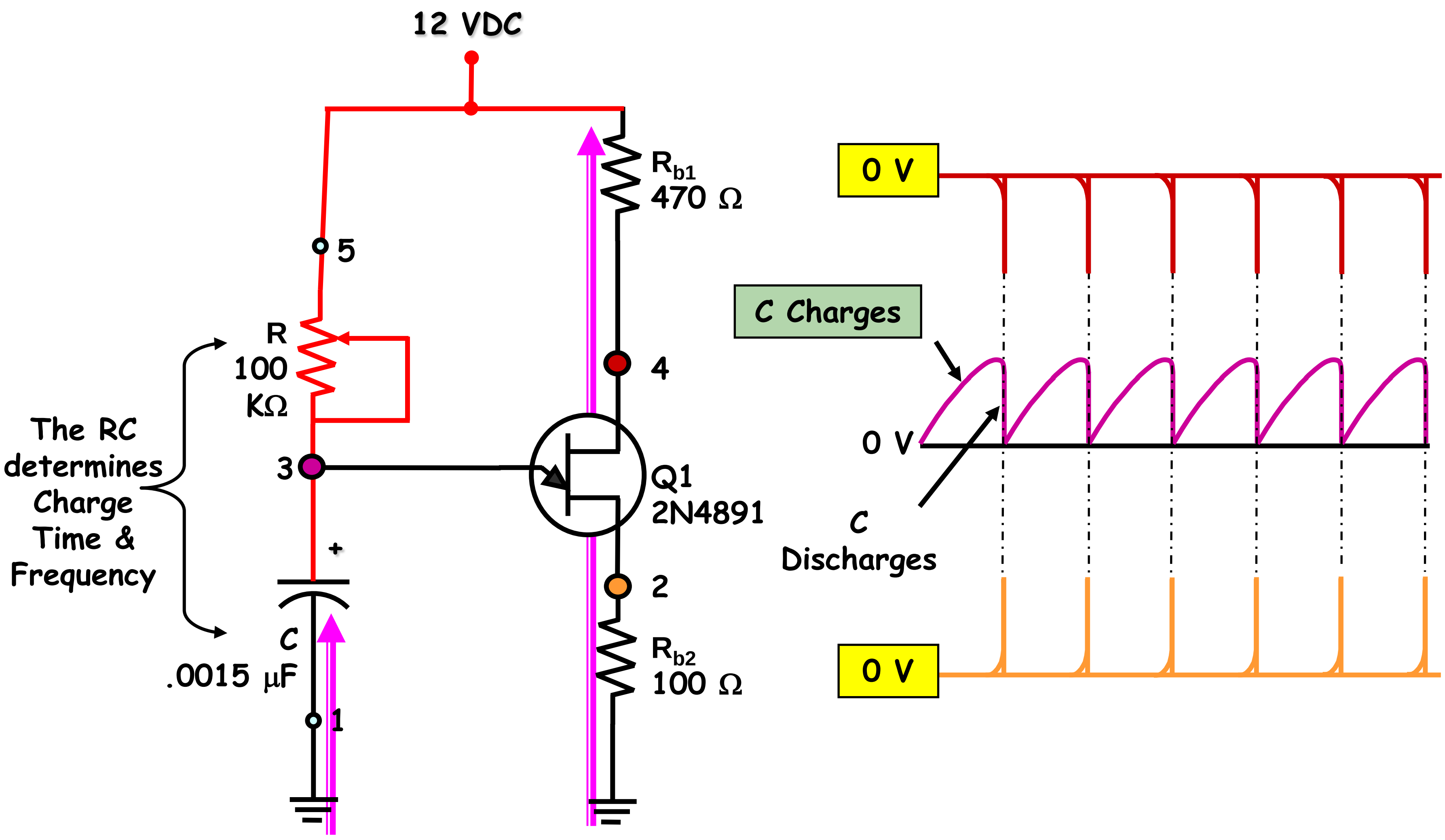
$$\tau = RC$$



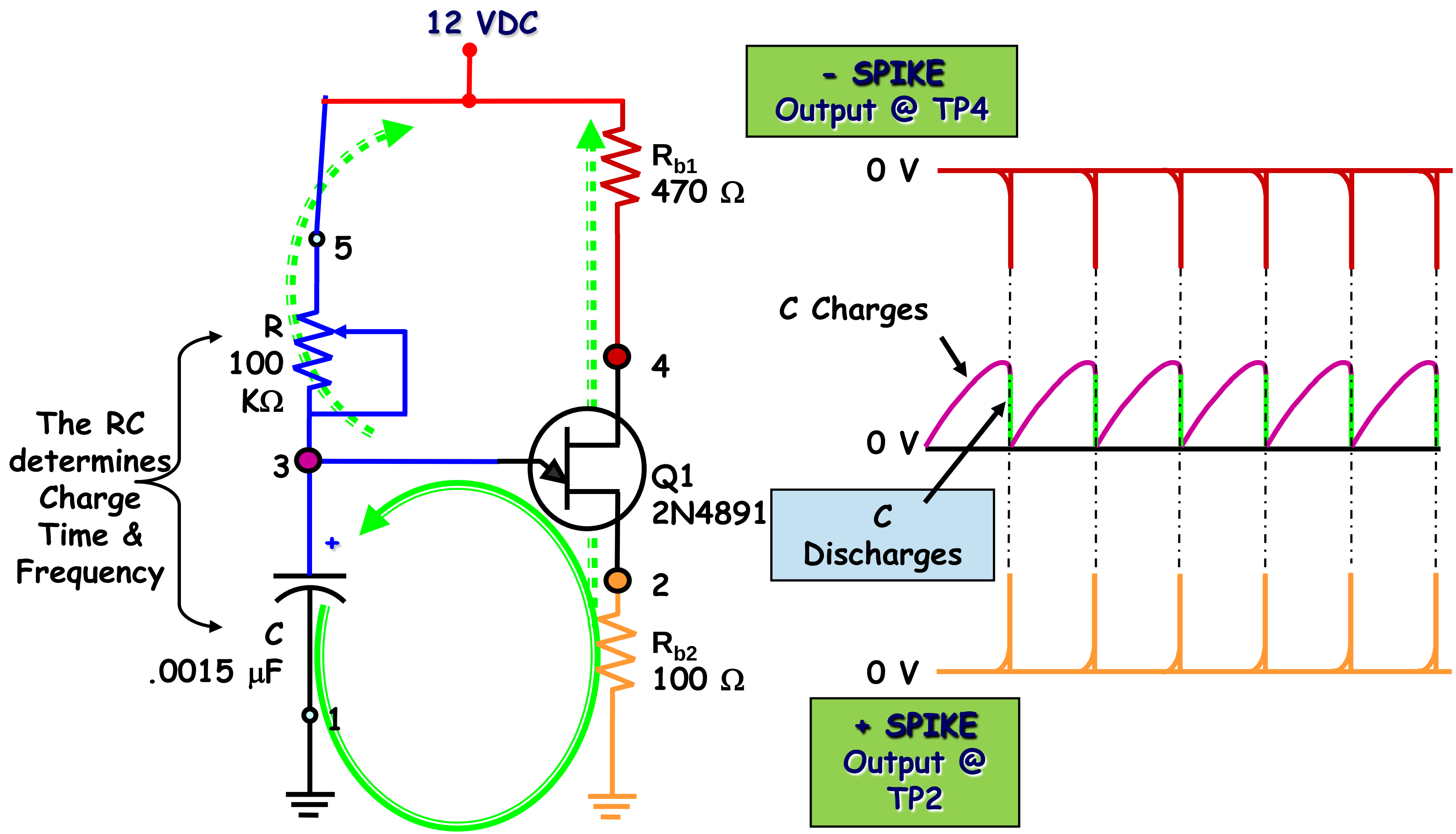
Exponential sawtooth



Operation of UJT Relaxation Oscillator: *C* charging



Operation of UJT Relaxation Oscillator: *C* discharging



Output; + & - Spikes and Sawtooth at TP3

Theory of operation as an oscillator

- For satisfactory operation, two conditions for the turn-on and turn-off of the UJT must be met as:

$$\frac{V_{BB} - V_V}{I_V} < R < \frac{V_{BB} - V_P}{I_P}$$

- The time period and, therefore, frequency of oscillation can be derived as below.

The voltage across the capacitor during charging is given as:

$$V_C = V_{BB}\left(1 - e^{-t/RC}\right)$$

$$V_P = V_D + \eta\, V_{BB}$$
$$= V_C = V_{BB}\left(1 - e^{-t/RC}\right)$$

$$V_{swing} = V_P - V_V$$

$$T = \frac{1}{f} = RC \ln\left(\frac{1}{1-\eta}\right)$$
$$= 2.3RC \log_{10}\left(\frac{1}{1-\eta}\right)$$

Example 1 Design a 1kHz relaxation oscillator using UJT with the following specifications: V_{BB} = 10V, η = 0.56 ~ 0.75, I_P = 5μA, and I_V = 4mA.

Solution:

The value of ***R*** can be found using the following condition:

$$\frac{V_{BB}-V_V}{I_V} < R < \frac{V_{BB}-V_P}{I_P}$$

$$\therefore R_{max} = \frac{V_{BB}-V_P}{I_P} \quad \text{and} \quad R_{min} = \frac{V_{BB}-V_V}{I_V}$$

$$V_P = 0.7 + \eta V_{BB}$$

$$V_V \simeq 0.1\, V_{BB} = 1\text{V}$$

For the given UJT, the average value of η can be found as: $\eta = \frac{0.56+0.75}{2} = 0.655$

$$\Rightarrow V_P = 0.7 + 0.655 \times 10 = 7.25\text{V}$$

$$\Rightarrow R_{max} = \frac{10-7.25}{5\times10^{-6}} = 550\text{k}\Omega \qquad \Rightarrow R_{min} = \frac{10-1}{4\times10^{-3}} = 2.25\text{k}\Omega$$

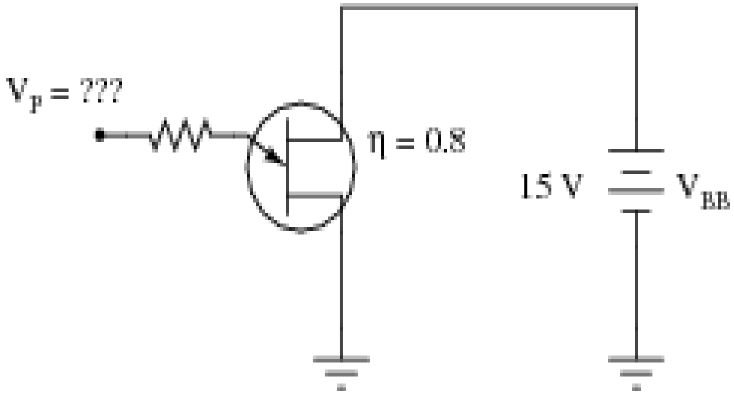
Suitable value of ***R*** can be selected between $2.25\text{k}\Omega$ and $550\text{k}\Omega$ as: $R = 10\text{k}\Omega$

$$f = \frac{1}{RC\ln\left(\frac{1}{1-\eta}\right)}$$

$$\Rightarrow C = \frac{1}{1\times10^{3}\times10\times10^{3}\times\ln\left(\frac{1}{1-0.655}\right)} \approx 94\text{nF}$$

Example 2 A given UJT with an intrinsic standoff ratio ($\eta = 0.8$) is powered by a 15 volt DC source. Calculate the emitter voltage (V_P) needed to "trigger" this UJT into its conductive state.

Solution:



$$V_P = V_D + \eta V_{BB} = 0.7 + \eta V_{BB}$$

$$\Rightarrow V_P = 0.7 + 0.8 \times 15 = 12.7\text{V}$$

Example 3 A given UJT relaxation oscillator has the following: V_{BB} = 30V, V_D = 0.6V, R = 100kΩ, and η = 0.6. What is the value of C that gives 1kHz output frequency? Calculate the maximum value of capacitor voltage V_C.

Solution:

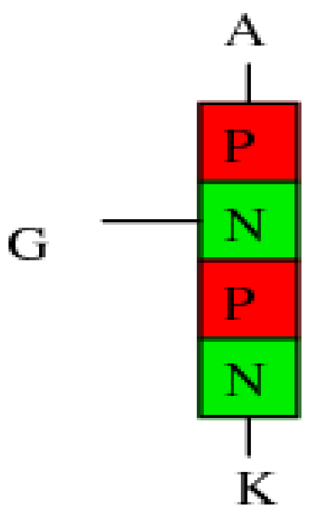
$$V_P = V_D + \eta V_{BB}$$ $$\Rightarrow V_P = 0.6 + 0.6 \times 30 = 18.6\text{V}$$

$$f = \frac{1}{RC \ln\left(\frac{1}{1-\eta}\right)}$$ $$\Rightarrow C = \frac{1}{1\times10^3 \times 100\times10^3 \times \ln\left(\frac{1}{1-0.6}\right)} = 1.09\times10^{-6} F = 1.09 \mu F$$

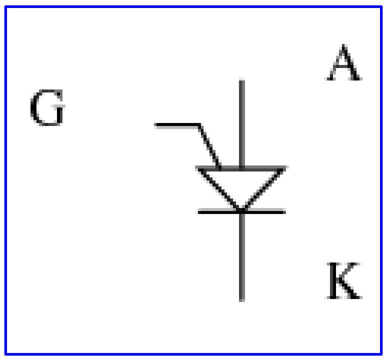
$$(V_C)_{\max} = V_P$$ $$\Rightarrow (V_C)_{\max} = 18.6\text{V}$$

Programmable Unijunction Transistor (PUT):

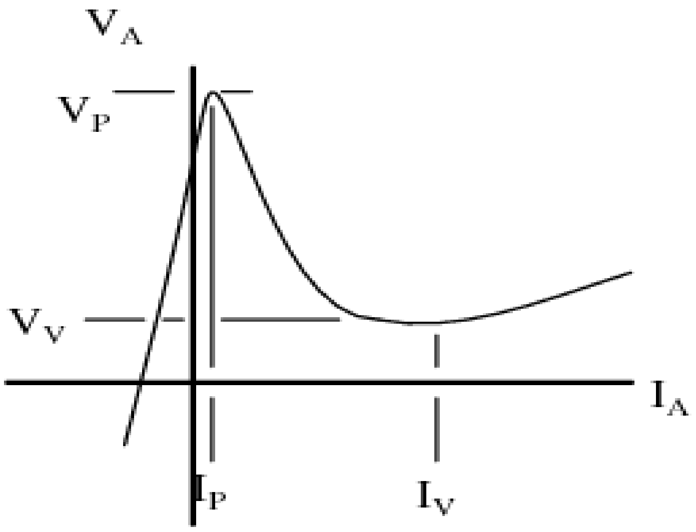
- Although UJT is expensive if obtained, the PUT is inexpensive and in production.
- PUT serves similar to the UJT, But it has three terminals as the thyristor.
- The PUT shares the four-layer structure typical of thyristors as shown.
- The gate, an N-type layer near the anode, is known as an “anode gate”.
- The gate lead on the schematic symbol is attached to the anode end of the symbol.



PUT Configuration

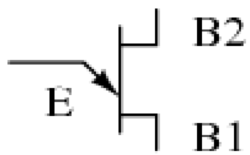


Symbol

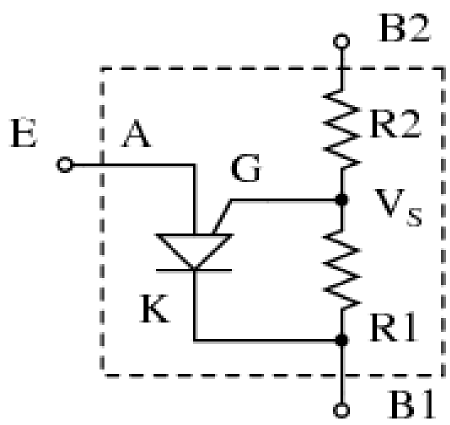


Input forward characteristics

PUT Equivalent Circuit



Unijunction



PUT equivalent

$$R_{BB0} = R1 + R2$$

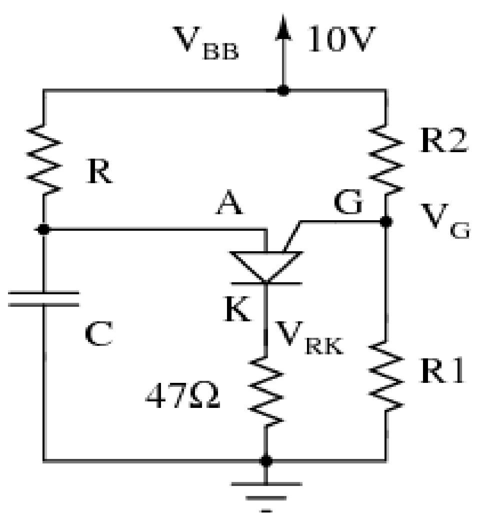
$$\eta = \frac{R1}{R1 + R2}$$

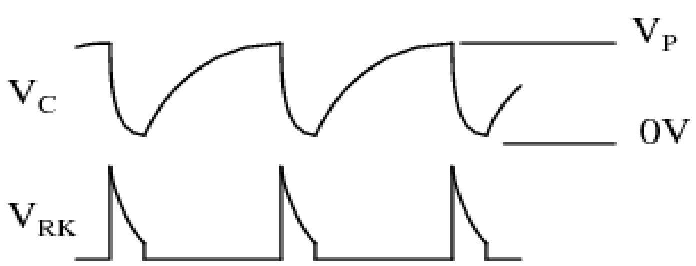
$$V_S = \eta V_{BB}$$

$$V_P = V_T + V_S$$

$$R_G = \frac{R1 \cdot R2}{R1 + R2}$$

PUT Relaxation Oscillator





Problems and Solutions

Question 1 For a given UJT of 180 mw power and inter-base resistor R_{BB} = 4kΩ , find the estimated value of maximum source voltage V_{BB}.

Solution:

$$\text{UJT Power} = \frac{V_{BB}^{2}}{R_{BB}}$$

$$\Rightarrow V_{BB}^{2} = 180\times10^{-3}\times4\times10^{3} = 720$$

$$\therefore\ V_{BB} = \sqrt{720} = 26.8\text{Volt}$$

Question 2 For a given UJT, V_{BB} = 10 volts, η_{min} = 0.6, and η_{max} = 0.8, find the possible values of V_P.

Solution:

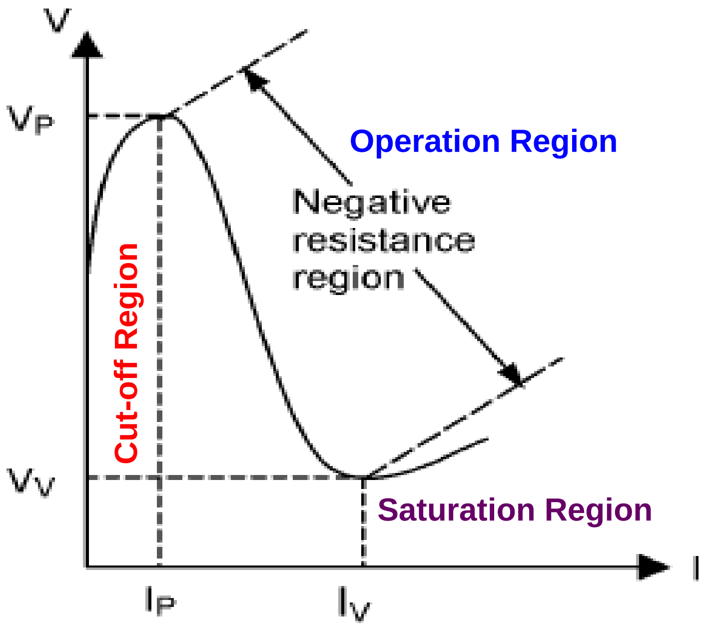
$$V_P = V_D + \eta V_{BB} = 0.7 + \eta V_{BB}$$

$$\Rightarrow (V_P)_{\min} = 0.7 + \eta_{\min} V_{BB} = 0.7 + 0.6\times10 = 6.7\text{Volt}$$

$$\Rightarrow (V_P)_{\max} = 0.7 + \eta_{\max} V_{BB} = 0.7 + 0.8\times10 = 8.7\text{Volt}$$

Question 3 Sketch the input forward characteristics curve of UJT and indicate the regions of operation.

Solution:



Input forward characteristics curve of UJT

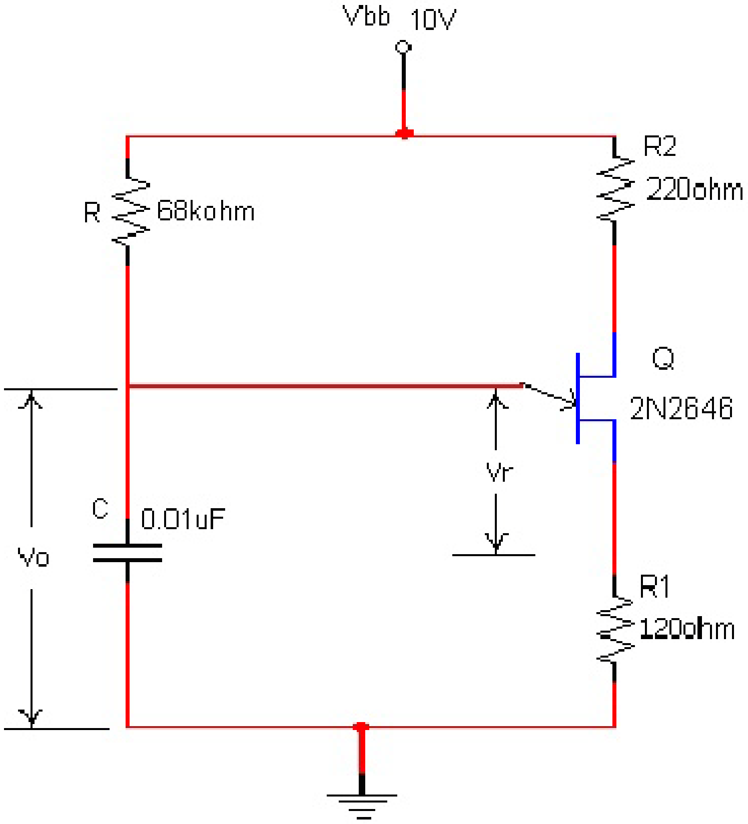
Question 4 Why a UJT oscillator called relaxation oscillator?

Solution:

Because the charging time constant ***RC*** is much greater than the discharging time. So the capacitor relax in charging state, and hence the name of relaxation oscillator.

Question 5 For the given relaxation oscillator: 1) find the output frequency if $\eta = 0.65$, 2) the maximum value of V_C , and 3) the value of V_C at half value of periodic time T.

Solution:



1)

$$f = \frac{1}{RC\ln\left(\frac{1}{1-\eta}\right)}$$

$$\therefore f = \frac{1}{68\times10^{3}\times0.01\times10^{-6}\times\ln\left(\frac{1}{1-0.65}\right)} = 1.4\text{kHz}$$

2)

$$V_P = V_D + \eta V_{BB} = 0.7 + \eta V_{BB}$$

$$\Rightarrow V_P = 0.7 + 0.65\times10 = 7.2\text{V}$$

$$\left(V_C\right)_{\max} = V_P \quad \Rightarrow \left(V_C\right)_{\max} = 7.2\text{V}$$

3)

$$V_C = V_{BB}\left(1-e^{-t/RC}\right)$$

$$\text{at } t = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2\times1.4\times10^{3}} = 0.357m\sec \Rightarrow V_C = 10\left(1-e^{\frac{-0.357\times10^{-3}}{\left(68\times10^{3}\times0.01\times10^{-6}\right)}}\right) = 4\text{V}$$

Question 6 A given UJT relaxation oscillator has the following: V_{BB} = 20V, I_V = 10mA, I_P = 40 μA, C = 1μF, and η = 0.65. Find the value of maximum and minimum frequencies.

Solution:

$$V_P = 0.7 + \eta V_{BB} \quad \Rightarrow V_P = 0.7 + 0.65 \times 20 = 13.7\text{V}$$

$$V_V \simeq 0.1\, V_{BB} = 2\text{V}$$

$$f = \frac{1}{RC \ln\left(\frac{1}{1-\eta}\right)} \quad \Rightarrow f_{\max} = \frac{1}{R_{\min} C \ln\left(\frac{1}{1-\eta}\right)} \quad \text{and} \quad f_{\min} = \frac{1}{R_{\max} C \ln\left(\frac{1}{1-\eta}\right)}$$

$$\because R_{\max} = \frac{V_{BB} - V_P}{I_P} \quad \text{and} \quad R_{\min} = \frac{V_{BB} - V_V}{I_V}$$

$$\Rightarrow R_{\max} = \frac{20 - 13.7}{40 \times 10^{-6}} = 157.5\text{k}\Omega \qquad \Rightarrow R_{\min} = \frac{20 - 2}{10 \times 10^{-3}} = 1.8\text{k}\Omega$$

$$\therefore f_{\max} = \frac{1}{1.8 \times 10^3 \times 1 \times 10^{-6} \times \ln\left(\frac{1}{1 - 0.65}\right)} = 529\text{Hz}$$

$$\therefore f_{\min} = \frac{1}{157.5 \times 10^3 \times 1 \times 10^{-6} \times \ln\left(\frac{1}{1 - 0.65}\right)} = 6\text{Hz}$$

Question 7 For the given UJT relaxation oscillator, calculate the maximum and minimum frequencies if $\eta = 0.6$

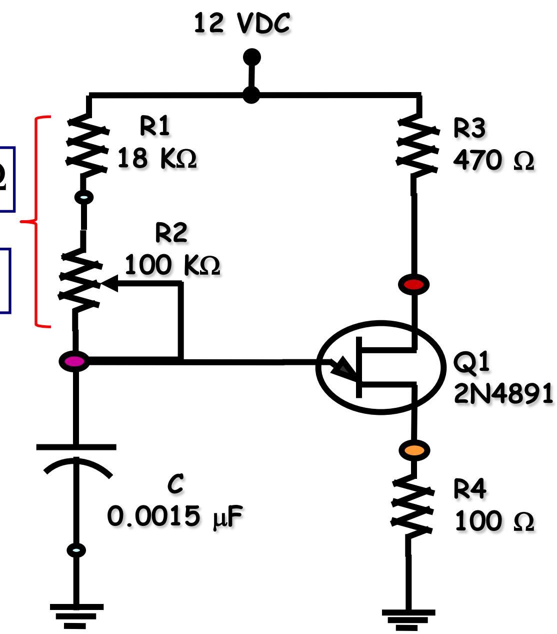
Solution:

$$V_P = 0.7 + \eta V_{BB} \quad \Rightarrow V_P = 0.7 + 0.6 \times 12 = 7.9\text{V}$$

$$V_V \simeq 0.1\, V_{BB} = 1.2\text{V}$$

$$R_{max} = R_1 + R_2 = 118\text{k}\Omega$$

$$R_{min} = R_1 + 0 = 18\text{k}\Omega$$



$$f = \frac{1}{RC \ln\left(\frac{1}{1-\eta}\right)}$$

$$\Rightarrow f_{max} = \frac{1}{R_{min} C \ln\left(\frac{1}{1-\eta}\right)}$$

$$= \frac{1}{18 \times 10^3 \times 0.0015 \times 10^{-6} \times \ln\left(\frac{1}{1-0.6}\right)} = 40\text{kHz}$$

$$\Rightarrow f_{min} = \frac{1}{R_{max} C \ln\left(\frac{1}{1-\eta}\right)} = \frac{1}{118 \times 10^3 \times 0.0015 \times 10^{-6} \times \ln\left(\frac{1}{1-0.6}\right)} = 6.16\text{kHz}$$

Question 8 For the given UJT relaxation oscillator with V_{BB} = 30V and η = 0.6, calculate the output frequency and the voltage swing.

Solution:

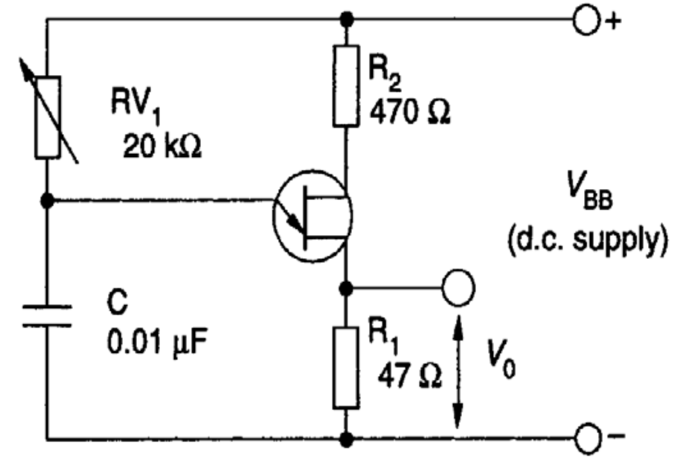
$$V_P = 0.7 + \eta V_{BB}$$

$$\Rightarrow V_P = 0.7 + 0.6 \times 30 = 18.7\text{V}$$

$$V_V \simeq 0.1\, V_{BB} = 3\text{V}$$

$$f = \frac{1}{RC \ln\left(\frac{1}{1-\eta}\right)}$$

$$\Rightarrow f = \frac{1}{20\times10^3 \times 0.01\times10^{-6} \times \ln\left(\frac{1}{1-0.6}\right)} = 545\text{kHz}$$



$$V_{swing} = V_P - V_V$$

$$\Rightarrow V_{swing} = 18.7 - 3 = 15.7\text{V}$$

Question 9 **Fill in the following blanks with suitable words.**

1) UJT is also called Double *Base Diode* .

2) Intrinsic standoff ratio is the coefficient of UJT that has typical values lie between *0.5* and *0.8*.

3) UJT interbase resistance of *N* block of several thousands of Ohms when positive voltage is applied to *B1* at *reverse biased condition of the pn junction*.

4) The condition of operation of UJT is: $V_E = 0.7 + \eta V_{BB}$.

5) The UJT has three operating regions as, *cut-off region*, *negative resistance region*, and *saturation region*.

6) The oscillation condition of UJT oscillator is: $\frac{V_{BB} - V_V}{I_V} < R < \frac{V_{BB} - V_P}{I_P}$

7) Capacitor voltage during charging time of UJT oscillator can be expressed as: $V_C = V_{BB}\left(1 - e^{-t/RC}\right)$

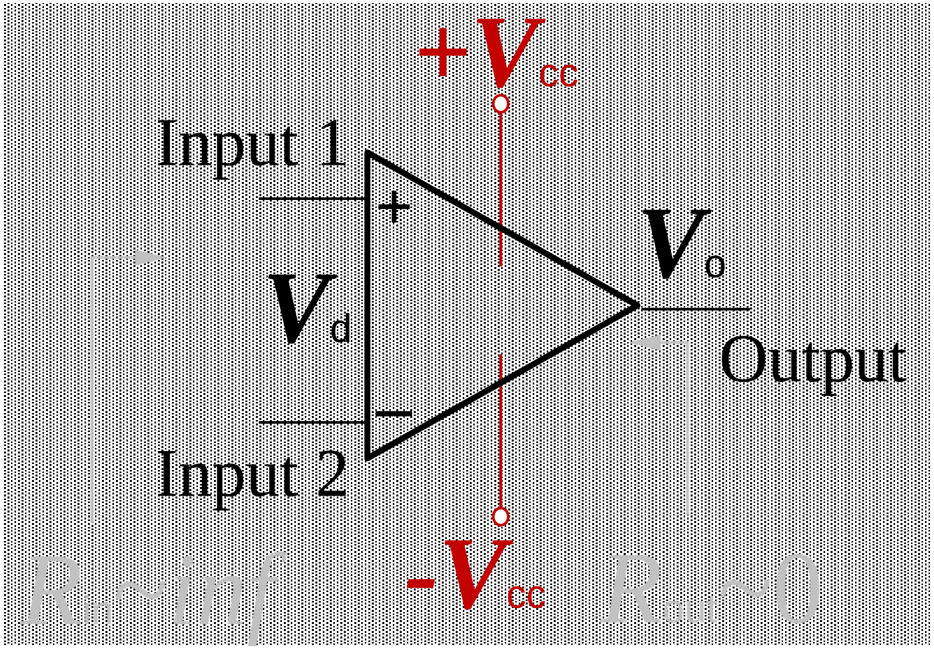
8) The negative resistance region in the characteristic of the *UJT* makes it useful for constructing *voltage oscillators*.

Chapter 4

The Operational Amplifier (Op-Amp)

The Operational Amplifier (Op-Amp)

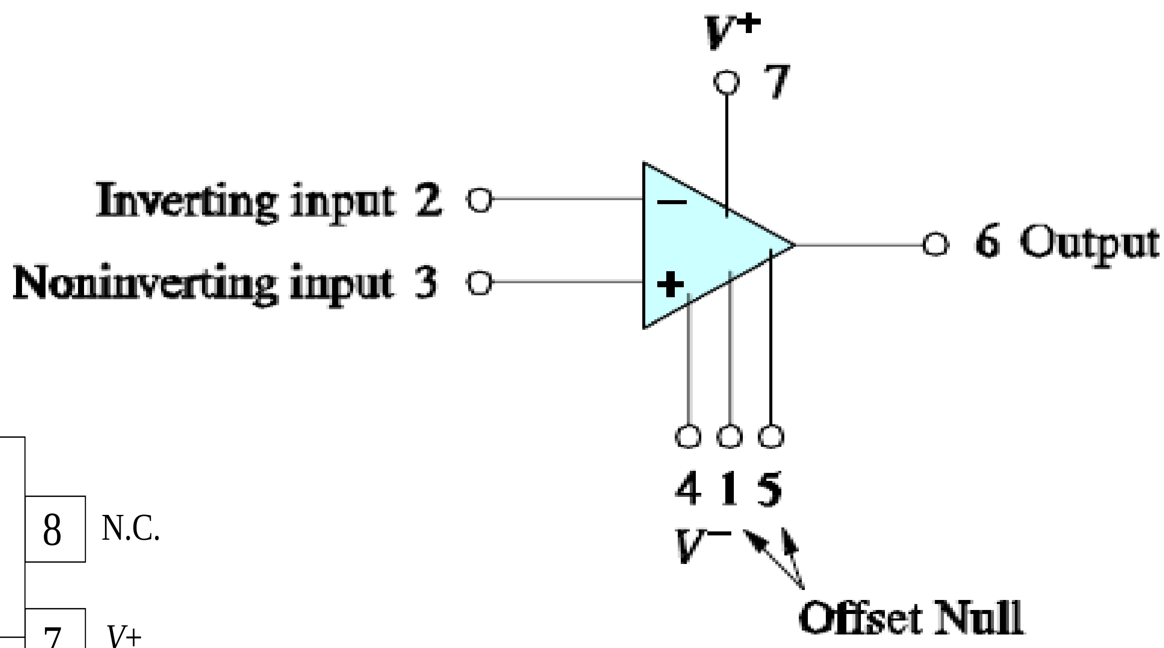
- Very high differential gain G_d .
- High input impedance R_{in} .
- Low output impedance $R_{out.}$
- Provide voltage changes (amplitude and polarity).
- Used in oscillator, filter and instrumentation.
- Accumulate a very high gain by multiple stages.

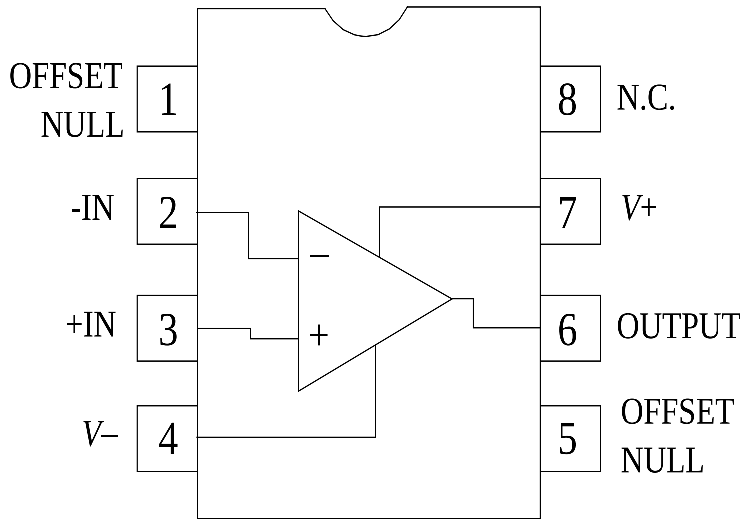


$$V_o = G_d V_d$$

G_d : differential gain normally very large about 10^5

The Op-Amp Symbol





DIP-741

Op-Amp Properties

(1) Infinite Open Loop gain

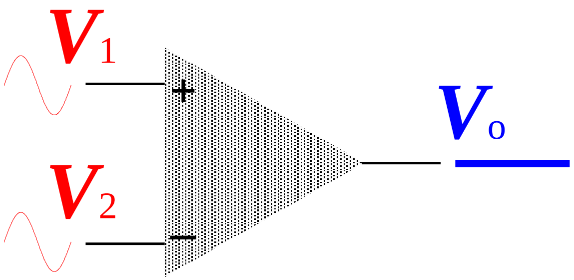
- The gain without feedback equal to G_d.
- Zero common-mode gain.
- Practically, G_d = 20,000 to 200,000

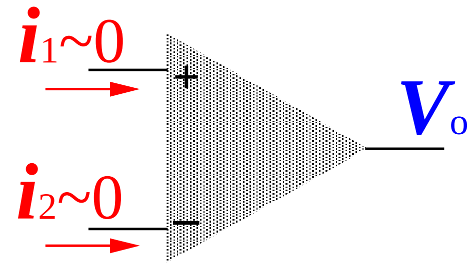
(2) Infinite Input impedance

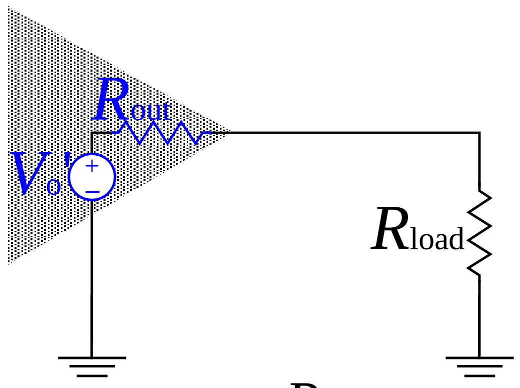
- Input current i_i , i_2 ~0A
- TΩ in high-grade op-amp
- mA input current in low-grade op-amp

(3) Zero Output Impedance

- act as perfect internal voltage source
- No internal resistance
- Output impedance in series with load
- Reducing output voltage to the load
- Practically, R_{out} ~ 20-100 Ω

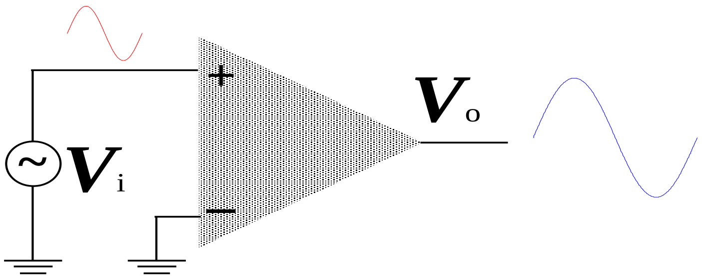




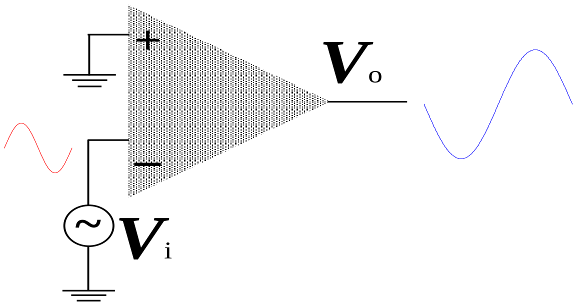


$$V_{load} = V_o' \frac{R_{load}}{R_{load} + R_{out}}$$

Single-Ended Input

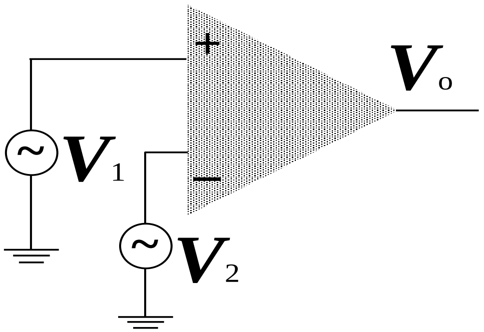


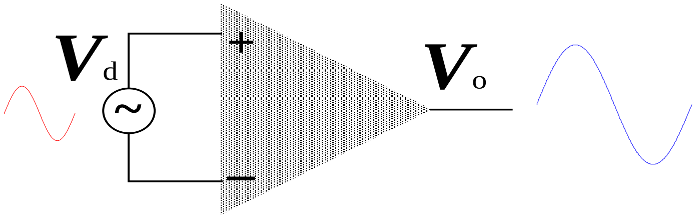
- + terminal : Source
- – terminal : Ground
- 0º phase change



- + terminal : Ground
- – terminal : Source
- 180º phase change

Double-Ended Input



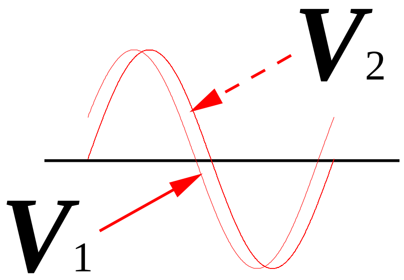


$$V_d = V_+ - V_-$$

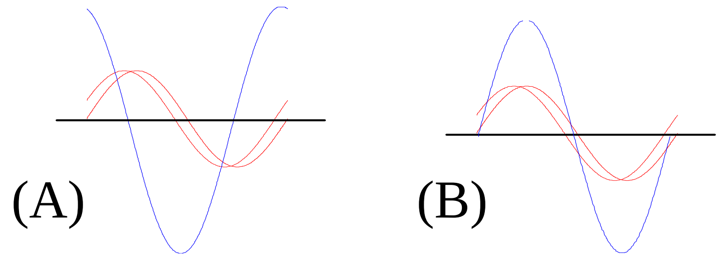
- Differential input V_d.
- 0º phase shift change between V_o and V_d

Question:

Which of the following (A or B) is the output voltage V_o if the input voltages is given as:



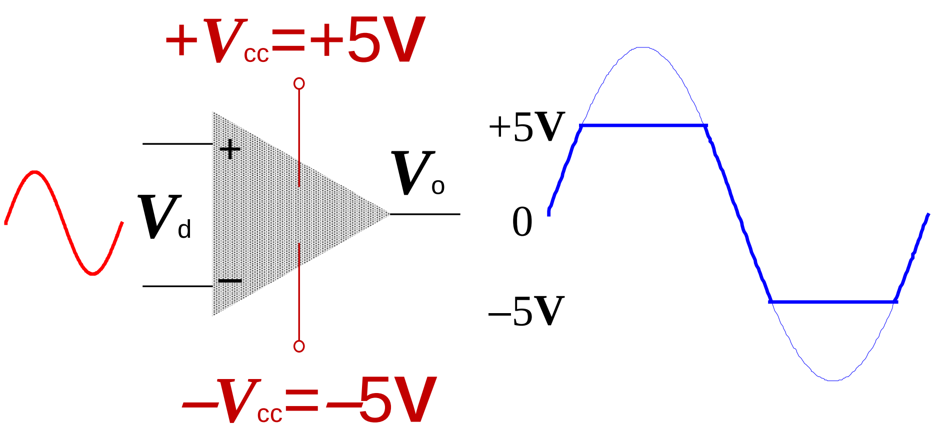
Input voltage



Output voltage (Aor B) ?

Op-Amp Distortion

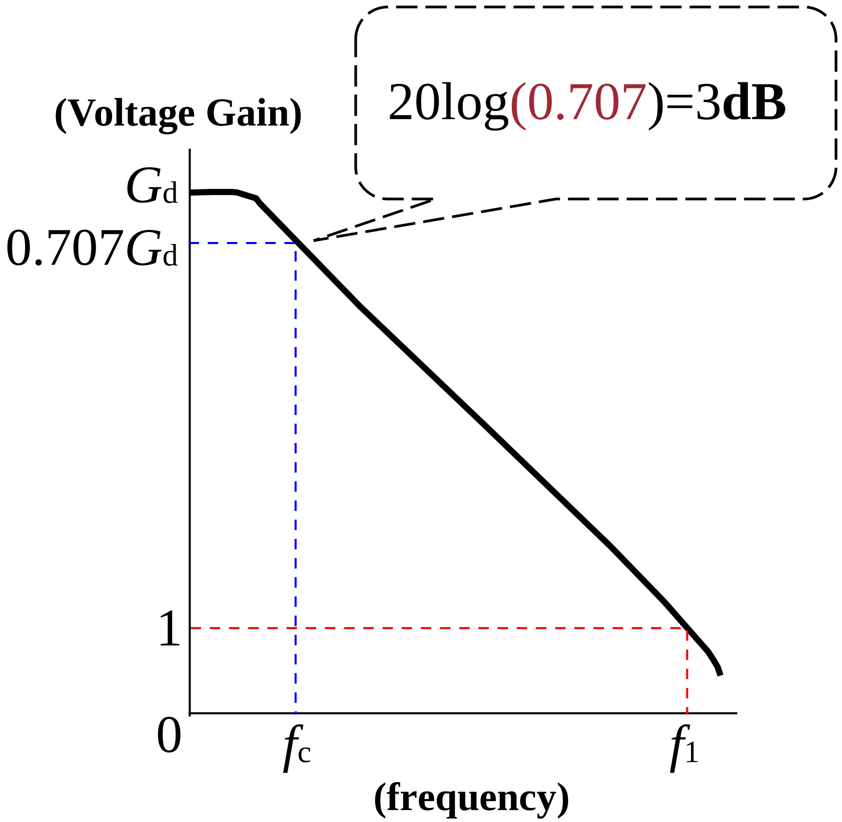
The output voltage never excess the DC biasing voltage of the Op-Amp.



Frequency-Gain Relation

- Ideally, signals are amplified from DC to the highest AC frequency
- Practically, bandwidth is limited
- 741 family op-amp have an limit bandwidth of few KHz.

- Unity Gain frequency f_1: the gain at unity
- Cutoff frequency f_c: the gain drop by 3dB from dc gain G_d
- Gain Bandwidth (GB) product is represented as:



$$\text{GB Product} : f_1 = G_d \, f_c$$

Example 1: Determine the cutoff frequency of an op-amp having a unit gain frequency f_1 = 10 MHz and voltage differential gain G_d = 20**V**/m**V.**

Solution:

Since f_1 = 10 MHz

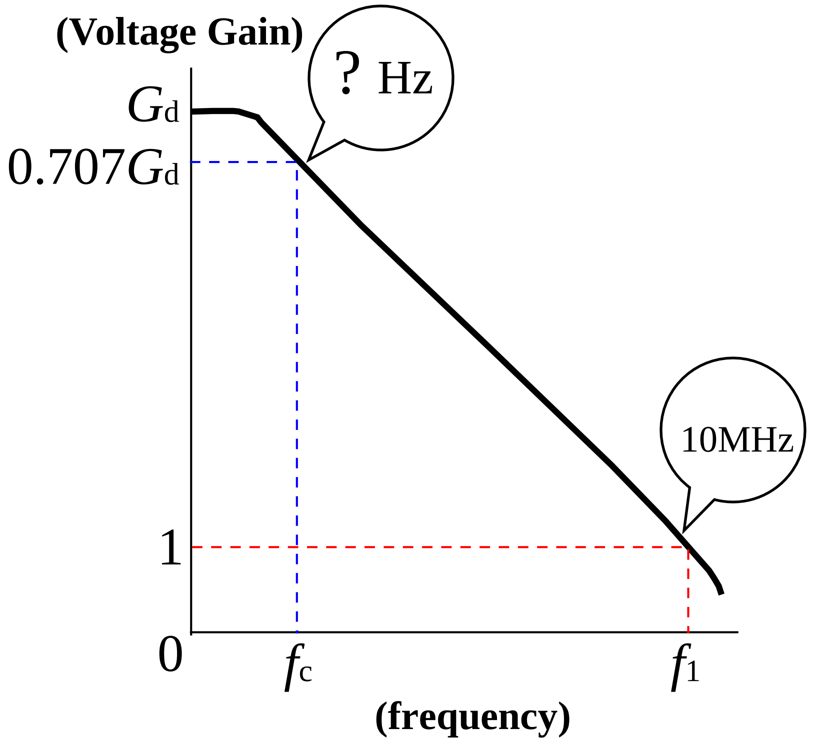
By using GB production equation

$$f_1 = G_d\, f_c$$

$$f_c = f_1 / G_d = 10 \text{ MHz} / 20 \text{ V/mV}$$

$$= 10 \times 10^6 / 20 \times 10^3$$

$$= 500 \text{ Hz}$$



Ideal Op-Amp Applications

Analysis Method :

Two ideal Op-Amp Properties:

(1) The voltage between V_+ and V_- is zero $V_+ = V_-$

(2) The current into both V_+ and V_- terminals is zero.

For ideal Op-Amp circuit:

(1) Write the Kirchhoff node equation at the non inverting terminal V_+

(2) Write the Kirchhoff node equation at the inverting terminal V_-

(3) Set $V_+ = V_-$ and solve for the desired closed-loop gain

Non-Inverting Amplifier

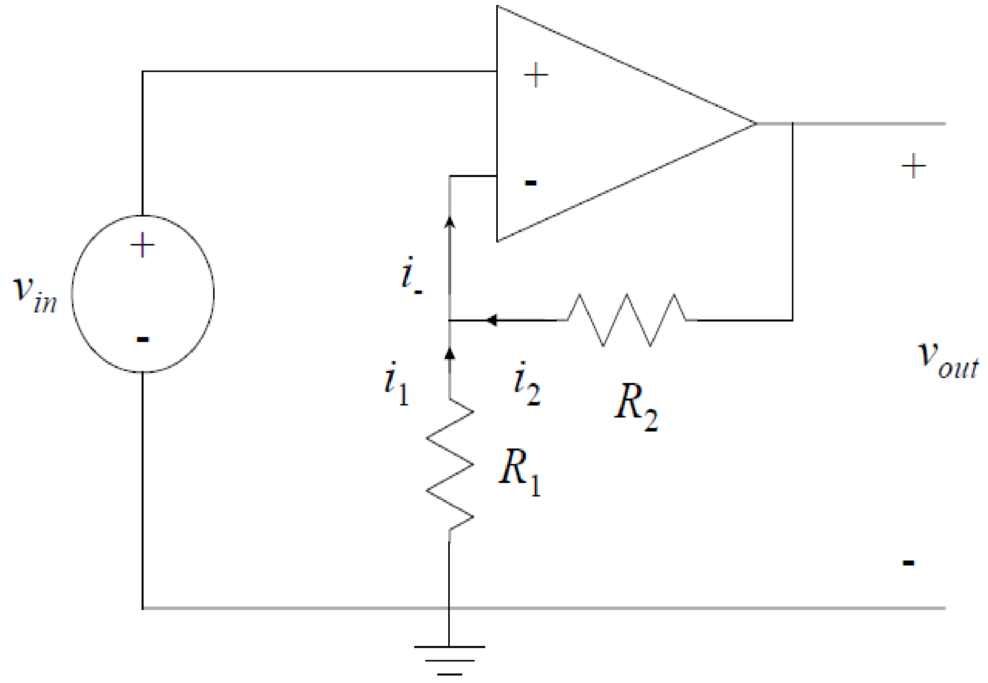
KCL at the Inverting Input

$$i_- = 0$$

$$i_1 = \frac{-v_-}{R_1} = \frac{-v_{in}}{R_1}$$

Since $v_- = v_+ = v_{in}$

$$i_2 = \frac{v_{out} - v_-}{R_2} = \frac{v_{out} - v_{in}}{R_2}$$

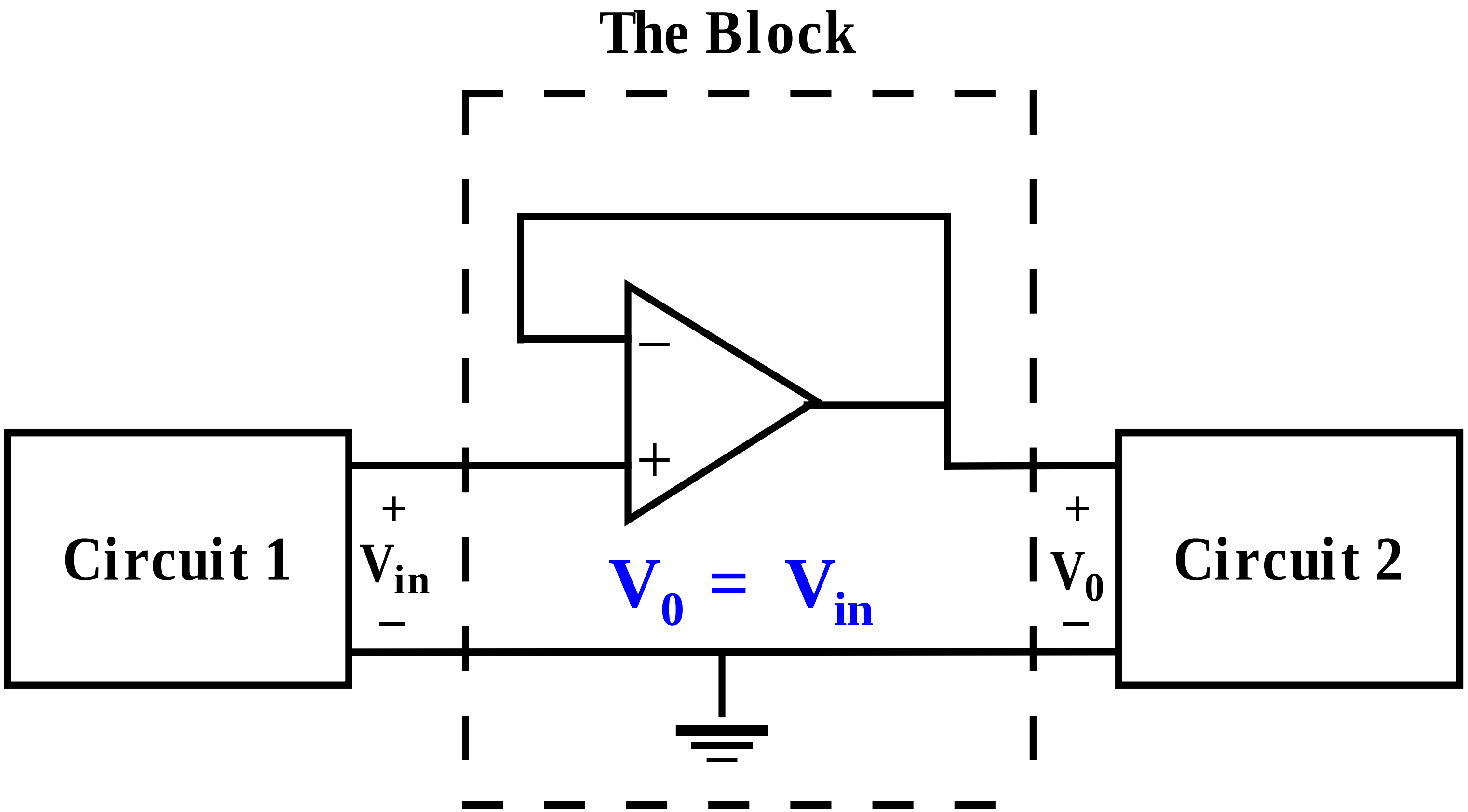


Solve for V_{out}

$$\frac{-v_{in}}{R_1} + \frac{v_{out} - v_{in}}{R_2} = 0$$

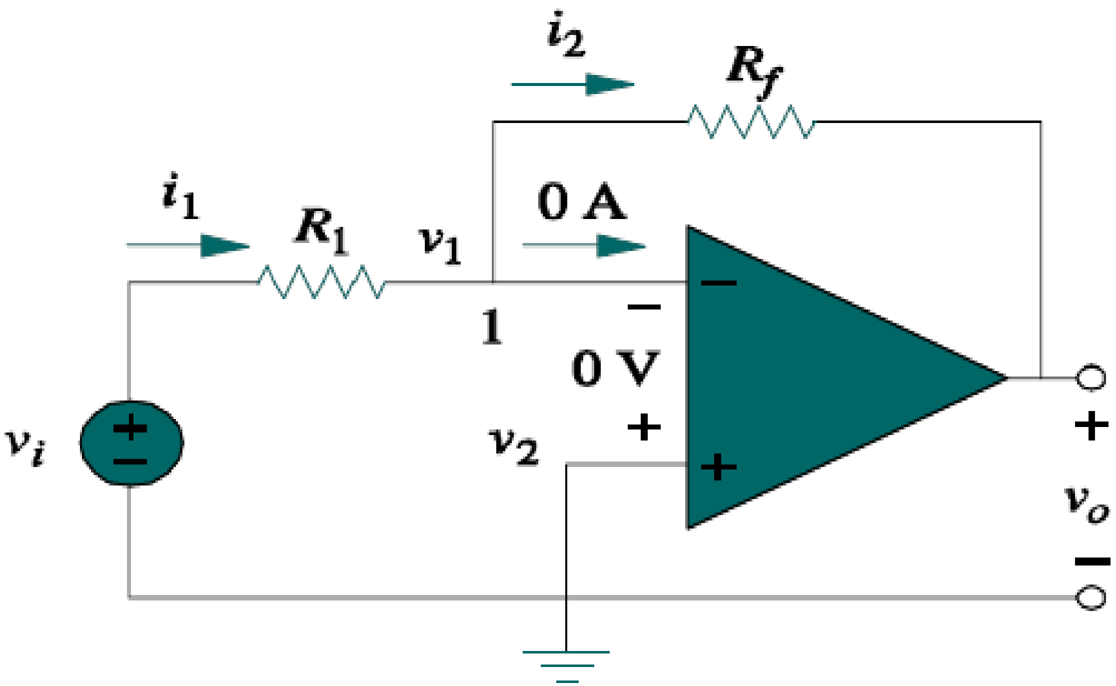
$$v_{out} = v_{in}\left(1 + \frac{R_2}{R_1}\right)$$

Isolation or Voltage Follower



Circuit isolation with an op-amp.

Inverting Amplifier



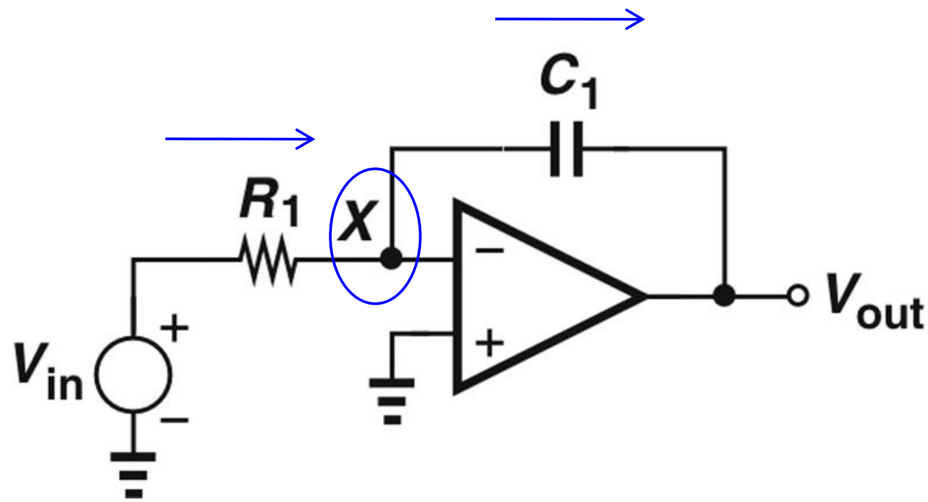
Since the noninverting terminal is grounded

$$v_1 = v_2 = 0$$

KCL at v_1: $i_1 = i_2 \Rightarrow \frac{v_i - 0}{R_1} = \frac{0 - v_0}{R_f}$ ⇒ $v_0 = -\frac{R_f}{R_1} v_i$

Op-Amp Integrator

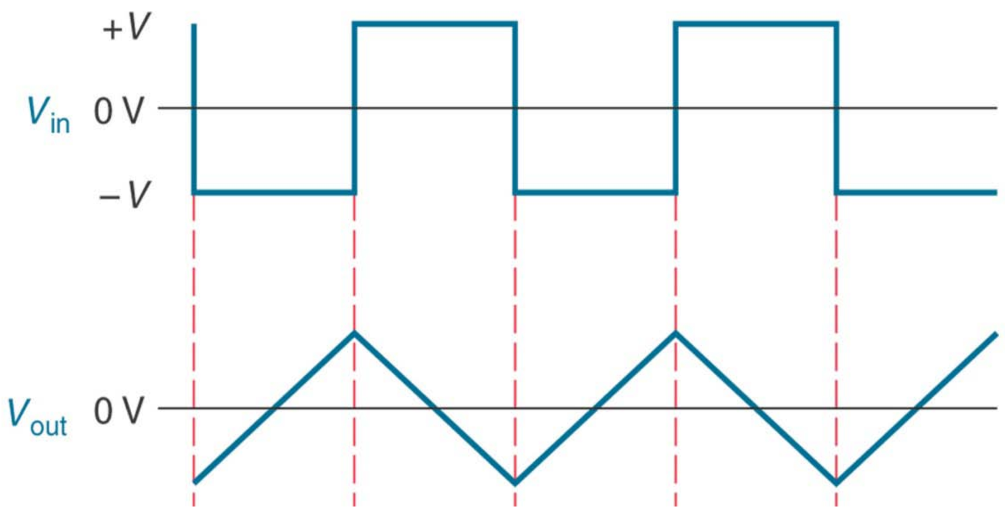
- The op-amp provides a constant-current source for the capacitor, causing it to charge at a linear rate.
- It is used as Low Pass Filter.



At point *X*

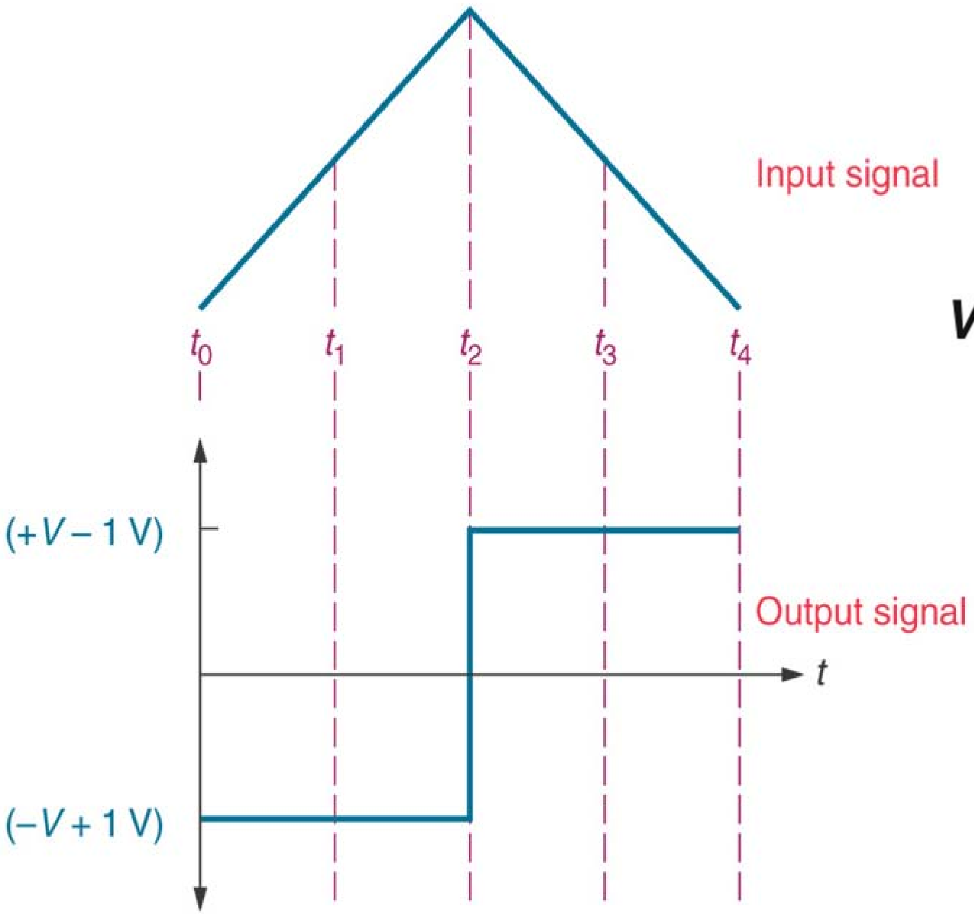
$$\frac{V_{in}}{R_1} = -C_1 \frac{dV_{out}}{dt}$$

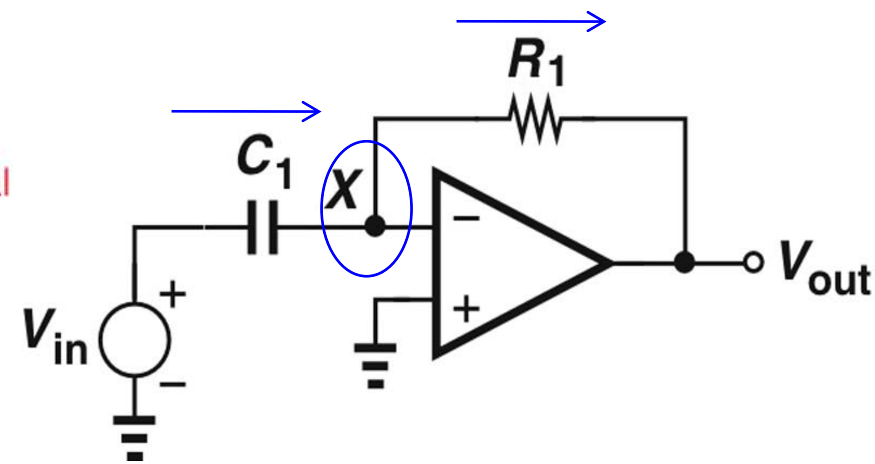
$$V_{out} = -\frac{1}{R_1 C_1} \int V_{in}\, dt$$



Op-Amp Differentiator

- The output signal is proportional to the rate of change of input signal.
- It is used as High Pass Filter.



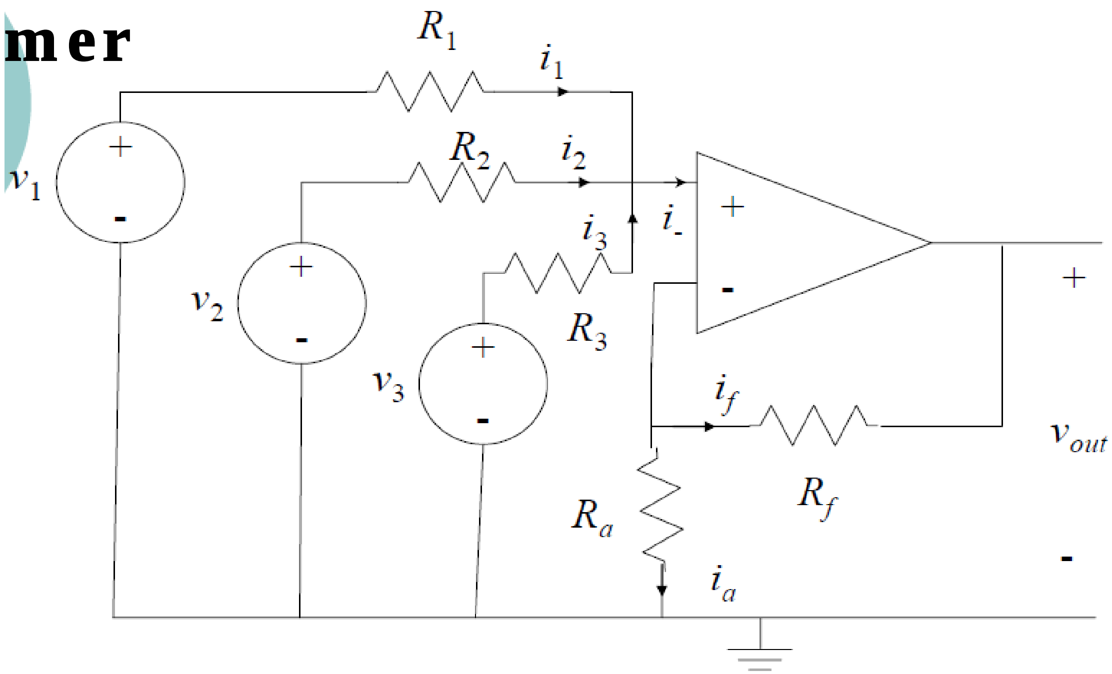


At point *X*

$$C_1 \frac{dV_{in}}{dt} = -\frac{V_{out}}{R_1}$$

Op-Amp with Multiple Inputs

Noninverting Summer



At the noninverting input:

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - v_+}{R_1}$$

$$i_2 = \frac{v_2 - v_+}{R_2} \qquad i_3 = \frac{v_3 - v_+}{R_3}$$

At the inverting input:

$$i_f + i_a = 0$$

$$i_f = \frac{v_- - v_{out}}{R_f}$$

$$i_a = \frac{v_-}{R_a}$$

$$v_- = \frac{R_a}{R_a + R_f} v_{out}$$

$$v_- = v_+$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_+$$

$$\frac{1}{R_T} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = \frac{1}{R_T} \frac{R_a}{R_a + R_f} v_{out}$$

$$v_{out} = \left(1 + \frac{R_f}{R_a}\right)\left(\frac{R_T}{R_1} v_1 + \frac{R_T}{R_2} v_2 + \frac{R_T}{R_3} v_3\right)$$

Inverting Summer

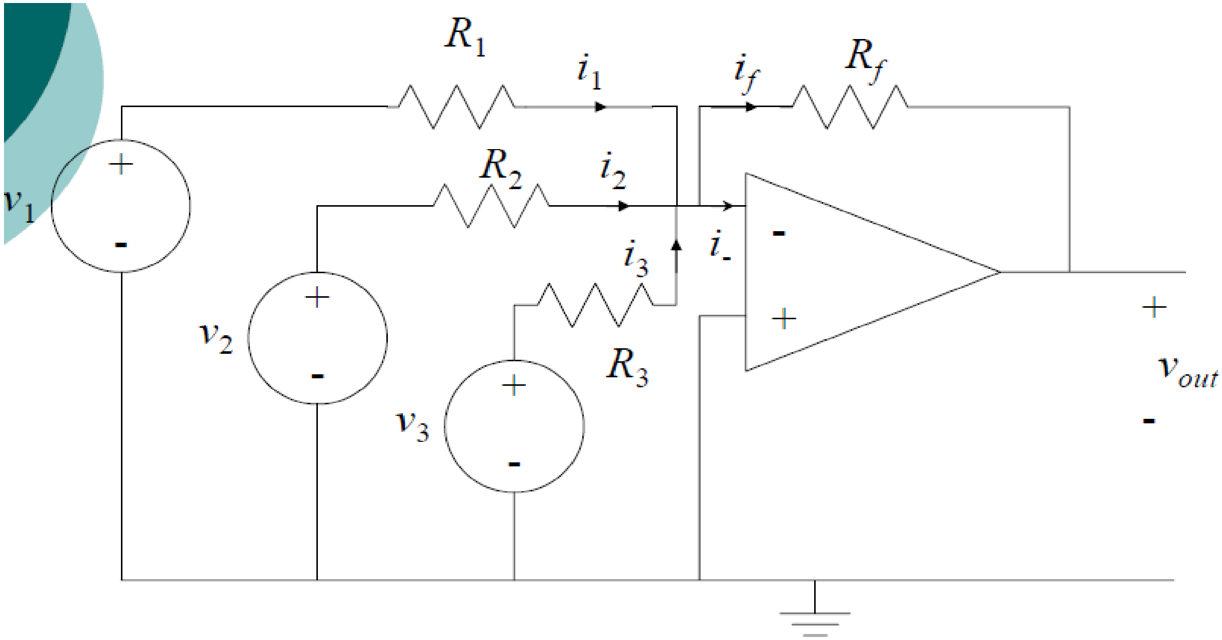
KCL at the Inverting Input

$$i_1 = \frac{v_1 - v_-}{R_1} = \frac{v_1}{R_1}$$

$$i_2 = \frac{v_2 - v_-}{R_2} = \frac{v_2}{R_2}$$

$$i_3 = \frac{v_3 - v_-}{R_3} = \frac{v_3}{R_3}$$

$$i_- = 0$$



$$i_f = \frac{v_{out} - v_-}{R_f} = \frac{v_{out}}{R_f}$$

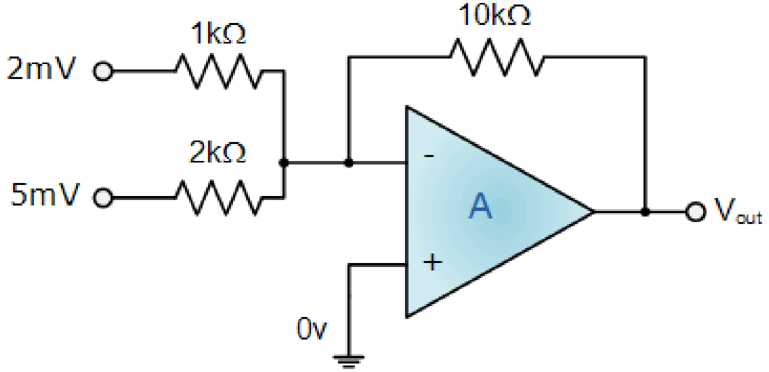
$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \frac{v_{out}}{R_f} = 0$$

$$v_{out} = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2 - \frac{R_f}{R_3} v_3$$

Example 2

Calculate V_{0ut} for the following summing circuit.

$$V_{out} = -R_f\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}\right)$$
$$= -10\times10^3\left(\frac{2\times10^{-3}}{1\times10^3}+\frac{5\times10^{-3}}{2\times10^3}\right)$$
$$= -10\times10^3\left(2\times10^{-6}+2.5\times10^{-6}\right) = 45\times10^{-3}\text{V} = 45\text{ mV}$$

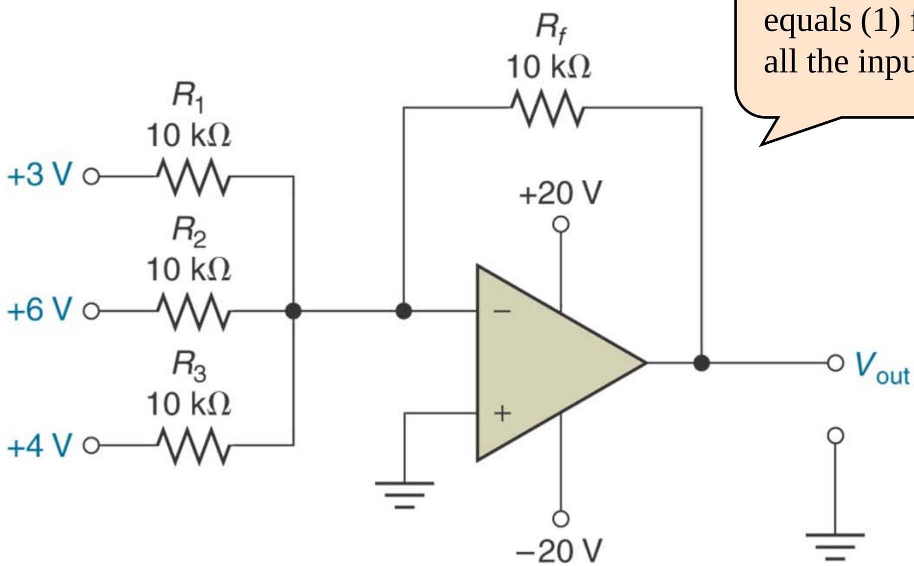


Example 3

Determine V_{0ut} for the following summing circuit.

Voltage gain equals (1) for all the inputs.

$$V_{out} = -R_f\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)$$
$$= -13\text{V}$$



Example 4

Design op-amp summing circuit to satisfy the following equation:
$V_{0ut} = 0.2\,V_1 + 0.25\,V_2 - V_3$. (Assume $R_f = 10k\Omega$)

Design steps:

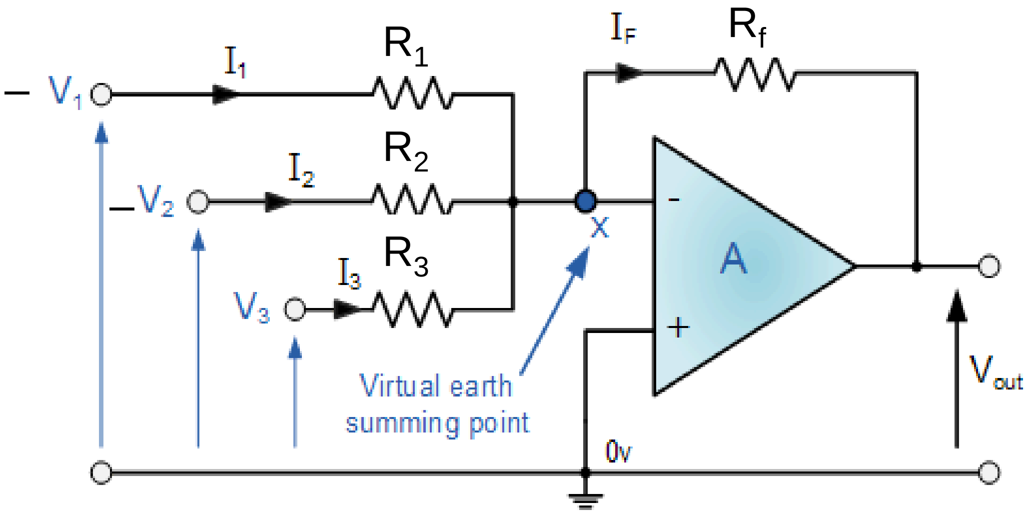
- Draw the summing circuit that has three input voltages and three resistances and reverse the sign of the three inputs as shown.
- Calculate the values of the three resistances by matching the summer equation with the given one as:

$$V_{out} = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2}V_2 - \frac{R_f}{R_3}V_3$$

$$\frac{R_f}{R_1} = 0.2 \Rightarrow R_1 = \frac{10k\Omega}{0.2} = 50k$$

$$\frac{R_f}{R_2} = 0.25 \Rightarrow R_2 = \frac{10k\Omega}{0.25} = 40k$$

$$\frac{R_f}{R_3} = 1 \Rightarrow R_3 = \frac{10k\Omega}{1} = 10k$$



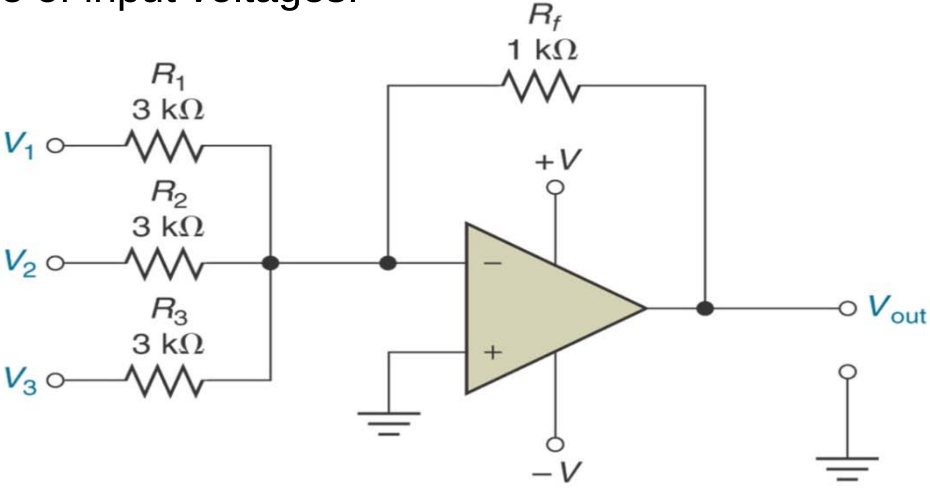
Averaging Amplifier

Averaging amplifier – is a summing amplifier with proper input and feedback resistors, that provides an output proportional to the average of input voltages.

Example 5

$$-V_{out} = \frac{1}{3}V_1 + \frac{1}{3}V_2 + \frac{1}{3}V_3$$

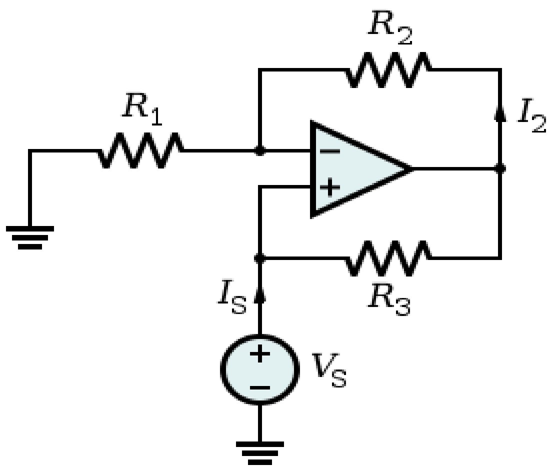
$$\therefore\ -V_{out} = \frac{V_1 + V_2 + V_3}{3}$$



Negative Impedance Converter NIC

- Creates a negative resistor for any signal generator.
- The ratio between input voltage and input current (input resistance) is given by:

$$R_{in} = -R_3\left(\frac{R_1}{R_2}\right)$$

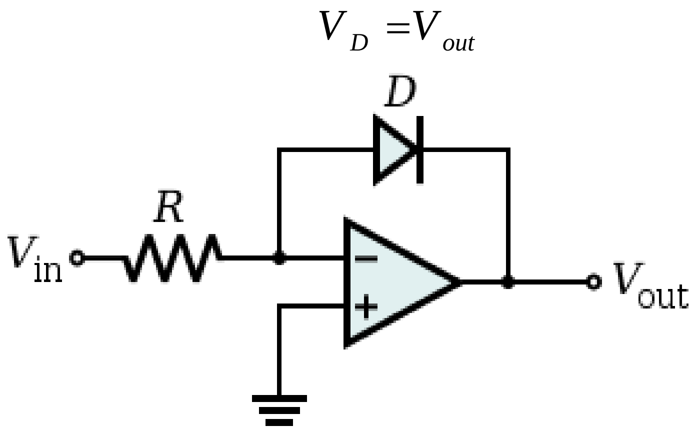


Logarithmic op-amp

$$\frac{V_{\text{in}}}{R} = I_R = I_D$$

$$I_D = I_S\left(e^{(V_D/V_T)} - 1\right) \simeq I_S e^{(V_D/V_T)}$$

$$\boxed{V_{\text{out}} = -V_{\text{T}} \ln\left(\frac{V_{\text{in}}}{I_S R}\right)}$$



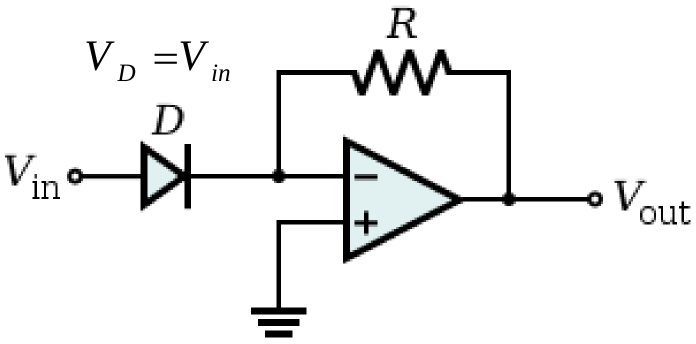
Where: I_s is the saturation current and V_T is the thermal voltage.

Op-amp Exponential Amplifier

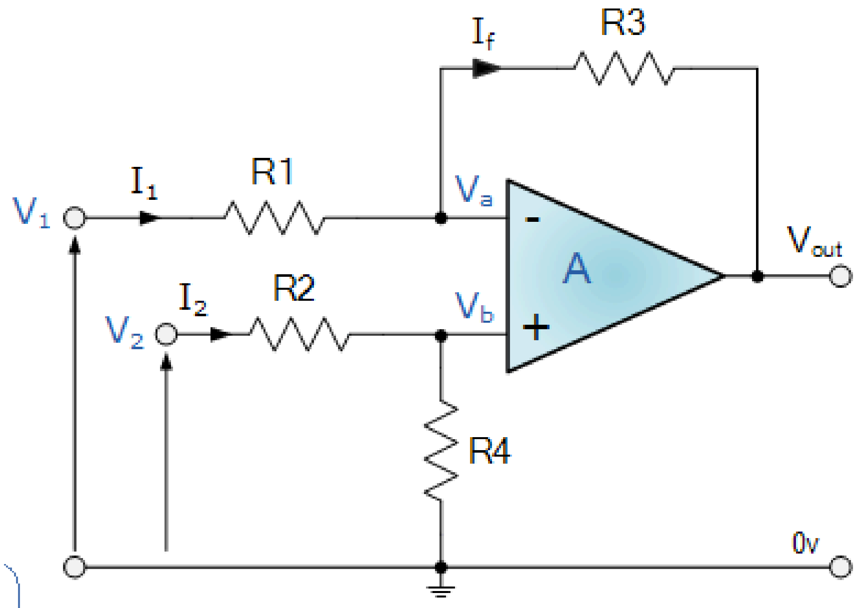
Considering ideal op-amp, the negative pin is virtually grounded, so the current through the diode is given by:

$$I_D = I_S\left(e^{(V_D/V_T)} - 1\right) \simeq I_S e^{(V_D/V_T)}$$

$$\boxed{V_{\text{out}} = -RI_D = -RI_S e^{(V_{in}/V_T)}}$$



Voltage Subtractor



$$I_1 = \frac{V_1 - V_a}{R_1}, \quad I_2 = \frac{V_2 - V_b}{R_2}, \quad I_f = \frac{V_a - (-V_{out})}{R_3}$$

Summing point $V_a = V_b$

If $V_b = 0$, then: $V_{out(a)} = -V_1\left(\frac{R_3}{R_3+R_1}\right) = -V_1\left(\frac{R_3}{R_1}\right)$

If $V_a = 0$, then: $V_{out(b)} = V_2\left(\frac{R_4}{R_2+R_4}\right)\left(1+\frac{R_3}{R_1}\right)$

$$V_{out} = V_{out(a)} + V_{out(b)}$$

$$\therefore\ V_{out} = -V_1\left(\frac{R_3}{R_1}\right) + V_2\left(\frac{R_4}{R_2+R_4}\right)\left(1+\frac{R_3}{R_1}\right)$$

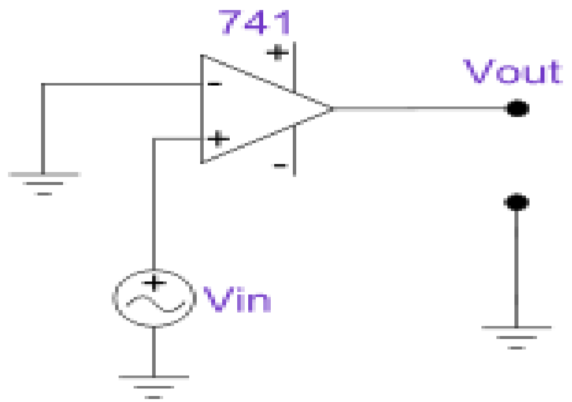
If $R_1 = R_2$ and $R_3 = R_4$

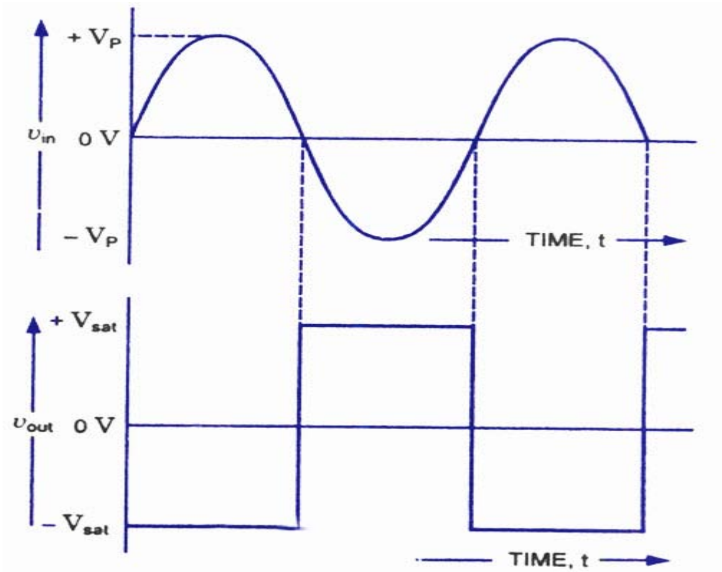
$$V_{out} = \frac{R_3}{R_1}(V_2 - V_1)$$

If $R_1 = R_2 = R_3 = R_4$

$$V_{out} = V_2 - V_1$$

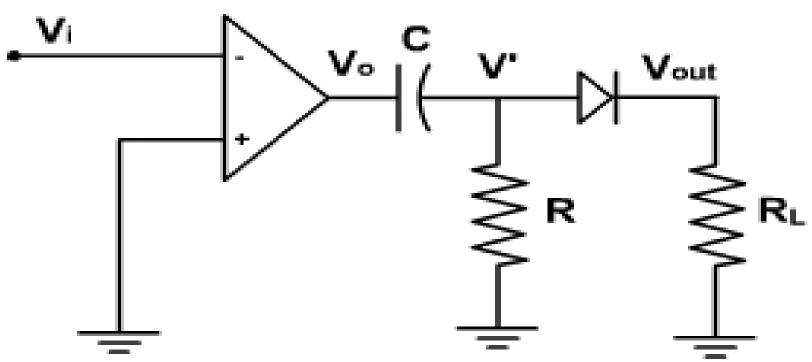
Zero Crossing Detector

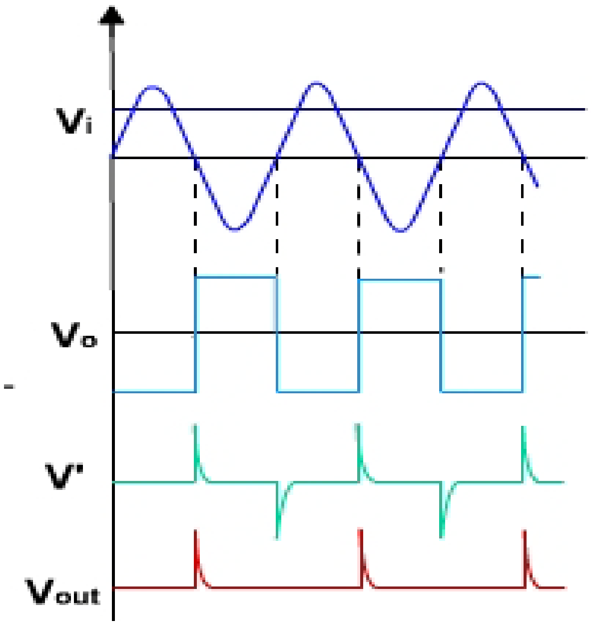




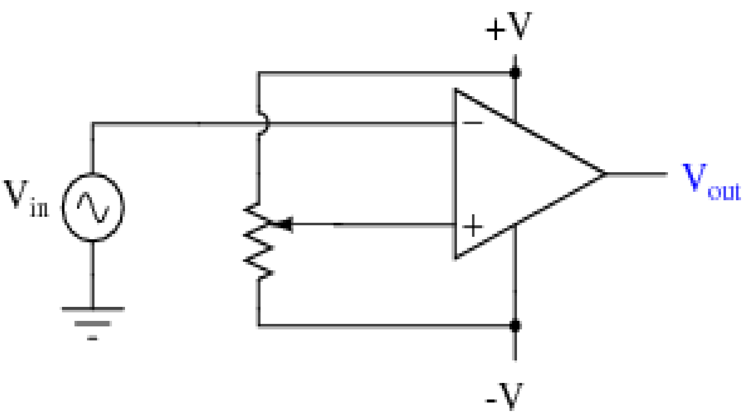
Input and Output Waveforms

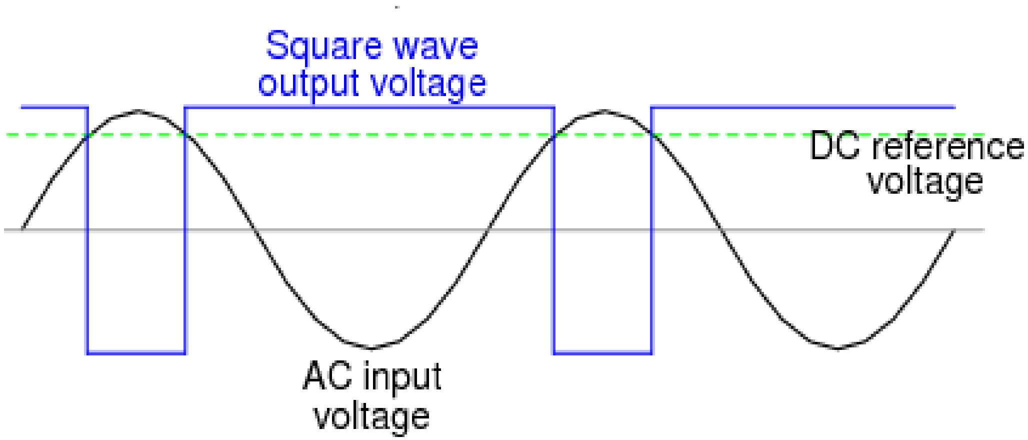
Zero crossing detector supplies trigger pulses





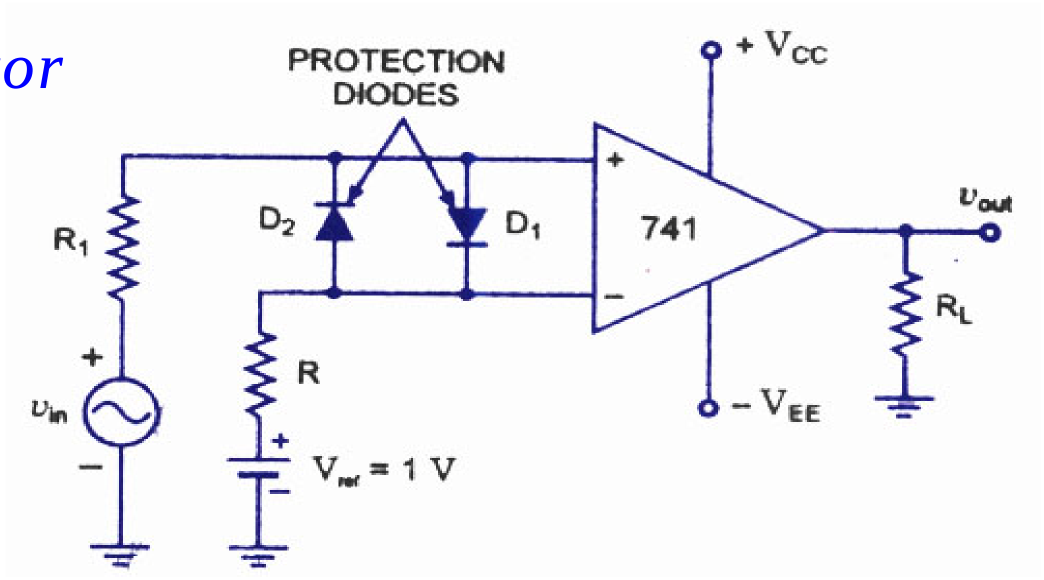
Inverting Comparator





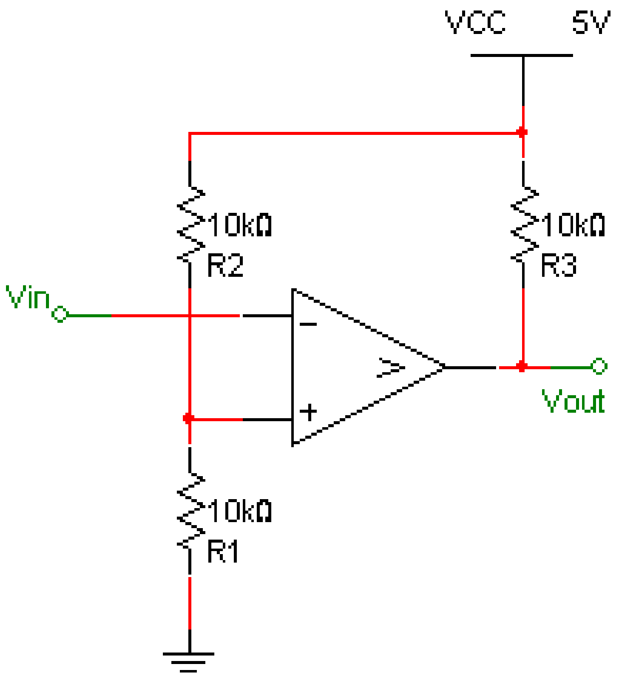
A "clean" AC input waveform produces predictable transition points on the output voltage square wave

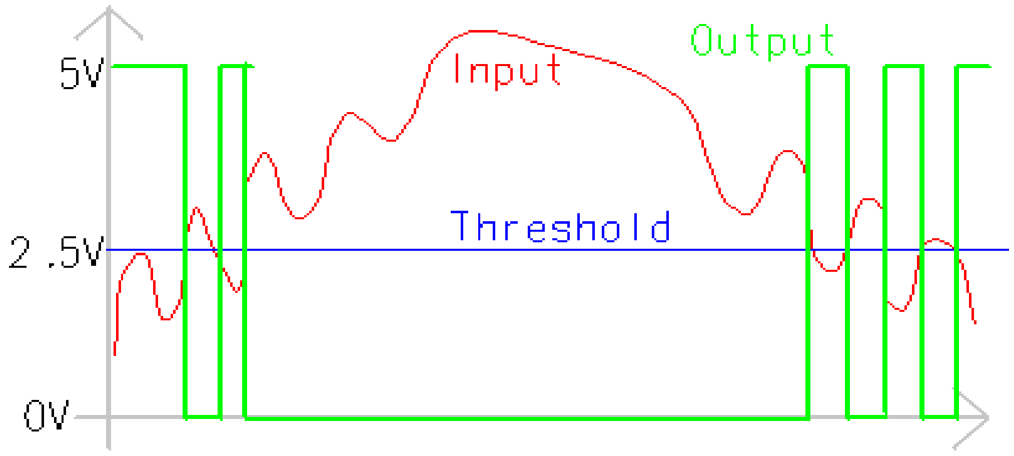
Noninverting Comparator



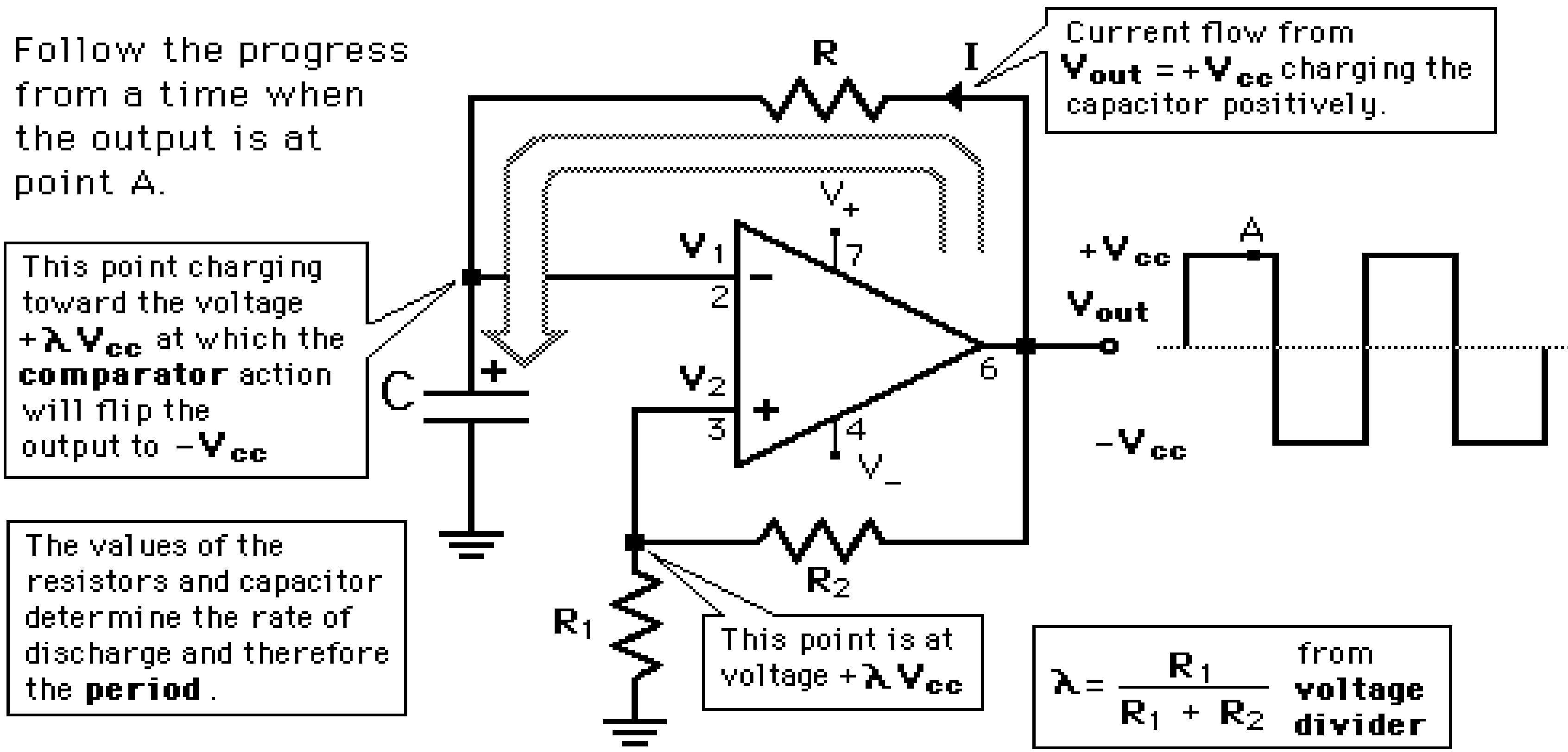
Example 6

Inverting comparator circuit using op-amp and threshold voltage of 2.5 volt.

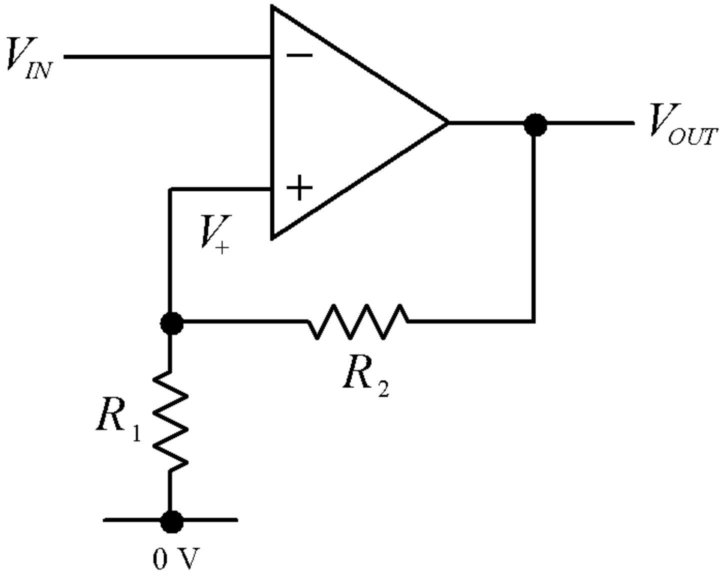




Square Wave Generator



Schmitt Trigger



Switching occurs when:

$$V_{IN} = V_- = V_+ = V_{OUT}\frac{R_1}{R_1 + R_2}$$

But,

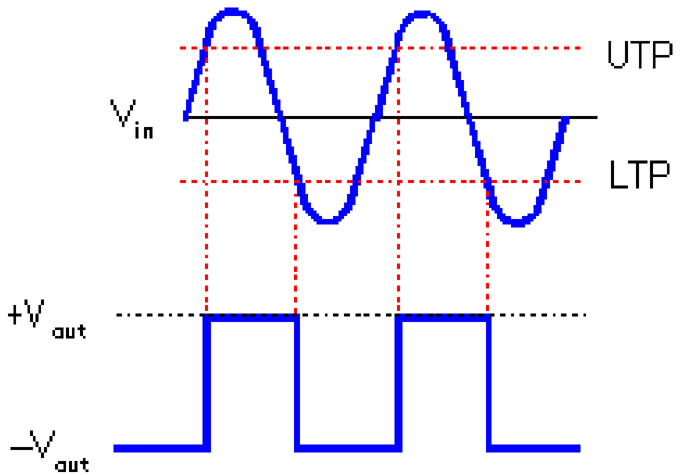
$$V_{OUT} = \pm V_{SAT}$$

$\therefore$ Threshold input voltage

for upper trigger point (UTP)

and lower trigger point (LTP)

$$V_{IN} = \pm V_{SAT}\frac{R_1}{R_1 + R_2}$$

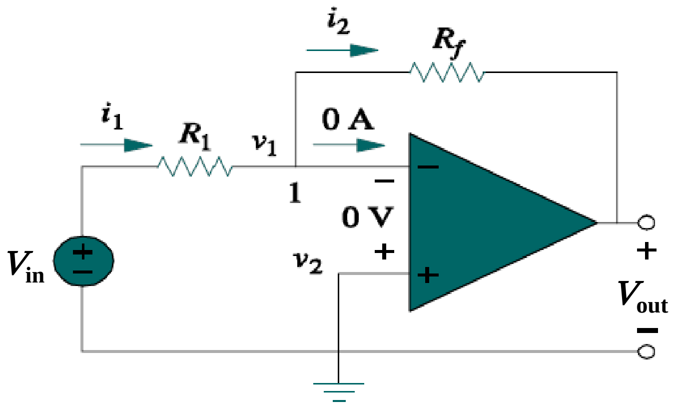


Problems and Solutions

Question 1

(a) Derive an expression for (V_{out} / V_{in}) for the inverting op-amp amplifier in terms of R_1 and R_f .
(b) Inverting op-amp amplifier with 1kHz sine wave input of 2 volts peak-to-peak, $R_1 = R_f = 10k\Omega$.
1. Calculate V_{out}. 2. Draw the input and output waveforms (V_{in} and V_{out}).

Solution:



(a) Since the non-inverting terminal is grounded:

$$v_1 = v_2 = 0$$

Also for the op-amp, we have: $i_- = i_+ = 0$

Using KCL at the node 1 we get:

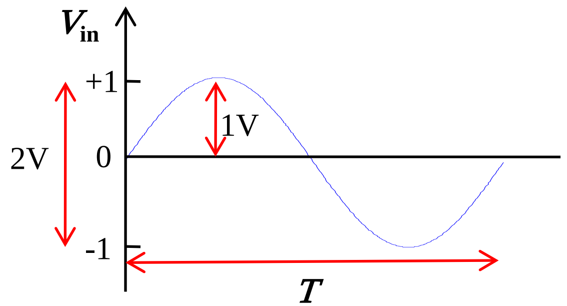
$$i_1 = i_2 \Rightarrow \frac{V_{in} - 0}{R_1} = \frac{0 - V_{out}}{R_f} \Rightarrow \boxed{\therefore \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_1}}$$

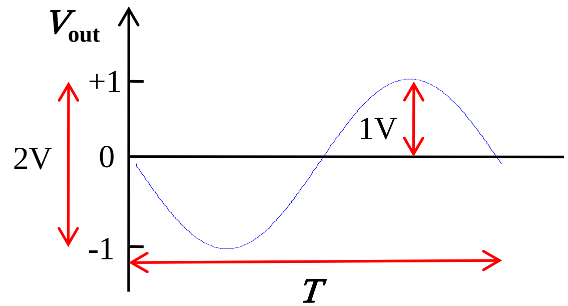
(b)

$$V_{out} = -\frac{R_f}{R_1} V_{in}$$

$$V_{out} = -\frac{10\times10^3}{10\times10^3}\times 2$$

$$V_{out} = -2 \ volt$$

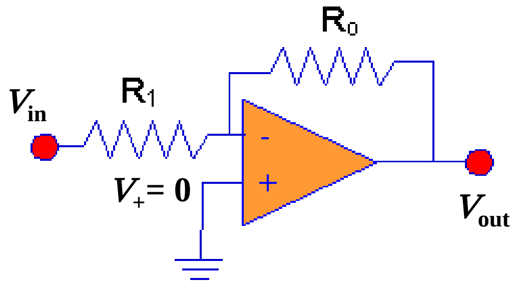




$$T = \frac{1}{f} = \frac{1}{1\times10^3} = 1m\sec$$

Question 2 For the following op-amp circuit with gain of A = -10, find the output voltage V_{out} if the input voltage V_{in} = 0.24 volt .

Solution:



$$V_{out} = -\frac{R_0}{R_1} V_{in} = -10 \times 0.24 = -2.4 \ volt$$

Question 3 Inverting amplifier with R_f = 4700Ω , find the value of R_1 that produce gain of A = -5.

Solution:

The gain of inverting amplifier is: $A = -\frac{R_f}{R_1} \Rightarrow R_1 = -\frac{R_f}{A} = -\frac{4700}{-5} = 940\Omega$

Question 4 An op-amp has a differential gain G_d = 250,000. If the output voltage V_{out} = 3.75 volt, find the difference between input voltages V_d.

Solution:

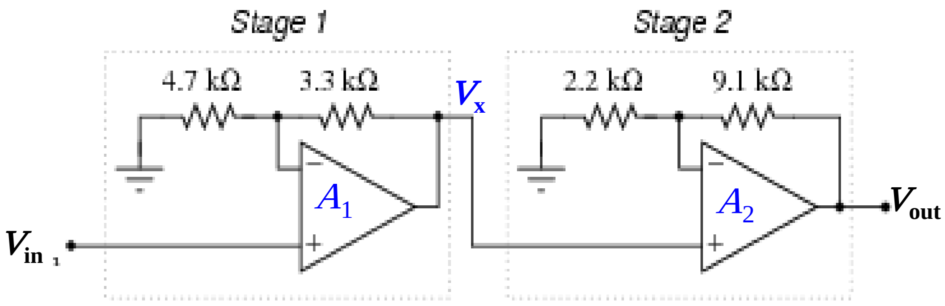
$$V_{out} = G_d V_{in}$$

$$V_{in} = \frac{V_{out}}{G_d} = \frac{3.75}{250{,}000} = 15 \times 10^{-6} \ volt = 15 \mu V$$

Question 5 Calculate the voltage gain for each stage (A_1 and A_2) of the given amplifier circuit and then find the overall voltage gain A.

Solution:

The gain of any single stage non-inverting amplifier is:



$$A = \frac{V_{out}}{V_{in}} = \left(1 + \frac{R_f}{R_1}\right)$$

The gain of stage 1 is:

$$A_1 = \frac{V_x}{V_{in}} = \left(1 + \frac{3300}{4700}\right) = 1.702$$

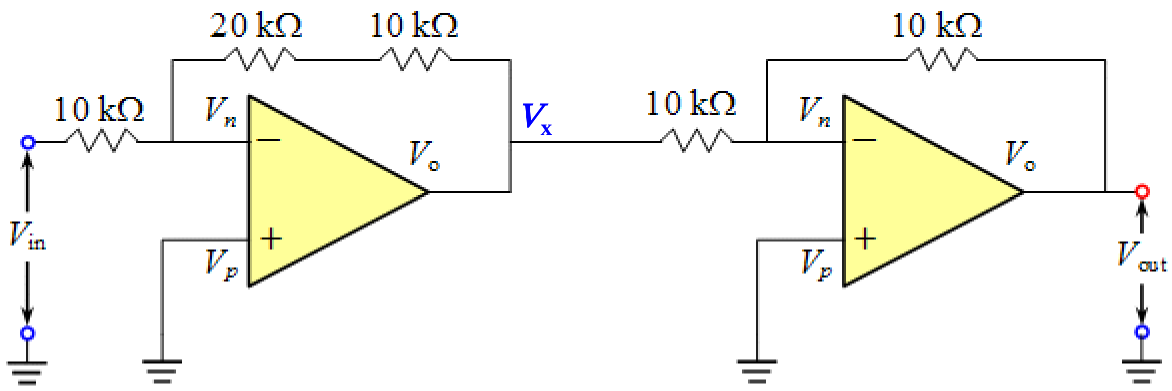
The gain of stage 2 is:

$$A_2 = \frac{V_{out}}{V_x} = \left(1 + \frac{9100}{2200}\right) = 5.136$$

Thus, the overall gain is:

$$A = \frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}} \times \frac{V_{out}}{V_x} = A_1 \times A_2 = 1.702 \times 5.136 = 8.74$$

Question 6 For the given circuit, calculate the output voltage V_{out} if the input voltage V_{in} = 1.5 volt .



Solution:

The gain of any single stage inverting amplifier is:

$$A = -\frac{R_f}{R_1}$$

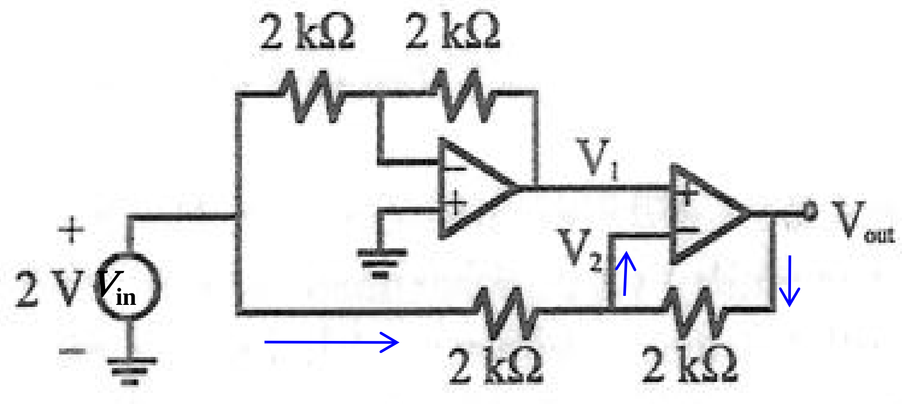
When V_{in} = **1.5 V**,The output voltage of first stage is:

$$V_x = \left(-\frac{20\times10^3 + 10\times10^3}{10\times10^3}\right)\times V_{in} = (-3)\times1.5 = -4.5\text{Volt}$$

Therefore, the output voltage V_{out} at second stage is:

$$V_{out} = \left(-\frac{10\times10^3}{10\times10^3}\right)\times V_x = (-1)\times(-4.5) = 4.5\text{Volt}$$

Question 7 For the given circuit, determine V_1 and V_2, and the output voltage V_{out}.



Solution:

The gain of any single stage inverting amplifier is:

$$A = -\frac{R_f}{R_1}$$

$$\therefore \; V_1 = \left(-\frac{2\times10^3}{2\times10^3}\right)\times V_{in} = (-1)\times 2 = -2\text{Volt}$$

For ideal op-amp we have: $V_1 = V_2 = -2\text{Volt}$

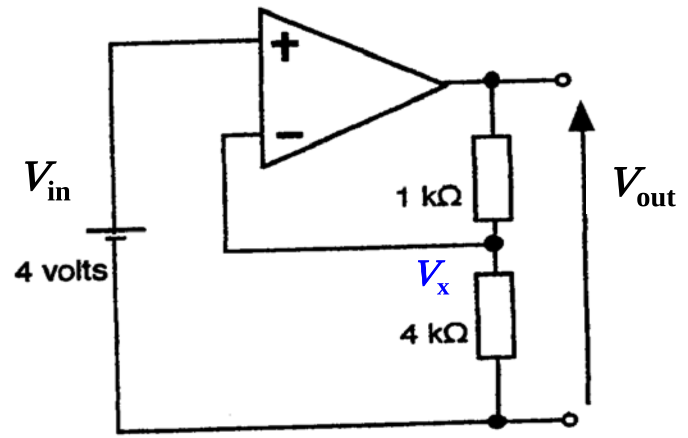
Also for the op-amp, we have: $i_+ = i_- = 0$

Using KCL at node V_2, we get:

$$\frac{V_{in} - V_2}{2\text{k}\Omega} + \frac{V_{out} - V_2}{2\text{k}\Omega} = i_- = 0$$

$$\therefore \; V_{out} = V_2 + V_2 - V_{in} = -2 - 2 - (2) = -6\text{Volt}$$

Question 8 For the given non-inverting op-amp, find the output voltage V_{out} .



Solution: $i_+ = i_- = 0$

$$v_x = v_- = v_+ = V_{in} = 4\text{Volt}$$

Using the voltage divider equation at node "V_x":

$$V_x = \left(\frac{4\text{k}\Omega}{4\text{k}\Omega + 1\text{k}\Omega}\right) V_{out}$$

$$\therefore \quad V_{out} = \frac{5}{4} \times V_x = \frac{5}{4} \times 4 = 5\text{Volt}$$

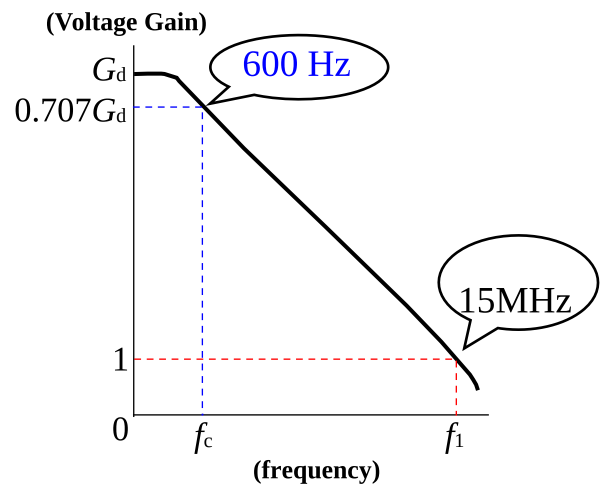
Question 9 Determine the cutoff frequency f_c of an op-amp having a unit gain frequency f_1 = 15MHz and voltage differential gain G_d = 25V/mV. Draw the frequency response.

Solution:

$$G_d = 25\text{V/mV} = \frac{25\text{V}}{10^{-3}\text{V}} = 25 \times 10^3$$

$$f_1 = G_d f_c$$

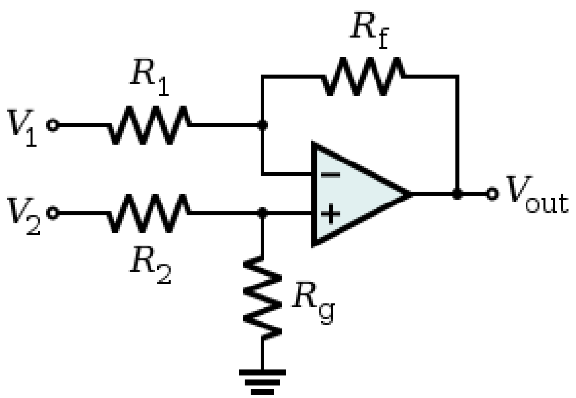
$$\therefore \quad f_c = \frac{f_1}{G_d} = \frac{15 \times 10^6}{25 \times 10^3} = 600\text{Hz}$$



Question 10 Design op-amp subtractor circuit to satisfy the following equation: $V_{out} = 2(V_2 - V_1)$ Assume the feedback resistance $R_f = 10\text{k}\Omega$.

Solution:

The circuit diagram of op-amp subtractor is shown as:



For good design $R_2 = R_1$ and $R_f = R_g$.

Then, the output voltage V_{out} is given as:

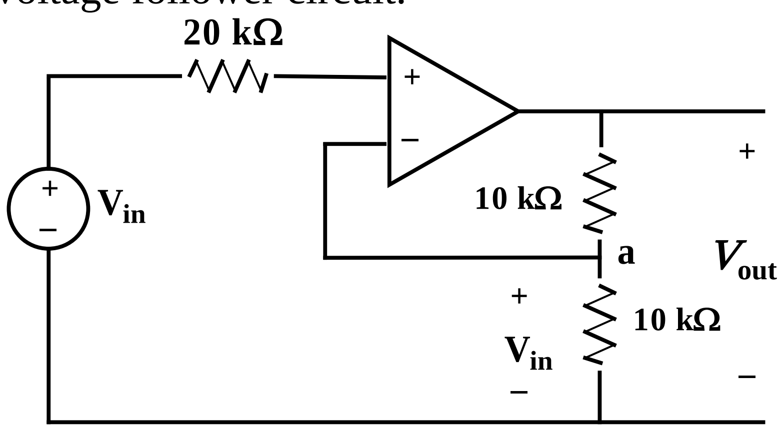
$$V_{out} = \frac{R_f}{R_1}(V_2 - V_1)$$

And we have from the given question $V_{out} = 2(V_2 - V_1)$

Then by similarity of both equations, we get $\frac{R_f}{R_1} = 2 \Rightarrow \therefore R_1 = \frac{R_f}{2} = \frac{10\text{k}\Omega}{2} = 5\text{k}\Omega$

Question 11 Calculate the voltage gain A for the voltage follower circuit.

Solution:



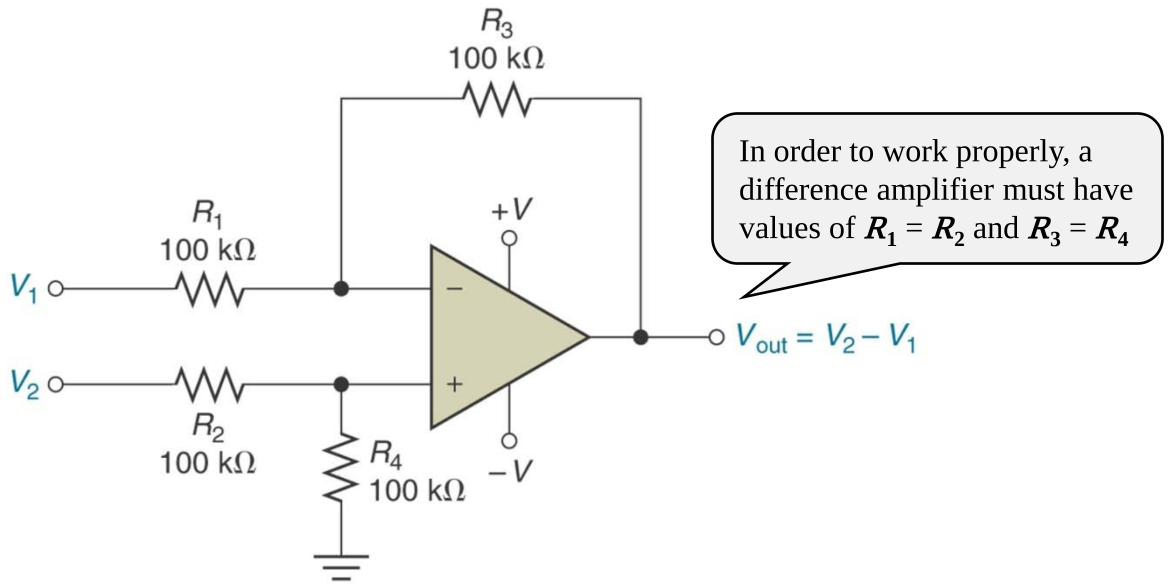
$$i_+ = i_- = 0$$

$$v_a = v_- = v_+ = V_{in}$$

Using the voltage divider equation at node "a":

$$V_{in} = \left(\frac{10\text{k}\Omega}{10\text{k}\Omega + 10\text{k}\Omega}\right)V_{out} \Rightarrow \therefore \quad A = \frac{V_{out}}{V_{in}} = 2$$

Question 12 For the given difference amplifier (subtractor), write the equation of output voltage V_{out} .



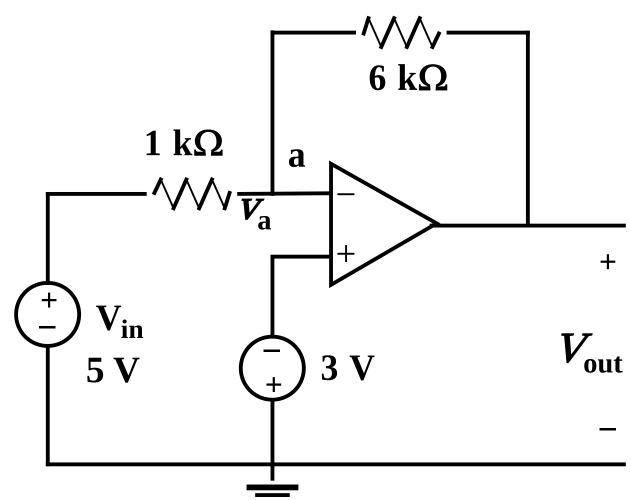
Solution:

$$V_{out} = \frac{R_3}{R_1}(V_2 - V_1)$$

$$V_{out} = \frac{100k\Omega}{100k\Omega}(V_2 - V_1)$$

$$\therefore V_{out} = V_2 - V_1$$

Question 13 Find V_{out} for the given op-amp circuit.



Solution: $i_+ = i_- = 0$

Since the non-inverting terminal is connected to +3 volts, we get

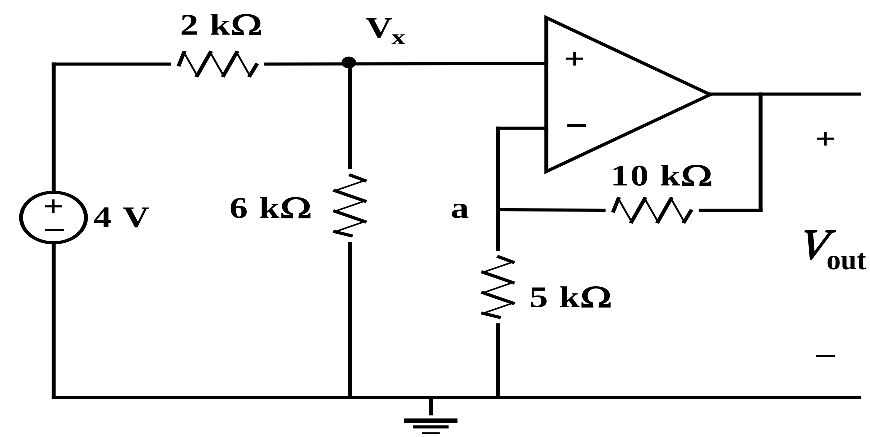
$$v_a = v_- = v_+ = -3 \text{ Volt}$$

Using KCL at the non-inverting node “a”:

$$\frac{V_{in} - v_a}{1k\Omega} = \frac{v_a - V_{out}}{6k\Omega} + i_-$$

$$\therefore V_{out} = -51\text{Volt}$$

Question 14 For the given non-inverting op-amp, find the output voltage V_{out} .



Solution: $i_+ = i_- = 0$

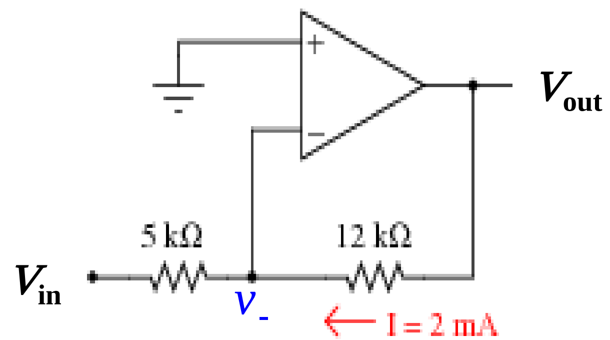
$$v_a = v_- = v_+ = V_x$$

$$V_x = \left(\frac{6\text{k}\Omega}{6\text{k}\Omega + 2\text{k}\Omega}\right) \times 4 = 3\text{Volt}$$

Using the voltage divider equation at node "a":

$$V_a = \left(\frac{5\text{k}\Omega}{5\text{k}\Omega + 10\text{k}\Omega}\right) \times V_{out} \Rightarrow V_{out} = 3 \times V_a = 9\text{Volt}$$

Question 15 Determine both of input voltage V_{in} and output voltage V_{out} for given circuit.



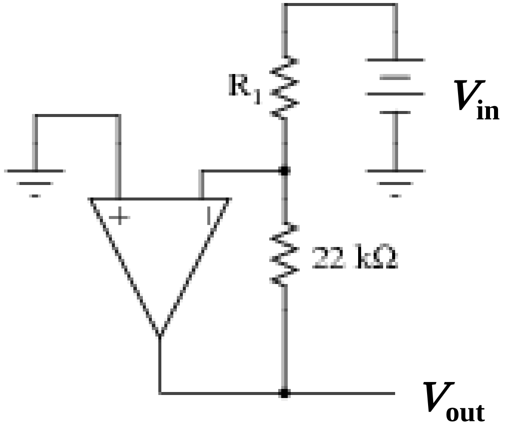
Solution: $i_+ = i_- = 0$

$$v_- = v_+ = 0 \quad \Rightarrow V_{out} = 2\text{mA} \times 12\text{k}\Omega = 24\text{Volt}$$

For the inverting op-amp we have: $V_{out} = -\dfrac{R_f}{R_1} V_{in}$

$$\therefore \; V_{in} = -\frac{R_1}{R_f} V_{out} = -\frac{5\text{k}\Omega}{12\text{k}\Omega} \times 24 = -10\text{Volt}$$

Question 16 Calculate $\boldsymbol{R_1}$ to produce a voltage gain of $\boldsymbol{A} = \mathbf{15}$ in the given op-amp circuit.



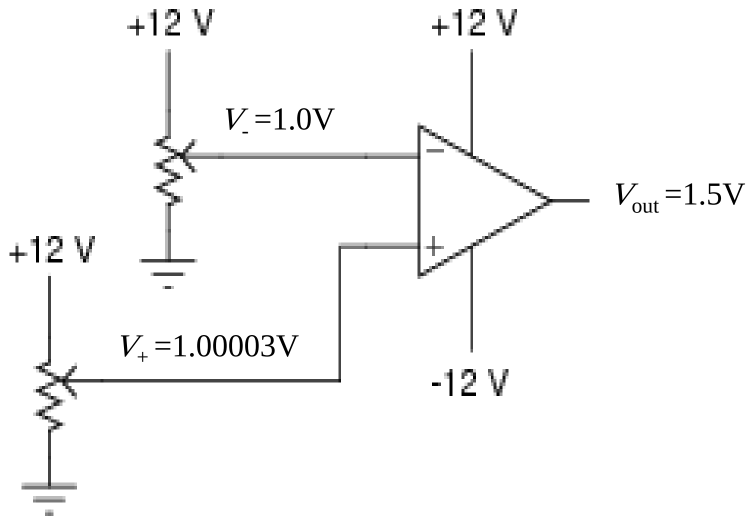
Solution:

The voltage gain of inverting op-amp is:

$$A = \frac{R_f}{R_1}$$

$$\Rightarrow R_1 = \frac{R_f}{A} = \frac{22\text{k}\Omega}{15} = 1.46\text{k}\Omega$$

Question 17 For the given op-amp circuit with two different input voltages. Determine the differential voltage gain $\boldsymbol{G_d}$ of this op-amp.



Solution:

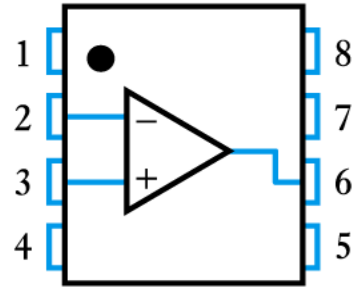
$$V_{out} = G_d V_d$$

$$V_d = V_+ - V_- = 1.00003 - 1 = 30 \times 10^{-6}\,\text{Volt}$$

$$\Rightarrow G_d = \frac{V_{out}}{V_d} = \frac{1.5}{30 \times 10^{-6}} = 50000$$

Question 18 Choose the correct alternative for the following multiple choice questions.

(1) What signal corresponds to pin 3 of this operational amplifier?



A- The non-inverting output. B- The positive supply voltage.

C- The non-inverting input. D- The inverting input

(2) What characteristics would characterize an ideal operational amplifier?

A- An infinite voltage gain, an infinite input resistance and an infinite output resistance.

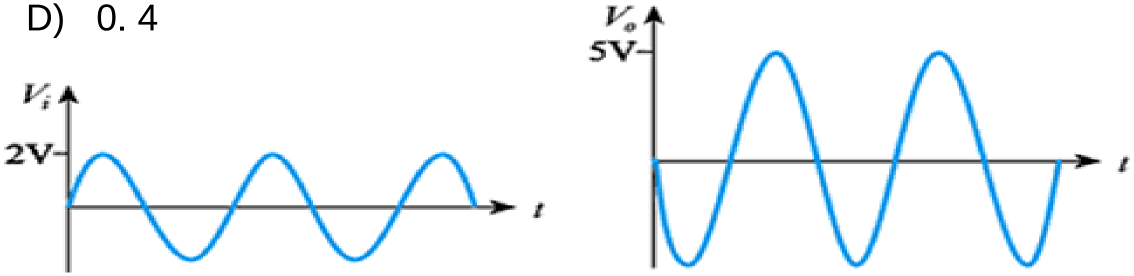
B- An infinite voltage gain, zero input resistance and an infinite output resistance.

C-. An infinite voltage gain, zero input resistance and zero output resistance.

D- .An infinite voltage gain, an infinite input resistance and zero output resistance

(3) The graphs show input and output waveforms of an op-amp.
What is the gain of this circuit?

A) - 0.4 B) -2.5 C) 2.5 D) 0. 4

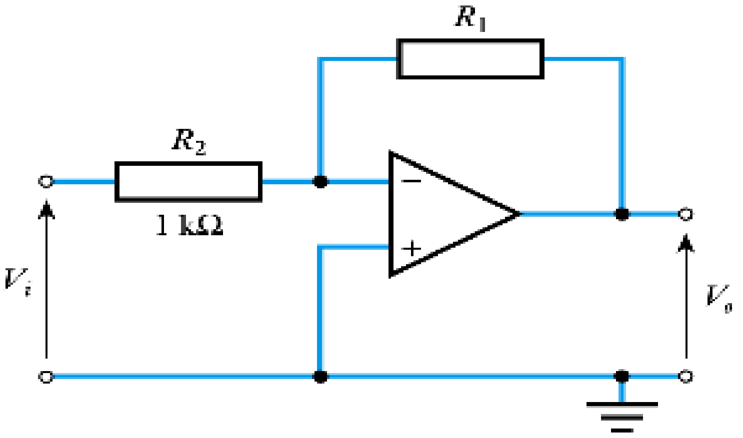


(4) A differential amplifier amplifies the ---------- between two input signals.

A- Addition B- subtraction C- multiplication

(5) In the given circuit, what value of R_1 is required to give a voltage gain of -50 ?

A) 49 kΩ B) 50 kΩ C) -50 kΩ D) -49 kΩ



(6) Frequency response of the differentiator is same as that of …………………

a) high pass filter b) low pass filter c) band pass filter

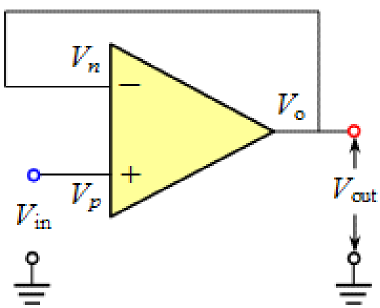
(7) What is the voltage gain of the unity follower?

a) 0 b) 1 c) –1 d) infinity

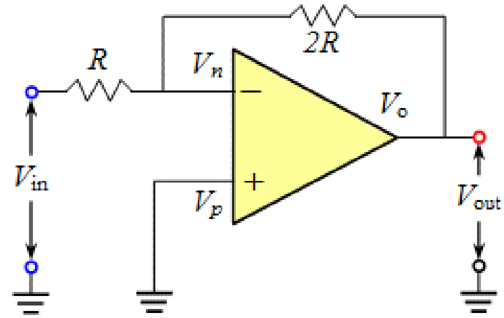
(8) What is the scale multiplier (factor) of a basic integrator?

a) R/C b) C/R c) –RC d) –1/RC

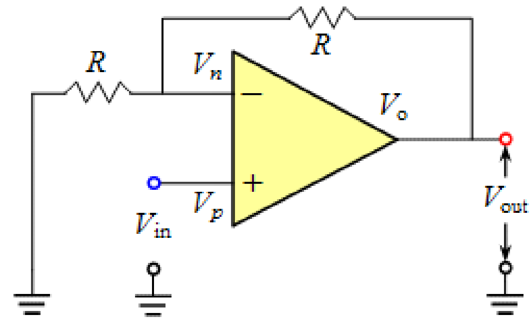
(9) Choose the circuit below that would take a DC input voltage V_{in} = 1 V and produce an output voltage V_{out} = 2 V.



(a)

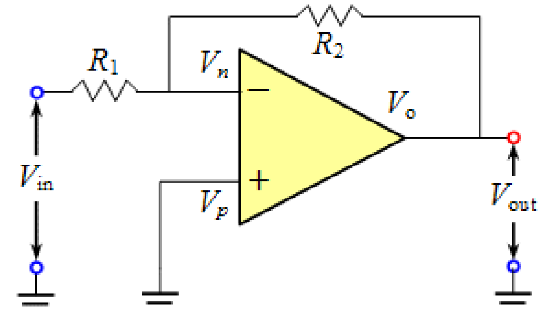


(b)



(c)

(10) If R_1 = 10 kΩ, and R_2 = 10 k Ω, this circuit would be……………..

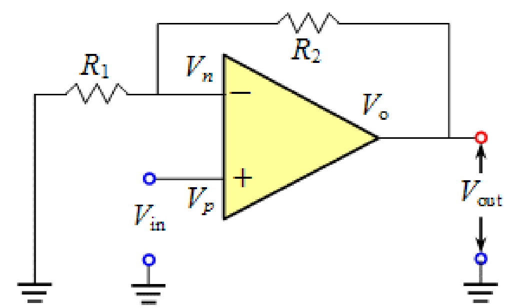


A) buffer B) inverting summer C) inverter

D) inverting amplifier E) noninverting amplifier

F) low-pass filter G) high-pass filter

(11) If R_1 = 10 kΩ, and R_2 = 20 k Ω, this circuit would be……………..



A) buffer B) inverting summer C) inverter

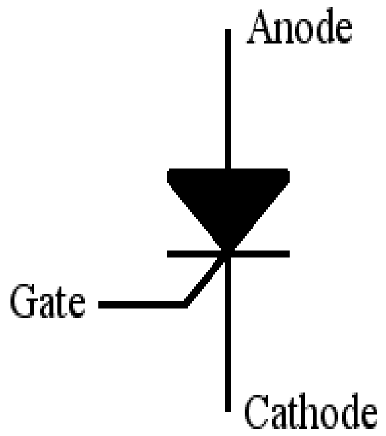
D) inverting amplifier E) noninverting amplifier

Chapter 5

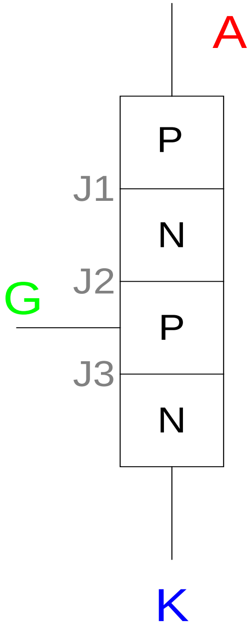
Silicon Controlled Rectifiers (SCRs)

Silicon Controlled Rectifier (SCR)

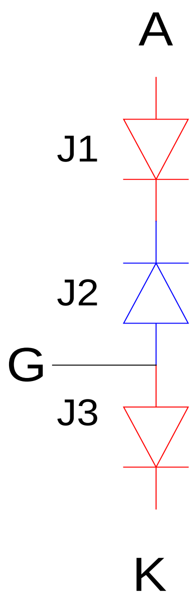
- **S**ilicon **C**ontrolled **R**ectifier (or **S**emiconductor **C**ontrolled **R**ectifier) is a four layer solid state device that controls current flow.
- The name "silicon controlled rectifier" is a trade name for the type of **Thyristor** commercialized at General Electric in 1957.



- Anode and Cathode as in conventional pn junction diode.
- Gate terminal is used for controlling input signal.



Structure



Diode Equivalent Circuit

Two-Transistor Model of Thyristors

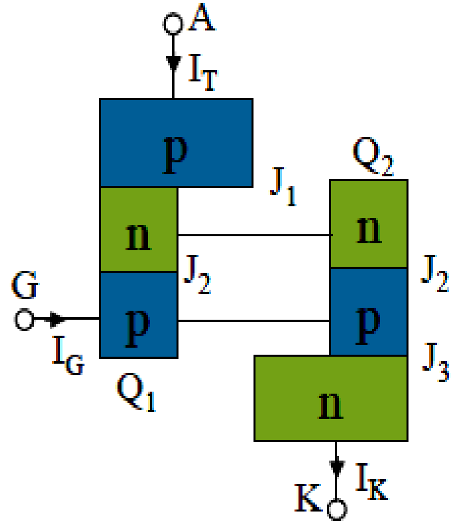
Collector-base junction leakage current

$$I_C = \alpha I_E + I_{CBO}$$

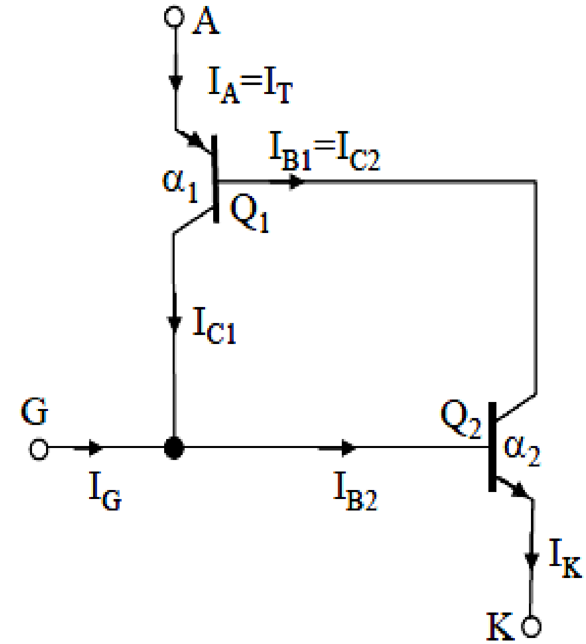
Collector current — Collector Emitter current gain — Emitter current

Q1 → $I_{C1} = \alpha_1 I_A + I_{CBO1}$

Q2 → $I_{C2} = \alpha_2 I_K + I_{CBO2}$



Basic Structure

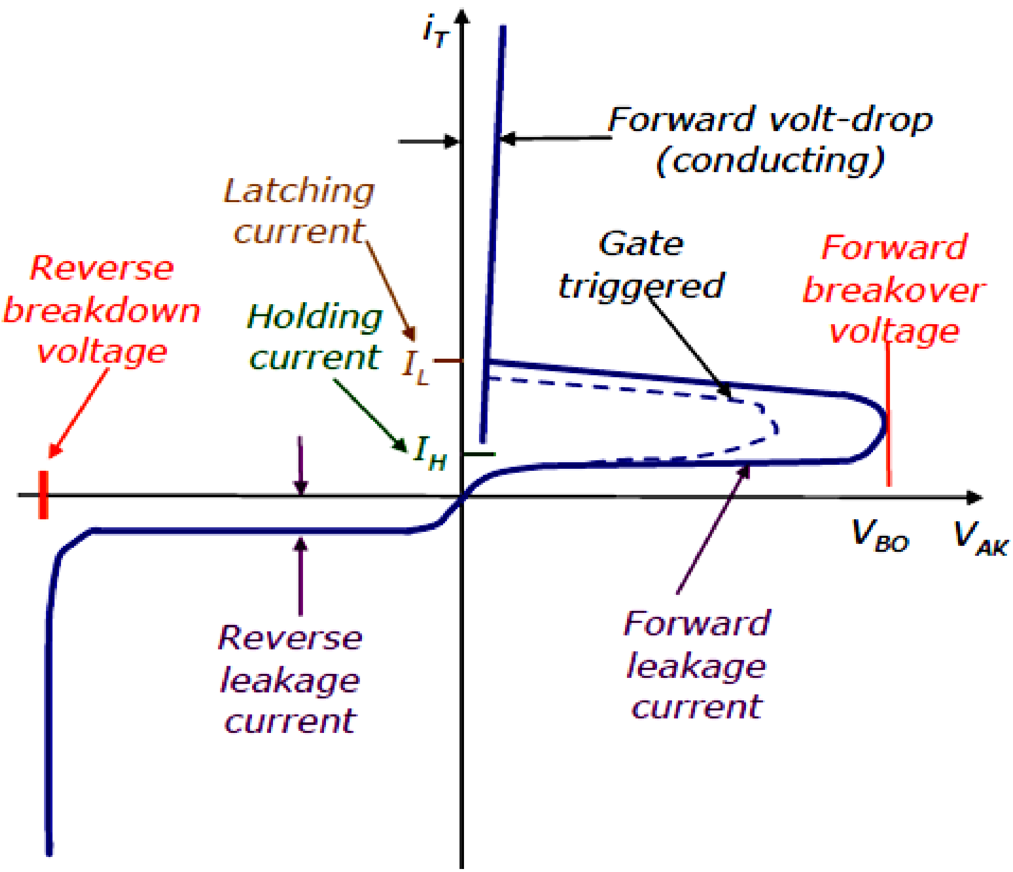


Equivalent Circuit

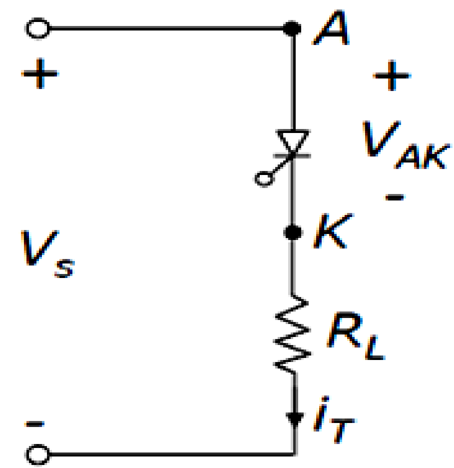
$$I_A = I_{C1} + I_{C2} = \alpha_1 I_A + I_{CBO1} + \alpha_2 I_K + I_{CBO2}$$

$$I_K = I_A + I_G \Rightarrow I_A = \frac{\alpha_2 I_G + I_{CBO1} + I_{CBO2}}{1-(\alpha_1+\alpha_2)}$$

Characteristics of Thyristors



V-I Characteristics



Circuit

Turning ON Conditions:

$$\begin{Bmatrix} V_{AK} > 0 \\ I_G > 0 \\ I_T > I_L \end{Bmatrix}$$

Thyristor turns ON and remains in this state even if I_G is just one pulse.

Turning OFF Conditions:

$$\{I_T < I_H\}$$

Thyristor turns OFF and remains in this state if turning ON conditions are not satisfied

Thyristor Operating modes

Thyristors have three modes of operation:

Forward blocking mode:

- Anode is positive w.r.t cathode, but the anode voltage is less than the break over voltage V_{BO}.
- Only leakage current flows, so the thyristor is not conducting.

Forward conducting mode:

- When anode voltage becomes greater than V_{BO} , thyristor switches from forward blocking to forward conduction state, and a large forward current flows.
- If $I_G = I_{G1}$, thyristor can be turned ON even when the anode voltage is less than V_{BO}. The current must be more than the latching current I_L.
- Once the SCR is turned ON, it remains ON even after removal of gate signal, as long as the Holding Current (I_H), is maintained.
- If the current reduced less than the holding current I_H, thyristor switches back to forward blocking state.

Reverse blocking mode:

When cathode is more positive than anode , small reverse leakage current flows. However if cathode voltage is increased to reverse breakdown voltage , Avalanche breakdown occurs and large current flows.

Thyristor Turn ON methods

- Thyristor turning **ON** is also known as **Triggering**.

- With **Anode** positive with respect to **Cathode**, a thyristor can be turned **ON** by any one of the following techniques :

  - **Forward Voltage Triggering**
  - **Gate Triggering**
  - ***dv/dt* Triggering**
  - **Temperature Triggering**
  - **Light Triggering**

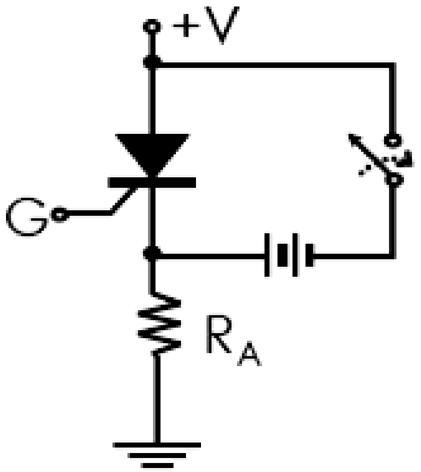
Thyristor Commutation

- **Commutation:** Process of turning **OFF** a conducting thyristor.
  - Current Commutation
  - Voltage Commutation
- **SCR** cannot be turned **OFF** via gate terminal.
- Thyristor will turn Off only after the anode current is negated either naturally or using forced commutation techniques.
- Methods of turning Off do not refer to the case when the anode current is gradually reduced below the Holding Current (I_H) manually or through a slow process.
- **Commutation Classification:**
  - Natural commutation
  - Forced commutation
- **Natural Commutation of Thyristor takes place in:**
  - AC Voltage Regulators
  - Phase Controlled Rectifiers
  - Cycloconverters

Turning The SCR Off:

Forced Commutation

- Forced commutation uses an external circuit to momentarily force current in the opposite direction to forward conduction.
- SCRs are commonly used in ac circuits, which forces the SCR out of conduction when the ac reverses.
- Applied to DC circuits.
- Applied to choppers and inverters.
- **Forced Commutation methods:**
  - ✓ Resonant Load
  - ✓ Self commutation
  - ✓ Auxiliary commutation
  - ✓ Capacitive commutation
  - ✓ External pulse commutation



Forced Commutation

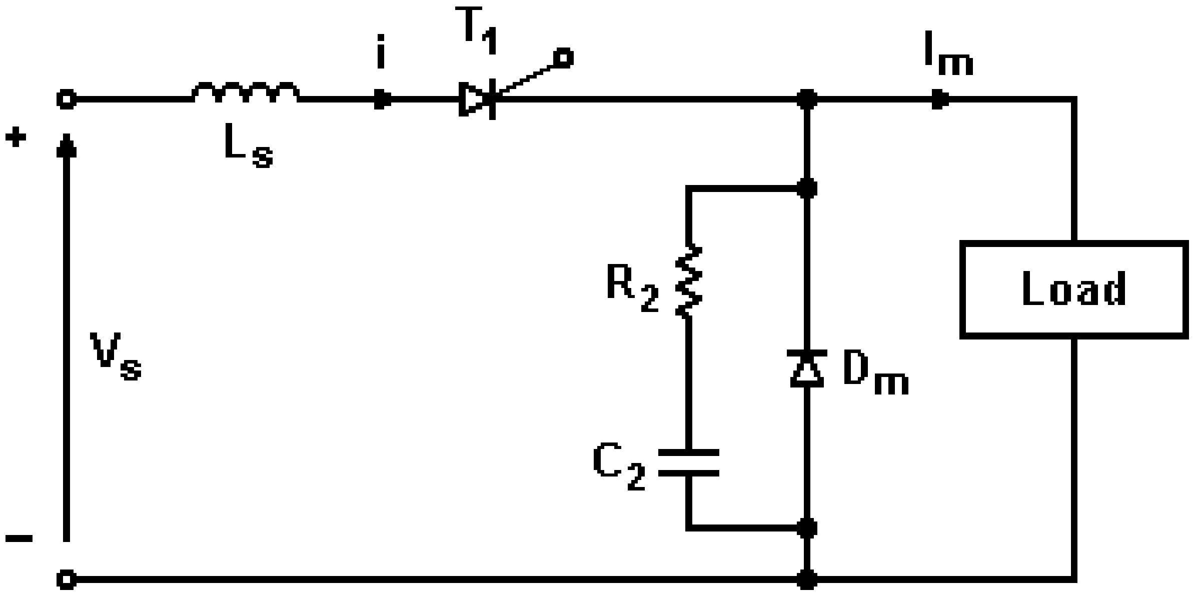
Thyristor Protection Circuits

- Reliable operation of a thyristor demands that its specified ratings are not exceeded.
- Various techniques are adopted for the protection of SCRs:
  - ***di/dt* protection**
  - ***dv/dt* protection**
  - **Over-Voltage protection**
  - **Over-Current protection**
  - **Thermal protection**
  - **Gate protection**

di/dt Protection

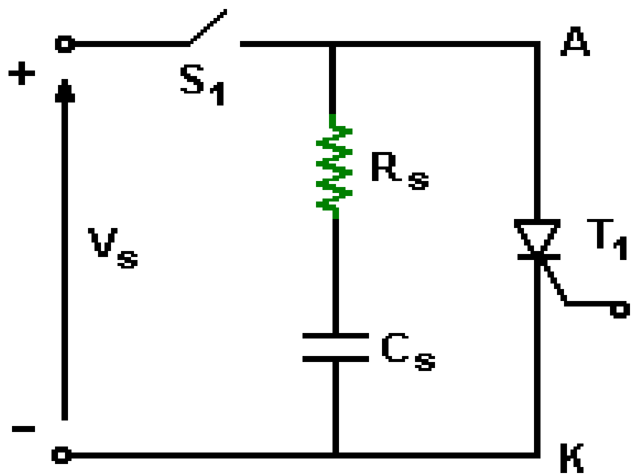
- A thyristor requires a minimum time to spread the current conduction uniformly throughout the junctions.
- Otherwise, a localized “hot-spot” heating may occur due to high current density.

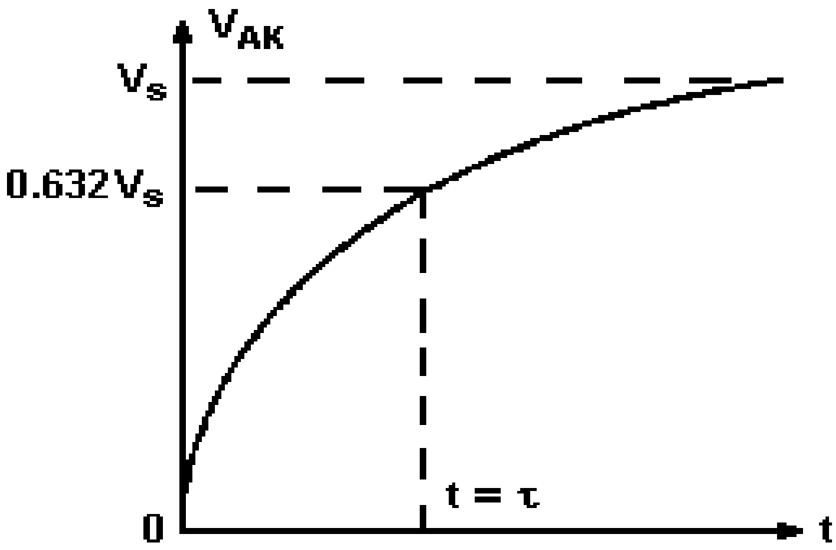
$$\frac{di}{dt} = \frac{V_s}{L_s}$$



dv/dt **Protection** (Snubber Circuit)

- A snubber circuit consists of a series combination of resistance Rs and capacitance Cs in parallel with the thyristor.
- A capacitor Cs in parallel with the device is sufficient to prevent unwanted dv/dt triggering of the SCR.
- Typical values of dv/dt are 20 – 500 V/µsec



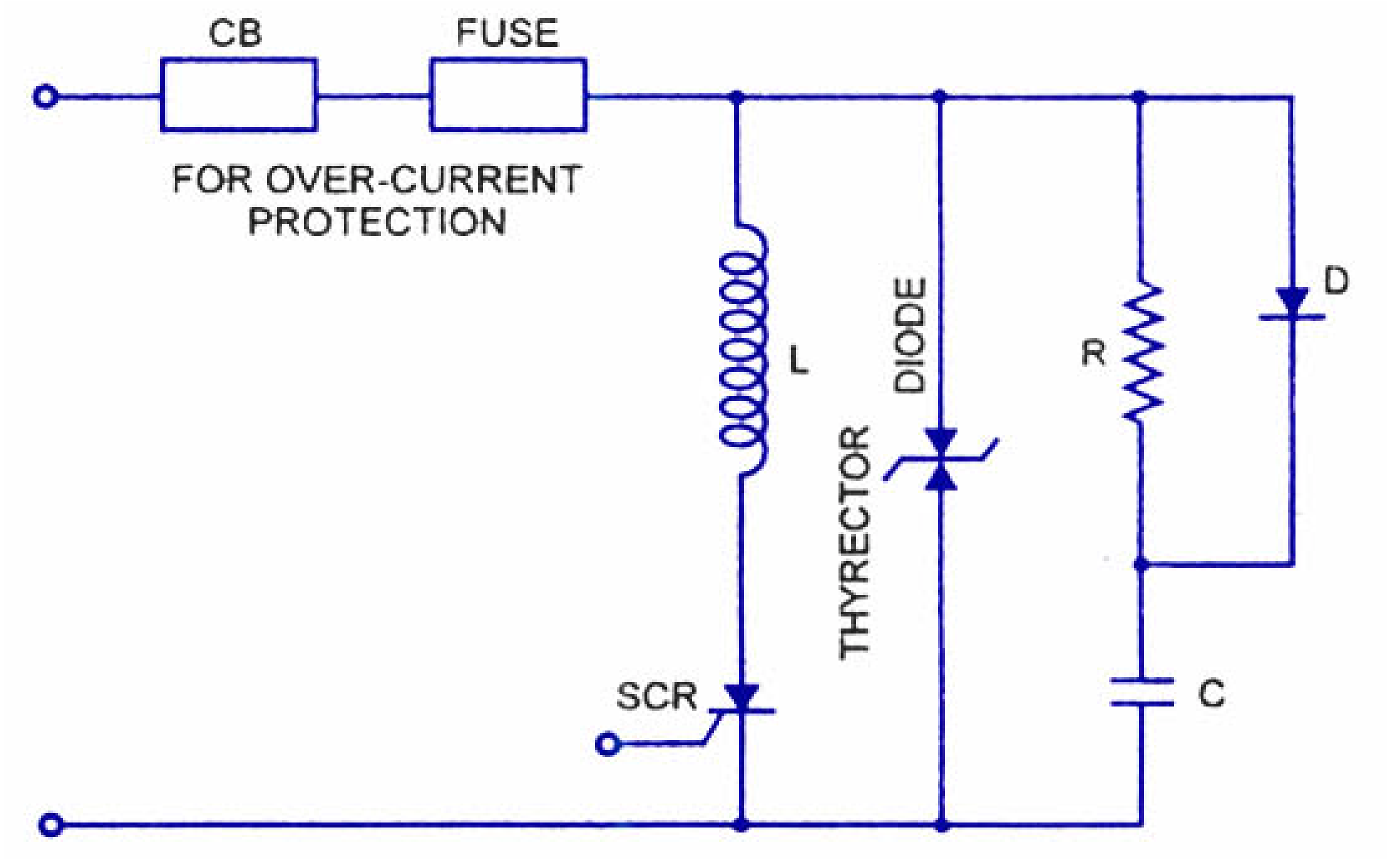


$$\frac{dv}{dt} = \frac{0.632\,V_s}{R_s C_s}$$

$$V_c = V_s\left(1 - e^{-t/R_s C_s}\right)$$

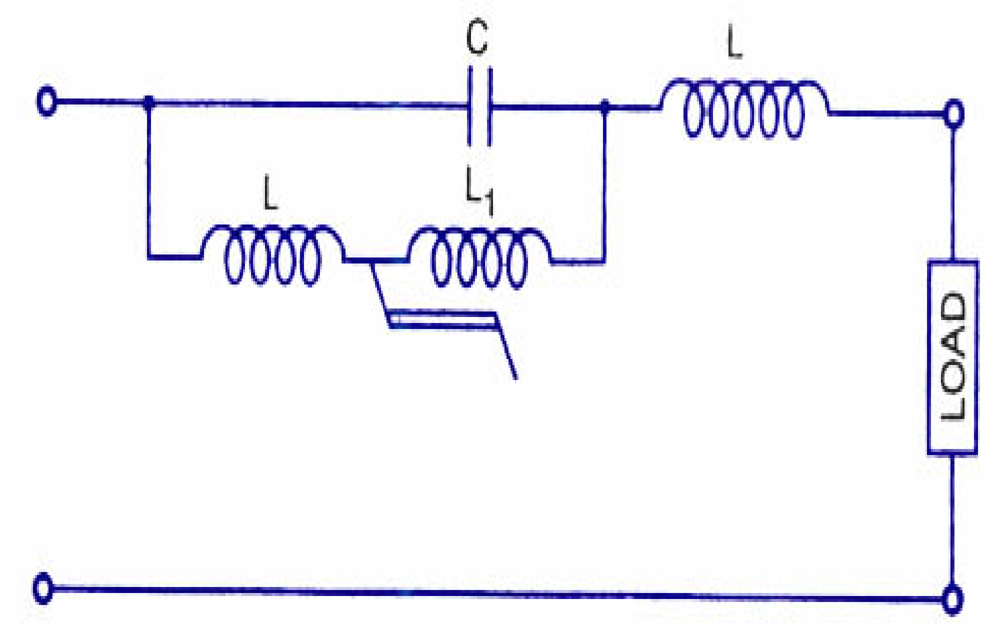
Over-Voltage Protection

High forward voltage protection is inherent in SCRs. The SCR will breakdown and start conducting before the peak forward voltage is attained so that high voltage is transferred to the load.



Over-Current Protection

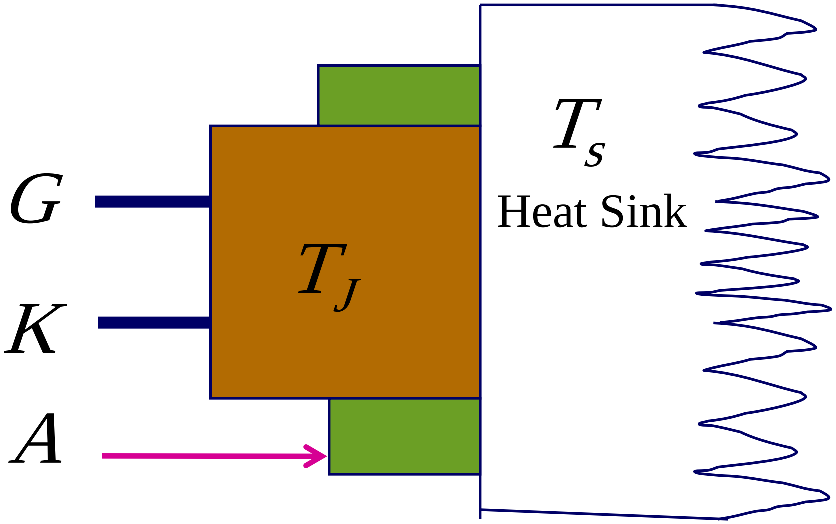
- If the output to the load is alternating current, LC resonance provides over-current protection as well as filtering.
- A current limiting device employing a saturable reactor L1 .
- With normal currents, L1 offers high impedance and C and L are in series to provide zero impedance to the flow of current of fundamental harmonic.
- An over-current saturates L1 and so gives negligible impedance. There is LC parallel resonance and hence infinite impedance to the flow of current at the resonant frequency.



Temperature protection

- For **10cm** square plate of copper or aluminium with length side **L** (in meter) to air, heat sink is estimated from the following equation: $R_{sa} = 0.5/L$
- Thermal power loss: $$P = \frac{T_{j\max} - T_a}{R_{ja}}$$
- R_{sa} = thermal sink to air resistance,
- R_{ja} = total junction to air resistance.
- The transient thermal impedance Z_{js} decreases in value as the temperature rises and result in high power dissipation.

$$Z_{js} = \frac{Temprature\ Rise}{Power\ Loss}$$



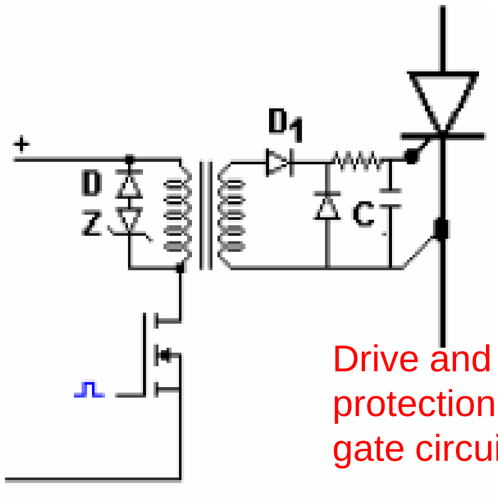
Gate protection circuits

(a) 1-Increase dv/dt capability, 2-keep gate clamped , 3-lower t_g time, and 4-noise immunity

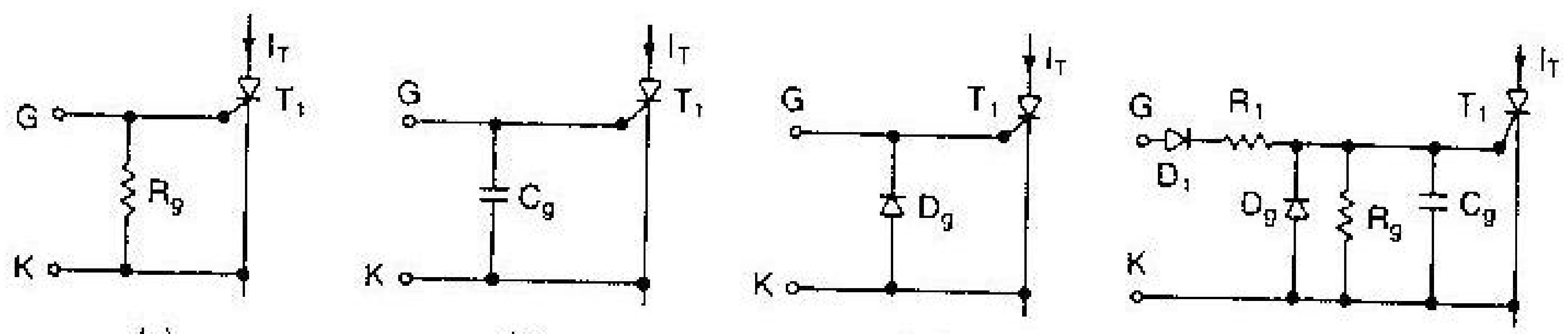
(b) 1- Increase dv/dt capability, 2 remove high frequency noise.

(c) D_g Provide fixed negative bias, 2-protect gate and gate supply from reverse transient , and 3- lower toff.

(d) D1 provide positive triggring.

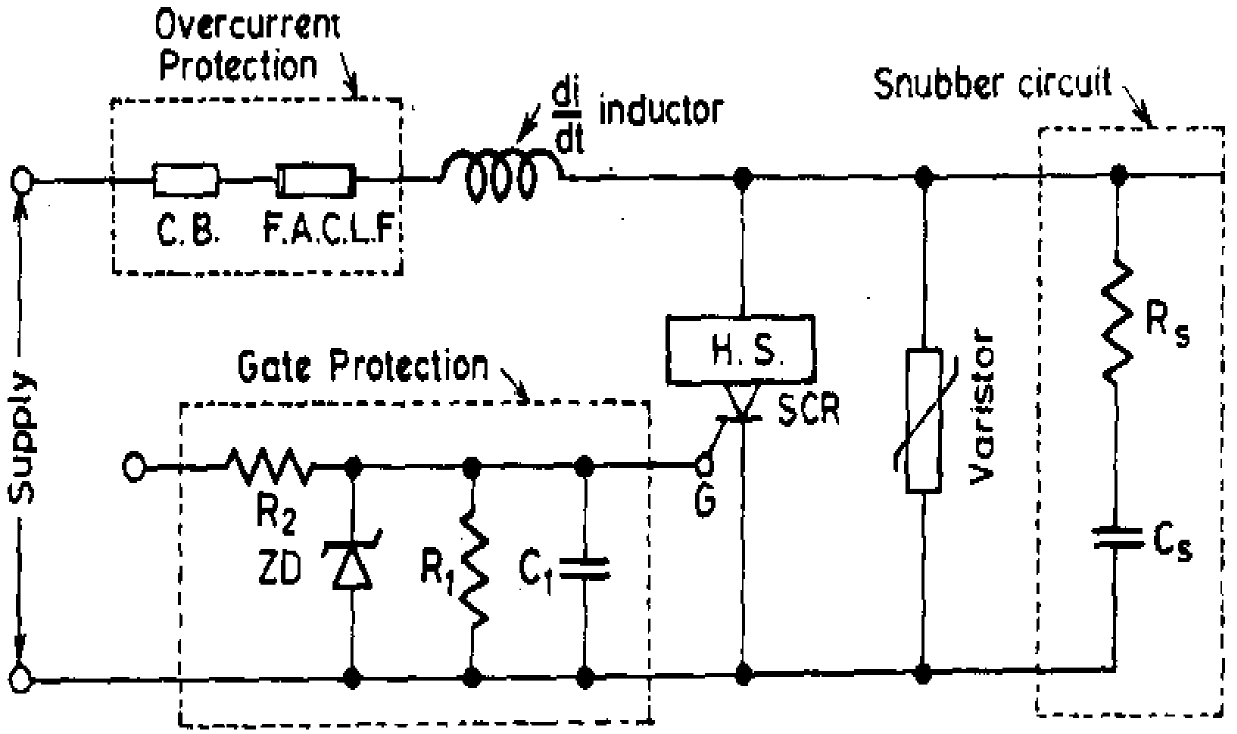


Drive and protection gate circuit



(a) (b) (c) (d)

Complete protection circuit

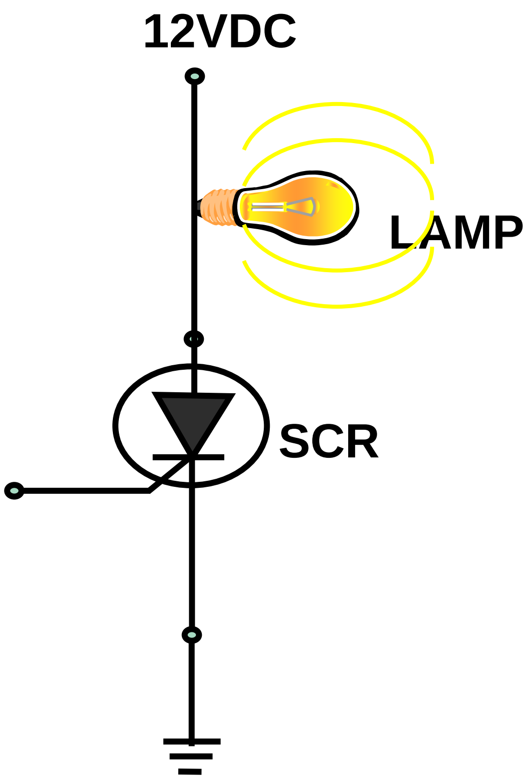


C.B.—Circuit breaker ; F.A.C.L.F.—Fast acting current limiting fuse ; H.S.—Heat sink ; ZD—Zener diode.

Example 1

An SCR with 1V voltage drop across the anode–cathode terminals is connected to 12V source and 50 mA Lamp. Determine the power in:

a) the lamp
b) the SCR



P = I X E

$P_{LAMP} = I_{LAMP} \text{ X } E_{LAMP}$

$P_{LAMP} = 50 \text{ mA X } 11\text{V}$

$P_{LAMP} = 550 \text{ mW}$

$P_{SCR} = I_{SCR} \text{ X } E_{SCR}$

$P_{SCR} = 50\text{mA X } 1\text{V}$

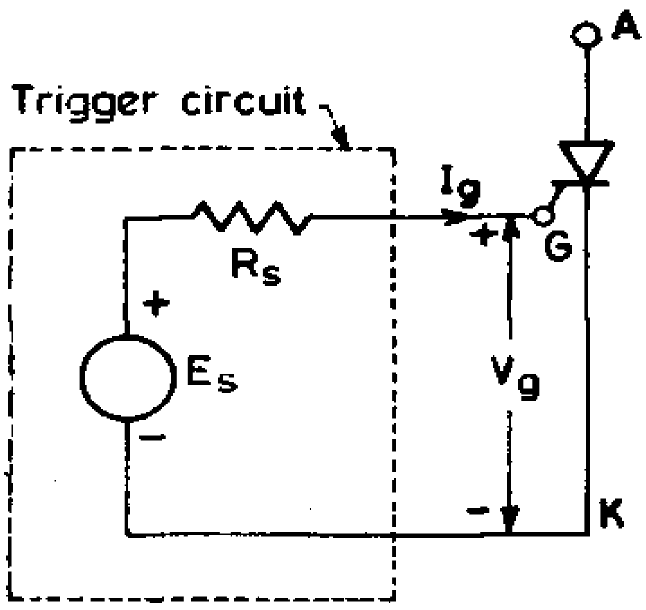
$P_{SCR} = 50 \text{ mW}$

Gate Triggering Circuits

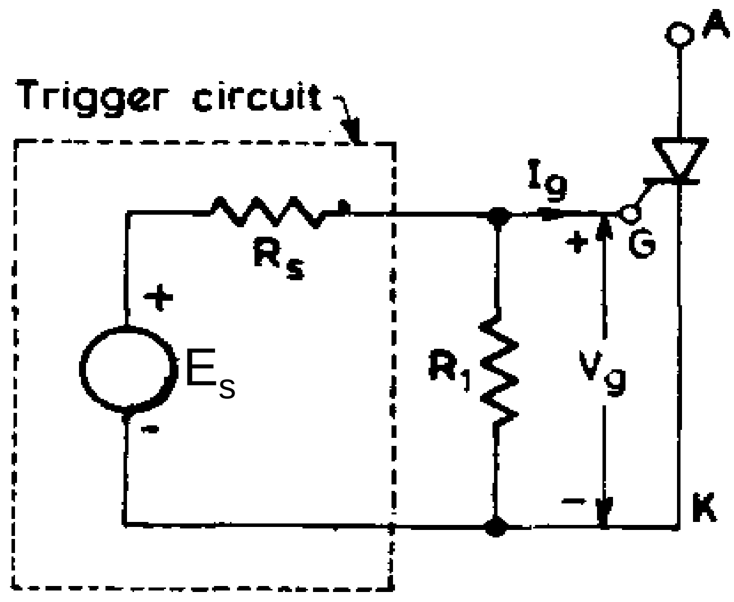
- It represent efficient and reliable methods for turning ON the SCR with different types as:

  - **DC-Triggering circuits**
  - **AC-Triggering circuits**
    - R-Triggering circuit
    - RC-Triggering circuit
    - LC-Triggering circuit
  - **Pulse-Triggering circuits** [UJT or BJT circuits]

DC-Triggering Circuits



(a)



(b)

$$E_s = V_g + I_g R_s$$

$$E_s = V_g + \left(I_g + \frac{V_g}{R_1} \right) R_s$$

Example 2

Design a suitable gate triggering circuit for an SCR with 800V, 110A, connected with 6V DC power supply of maximum permissible current $I_{dc\text{-}MAX}$ = 0.2A and short circuit current $I_{S/C}$ =0.5A. The SCR has the following gate parameters:-
V_G =2.5v , I_G = 50mA , V_{GM} =3V , I_{GM} =0.1A, P_{GM} = 0.5W

Solution:

1) For a Gate drive circuit without R_g , we have

$$R_S = \frac{E_S}{I_{S/C}} = \frac{6}{0.5} = 12\Omega$$

To protect the source from excessive current, the minimum value of R_1 in series with R_S is:

$$R_S + R_1 \geq \frac{E_S}{I_{dc-MAX}} = \frac{6}{0.2} = 30\Omega \quad \Rightarrow \quad \therefore R_1 \geq 30 - 12 = 18\Omega$$

Also, to protect the gate-cathode junction of the SCR, the minimum value of R_1 in series with R_S is given by:

$$R_S + R_1 \geq \frac{E_S}{I_{GM}} = \frac{6}{0.1} = 60\Omega \quad \Rightarrow \quad \therefore R_1 \geq 60 - 12 = 48\Omega$$

Therfore, R_1 should be greater than 48Ω

Solution: (cont.)

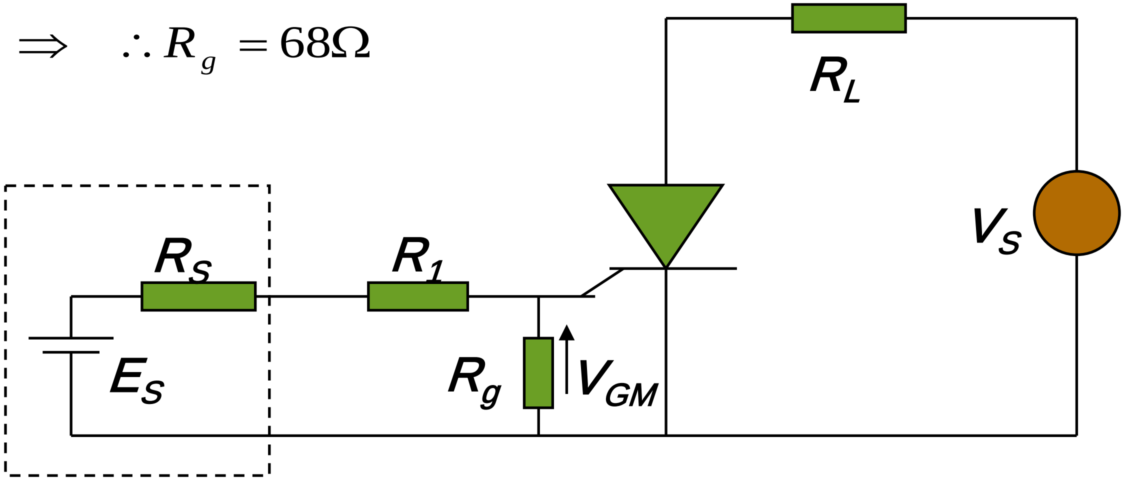
Now, corresponding to the typical values of $V_G = 2.5V$ and $I_G = 50mA$, the maximum value of R_1 in series with R_S is given by:

$$R_S + R_1 \leq \frac{E_S - V_G}{I_G} = \frac{6 - 2.5}{0.05} = 70\Omega \quad \Rightarrow \quad \therefore R_1 \leq 70 - 12 = 58\Omega$$

Therfore, $48\Omega \geq R_1 \geq 58\Omega$, and we will choose $R_1 = 56\Omega$

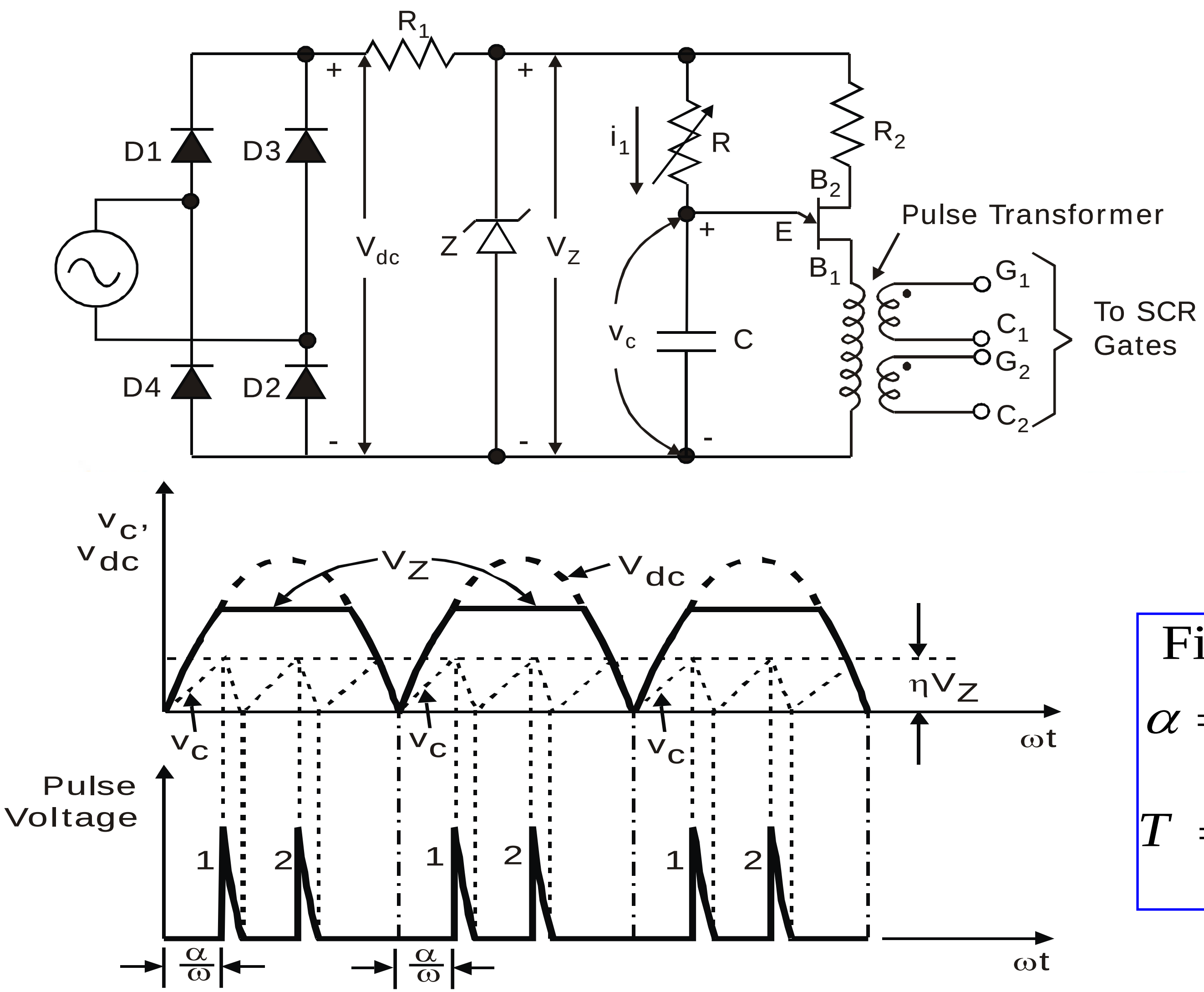
The value of R_g is given by the voltage divider as:

$$E_S \times \frac{R_g}{R_g + R_S + R_1} \leq V_{GM} \quad \Rightarrow \quad \therefore R_g = 68\Omega$$



The complete design of DC trigger circuit

UJT-Trigger Circuit



Firing angle:

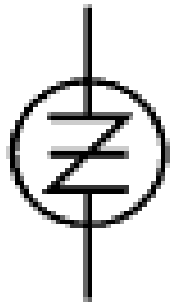
$$\alpha = \omega T = 2\pi f$$

$$T = R_E C \ln\left(\frac{1}{1-\eta}\right)$$

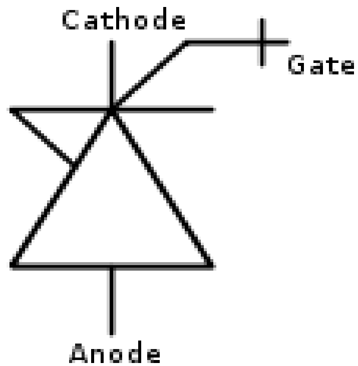
Voltage waveforms of UJT trigger circuit

Thyristor Family Members

- SCR: Silicon Controlled Rectifier
- DIAC: Diode on Alternating Current
- TRIAC : Triode for Alternating Current
- SCS: Silicon Control Switch
- SUS: Silicon Unilateral Switch (*Shockley Diode*)
- SBS: Silicon Bidirectional Switch
- SIS: Silicon Induction Switch
- LASCS: Light Activated Silicon Control Switch
- LASCR: Light Activated Silicon Control Rectifier or LTT
- SITh : Static Induction Thyristor
- RCT: Reverse Conducting Thyristor
- GTO : Gate Turn-Off Thyristor
- MCT: MOSFET Controlled Thyristor
- ETOs: Emitter Turn ON Thyristor
- IGCT: Integrated Gate-Commutated Thyristor
- SIDAC: Silicon Diode for Alternating Current
- QUADRACS: DIAC+TRIAC in single package where the DIAC is a triggering device for the TRAIC



SIDAC



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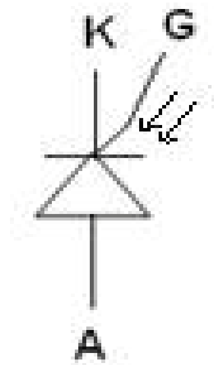
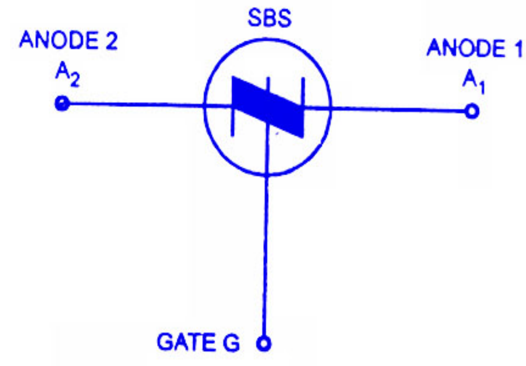
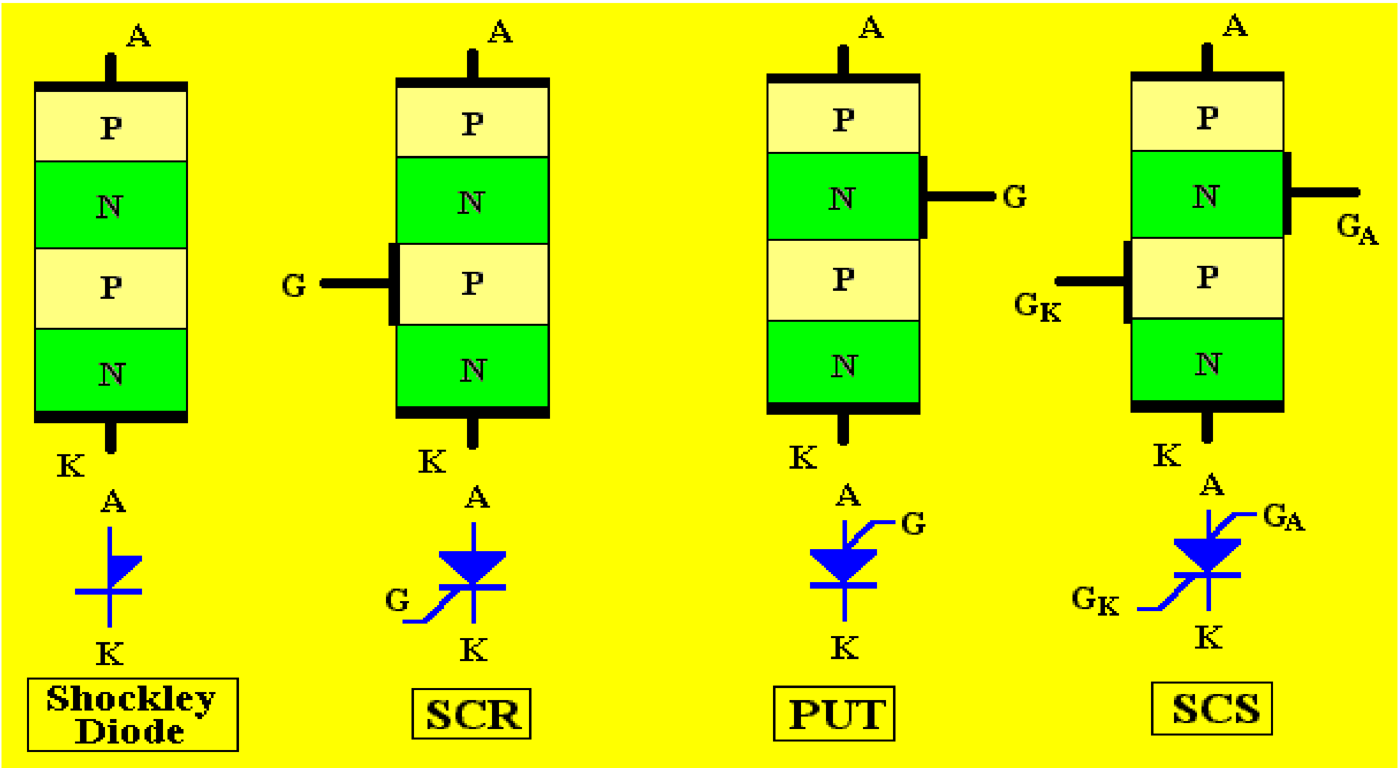


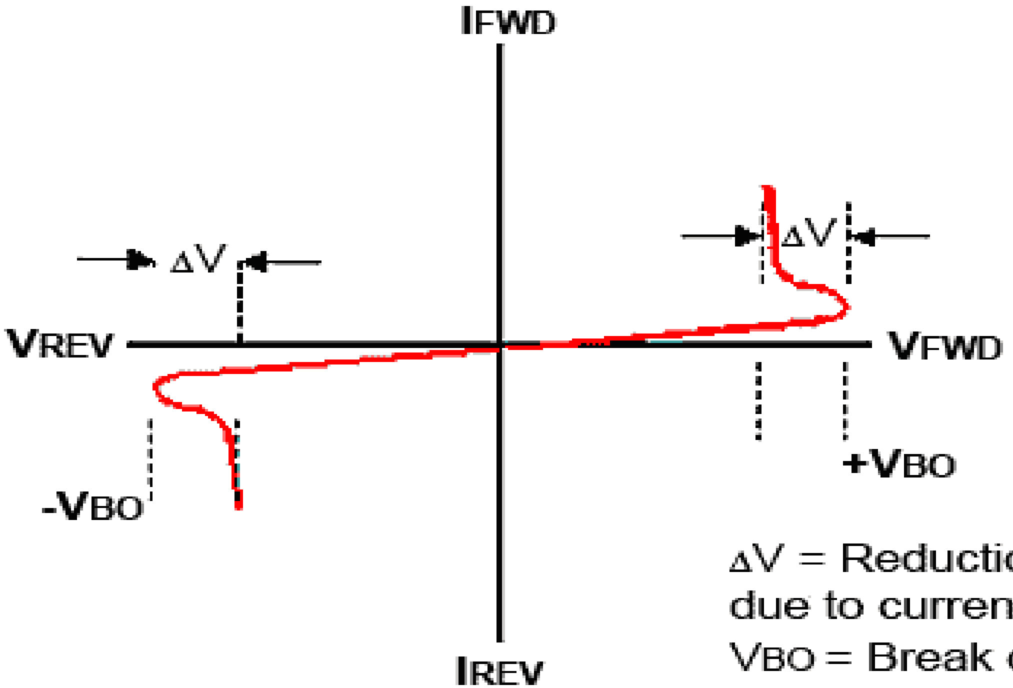
LASCR

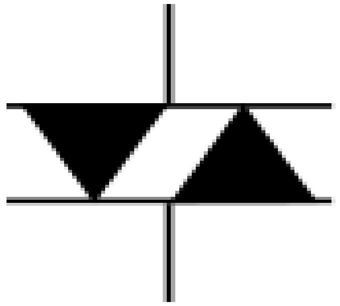


SBS



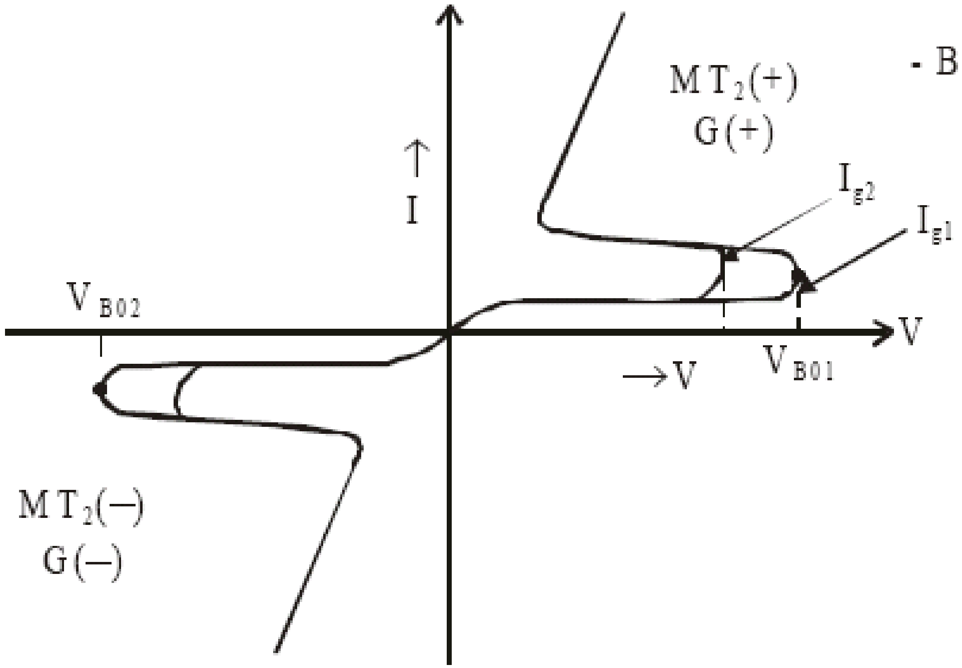
DIAC Thyristor





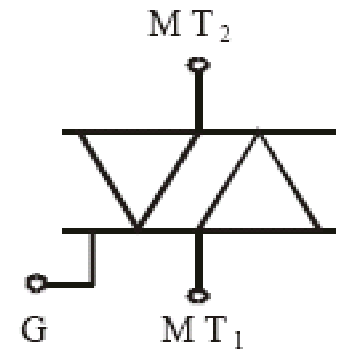
DIAC Characteristics

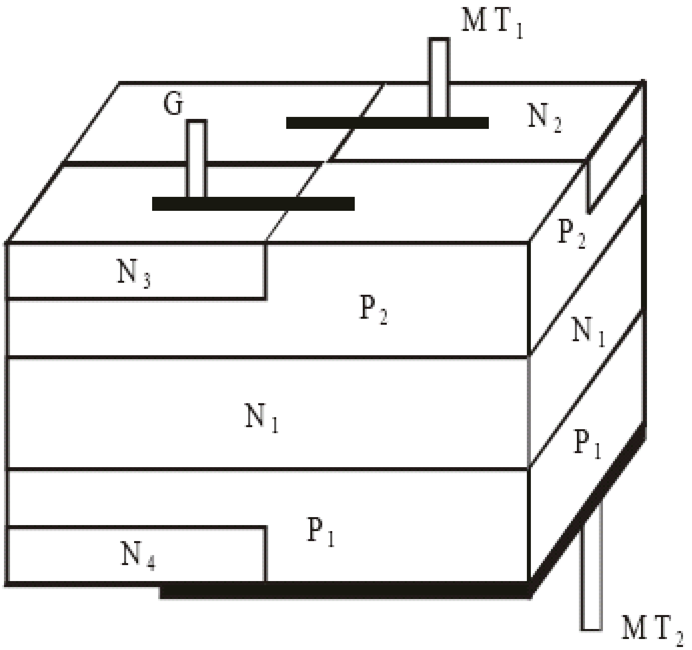
Bidirectional Triode Thyristors (TRIAC)



V_{B01}, V_{B01}
- Breakover voltages

$I_{g2} > I_{g21}$



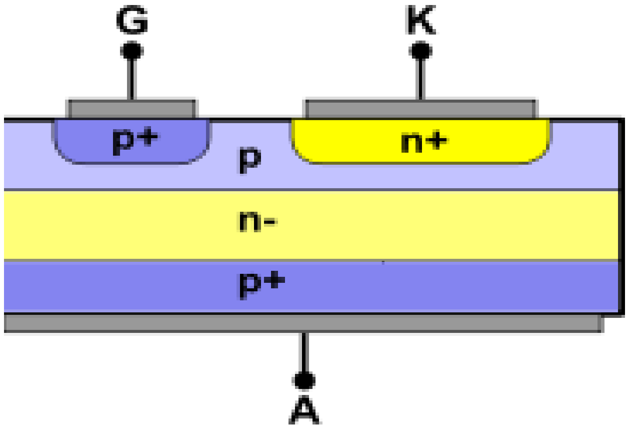


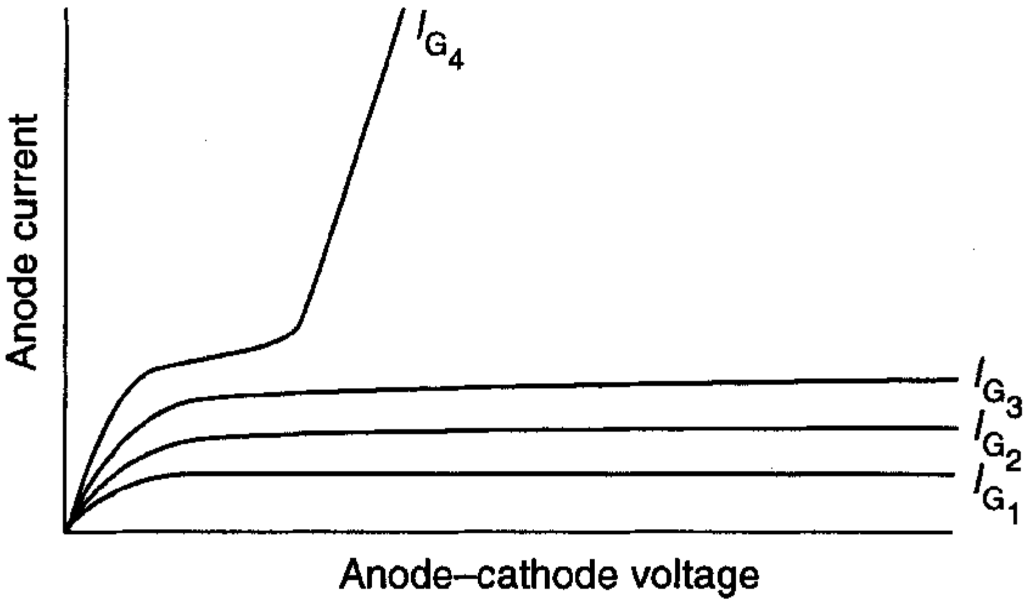
TRIAC Characteristics

GTO Thyristor

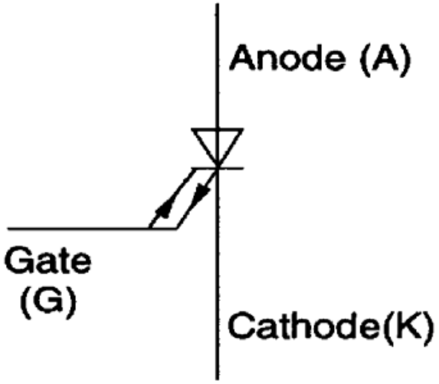
can be turned-on by a gate signal, and can also be turned-off by a gate signal of negative polarity

The main applications of GTO are in variable speed motor drives, high power, inverters and traction





GTO Characteristics



Problems and Solutions

Question 1 What are the different operation regions of the SCR?

Solution:

SCR will have three regions of operations based on the mode in which the device is connected as:

Reverse blocking region: When the cathode of is made positive with respect to the anode and no gate signal is applied. In this region, SCR exhibits reverse blocking characteristics similar to the diode.

Forward blocking region: the anode is made positive with respect to the cathode and no gate signal is applied to the thyristor. A small leakage current flow in this mode of operation.

Forward conduction region: when the forward voltage applied between the anode and cathode increases at particular break over voltage, avalanche breakdown takes place and thyristor starts conducting current in forward direction. By this type of triggering the device may damages the SCR. Hence a gate signal is applied before the forward break over voltage to trigger the SCR.

Question 2 What is the Latching current (I_L) ?

Solution:

It is the minimum anode current to keep the thyristor in conducting state after the gate pulse is removed.

Question 3 What is the Holding current (I_H) ?

Solution:

It is the minimum anode current at which the thyristor continue to conduct. If the anode current becomes less than I_H, the thyristor will be turned off.

Question 4 What is the relation between the gate signal and forward break over voltage (V_{BO})?

Solution:

Thyristor can be triggered by increasing the forward voltage between the anode and cathode. At forward break over voltage, it starts conducting. However this process may damage the thyristor. So it is usually triggered on through a gate pulse before reaching the break over voltage. Forward voltage at which the thyristor triggered on depends on the magnitude of gate current. Higher is the gate current lower is the forward break over voltage.

Question 5 What are the different losses that occur in thyristor while operating?

Solution:

- Forward conduction losses during conduction of the thyristor.
- Loss due to leakage current during forward and reverse blocking.
- Power loss at gate or Gate triggering loss.
- Switching losses at turn-on and turn-off..

Question 6 What are the different turns ON methods of the SCR?

Solution:

- Forward voltage triggering.
- Gate triggering.
- dv/dt triggering.
- Temperature triggering.
- Light triggering

Question 7 What is the thyristor commutation?

Solution:

It is the process to **turn-off** the thyristor by changing the direction of current flow.

Question 8 What is the snubber circuit?

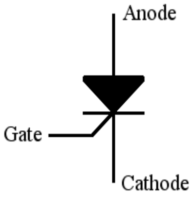
Solution:

It is a series combination of resistor and capacitor and used for dv/dt protection of the SCR.

Question 9 Define the SCR or thyristor and draw its symbol.

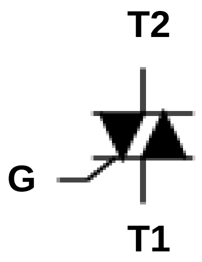
Solution:

SCR stands for Silicon Controlled Rectifier. It is a four layer (p-n-p-n) semiconductor and three terminals (anode, cathode, and gate) device.



Question 10 Label the terminals on a TRIAC with their proper designations:

Solution:



Question 11 Identify at least three different types of thyristors (besides SCRs)

Solution:

- **RCT**: Reverse Conducting Thyristor
- **GTO**: Gate Turn-Off Thyristor
- **MCT**: MOSFET Controlled Thyristors

Question 12 Outline the successful gate trigger conditions for an SCR.

Solution:

1- V_G and I_G should be within the trigger zone.

2- The gate pulse should be timed properly (synchronization).

3- Minimum gate power dissipation.

Question 13 **Choose the correct answer for the following Questions**

1) The SCR is considered to be a semi-controlled device because……………

A. It can be turned OFF but not ON with a gate pulse.
B. It conducts only during one half-cycle of an alternating current wave
C. It can be turned ON but not OFF with a gate pulse.
D. It can be turned ON only during one half-cycle of an alternating voltage wave.

---

2) The Silicon Controlled Switch (SCS) is similar in construction to the..………

A. Triac. B. Diac. C. SCR. D. Four-Layer Diode

---

3) The SCR can be triggered ON by a pulse at the..………

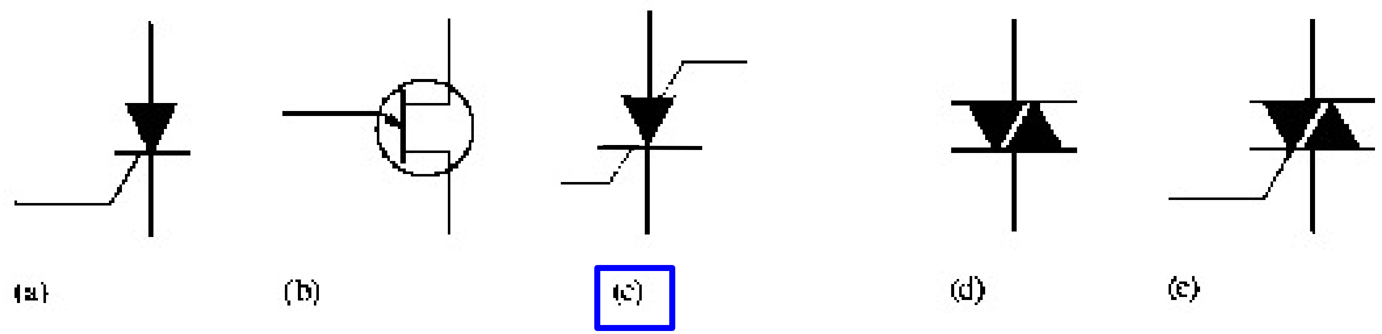
A. Anode. B. Cathode. C. Gate. D. None of the above.

---

4) How many semiconductor layers are thyristors constructed with?

A. 2 B. 3 C. 4 D. 5

5) Which symbol represents the SCS?



6) Holding current of the thyristor is……….

A. Less than latching current
B. More than latching current
C. Equal to latching current
D. Zero

7) For an SCR, dv/dt protection is achieved through the use of……….

A. RL in series with SCR
B. RC across SCR
C. L in series with SCR
D. RC in series with SCR

8) The procedure of switching off an SCR is called:………………

A. Forcing
B. Commutation
C. Triggering
D. Latching

9) Anode current in an thyristor is made up of…………………

A. electrons only B. electrons or holes C. electrons and holes D. holes only

---

10) An SCR can only be turned off via it's……………

A. cathode B. anode C. gate D. none of the above

---

11) In a thyristor, di/dt protection is achieved by the use of

(A) An inductance L in series with the thyristor

(B) A resistor in series with the thyristor

(C) RC in series with the thyristor

(D) RL in series with the thyristor

Question 14 Fill in the blanks with the appropriate word(s)

i. Forward break over voltage of a thyristor decreases with increase in the…………….. current.

ii. Reverse………… voltage of a thyristor is ……………..of the gate current.

iii. Reverse saturation current of a thyristor ……………… with gate current.

iv. In the pulsed gate current triggering of a thyristor ,the gate current pulse width should be larger than the………………….. time of the device.

v. To prevent unwanted turn ON of a thyristor, all spurious noise signals between the gate and the cathode must be less than the gate ……………………voltage.

Answers: (i) gate; (ii) break down, independent; (iii) increases; (iv) Turn ON; (v) non- trigger.

Question 15 Fill in the blanks with the appropriate word(s)

i. A thyristor is turned on by applying a………………gate current pulse when it is…………….biased.

ii. Total turn on time of a thyristor can be divided into………….. time……………time, and……….. time.

iii. During rise time the rate of rise of anode current should be limited to avoid creating local…………

iv. A thyristor can be turned off by bringing its anode current below……………current and applying a reverse voltage across the device for duration larger than the……………… time of the device.

v. Reverse recovery charge of a thyristor depends on the……………..…of the forward current just before turn off and its…………………..

vi. Inverter grade thyristors have……………turn off time compared to a converter grade thyristor.

Answers: (i) positive, forward; (ii) delay, rise, spread; (iii) hot spots (iv) holding, turn off; (v) magnitude, rate of decrease (vi) faster

Question 16 Fill in the blanks with the appropriate word(s)

i. Peak non-repetitive over voltage may appear across a thyristor due to…………….. or …………….. surges in a supply network.

ii. VRSM rating of a thyristor is greater than the…………… rating but less than the……………. rating.

iii. Maximum average current a thyristors can carry depends on the…………of the thyristor and the………….... of the current wave form.

iv. The ISM rating of a thyristor applies to current waveforms of duration ……..….than half cycle of the power frequency where as the $\int i^2 dt$ rating applies to current durations……………than half cycle of the power frequency.

v. The gate non-trigger voltage specification of a thyristor is useful for avoiding unwanted turn on of the thyristor due to……………..voltage signals at the gate.

Answers: (i) switching, lightning; (ii) VRRM, VBRR; (iii) case temperature, conduction angle; (iv) greater, less; (v) noise

Question 17 Fill in the blanks with the appropriate word(s)

i. A Triac is a…………………………minority carrier device.

ii. A Triac behaves like two……………………..connected thyristors.

iii. The gate sensitivity of a triac is maximum when the gate is………………with respect to MT1 while MT2 is positive with respect to MT1 or the gate is……………………with respect to MT1 while MT2 is negative with respect to MT1 .

iv. A Triac operates either in the………………..or the……………..…. quadrant of the I-V characteristics.

v. In the………………quadrant, the triac is fired with……………. gate current while in the………… quadrant the gate current should be……………………...

vi. The maximum possible voltage and current rating of a Triac is considerably………….. compared to thyristor due to………..……….of the two current carrying paths inside the structure of the triac.

vii. To avoid unwanted turn on of a triac due to large dv/dt………………… are used across triacs.

viii. For "clean turn ON" of a triac, the……………of the gate current pulse should be as………………. as possible.

Answers: (i) bidirectional; (ii) anti parallel; (iii) positive, negative; (iv) first, third; (v) first, positive, third, negative (vi) lower, interaction; (vii) R-C snubbers; (viii) rise time, small.

Question 18 **Fill in the blanks with the appropriate word(s)**

i. A thyristor is a ________________ carrier semi controlled device.

ii. A thyristor can conduct current in ________________ direction and block voltage in ________________ direction.

iii. A thyristor can be turned ON by applying a forward voltage greater than forward ________________ voltage or by injecting a positive ________________ current pulse under forward bias condition.

iv. To turn OFF a thyristor the anode current must be brought below ________________ current and a reverse voltage must be applied for a time larger than ________________ time of the device.

v. A thyristor may turn ON due to large forward ________________.

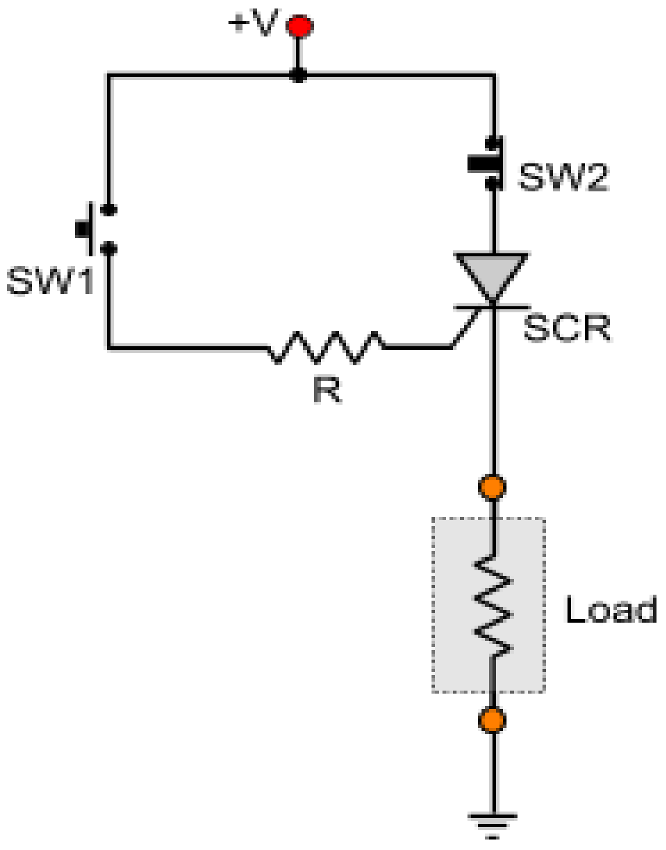
Answers: (i) minority; (ii) one, both; (iii) break over, gate; (iv) holding, turn off; (v) dv/dt

Question 19

Calculate the protective resistance (R) shown in the given circuit for voltage gate $V_G = 3V$ and current gate $I_G = 150$ mA, where the supply voltage $V = 12V$.

Solution:

$$R = \frac{V - V_G}{I_G} = \frac{12-3}{0.15} = 60\Omega$$



Question 20 UJT relaxation oscillator is used to trigger single SCR in AC circuit [220V, 50Hz] , for $\alpha = 90^\circ$ and $\eta = 0.6$, R_E=1kΩ , find the value of C.

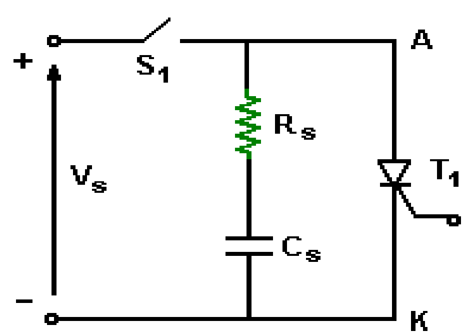
Solution:

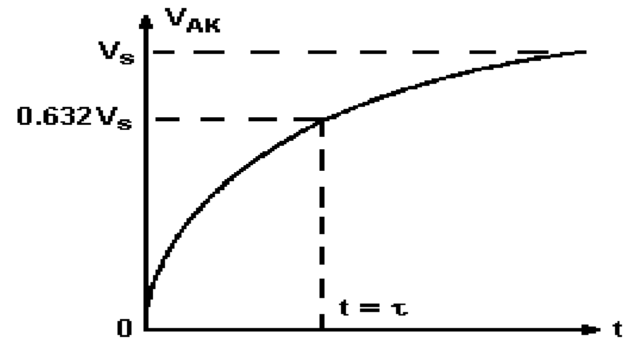
$$\alpha = 90^o = \frac{\pi}{2} = \omega T \Rightarrow T = \frac{\alpha}{\omega} = \frac{\pi/2}{2\pi f_i} = \frac{1}{4f_i} = \frac{1}{200} = 5m\sec$$

$$C = \frac{T}{R\ln\left(\frac{1}{1-\eta}\right)} \Rightarrow C = \frac{5\times10^{-3}}{1\times10^3\ln\left(\frac{1}{1-0.6}\right)} = 1.09\mu F$$

Question 21 The circuit shown below, if Vs = 600V and [dv/dt]max =100 V/ µs , determine the snubber time constant .

Solution:





$$\frac{dv}{dt} = \frac{0.632\ V_S}{R_S C_S}$$

$$\therefore \text{ Snubber time constant, } T_S = R_S C_S = 0.632\times\frac{V_S}{\left(\frac{dv}{dt}\right)_{\max}} = 0.632\times\frac{600}{100} = 3.8\mu\sec$$

Question 22

A thyristor has a load resistance of 15Ω and operates from 240V supply. If the thyristor has a [di/dt]max rating of 12A/μsec, calculate the series inductance that must be included in the circuit to protect it.

Solution:

$$\frac{di}{dt} = \frac{V_S}{L_S} \quad \Rightarrow \quad L_S = \frac{240}{12} = 20\mu H$$

Question 23

A 200 V, 10A thyristor has a thermal resistance of 1°C/W. What is the steady state power loss if the working junction temperature is 100°C and the ambient temperature is 15°C.

Solution:

$$P_{loss} = \frac{T_{j\max} - T_a}{R_{ja}} \quad \Rightarrow \quad P_{loss} = \frac{100 - 15}{1C^o/W} = 85 \text{ Watt}$$

Question 24

A thyristor has thermal resistance of 1.5°C/W from junction to air (including the heat sink). The ambient temperature is 25°C. If T_{Jmax} = 125°C, calculate the maximum power loss of the thyristor.

Solution:

$$P_{loss} = \frac{T_{j\max} - T_a}{R_{ja}} \quad \Rightarrow \quad P_{loss} = \frac{125 - 25}{1.5C^o/W} = 66.7 \text{ Watt}$$

Chapter 6

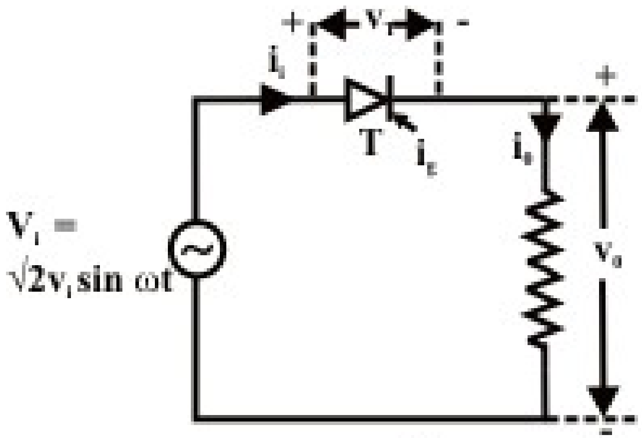
AC to DC Convertors

AC to DC Convertors

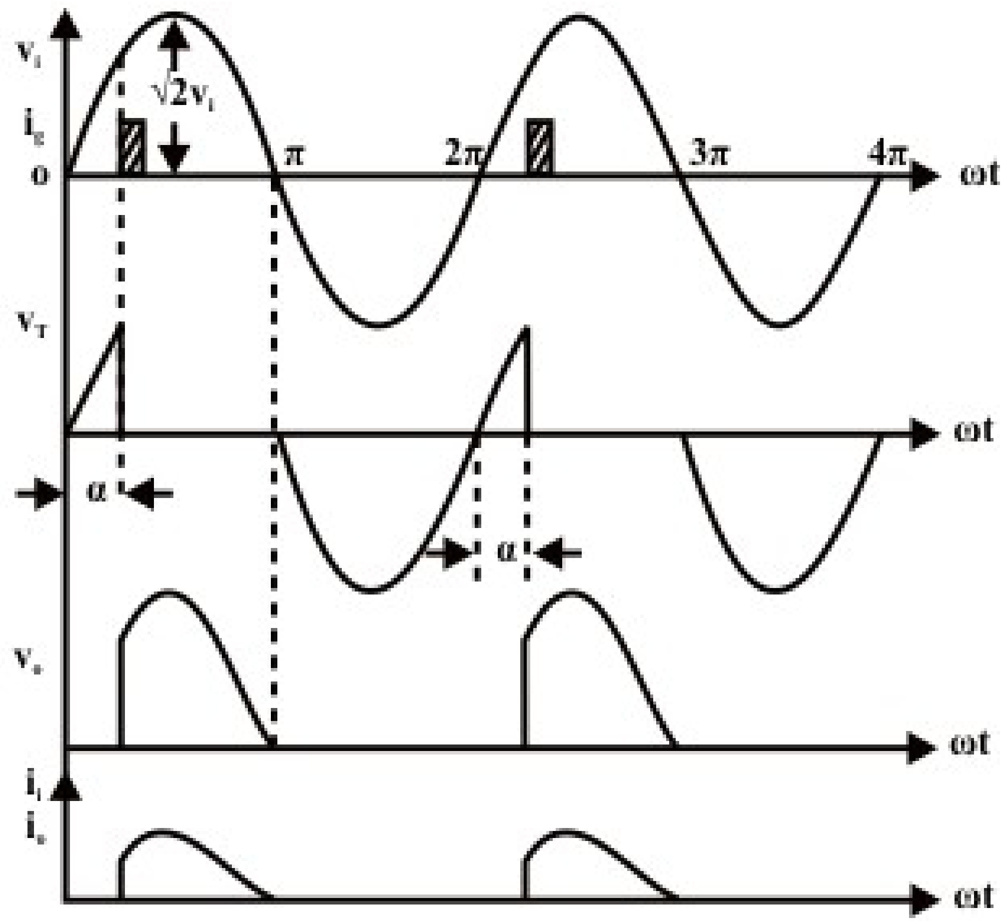
(Controlled Rectifications)

- Rectifiers can be classified as: **uncontrolled** and **controlled**.
- Controlled rectifiers are divided as: **semi-controlled** and **fully-controlled**.
- Uncontrolled rectifier circuits are built with diodes.
- Fully controlled rectifier circuits are built with SCR's.
- Both diodes and SCR's are used in semi-controlled rectifier circuits.

Single Phase Half Wave Controlled Rectifier with Resistive Load



(a)



For $\alpha < \omega t \leq \pi$

$$v_0 = v_i = \sqrt{2}\, V_i \sin\omega t$$

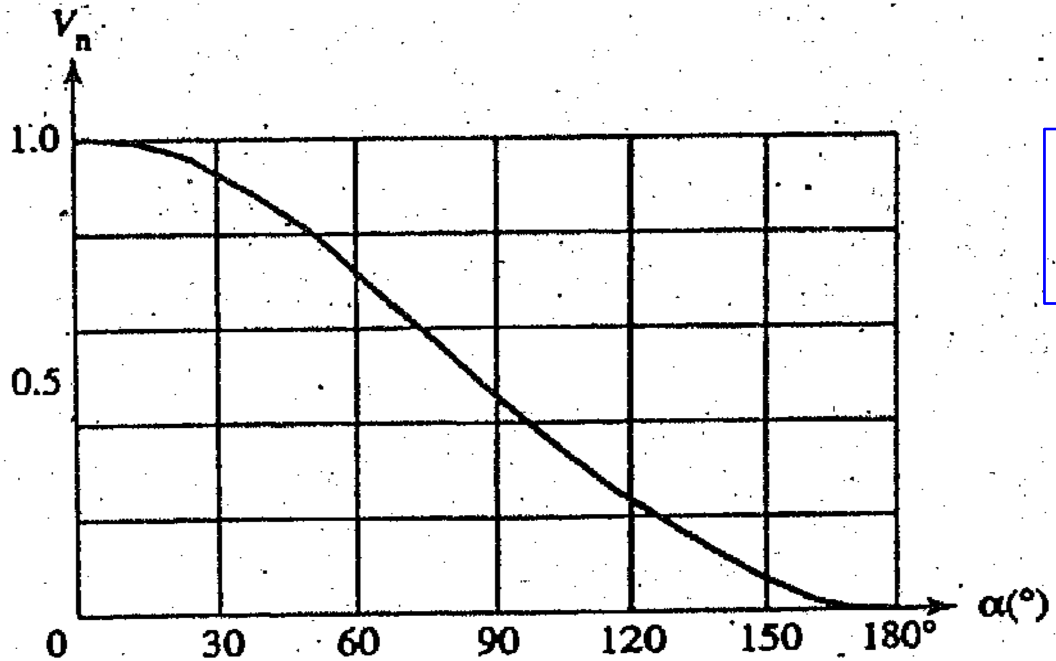
$$i_0 = \frac{v_0}{R} = \sqrt{2}\frac{V_i}{R} \sin\omega t$$

$$V_{o(avg)} = \frac{V_m\left(1+\cos\alpha\right)}{2\pi}$$

These equations tell us that the magnitude of the output voltage is controlled by the firing angle. Increasing α by firing the SCR later in the cycle lowers the voltage, and vice versa. The maximum output voltage, $V_{do} = V_m/\pi$, occurs when $\alpha = 0°$. This is the same voltage as for a half-wave diode circuit. Therefore, if the SCR is fired at $\alpha = 0°$, the circuit acts like a diode rectifier.

The normalized average voltage is:

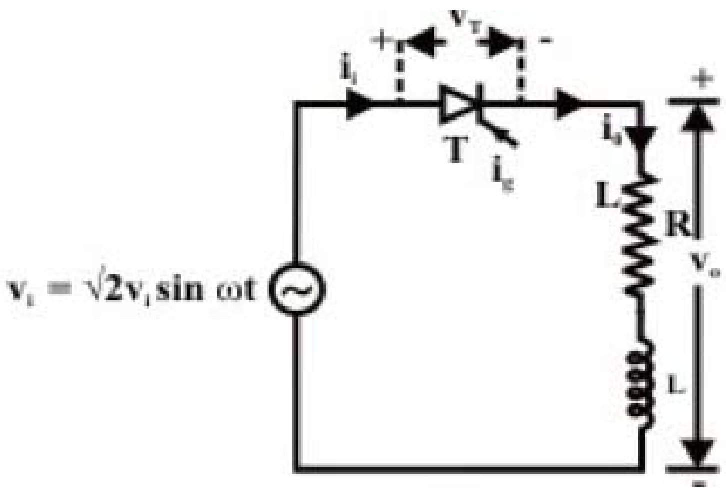
$$V_n = \frac{V_{o(avg)}}{V_{do}} = \frac{V_m(1+\cos\alpha)/2\pi}{V_m/\pi} = \frac{1+\cos\alpha}{2}$$

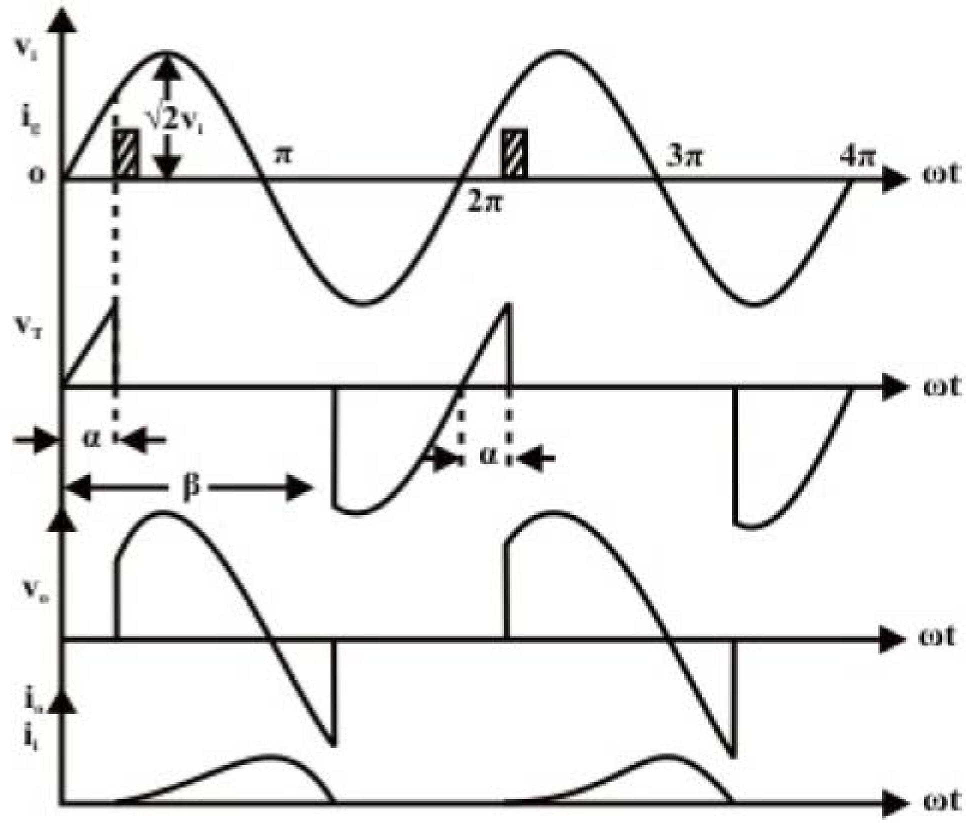


Delay (firing) angle: α
Conduction angle: 180-α

V_n as a function of α is known as the control characteristic of the rectifier

Single Phase Half Wave Controlled Rectifier with Resistive - Inductive Load



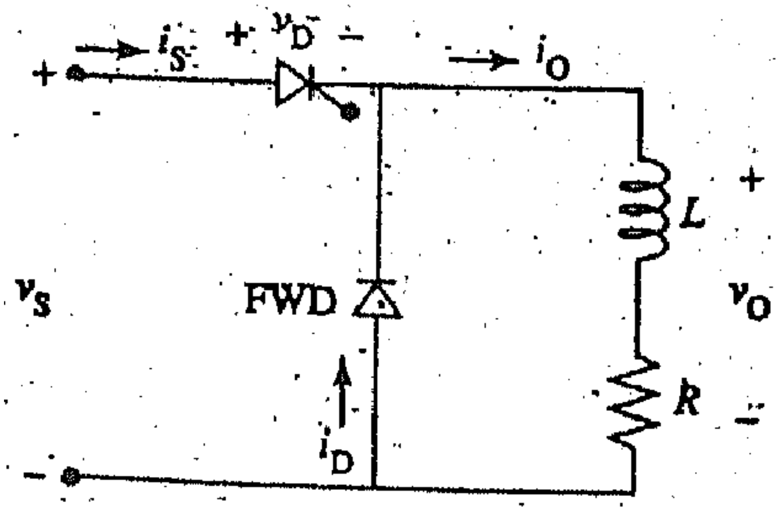


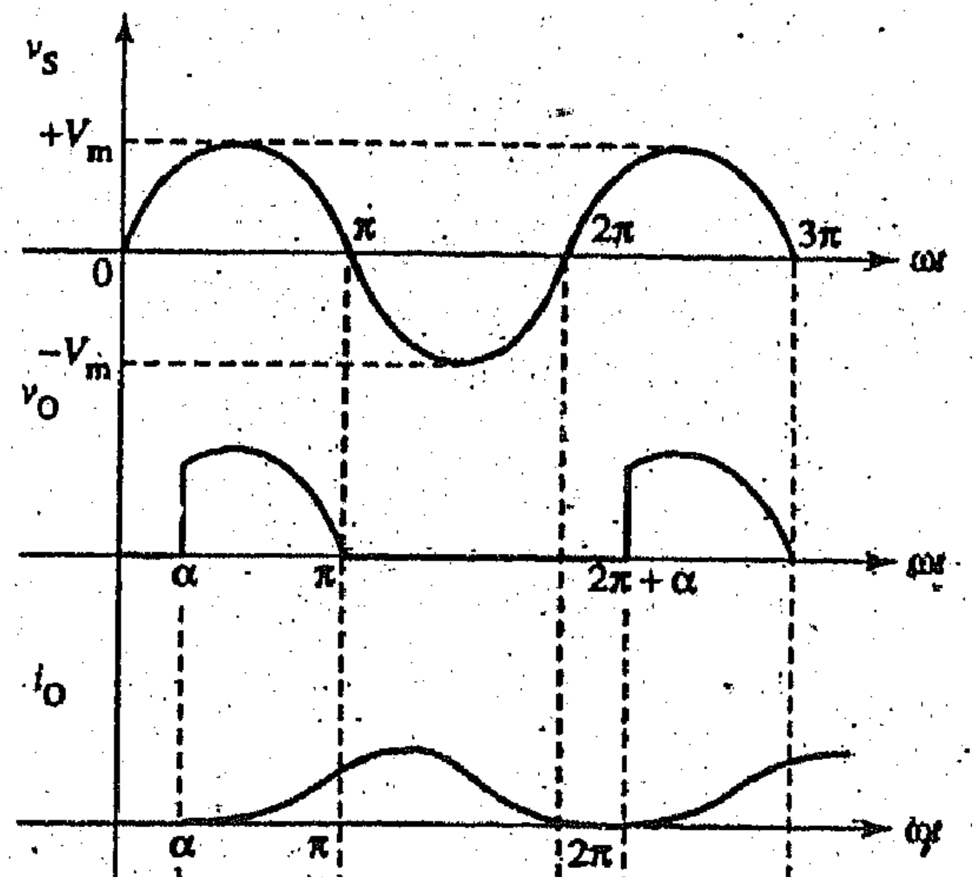
For $\alpha \le \omega t \le \beta$

$$v_0 = v_i = \sqrt{2}\, V_i \sin\omega t$$

$$v_0 = 0 \text{ otherwise}$$

Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode (FWD)





$$V_{o(avg)} = \frac{V_m(1+\cos\alpha)}{2\pi}$$

Example 1

A half-wave controlled rectifier is supplied from a 120 V source. If the load resistance is 10 Ω, find the load voltage and power to the load for the following delay angles:

a) $\alpha = 0°$
b) $\alpha = 45°$

Solution:

$$\text{peak load voltage} = V_m = \sqrt{2}\ V_S = 1.414 * 120 = 170\ \text{V}$$

$$\text{average load voltage} = \frac{V_m(1+\cos\alpha)}{2\pi}$$

a) For $\alpha = 0°$, we get

$$V_{o(avg.)} = \frac{170\,(1+\cos 0°)}{2\pi} = 54.0\ \text{V}$$

$$P_L = V^2_{o(avg.)}/R = 54.0^2/10 = 293\ \text{W}$$

b) For $\alpha = 45°$, we get

$$V_{o(avg.)} = \frac{170\,(1+\cos 45°)}{2\pi} = 46.2\ \text{V}$$

$$P_L = V^2_{o(avg.)}/R = 46.2^2/10 = 213\ \text{W}$$

Example 2

Half wave controlled rectifier connected to 150 V, 60Hz source is supplying a resistive load of 10Ω .if the delay angle α is 30°, find:-

a- The maximum load current, b- The average load voltage, c- The average load current,
d-The RMS load current, e- The power supplied to the load , f- The conduction angle
g- The ripple frequency, h- The power factor

Solution:

$V_m = \sqrt{2}\ V_s = 1.414 \times 150 = 212$ V

a) maximum load current

$$I_m = \frac{V_m}{R} = \frac{212}{10} = 21.2 \text{ A}$$

b) average load voltage $= \dfrac{V_m(1 + \cos\alpha)}{2\pi} = \dfrac{(212)(1 + \cos 30^\circ)}{2\pi} = 63$ V

c) average load current $= \dfrac{(I_m)(1 + \cos\alpha)}{2\pi} = \dfrac{(21.2)(1 + \cos 30^\circ)}{2\pi} = 6.3$ A

d) RMS load current

$$I_{RMS} = \left(\frac{I_m}{2}\right)\sqrt{\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]}$$

$$= \frac{(21.2)}{2}\sqrt{\left[1 - \frac{30}{180} + \frac{\sin 60}{2\pi}\right]}$$

$$= 10.5 \text{ A}$$

e) power supplied to the load $= I_{RMS}^2 R = 10.5^2(10) = 1094$ W

f) conduction angle

$$\theta = 180° - \alpha = 180° - 30° = 150°$$

g) ripple frequency

$$f_r = \text{input supply frequency} = 60 \text{ Hz}$$

h) $S = V_S * I_{RMS} = 150 * 10.5 = 1575$ VA

$$PF = \frac{P}{S} = \frac{1094}{1575} = 0.69$$

Example 3 A half wave controlled rectifier is connected to a 120 V source , Calculate the firing angle necessary to deliver 150 W of power to 10Ω load.

Solution:

$$V_{o(avg.)} = \frac{V_m(1 + \cos\alpha)}{2\pi}$$

Rearranging.

$$V_m(1 + \cos\alpha) = 2\pi V_{o(avg.)}$$

$$1 + \cos\alpha = \frac{2\pi V_{o(avg.)}}{V_m}$$

$$\cos\alpha = \frac{2\pi V_{o(avg.)}}{V_m} - 1$$

$$\alpha = \cos^{-1}\left\{\frac{2\pi V_{o(avg.)}}{V_m} - 1\right\}$$

$$V_m = \sqrt{2} * 120 = 170 \text{ V}$$

$$P_{avg.} = \frac{V_{o(avg.)}^2}{R}$$

$$V_{o(avg.)}^2 = P_{avg.} * R = 150 * 10 = 1500$$

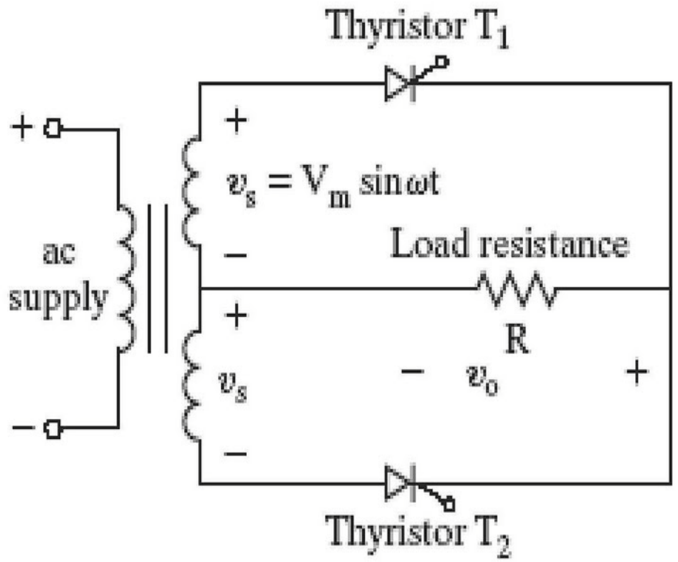
$$V_{o(avg.)} = \sqrt{1500} = 38.7 \text{ V}$$

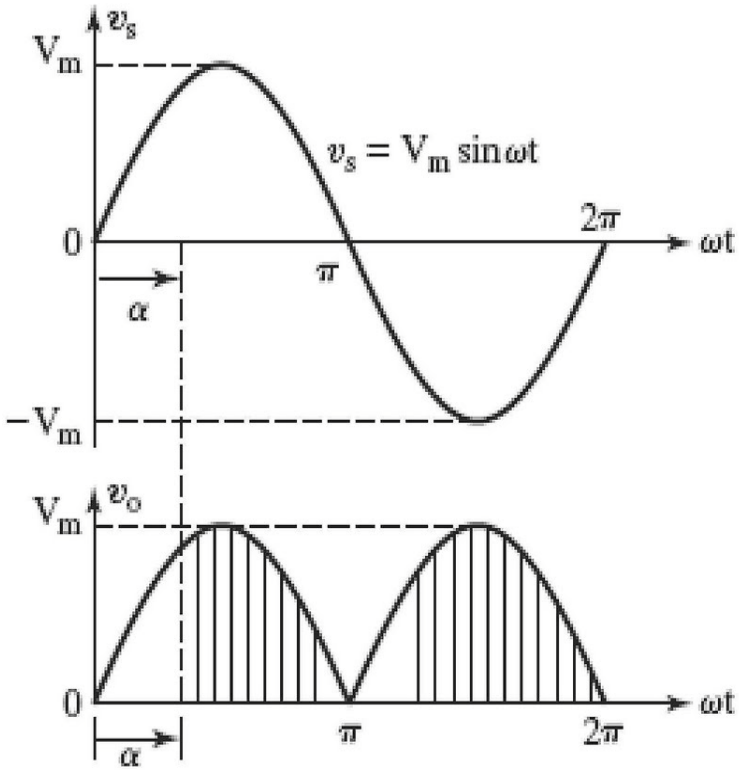
Therefore

$$\alpha = \cos^{-1}\left\{\frac{2\pi\, 38.7}{170} - 1\right\}$$

$$= 64.5°$$

Single Phase Full Wave Controlled Rectifier with Center-Tap Transformer and R Load





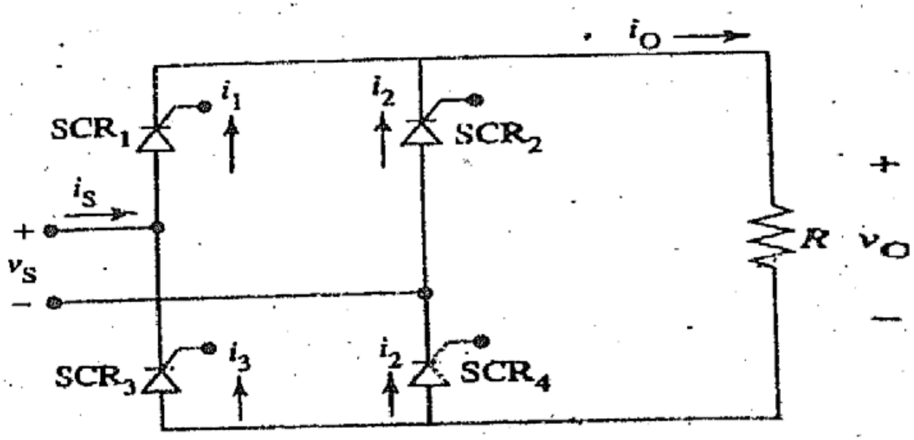
Delay (firing) angle: α
Conduction angle: 180-α

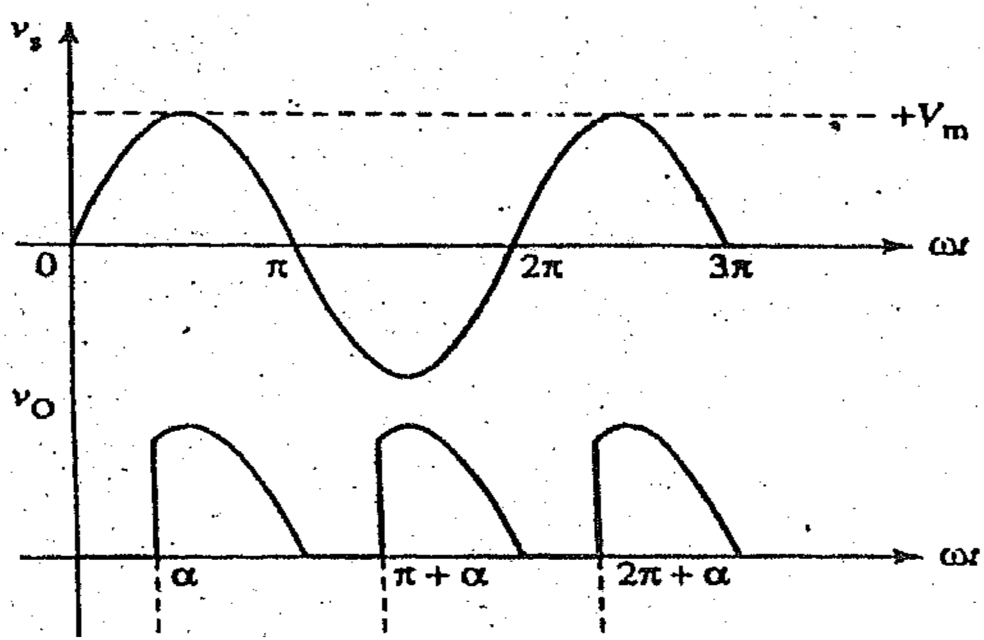
$$V_{o,DC} = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$V_{o,RMS} = V_m\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

$$I_{o,RMS} = \frac{V_{o,RMS}}{R}$$

Single Phase Full Wave Bridge Controlled Rectifier with R Load





Delay (firing) angle: α
Conduction angle: 180-α

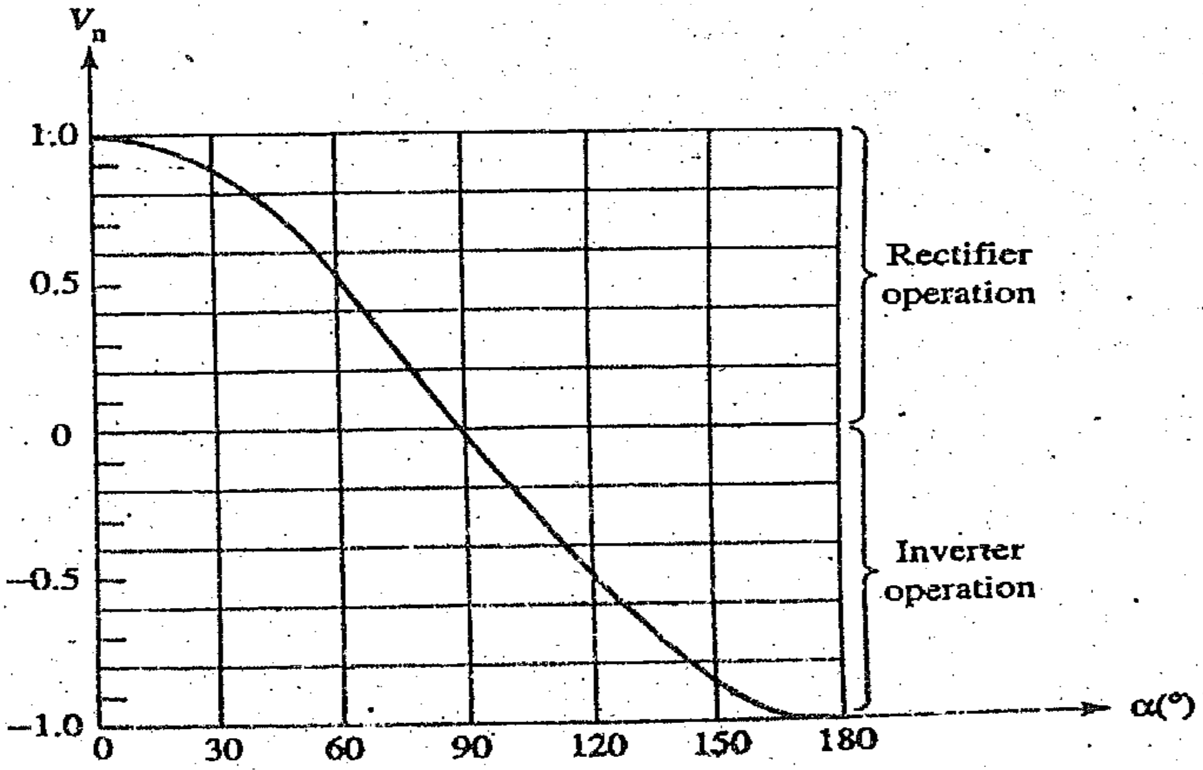
$$V_{o,DC} = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$V_{o,RMS} = V_m\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

$$I_{o,RMS} = \frac{V_{o,RMS}}{R}$$

$$I_{o(avg)} = \frac{V_{o,DC}}{R} = \frac{I_m(1+\cos\alpha)}{\pi}$$

Control Characteristics for the Bridge Rectifier



- When α becomes larger than 90°, the average value of output voltage becomes negative. This means that 90° to 180°m power flows from the DC load side to the AC source side and the circuit operates as an inverter.
- When rectification and inversion are obtained from one converter, the process is called two-quadrant operation and the converter is called a full converter.

Example 4

Full wave controlled rectifier connected to 150 V, 60Hz source is supplying a resistive load of 10Ω .if the delay angle α is 30°, find: a- The average load voltage , b-The average load current c-The maximum load current , d-The RMS load current, e- The power supplied to the load, f- The conduction angle, g- The ripple frequency , h- The power factor.

Solution:

$$V_m = \sqrt{2}\,(150) = 212\ \text{V}$$

a) average load voltage $= \dfrac{V_m(1+\cos\alpha)}{\pi} = \dfrac{(212)(1+\cos 30°)}{\pi} = 126\ \text{V}$

b) average load current $= \dfrac{(V_m)\,(1+\cos 30°)}{R\pi} = 12.6\ \text{A}$

c) maximum load current

$$I_m = \frac{V_m}{R} = \frac{212}{10} = 21.2\ \text{A}$$

d) RMS load current

$$I_{RMS} = \frac{I_m}{\sqrt{2}}\sqrt{\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]}$$
$$= 14.8\ \text{A}$$

e) Power supplied to the load = I^2rms . R = $14.8^2(10)$ = 2182 W

f) The conduction angle = 180° - α = 150°

g) Ripple frequency fr = 2 x input supply frequency = 2 x 60 = 120 Hz

h) S = Vs x Irms = 150 x 14.8 = 2220 VA

P.F = P / S = 2182 / 2220 = 0.98

Problems and Solutions

Question 1 A full-wave center-tap rectifier is fed from a transformer with a secondary voltage of 120 V (center-tap to line). If it is used to charge a 12 V battery having an internal resistance of 0.1 Ω, find

(a) the firing angle necessary to produce a charging current of 10 A

(b) the average SCR current

(c) the PIV rating of the SCR

Solution:

a) $I_{dc} = \dfrac{V_{dc} - E}{R} \Rightarrow 10 = \dfrac{V_{dc} - 12}{0.1} \Rightarrow V_{dc} = 13 \text{ Volt}$

$$V_{dc} = \frac{V_m (1 + \cos\alpha)}{\pi}$$

$$13 = \frac{(1.414 \times 120)(1 + \cos\alpha)}{\pi} \Rightarrow \cos\alpha = -0.16 \Rightarrow \therefore \alpha = 99.2^o$$

b) $I_{SCR(avg)} = \dfrac{I_{dc}(180 - \alpha)}{2\pi} = \dfrac{10(180 - 99.2)}{360} = 2.24 \text{ A}$

c) $\text{PIV} = 2V_m = 2 \times (1.414 \times 120) = 340 \text{ Volt}$

Question 2 A half-wave controlled rectifier is supplied from a 120 V source is used to control the power to a 15Ω load. If the delay angle is 75°, find:-
a) the power supplied to the load , b) the maximum SCR current
c) the average SCR current, d) the SCR maximum reverse voltage

Solution:

$$\text{a)}\quad P_s = \frac{V_{o,RMS}^{2}}{R} \Rightarrow \quad \boxed{V_{o,RMS}^{2} = \left(\frac{V_m}{2}\right)^2 \left(1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}\right)}$$

$$V_{o,RMS}^{2} = \left(\frac{1.414\times 120}{2}\right)^2 \left(1-\frac{75}{180}+\frac{\sin(2\times 75)}{2\times 3.14}\right) = 4761\ \mathrm{V}^2$$

$$\therefore\quad P_s = \frac{4761}{15} = 317.7\ \ \text{Watt}$$

$$\text{b)}\quad I_{SCR(MAX)} = I_m = \frac{V_m}{R} = \frac{1.414\times 120}{15} = 11.3\ \mathrm{A}$$

$$\text{c)}\quad I_{SCR(avg)} = \frac{V_{dc}}{R} = \frac{V_m(1+\cos\alpha)/2\pi}{R} = \frac{(1.414\times 120)(1+\cos 75)}{2\times 3.14\times 15} = 2.3\ \mathrm{A}$$

$$\text{d)}\quad \mathrm{PIV} = V_m = 1.414\times 120 = 170\ \text{Volt}$$

Question 3 A full - wave fully controlled rectifier with R load is fed from 200V source supplies an average current of 10A. If the firing angle is 30°, find:
a) the average power delivered to the load, b) the average SCR current,
c) the maximum SCR current, d) the SCR peak reverse voltage, e) the load resistance

Solution:

a) $P_{avg} = V_{avg} I_{avg} \quad \Rightarrow \quad \boxed{V_{avg} = V_{dc} = \frac{V_m (1+\cos\alpha)}{\pi}}$

$$V_{avg} = \frac{(1.414\times 200)(1+\cos 30)}{3.14} = 168 \text{ Volt}$$

b) $I_{SCR(avg)} = \frac{I_{dc}(180-\alpha)}{2\pi} = \frac{10(180-30)}{360} = 8.33 \text{ A}$

c) $I_m = \frac{V_m}{R} = \frac{1.414\times 200}{(168/10)} = 16.8 \text{ A}$

d) $\text{PIV} = V_m = 1.414\times 200 = 282.8 \text{ Volt}$

e) $R = \frac{V_{avg}}{I_{avg}} = \frac{168}{10} = 16.8\Omega$

Question 4 A single phase Half-Wave converter is fed by transformer whose secondary voltage is 325sinωt. It supplies a resistance load of 20Ω for firing angle of 45°, calculate:
a) average values of load voltage and current, b) rms values of load voltage and current,
c) P_{dc} , P_{ac} and rectification efficiency η, d) form factor (FF), e) the VA rating S of the transformer and utilization factor UTF, g) PIV of the thyristor.

Solution:

a) $$V_{avg} = \frac{V_m(1+\cos\alpha)}{2\pi} = \frac{325(1+\cos 45)}{2\times 3.14} = 88.5 \text{ Volt}$$

$$I_{avg} = V_{avg}/R = 88.5/20 = 4.4 \text{ A}$$

b) $$V_{oRMS} = \left(\frac{V_m}{2}\right)\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}} = \left(\frac{325}{2}\right)\sqrt{1-\frac{45}{180}+\frac{\sin 90}{2\times 3.14}} = 155 \text{ Volt}$$

$$I_{oRMS} = V_{oRMS}/R = 155/20 = 7.75 \text{ A}$$

c) $$P_{dc} = V_{dc}^2/R = (88.5)^2/20 = 387.2 \text{ W}$$

$$P_{ac} = V_{oRMS}^2/R = (155)^2/20 = 1201 \text{ W}$$

$$\eta = (P_{dc}/P_{ac})\times 100\% = (387.2/1201)\times 100\% = 32\%$$

d) $$FF = V_{oRMS}/V_{dc} = 155/88.5 = 1.75$$

e) $$VA = S = V_{RMS} I_{RMS} = \left(325/\sqrt{2}\right)\times 7.75 = 1781.3 \text{ W}$$

$$TUF = (P_{dc}/S)\times 100\% = (387.2/1781.3)\times 100\% = 21.7\%$$

g) $$PIV = V_m = 325 \text{ Volt}$$

Question 5

A single phase Half-Wave converter is used to control 1kw power of 10Ω dc load , for 240V, 50Hz supply, calculate the firing angle α

Solution:

$$P_{dc} = V_{avg}^{2} / R = 1000 \quad \Rightarrow V_{avg} = \sqrt{1000 \times 10} = 100 \text{ Volt}$$

$$\boxed{V_{avg} = \frac{V_m (1+\cos\alpha)}{2\pi}} \quad \Rightarrow V_{avg} = \frac{(1.414 \times 240)(1+\cos\alpha)}{2 \times 3.14} = 100 \quad \Rightarrow \alpha = 31.7^{o}$$

Question 6

A single phase Half-Wave converter is connected to 15Ω load and supply voltage of 230V,50Hz. For firing angle α = 30˚ and 60˚, calculate: V_{dc} and V_{SCR}.

Solution:

$$V_m = 1.414 \times 230 = 325 \text{ Volt}$$

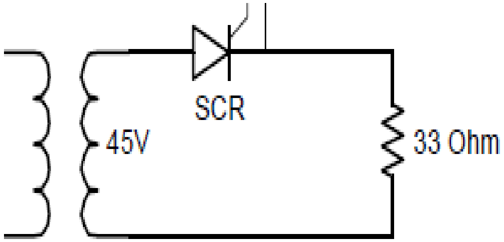
$$\boxed{V_{avg} = \frac{V_m (1+\cos\alpha)}{2\pi}} \qquad \boxed{V_{SCR} = \frac{V_m (1-\cos\alpha)}{2\pi}}$$

$$\text{For } \alpha = 30^{o} \Rightarrow V_{avg} = \frac{325(1+\cos 30)}{2 \times 3.14} = 96.7 \text{ Volt}$$

$$V_{SCR} = \frac{325(1-\cos 30)}{2 \times 3.14} = 6.7 \text{ Volt}$$

$$\text{Similarly for } \alpha = 60^{o} \Rightarrow V_{avg} = 77.7 \text{ Volt, and } V_{SCR} = 25.9 \text{ Volt}$$

Question 7 For the given single phase Half-Wave converter, the delay angle α = 90°. Calculate the: a) efficiency η, b) form factor (FF), c) transformer utilization factor UTF, d) SCR current.



Solution:

$$V_m = 1.414 \times 45 = 63.63 \text{ Volt}$$

$$V_{avg} = \frac{V_m(1+\cos\alpha)}{2\pi} = \frac{63.63(1+\cos 90)}{2\times 3.14} = 10.7 \text{ Volt}$$

$$I_{avg} = V_{avg}/R = 10.7/33 = 0.324 \text{ A}$$

$$V_{oRMS} = \left(\frac{V_m}{2}\right)\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}} = \left(\frac{63.63}{2}\right)\sqrt{1-\frac{90}{180}+\frac{\sin 180}{2\times 3.14}} = 22.5 \text{ Volt}$$

$$I_{oRMS} = V_{oRMS}/R = 22.5/33 = 0.681 \text{ A}$$

a) $\eta = (P_{dc}/P_{ac})\times 100\% = (V_{dc}^2/V_{oRMS}^2)\times 100\% = \left((10.7)^2/(22.5)^2\right)\times 100\% = 20\%$

b) $FF = V_{oRMS}/V_{dc} = 22.5/10.7 = 2.23$

c) $P_{dc} = V_{avg} I_{avg} = 10.7\times 0.324 = 3.47 \text{ W}$

$\text{VA} = S = V_{RMS} I_{RMS} = 22.5\times 0.681 = 15.3 \text{ W}$

$\text{TUF} = (P_{dc}/S)\times 100\% = (3.47/15.3)\times 100\% = 10.15\%$

d) $I_{SCR} = I_{avg} = 0.324 \text{ A}$

Question 8 For the given single phase Half-Wave converter, the delay angle α = 30°. Calculate: a) I_{dc} , b) draw the waveforms of V_{Load}, and I_{Load} .

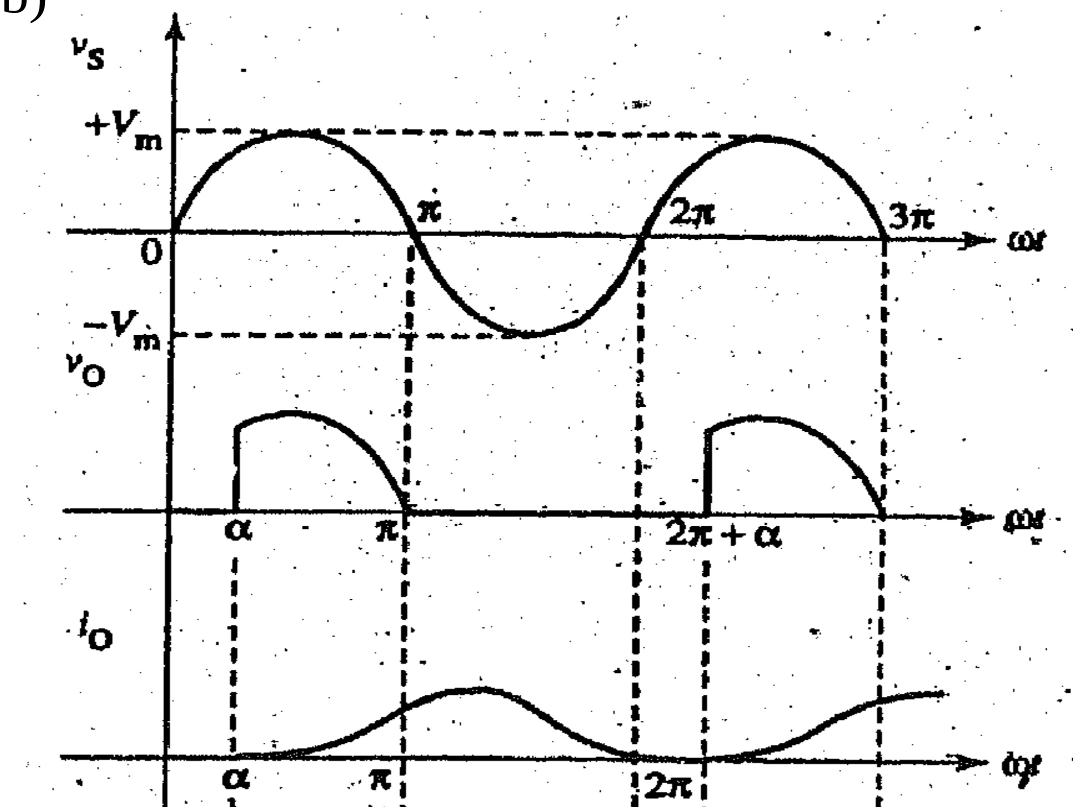
Solution:

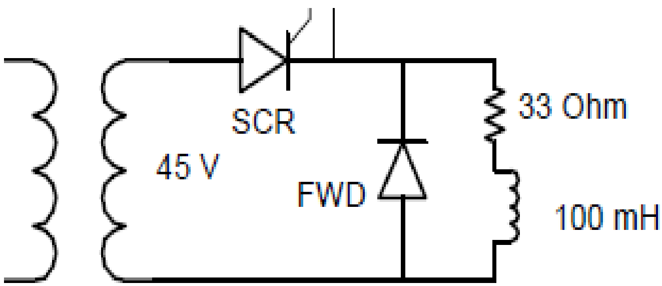
a) $V_m = 1.414 \times 45 = 63.63$ Volt

$$V_{avg} = \frac{V_m(1+\cos\alpha)}{2\pi} = \frac{63.63(1+\cos 30)}{2\times 3.14} = 15.2 \text{ Volt}$$

$$I_{dc} = I_{avg} = V_{avg}/R = 15.2/33 = 0.46 \text{ A}$$

b)





Question 9 A 1-Φ full wave converter is operated at 230V, 50Hz for a resistive load of 12Ω. If the average output voltage is 25% of maximum possible average output voltage, find (i) delay angle (ii) average and rms output currents (iii) average and rms thyristors currents.

Solution:

i) $V_{avg} = 0.25 \times V_{avg\,(\max)} = 0.25 \times \frac{2V_m}{\pi} = \frac{V_m(1+\cos\alpha)}{\pi}$

$\Rightarrow \cos\alpha = -0.5 \quad \Rightarrow \alpha = 120^o$

ii) $I_{avg} = \frac{V_{avg}}{R} = \frac{V_m(1+\cos\alpha)}{\pi R} = \frac{(1.414\times 230)(1+\cos 120)}{3.14\times 12} = 4.3 \text{ A}$

$V_{oRMS} = \left(\frac{V_m}{2}\right)\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}} = \left(\frac{1.414\times 230}{2}\right)\sqrt{1-\frac{120}{180}+\frac{\sin 240}{2\times 3.14}} = 100.8 \text{ Volt}$

$I_{oRMS} = V_{oRMS}/R = 100.8/12 = 8.4 \text{ A}$

iii) $I_{SCRavg} = I_{avg}/2 = 4.3/2 = 2.65 \text{ A}$

$I_{SCRrms} = I_{SCRavg}/\sqrt{2} = 2.65/1.414 = 1.87 \text{ A}$

Question 10 Choose the correct answer for the following Questions

1) A thyristor of half wave controlled converter has a supply voltage of 240V at 50Hz and a load resistance of 100 Ohm. When the firing angle is 30, the average value of load current is…………………

A. 126 A, **B.** 2.4 A, **C.** 126 mA, **D.** 24 A

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2) A half-controlled bridge converter is operating from an r.m.s input voltage of 120 V. Neglecting the voltage drops, what are the mean load voltage at a firing delay angle of 0° and 180°, respectively ?

(A) $\frac{120 \times 2\sqrt{2}}{\pi}$ V and 0 (C) $\frac{120\sqrt{2}}{\pi}$ V and 0

(B) 0 and $\frac{120 \times 2\sqrt{2}}{\pi}$ V (D) 0 and $\frac{120\sqrt{2}}{\pi}$ V

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3) In a single phase full wave controlled bridge rectifier, minimun output voltage and maximum output voltage are obtained at whicl conduction angles ?

(A) $0^\circ, 180^\circ$ respectively (C) $0^\circ, 0^\circ$ respectively

(B) $180^\circ, 0^\circ$ respectively (D) $180^\circ, 180^\circ$ respectively

Question 11 Fill in the Following Blanks with the Correct Word (s)

1) The controlled AC-DC converters are used to convert an ……… **AC** …..voltage of variable…**Amplitude**…… and frequency to a variable …**DC**…… voltage.

2) Single phase half wave controlled rectifier is constructed using one …… **SCR**……. .

3) … **α** …is the number of degrees from the beginning of the cycle until the SCR is gated ON.

4) ……… **180 - α** …….is the number of degrees that the SCR remains conducting.

5) …… **α** ….is the degree at which the SCR is turning off in full wave controlled rectifier.

6) A fully-controlled rectifier circuit contains only….. **SCR**……rectifiers, whereas a …… **semi-controlled rectifier** …….circuit is made up of both diodes and SCRs.

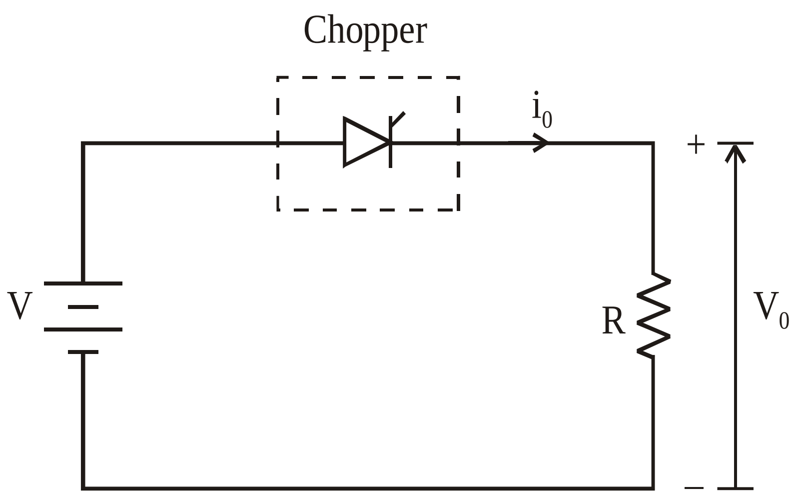
Chapter 7

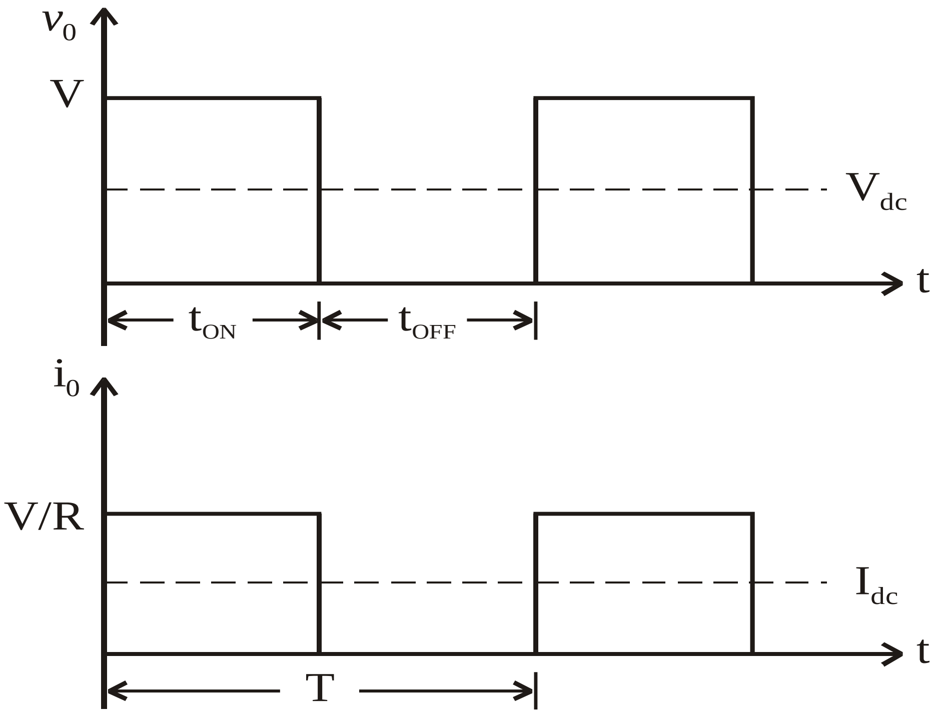
DC-DC Convertors
(DC Choppers)

DC Choppers
(DC to DC Convertors)

- Chopper is a static device and known as DC-to-DC converter.
- A variable DC voltage is obtained from a constant DC source.
- Widely used for motor control.
- Also used in regenerative braking.
- Thyristor converter offers greater efficiency, faster response, lower maintenance, smaller size and smooth control.
- Choppers are of two types:
  - *Step-Down Choppers*: output voltage is less than the input.
  - *Step-Up Choppers*: output voltage is more than the input.

Principle of Step-Down Chopper





A step-down chopper with resistive load

- The thyristor acts as a switch.
- When thyristor is ON, the supply voltage appears across the load.
- When thyristor is OFF, the voltage across the load will be zero.

V_{dc} = Average value of load voltage.

I_{dc} = Average value of load current.

t_{ON} = Time interval for which SCR conducts.

t_{OFF} = Time interval for which SCR is OFF.

$T = t_{ON} + t_{OFF}$ = Period of switching (chopping).

$f = 1/T$ = Freq. of chopper switching (chopping freq).

Average Output Voltage: $V_{dc} = V\left(\frac{t_{ON}}{t_{ON}+t_{OFF}}\right) = V\left(\frac{t_{ON}}{T}\right) = Vd$

where the duty cycle is: $d = (t_{ON}/T)$

and during t_{ON}, $v_o = V$ $\Rightarrow$ therefore RMS output voltage will be

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} V^2 dt} = \sqrt{\frac{V^2}{T} t_{ON}} = \sqrt{\frac{t_{ON}}{T}}\, V = \sqrt{d}\, V$$

Output power: $P_O = V_O I_O = (V_O^2/R) = (d\, V^2/R)$

Effective input resistance of chopper: $R_i = (V/I_{dc}) = R/d$

The output voltage can be varied by varying the duty cycle

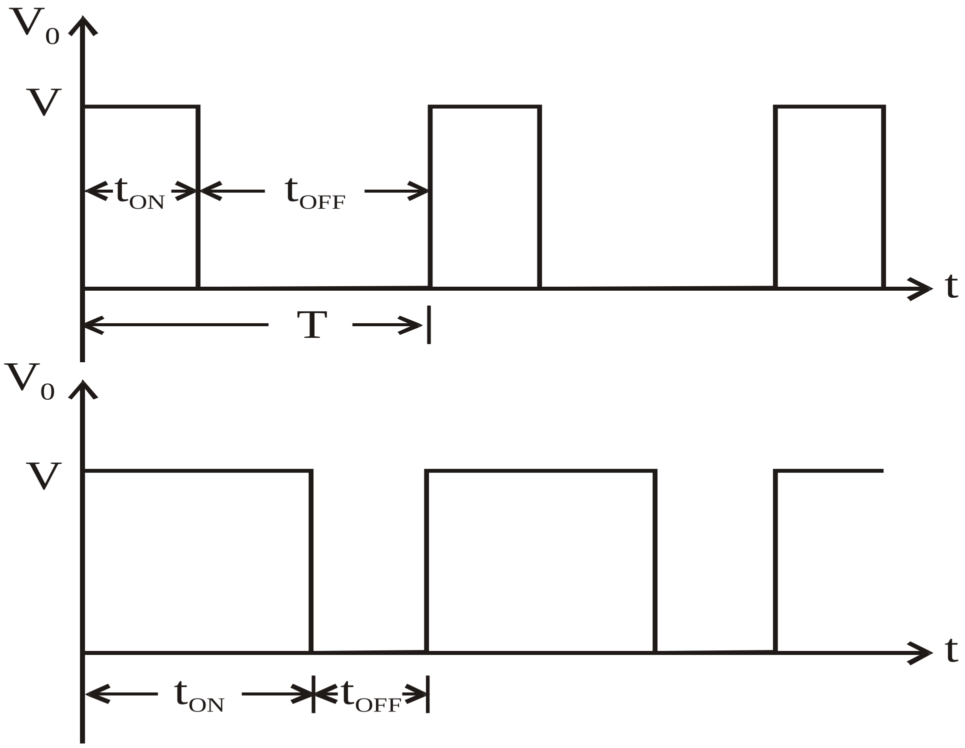
Methods of Control

Output DC voltage can be varied by the following methods:

- Pulse Width Modulation (PWM) control or constant frequency operation.
- Variable frequency control.

Pulse Width Modulation (PWM)

- t_{ON} is varied keeping chopping frequency f and chopping period T constant.
- Output voltage is varied by varying the ON time t_{ON}

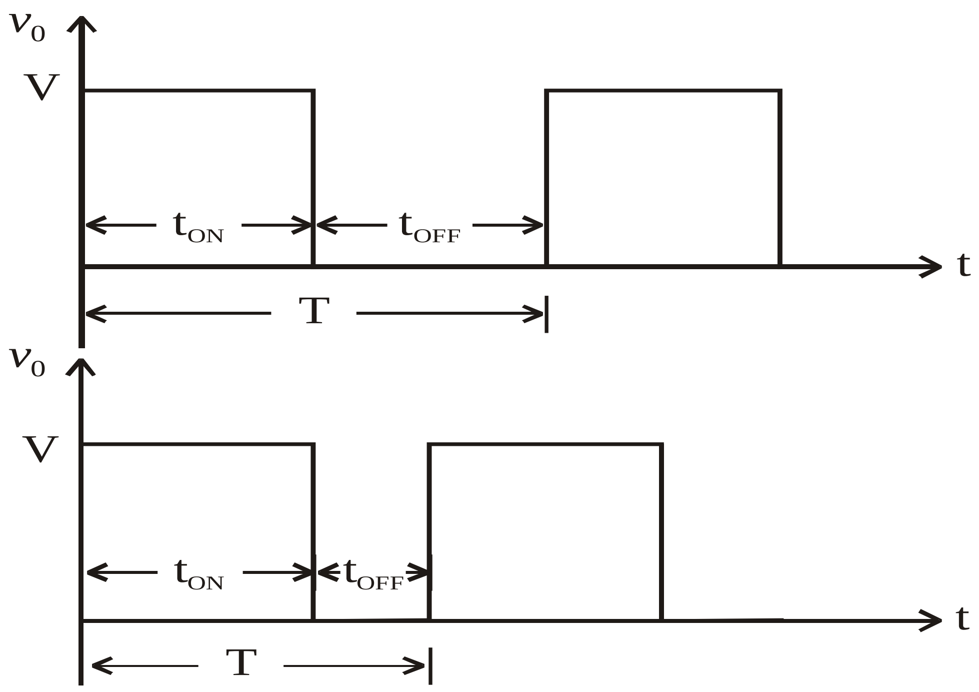


Variable Frequency Control

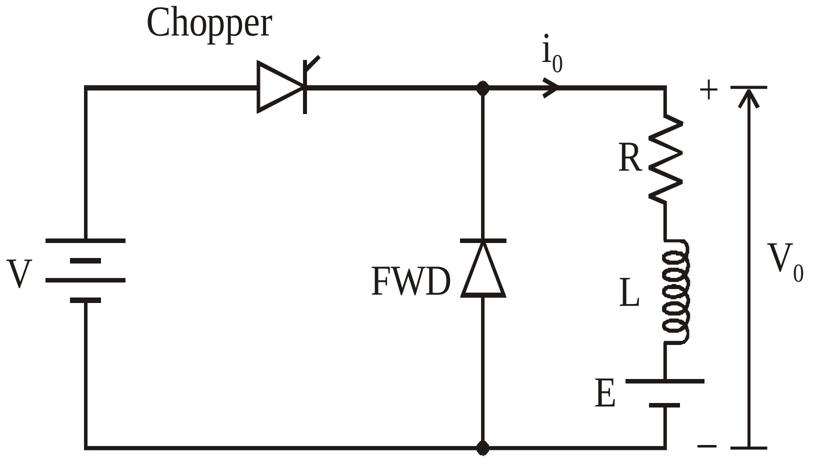
- Chopping frequency f is varied keeping either t_{ON} or t_{OFF} constant.

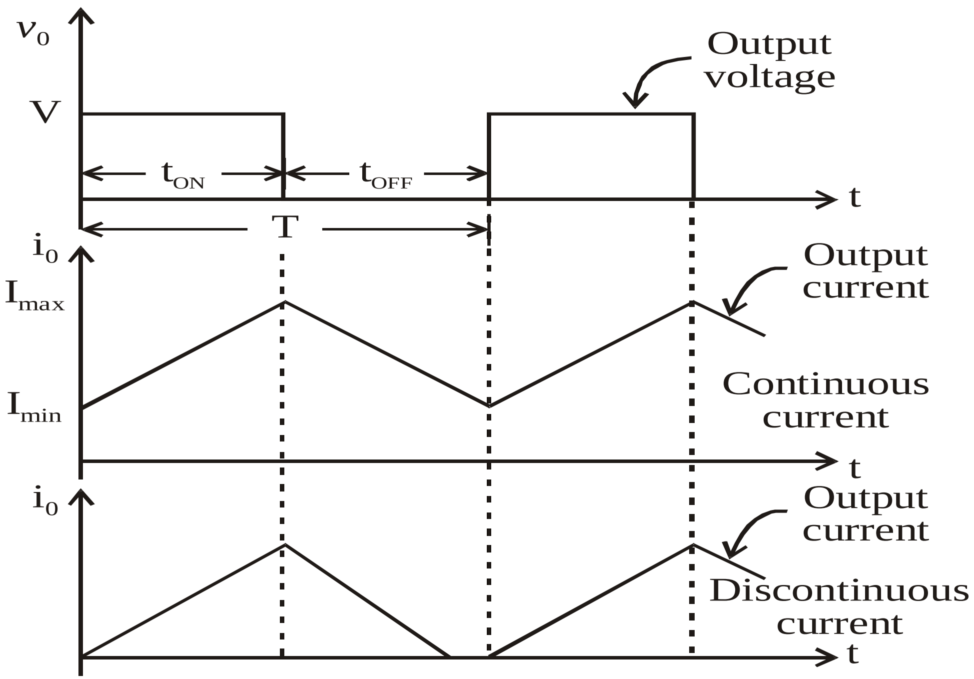
- To obtain full output voltage range, f has to be varied over a wide range.

- This method produces harmonics in the output and for large t_{OFF} , load current may become discontinuous.



Step-Down Chopper with *R-L* Load





- When chopper is ON, the current flows from the supply *V* to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in the inductor *L*.
- Load current can be continuous or discontinuous depending on *L* and duty cycle *d*.
- For continuous current operation, load current varies between I_{max} and I_{min}.
- When current becomes equal to I_{max} , the chopper is turned-off , and it is turned-on when the current reduces to I_{min}.

$$I_{min} = \left(I_{max} e^{-\frac{(1-d)RT}{L}} \right) - \frac{E}{R} \left(1 - e^{-\frac{(1-d)RT}{L}} \right)$$

$$I_{max} = \frac{V - E}{R} \left(1 - e^{-\frac{dRT}{L}} \right) + I_{min} e^{-\frac{dRT}{L}}$$

From the above equations, we get:

$$I_{max} = \frac{V}{R} \left(\frac{1 - e^{-\frac{dRT}{L}}}{1 - e^{-\frac{RT}{L}}} \right) - \frac{E}{R}$$

$$I_{min} = \frac{V}{R} \left(\frac{e^{\frac{dRT}{L}} - 1}{e^{\frac{RT}{L}} - 1} \right) - \frac{E}{R}$$

Steady state ripple $= I_{max} - I_{min}$

Peak-to-peak ripple current: $\Delta I = I_{max} - I_{min}$

Average output voltage: $V_{dc} = dV$

Average output current: $I_{dc(approx)} = \frac{I_{max} + I_{min}}{2}$

Example 1 A Chopper circuit is operating on TRC at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

Solution:

$$\text{Total time period of chopper} \Rightarrow T = \frac{1}{f} = \frac{1}{2\times10^{3}} = 0.5\ \text{msec}$$

$$V_{dc} = Vd = V\left(\frac{t_{ON}}{T}\right)$$

$$\therefore \text{Conduction period} \Rightarrow t_{ON} = \frac{T \times V_{dc}}{V} = \frac{0.5\times10^{-3}\times350}{460} = 0.38\ \text{msec}$$

Example 2 A dc chopper has a resistive load of 20Ω and input voltage V_S = 220V. When the chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper ON time.

Solution:

$$V_S = 220\text{V},\ R = 20\Omega,\ f = 10\ \text{kHz},\ d = \frac{t_{ON}}{T} = 0.80$$

$$\text{Voltage drop across chopper} \Rightarrow V_{ch} = 1.5\ \text{V}$$

$$\text{Average output voltage} \Rightarrow V_{dc} = \left(\frac{t_{ON}}{T}\right)(V_S - V_{ch}) = 0.80(220 - 1.5) = 174.8\ \text{V}$$

$$\text{Chopping period} \Rightarrow T = \frac{1}{f} = \frac{1}{10\times10^{3}} = 100\ \mu\text{sec}$$

$$\text{Chopper ON time} \Rightarrow t_{ON} = dT = 0.80\times100\times10^{-6} = 80\ \mu\text{sec}$$

Example 3 A battery powered vehicle uses dc motor drive controlled by DC chopper working at 10 kHz. The battery voltage is 220 V, the min. turn ON and OFF times of the chopper switch are 2μsec and 25μsec, respectively. Determine the min. and max. dc voltage that the chopper can deliver to the motor.

Solution:

$$\text{Total time period of chopper} \Rightarrow T = \frac{1}{f} = \frac{1}{10\times 10^3} = 100\ \mu\,\text{sec}$$

$$\text{Min. ON time} = 2\ \mu\,\text{sec}$$

$$\text{Min. duty cycle } d_{\min} = \frac{t_{ON\,(\min)}}{T} = \frac{2\mu\,\text{sec}}{100\mu\,\text{sec}} = 0.02$$

$$\therefore V_{dc\,(\min)} = d_{\min} \times V_S = 0.02\times 220 = 4.4\ \text{V}$$

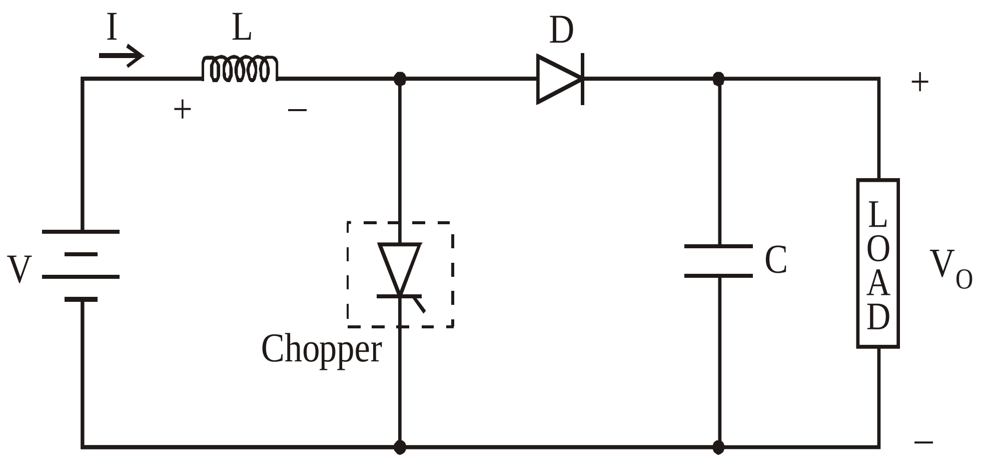
$$\text{Max. ON time} = T - t_{OFF} = 100\mu\,\text{sec} - 25\mu\,\text{sec} = 75\mu\,\text{sec}$$

$$d_{\max} = \frac{t_{ON\,(\max)}}{T} = \frac{75\mu\,\text{sec}}{100\mu\,\text{sec}} = 0.75$$

$$\therefore V_{dc\,(\max)} = d_{\max} \times V_S = 0.75\times 220 = 165\ \text{V}$$

$$\text{The output range of control is: } 4.4\text{V} \le V_{dc} \le 165\text{V}$$

Principle of Step-Up Chopper



- A large C connected across the load to provide continuous V_O .
- D prevents any current flow from C to the source.
- Step-Up choppers are used for regenerative braking of DC motors.

- Step-up chopper is used to obtain a load voltage V_O higher than the input voltage V.
- L and C are chosen depending upon the requirement of output voltage and current.
- When the chopper is ON, the inductor L is connected across the supply and its current I rises. L stores energy during the ON time of the chopper t_{ON}.
- When the chopper is OFF, inductor current I is forced to flow through D and load for a period, t_{OFF}.
- The current tends to decrease resulting in reversing the polarity of induced EMF in L. Therefore voltage across load is given by:

$$V_O = V + L\frac{dI}{dt} \Rightarrow \text{which means that } V_O > V$$

Expression for Output Voltage

Assume average current of L is I during ON and OFF time of chopper.

When the chopper is ON:

Voltage across $L = V \Rightarrow$ Energy stored in $L = VIt_{ON}$

When the chopper is ON: (energy is supplied by L to the load)

Voltage across $L = V_O - V \Rightarrow$ Energy supplied by $L = (V_O - V)It_{OFF}$

Neglecting losses,

Energy stored in L = Energy supplied by L

$$\therefore \; VIt_{ON} = (V_O - V)It_{OFF} \Rightarrow V_O = \frac{V(t_{ON} + t_{OFF})}{t_{OFF}} \Rightarrow V_O = V\left(\frac{T}{T - t_{ON}}\right)$$

where $T = t_{ON} + t_{OFF}$ is the chopping (switching) period.

$$\therefore \quad V_O = V\left(\frac{1}{1 - (t_{ON}/T)}\right) \Rightarrow V_O = V\left(\frac{1}{1-d}\right)$$

where $d = t_{ON}/T =$ duty cycle

For $0 < d < 1 \Rightarrow$ the output voltage will vary in the range $V < V_O < \infty$

Example 4 For step-up chopper, the input is 200V and the required output is 600V. If the conducting time of the thyristor is 200 μsec, 1) compute the chopping frequency. 2) If the pulse width is halved for constant frequency of operation, find the new output voltage.

Solution:

$$V = 200\text{ V},\ t_{ON} = 200\mu s,\ V_{dc} = 600\text{ V}$$

1)

$$V_{dc} = V\left(\frac{T}{T - t_{ON}}\right) \Rightarrow 600 = 200\left(\frac{T}{T - 200\times10^{-6}}\right) \Rightarrow T = 300\mu s$$

2)

$$\text{Chopping frequency } f = \frac{1}{T} = \frac{1}{300\times10^{-6}} = 3.33\text{KHz}$$

$$\text{Pulse width is halved} \Rightarrow t_{ON} = \frac{200\times10^{-6}}{2} = 100\mu s$$

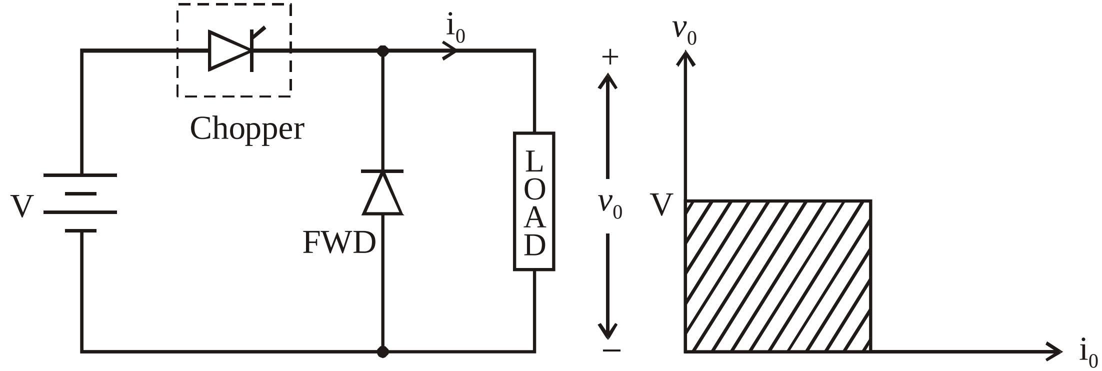
$$\therefore \text{Output voltage} = V\left(\frac{T}{T - t_{ON}}\right) = 200\left(\frac{300\times10^{-6}}{(300-100)10^{-6}}\right) = 300\text{V}$$

Classification of Choppers

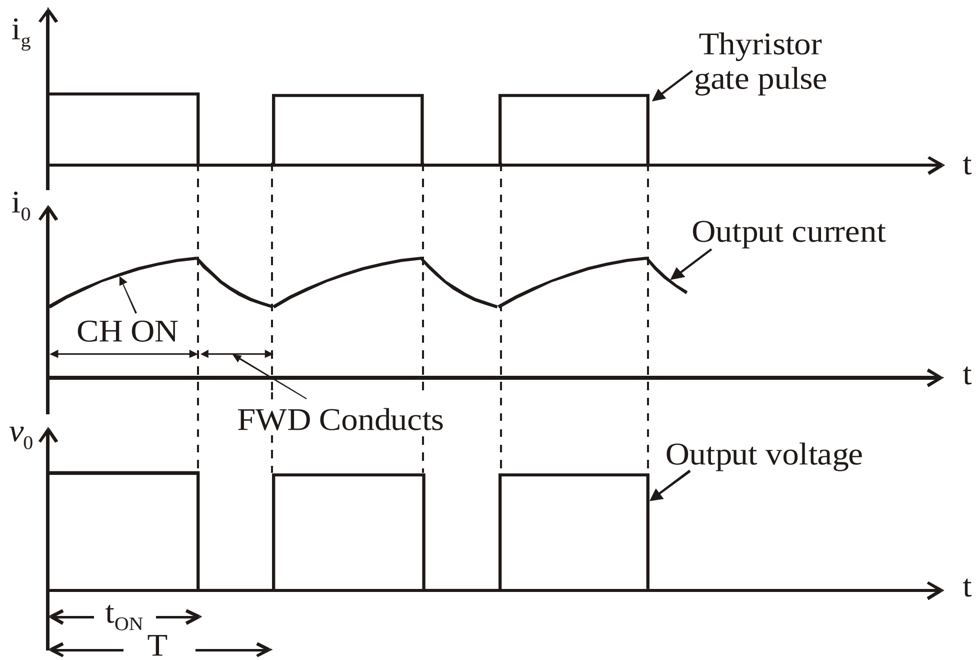
- Class A chopper.
- Class B chopper.
- Class C chopper.
- Class D chopper.
- Class E chopper.

Class A chopper

It is a step-down (first quadrant) chopper in which the power flows form the source to the load, and used DC motor speed control.



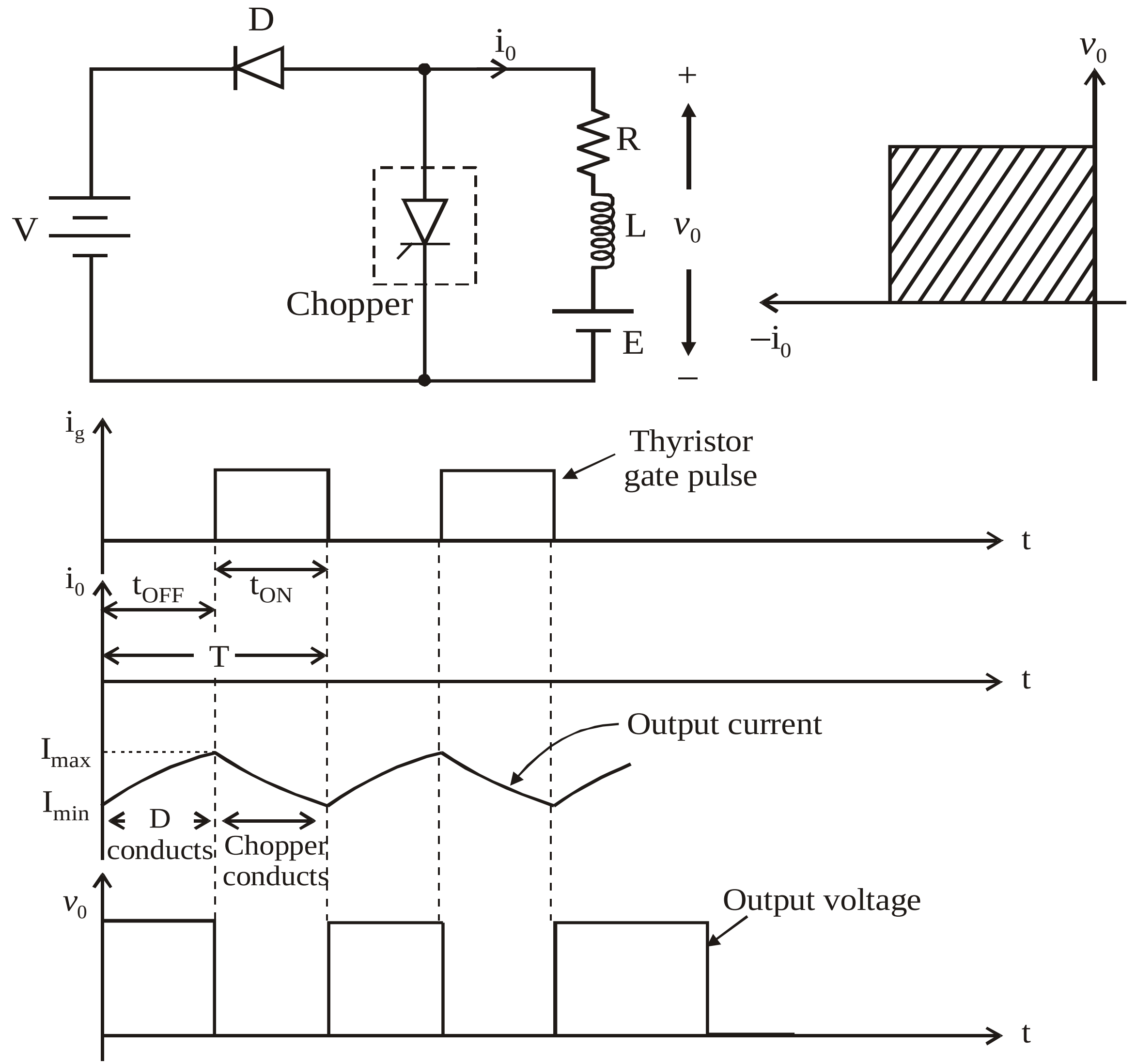
- When chopper is ON, supply voltage V is connected across the load.
- When chopper is OFF, $V_o = 0$ and the load current continues to flow in the same direction through the FWD.
- The average values of output voltage and current are always positive.
- The output current equations obtained in step-down chopper with *RL* load can be used to study the performance of Class A Chopper.



Class B chopper

It is a step-up (second quadrant) chopper in which the power flows form the load to the source, and used for regenerative braking of DC motors.

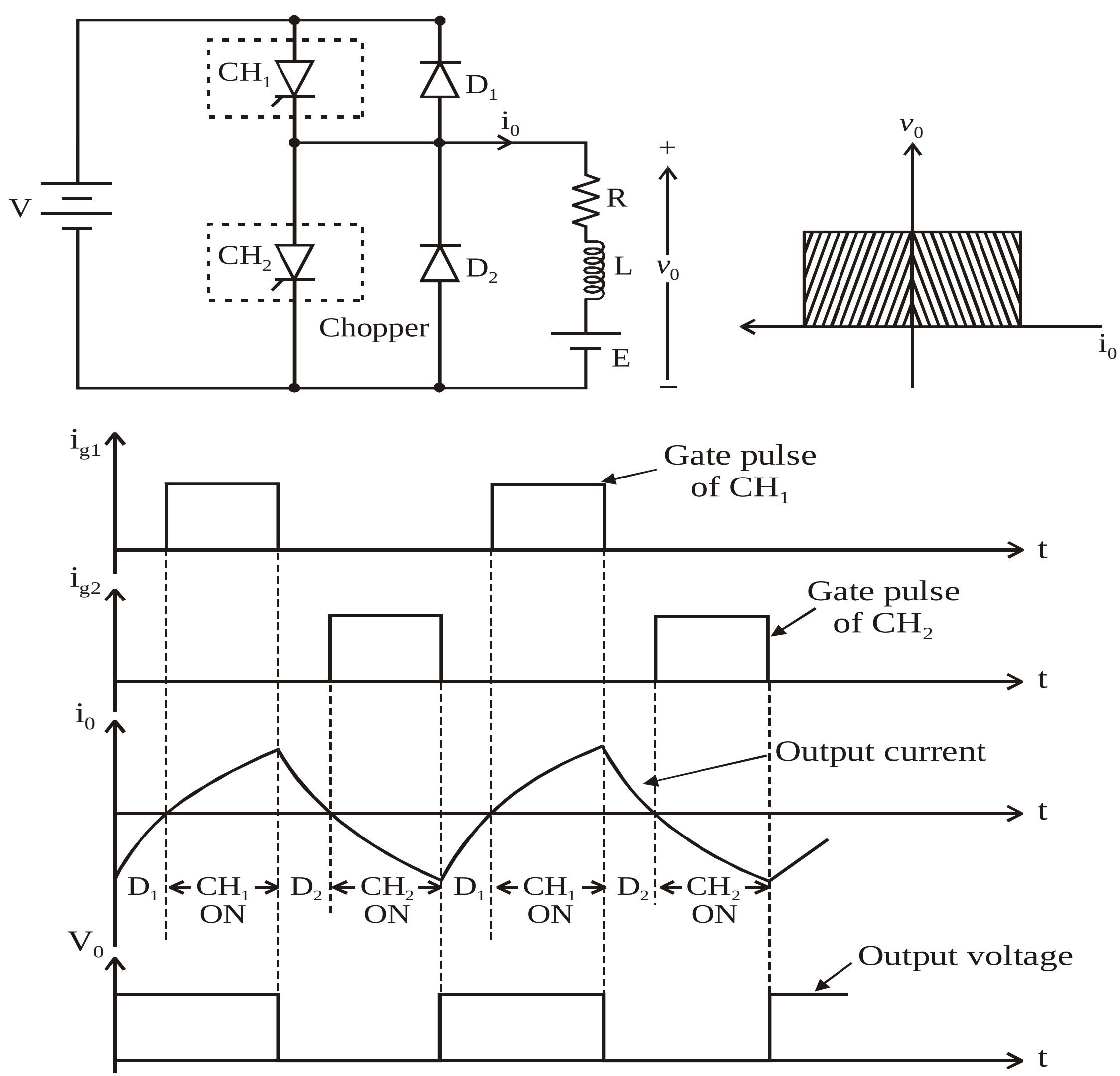
- When chopper is ON, *E* drives current through *L* and *R* in a direction opposite to that shown in figure.
- During the ON period, *L* stores energy.
- When chopper is OFF, *D* conducts and part of the energy stored in *L* is returned to the supply.
- Average output voltage is positive.
- Average output current is negative.



Class C chopper

It is a combination of class A and B choppers (step-up/step-down) and used for DC motors speed control and regenerative braking.

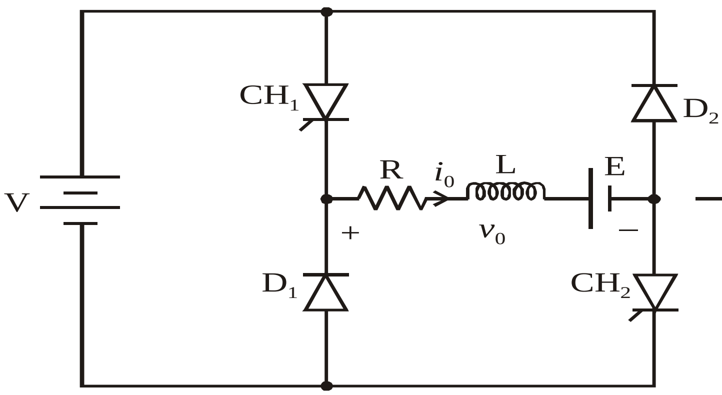
- For first quadrant operation, CH_1 is ON or D_2 conducts.
- For second quadrant operation, CH_2 is ON or D_1 conducts.
- When CH_1 is ON, load current is positive, output voltage equals *V*, and the load receives power from the source.
- When CH_1 is OFF, energy stored in *L* forces current to flow through D_2, output voltage is zero, and , load current is positive.
- When CH_2 is triggered, *E* forces current to flow in opposite direction through *L* and CH_2. The output voltage is zero.
- On turning OFF CH_2 , the energy stored in *L* drives current through D_1 and the supply. Output voltage is *V*, the input current becomes negative ,and power flows from the load to the source.
- Average output voltage is positive.
- Average output current can take both positive and negative values.
- Choppers CH_1 & CH_2 should not be turned ON simultaneously as it would result in short circuiting the supply.

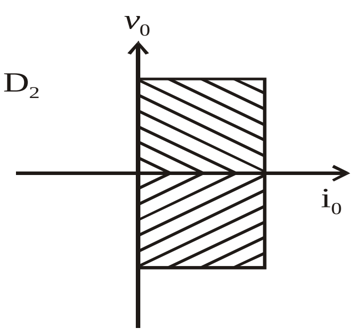


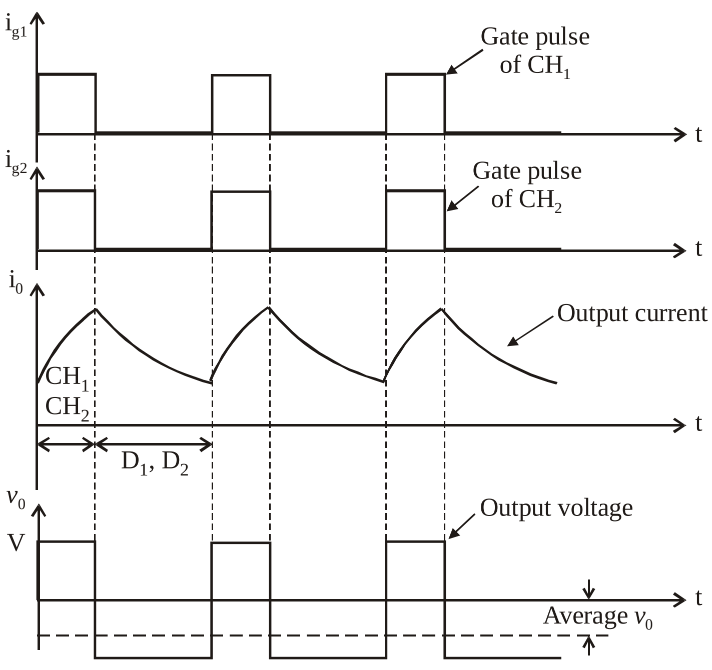
Class D chopper

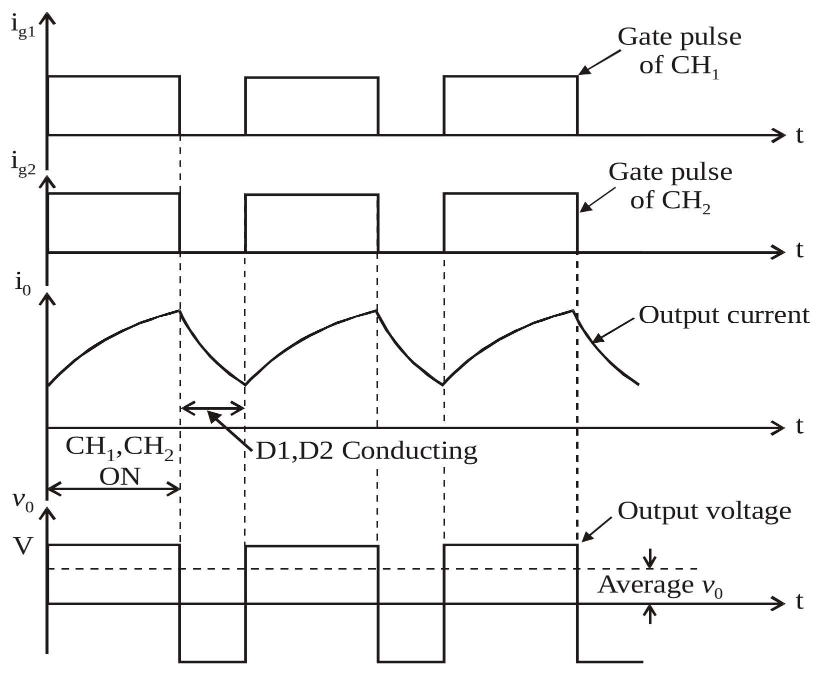
It is a two quadrant operation chopper where the current is always positive but the load voltage can be positive or negative.

- When both CH_1 and CH_2 are triggered simultaneously, the output voltage $v_O = V$ and output current flows through the load.
- When CH_1 and CH_2 are OFF, load current continues to flow in same direction through load, D_1 and D_2 , due to the energy stored L. Output voltage $v_O = - V$.
- Average load voltage is positive if $t_{ON} > t_{OFF}$
- Average output voltage becomes negative if $t_{ON} < t_{OFF}$.





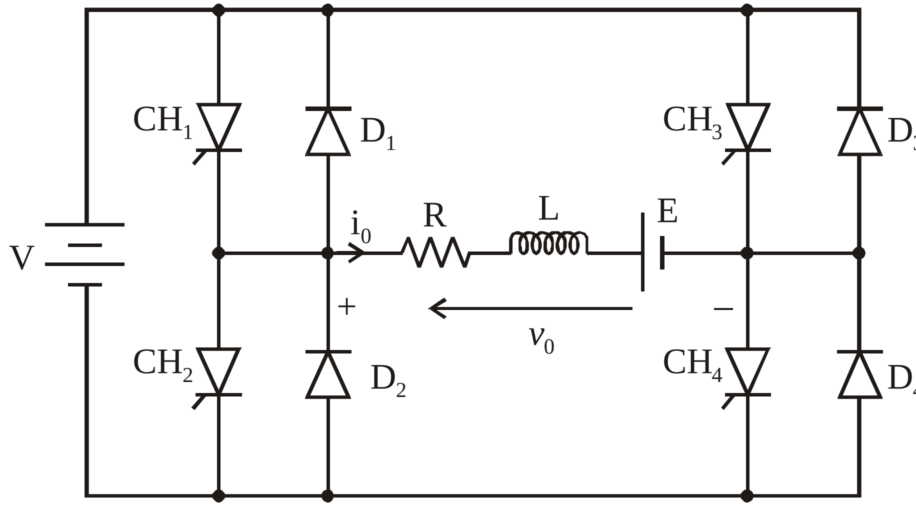


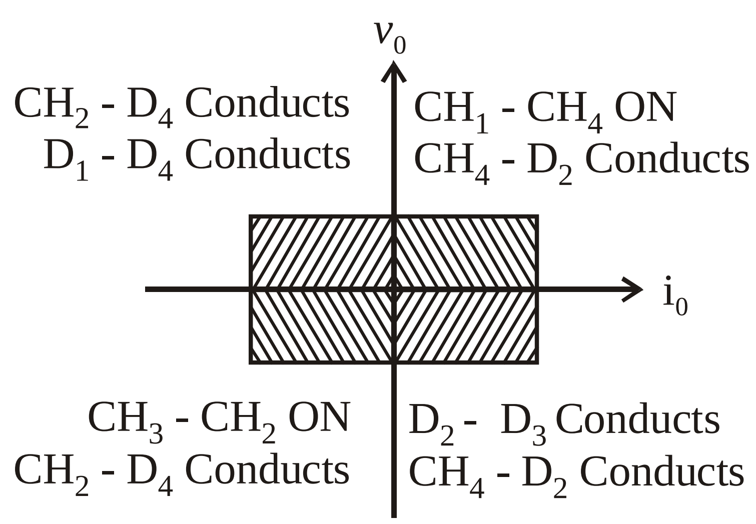


Class E chopper

It is a four quadrant operation chopper where the load current and voltage can be positive or negative.

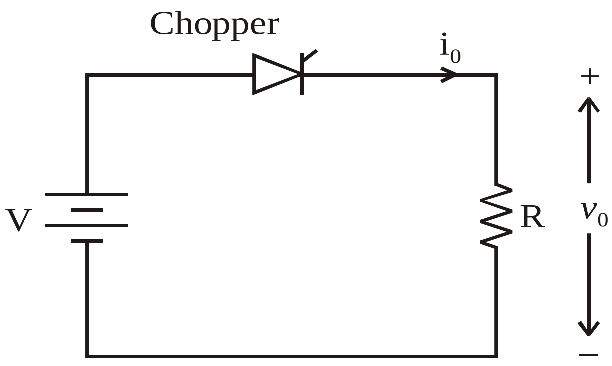
- When CH_1 and CH_4 are triggered, output current i_O flows in positive direction through CH_1 and CH_4, with output voltage $v_O = V$. This gives the first quadrant operation.
- When both CH_1 and CH_4 are OFF, the energy stored in L drives i_O through D_2 and D_3 in the same direction, but output voltage $v_O = -V$. Therefore the chopper operates in the fourth quadrant.
- When CH_2 and CH_3 are triggered, the load current i_O flows in opposite direction and output voltage $v_O = -V$. Since both i_O and v_O are negative, the chopper operates in third quadrant.
- When both CH_2 and CH_3 are OFF, the load current i_O continues to flow in the same direction D_1 and D_4 and the output voltage $v_O = V$. Therefore the chopper operates in second quadrant as v_O is positive but i_O is negative.





Problems and Solutions

Question 1 For the given dc chopper with resistive load R = 10Ω and input voltage V = 200 V. When the chopper is ON, its voltage drop is 2 V and chopping frequency is 1 kHz. If the duty cycle is 60%, determine 1) average output voltage, 2) RMS value of output voltage, 3) effective input resistance of the chopper, and 4) chopper efficiency.



Solution:

1) $$V_{dc} = d\left(V - V_{ch}\right) = 0.60\left(200-2\right) = 118.8 \text{ V}$$

2) $$\text{RMS of } V_O = \sqrt{d}\left(V - V_{ch}\right) = \sqrt{0.6}\left(200-2\right) = 153.3 \text{ V}$$

3) $$I_{dc} = \frac{V_{dc}}{R} = \frac{118.8}{10} = 11.88 \text{ A} \quad \Rightarrow R_i = \frac{V}{I_{dc}} = \frac{200}{11.88} = 16.83\Omega$$

4) $$P_O = \frac{1}{T}\int_0^{dT} \frac{v_0^2}{R} dt = \frac{1}{T}\int_0^{dT} \frac{\left(V - V_{ch}\right)^2}{R} dt = \frac{d\left(V - V_{ch}\right)^2}{R} = \frac{0.6\left(200-2\right)^2}{10} = 2352 \text{ W}$$

$$P_i = \frac{1}{T}\int_0^{dT} V i_O dt = \frac{1}{T}\int_0^{dT} \frac{V\left(V - V_{ch}\right)}{R} dt = \frac{dV\left(V - V_{ch}\right)}{R} = \frac{0.6\times 200\left(200-2\right)}{10} = 2376 \text{ W}$$

$$\text{Chopper efficiency} \quad \Rightarrow \eta = \frac{P_O}{P_i}\times 100 = \frac{2352}{2376}\times 100 = 99\%$$

Question 2 In a dc chopper, the average load current is 30 A, chopping frequency is 250 Hz, supply voltage is 110 V. Calculate the ON and OFF periods of the chopper if the load resistance is 2Ω.

Solution:

$$T = 1/f = 1/250 = 4\times10^{-3} = 4 \text{ msec}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{dV}{R} \quad \Rightarrow \quad d = \frac{I_{dc}R}{V} = \frac{30\times2}{110} = 0.545$$

$$\text{Chopper ON period} \quad \Rightarrow \quad t_{ON} = dT = 0.545\times4\times10^{-3} = 2.18 \text{ msec}$$

$$\text{Chopper OFF period} \quad \Rightarrow \quad t_{OFF} = T - t_{ON} = 4\times10^{-3} - 2.18\times10^{-3} = 1.82 \text{ msec}$$

Question 3 A chopper is supplying an inductive load with a free-wheeling diode. The load inductance is 5 H and resistance is 10Ω. The input voltage to the chopper is 200 volts and the chopper is operating at $f = 1$kHz. If the ON/OFF time ratio is 2:3. Calculate:

1) Maximum and Minimum values of load current in one cycle of the chopper.
2) Average load current.

Solution:

1)

$$L = 5 \text{ H}, \quad R = 10\Omega, \; f = 1 \text{ kHz}, \; V = 200\text{V}, \quad t_{ON} : t_{OFF} = 2:3$$

$$\text{Chopping period} \quad \Rightarrow \quad T = 1/f = 1/1000 = 1 \text{ msec}$$

$$\frac{t_{ON}}{t_{OFF}} = \frac{2}{3} \quad \Rightarrow \quad t_{ON} = \frac{2}{3}t_{OFF}$$

$$T = t_{ON} + t_{OFF} = \frac{2}{3}t_{OFF} + t_{OFF} = \frac{5}{3}t_{OFF} \quad \Rightarrow \quad t_{OFF} = \frac{3}{5}T = \frac{3}{5}\times1\times10^{-3} = 0.6 \text{ msec}$$

$$t_{ON} = T - t_{OFF} = (1-0.6)\times 10^{-3} = 0.4 \text{ msec}$$

$$\text{Duty cycle} \Rightarrow d = \frac{t_{ON}}{T} = \frac{0.4\times 10^{-3}}{1\times 10^{-3}} = 0.4$$

$$\text{Maximum load current:} \Rightarrow I_{\max} = \frac{V}{R}\left(\frac{1-e^{-\frac{dRT}{L}}}{1-e^{-\frac{RT}{L}}}\right) - \frac{E}{R}$$

Since there is no voltage source in the load circuit ($E = 0$),

$$\therefore \quad I_{\max} = \frac{200}{10}\left(\frac{1-e^{-\left(0.4\times 10\times 1\times 10^{-3}\right)/5}}{1-e^{-\left(10\times 1\times 10^{-3}\right)/5}}\right) - \frac{0}{10} = 8.0047\text{A}$$

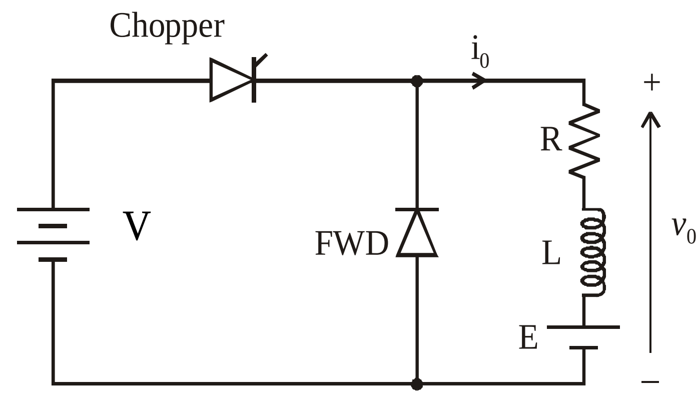
$$\text{Minimum load current with } E = 0: \Rightarrow I_{\min} = \frac{V}{R}\left(\frac{e^{\frac{dRT}{L}}-1}{e^{\frac{RT}{L}}-1}\right)$$

$$\therefore I_{\min} = \frac{200}{10}\left(\frac{e^{\left(0.4\times 10\times 1\times 10^{-3}\right)/5}-1}{e^{\left(10\times 1\times 10^{-3}\right)/5}-1}\right) = 7.995 \text{ A}$$

2) $$\text{Average load current} \Rightarrow I_{dc} = \frac{I_{\max}+I_{\min}}{2} = \frac{8.0047+7.995}{2} \approx 8 \text{ A}$$

Question 4 A chopper feeding on RL load is shown with $V = 200$ V, $R = 5\Omega$, $L = 5$mH, $f = 1$kHz, $d = 0.5$ and $E = 0$ V. Calculate : 1) Maximum and Minimum load currents, 2) average load current, and 3) effective input resistance.

Solution:



1) Chopping period: $T = \dfrac{1}{f} = \dfrac{1}{1\times10^3} = 1$ msec

Max. load current: $I_{max} = \dfrac{V}{R}\left(\dfrac{1-e^{-\frac{dRT}{L}}}{1-e^{-\frac{RT}{L}}}\right) - \dfrac{E}{R}$

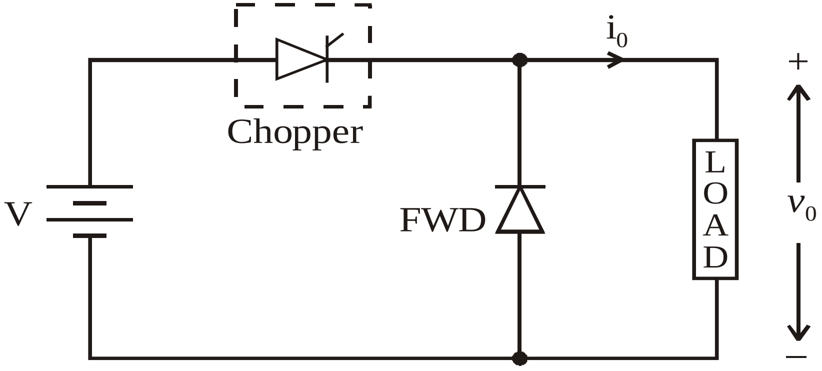
$$I_{max} = \frac{200}{5}\left(\frac{1-e^{-\left(0.5\times5\times1\times10^{3}\right)/5\times10^{-3}}}{1-e^{-\left(5\times1\times10^{3}\right)/5\times10^{-3}}}\right) - \frac{0}{5} = 24.9 \text{ A}$$

Min. load current: $I_{min} = \dfrac{V}{R}\left(\dfrac{e^{\frac{dRT}{L}}-1}{e^{\frac{RT}{L}}-1}\right) - \dfrac{E}{R} = \dfrac{200}{5}\left(\dfrac{e^{\frac{0.5\times5\times1\times10^{-3}}{5\times10^{-3}}}-1}{e^{\frac{5\times1\times10^{-3}}{5\times10^{-3}}}-1}\right) - \dfrac{0}{5} = 15.1$ A

2) Average load current: $I_{dc} = \dfrac{I_{max}+I_{min}}{2} = \dfrac{24.9+15.1}{2} = 20$ A

3) Effective input resistance: $R_i = \dfrac{V}{I_S} = \dfrac{V}{dI_{dc}} = \dfrac{200}{0.5\times20} = 20\Omega$

Question 5 For the given first quadrant chopper, express the following variables as functions of V, R and duty cycle d in case of resistive load:
1) Average output voltage and current, 2) Output current at the instant of commutation, 3) Average and RMS free wheeling diode current, 4) RMS value of output voltage, and 5) RMS and average thyristor currents.



Solution:

1) Average output voltage:

$$V_{dc} = \left(\frac{t_{ON}}{T}\right)V = dV$$

Average output current: $I_{dc} = \frac{V_{dc}}{R} = \frac{dV}{R}$

2) The thyristor is commutated at the instant $t = t_{ON}$ and the output voltage is V

$\therefore$ output current at the instant of commutation is $\Rightarrow V/R$

3) Free wheeling diode (FWD) will never conduct in a resistive load.

$\therefore$ Average & RMS free wheeling diode currents are zero.

4) $$V_{O(RMS)} = \sqrt{\frac{1}{T}\int_0^{t_{ON}} v_0^2 dt} = \sqrt{\frac{1}{T}\int_0^{t_{ON}} V^2 dt} = \sqrt{V^2\left(\frac{t_{ON}}{T}\right)} = \sqrt{d}V$$

5) RMS current of thyristor = RMS current of load $= V_{O(RMS)}/R = \sqrt{d}V/R$

Average current of thyristor = Average current of load $= dV/R$

Question 6 In a 100V dc chopper drive, the max. value of accelerating current is 425A, the lower limit of current pulsation is 180 A , the length of ON and OFF period is 14msec and 11msec, respectively . Determine: 1) the limit of current pulsation, 2) the chopping frequency, 3) the duty cycle ratio, and 4) the output voltage.

Solution:

1) $\text{Limit of current pulsation} = \text{Max. current} - \text{Min. current} = 425 - 180 = 245\ \text{A}$

2) $T = (14 + 11)\,\text{msec} = 25\ \text{msec} \Rightarrow f = 1/T = 1/\left(25 \times 10^{-3}\right) = 40\text{Hz}$

3) $d = t_{ON}/T = 14\text{msec}/25\text{msec} = 0.56$

4) $\text{Output voltage} = dV = 0.56 \times 100 = 56\text{V}$

Question 7 Step-down dc chopper with 100Ω load. For 220V dc supply, calculate the average load current if the OFF period = 6msec and the chopping frequency is 100Hz.

Solution:

$$T = \frac{1}{f} = \frac{1}{100} = 10\ \text{msec}$$

$$t_{ON} = T - t_{OFF} = 10\text{msec} - 6\text{msec} = 4\text{msec}$$

$$d = t_{ON}/T = 4\text{msec}/10\text{msec} = 0.4$$

$$I_{dc} = \frac{dV}{R} = \frac{0.4 \times 220}{100} = 0.88\text{A}$$

Question 8 A thyristor step-up chopper has the following data: V = 50V, chopping frequency = 250Hz, and t_{OFF} =2msec .Calculate V_O.

Solution:

$$T = 1/f = 1/250 = 4 \text{ msec}$$

$$V_0 = V \frac{T}{T - t_{ON}} = V \frac{T}{t_{OFF}} = 50 \times \frac{4\times 10^{-3}}{2\times 10^{-3}} = 100\text{V}$$

Question 9 A step-down chopper is connected to 60V dc supply and 10Ω resistive load. If the load voltage waveform has width of 2msec from 5msec periodic time, calculate : 1) I_{dc} , 2) R_{in} , and 3) Chopping frequency f .

Solution:

1) $$I_{dc} = Vd/R = V\left(t_{ON}/T\right)/R = 60\times\left(2\times 10^{-3}/5\times 10^{-3}\right)/10 = 0.24\text{A}$$

2) $$R_{in} = R/d = R/\left(t_{ON}/T\right) = 10/\left(2\times 10^{-3}/5\times 10^{-3}\right) = 25\Omega$$

3) $$f = 1/T = 1/5\times 10^{-3} = 200\text{Hz}$$

Question 10 A chopper circuit is operation on Time Ratio Control (TRC) principle at frequency f = 1kHz on a 220V dc supply. If the load voltage is 180V, calculate the conducting and blocking periods of the power device in each cycle.

Solution:

$$V_{dc} = Vd \quad \Rightarrow \; 180 = 200\times d \;\Rightarrow\; d = 0.8$$

$$t_{ON} = dT = d/f = 0.8/1\times 10^{3} = 0.8\text{msec}$$

$$t_{OFF} = T - t_{ON} = 1\text{msec} - 0.8\text{msec} = 0.2\text{msec}$$

Question 11

Choose the correct answer for the following Questions

1) The average value of output voltage for step-up chopper is …………………..

A. $dV/(1-d)$ B. $V/(1-d)$ C. dV D. $(1-d)V$

2) The duty ratio d is between …………………..

A. 1 and ∞ B. -1 and $+1$ C. 0 and 1 D. -1 and 0

3) DC to DC chopper operates from a 48V battery source into a resistive load of 24Ω. The frequency of the chopper is set to 250Hz. When chopper on-time is 1ms, the load power is ………………………..

A. 6W B. 12W C. 24W D. 48W

4) DC to DC chopper operates from a 48V source with a resistive load of 24Ω. The frequency of the chopper is 250Hz. When chopper on-time is 3ms, the RMS current is ………..

A. 1.5A B. 15mA C. 1.73A D. 173mA

5) An ideal chopper is operating at a frequency of 500 Hz from a 60 V battery input. It is supplying a load having 3 Ω resistance and 9 mH inductance. Assuming the load is shunted by a perfect commutating diode and assuming battery is low less, what is the mean load current at an on/off ratio of 1/1 ?

(A) 10 A (C) 20 A

(B) 15 A (D) None of the above

---

6) A two-quadrant d.c. to d.c. chopper can operate with which of the following load conditions ?

1. +ve voltage, +ve current
2. −ve voltage, +ve current
3. −ve voltage, −ve current

A. 1 only B. 1 and 2 C. 1 and 4 D. 3 and 4

---

7) For a step up d.c.-d.c. chopper with an input d.c. voltage of 220 volts, if the output voltage required is 330 volts and the non-conducting time of thyristor-chopper is 100 μs, the ON time of thyristor-chopper would be

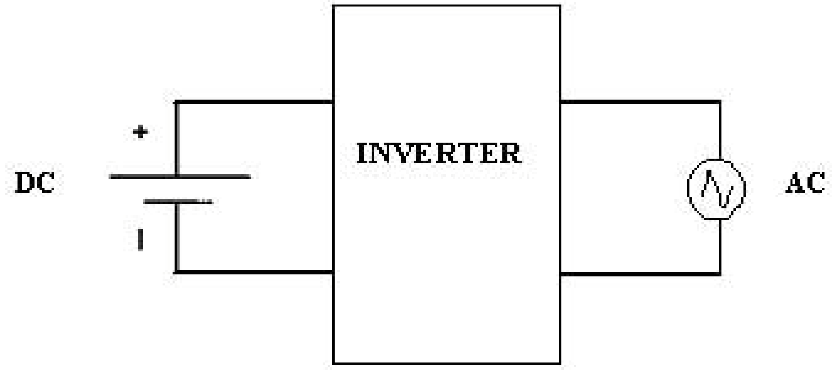
A. 66.6 μsec B. 100 μsec C. 50 μsec D. 200 μsec

Chapter 8

DC-AC Inverters

Inverters
(DC to AC Inverters)

- The converter that changes a DC to AC is called an inverter.
- Earlier inverters were built with SCR semiconductor devices.
- Since the circuitry to turn the SCR off is complex, other power devices such as BJT, MOSFETs, IGBT and MCTs.
- Currently SCRs are used in high power inverters (such as 500 kW or higher).
- Inverters are used in:
  - *Emergency lighting systems.*
  - *AC variable speed drives.*
  - *Uninterrupted power supplies UPS.*
  - *Frequency converters.*



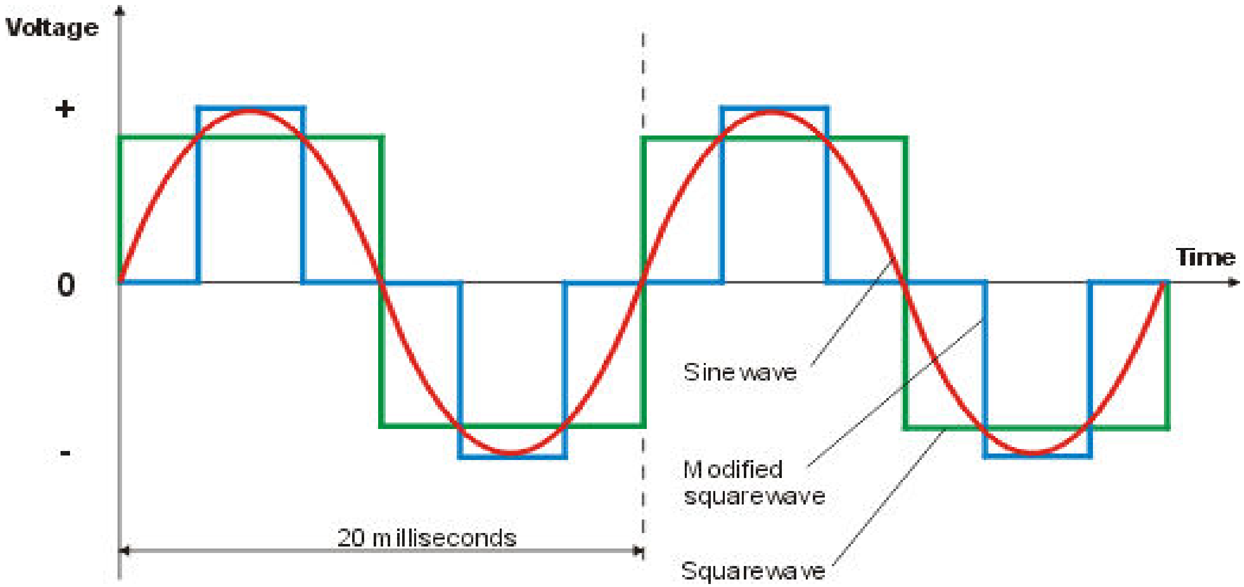
Types of Inverters

- **Single Phase Inverters:**
  - ✓ Mini inverters.
  - ✓ Parallel inverters.
  - ✓ Series inverters.
  - ✓ Bridge inverters.

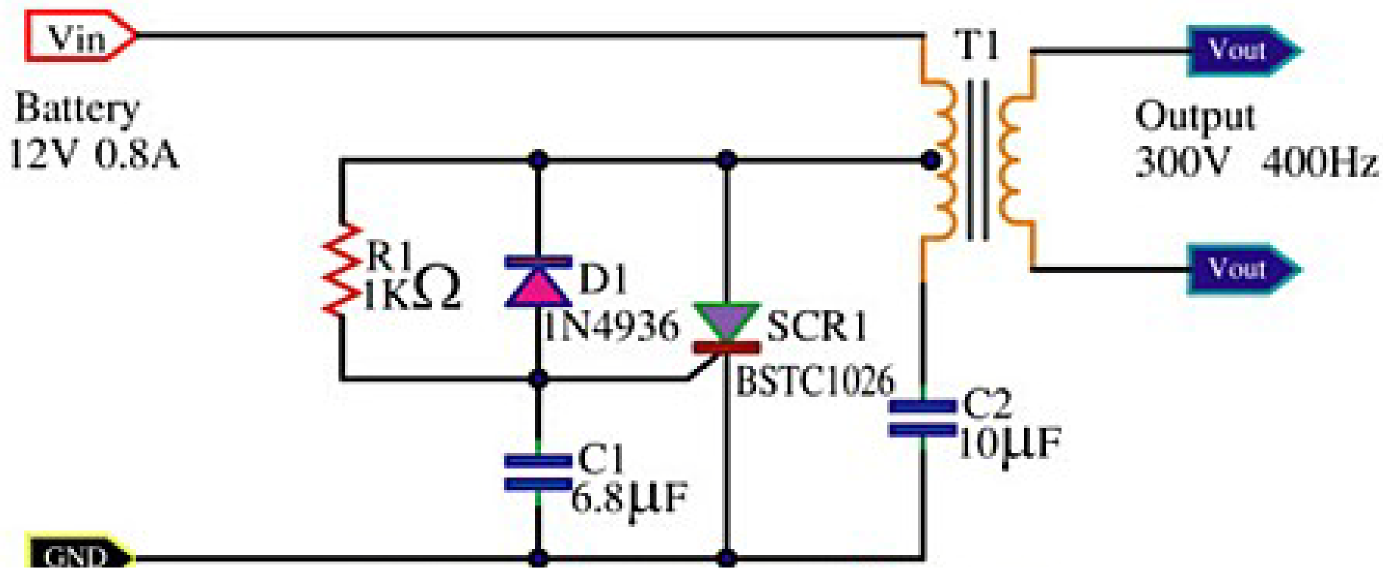
- **Three Phase Inverters:**
  - ✓ SCR bridge inverters.
  - ✓ IGBT bridge inverters.

Not all inverters outputs are the same:

- o Square Wave.
- o Modified Square Wave.
- o Stepped Sine Wave.
- o Pure Sine Wave.

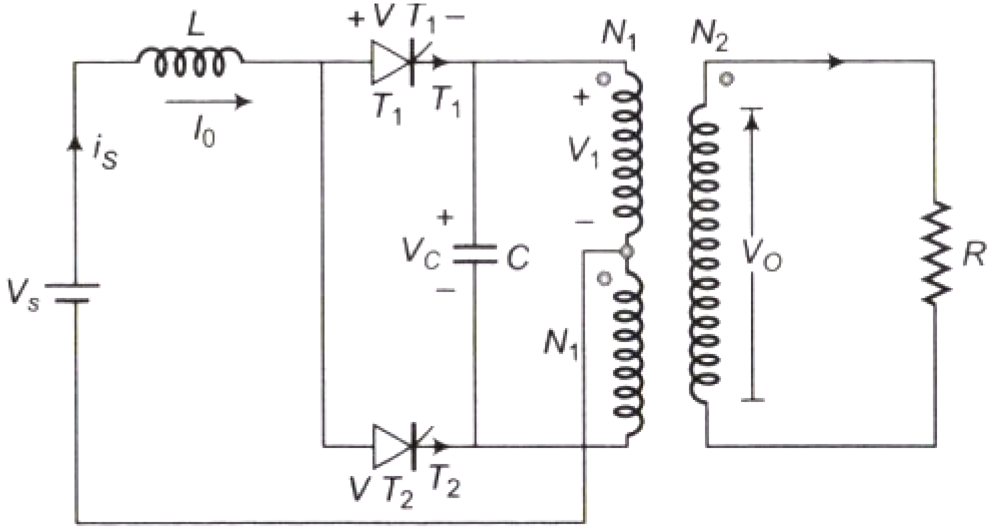


Mini Inverters



- The SCR provides 300 V of 400Hz from input voltage of 12V, 0.8A source.
- Drawback: it might latch in the conducting state if the load is too heavy or if there is a short circuit at the output. This requires some kind of protection.
- The transformer used is a 10W mains type with 6V+6V windings on the SCR side and a 110V+110V windings, in series, at the output.
- The efficiency is 50% and the ideal load is equivalent to a 22k resistor, 5W.
- The output waveform is vaguely sinusoidal at a frequency of 400Hz.

Parallel Inverters



$$f_{\max} = \frac{n^2}{48CR}$$

$$\text{Transformer ratio: } n = \frac{N_2}{N_1} = \frac{V_O}{V_S}$$

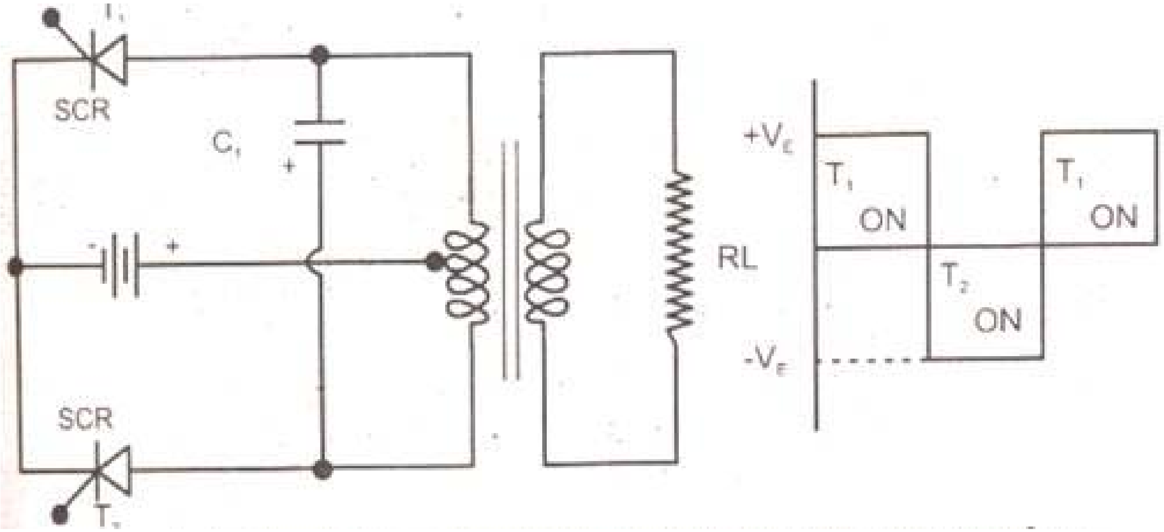
$$t_{ON} = \frac{4CR}{n^2}; \qquad t_{OFF} = \frac{Ln^2}{R}$$

$$0.25 < \frac{t_{ON}}{t_{OFF}} < 3.34$$

- The basic single phase parallel inverter circuit consists of two SCRs **T1** and **T2**, an inductor ***L***, an output transformer and a commutating capacitor ***C***.
- The function of ***L*** is to make the source current constant.
- During the work of this inverter, capacitor ***C*** comes in parallel with the load via the transformer (so it is called as a parallel inverter).

Example 1 Design a single phase parallel inverter of maximum frequency f_{max} = 400Hz using 12V battery to supply an AC load of [120W, 240V] .

Solution:



$$n = \frac{V_O}{V_S} = \frac{240}{12} = 20$$

$$P = \frac{(V_O)^2}{R} \Rightarrow 120 = \frac{240^2}{R}$$

$$\therefore \; R = 480\Omega$$

$$f_{max} = \frac{n^2}{48CR} \Rightarrow 400 = \frac{20^2}{48\times C\times 480} \Rightarrow C = 43\mu F$$

Let: $\frac{t_{ON}}{t_{OFF}} = 3 \Rightarrow t_{ON} = 3t_{OFF}$

$$\frac{4CR}{n^2} = 3\frac{Ln^2}{R} \Rightarrow \frac{4\times 43\times 10^{-6}\times 480}{20^2} = 3\frac{L\times 20^2}{480}$$

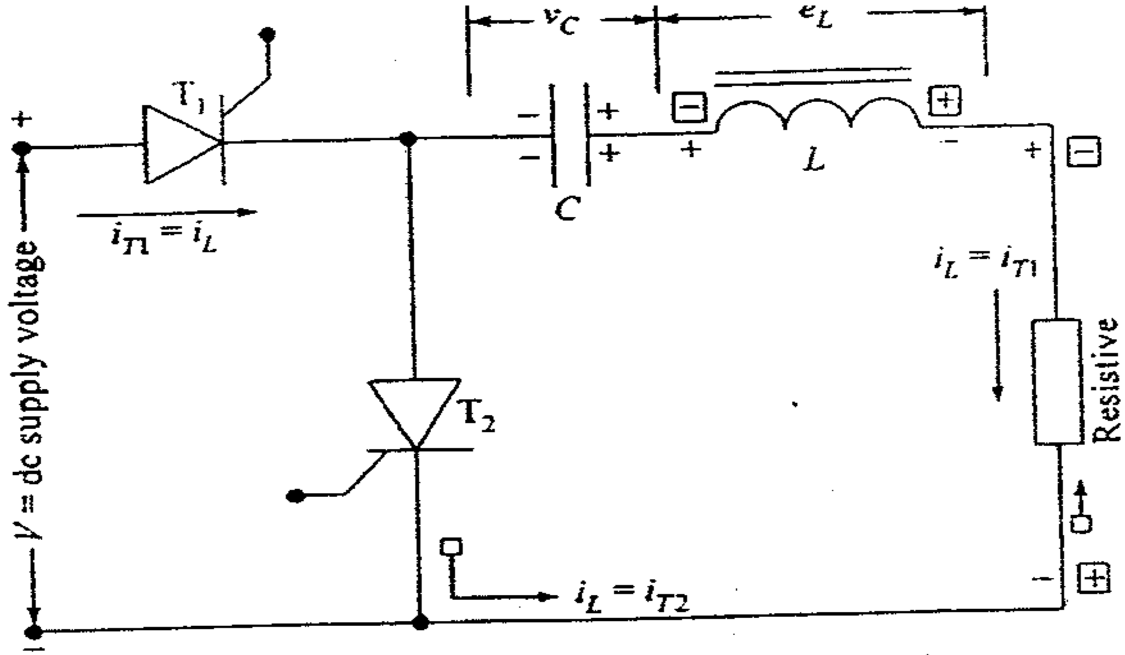
$$\therefore \; L = 90\mu H$$

Series Inverters

$$f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{4L}} \simeq \sqrt{\frac{1}{LC}}$$

$$f_{out} = \frac{1}{(T/2) + t_{Off}}$$

$$T = \frac{\pi}{f_r}$$

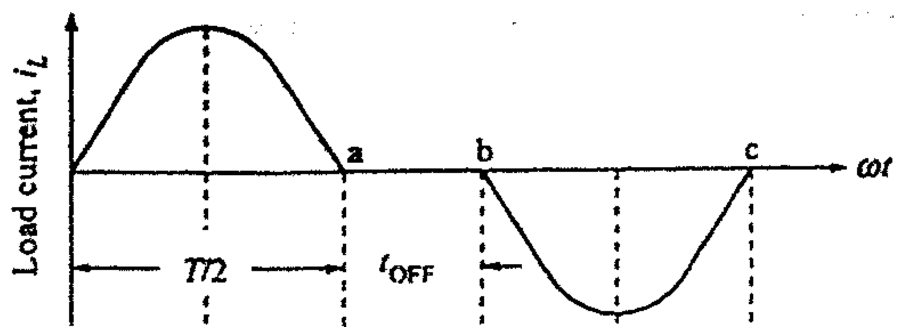


+ , − polarities correspond to the load current $i_L = i_{T1}$

⊞, ⊟ polarities correspond to the load current $i_L = i_{T2}$

⟶ Direction of the load current when T_1 conducts

o⟶ Direction of the load current when T_2 conducts



Example 2 For a series inverter of output frequency f_{out} = 50Hz , the time period between turn-off of one SCR and the turn-on of the other SCR is 0.012 sec. Find the resonant frequency (f_r) and time period of oscillation (T).

Solution:

$$f_{out} = \frac{1}{(T/2)+t_{Off}} \Rightarrow \quad 50 = \frac{1}{(T/2)+0.012} \Rightarrow \quad T = 0.016\sec$$

$$T = \frac{\pi}{f_r} \quad \Rightarrow f_r = \frac{\pi}{T} = \frac{\pi}{0.016} = 196.3\ Hz$$

Example 3 For a series inverter operating under resonant frequency of f_r = 2kHz . Calculate the required capacitor value if the used inductance is 6 mH.

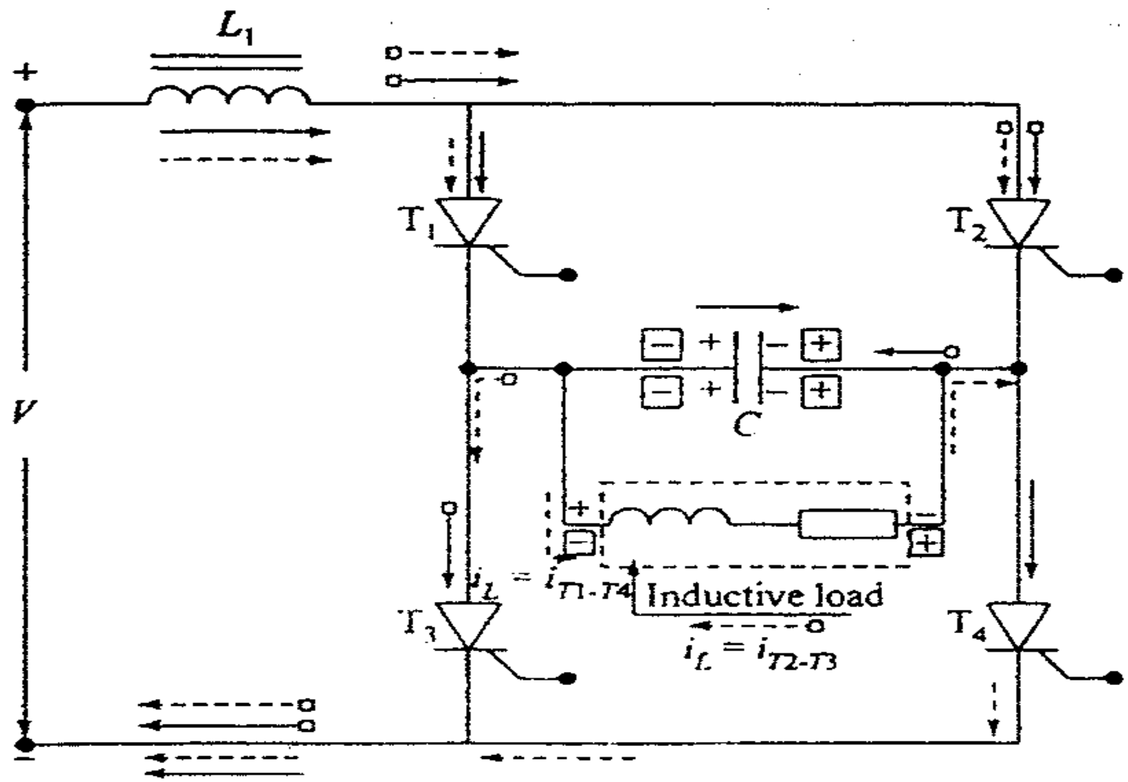
Solution:

$$f_r = \frac{1}{\sqrt{LC}}$$

$$2000 = \frac{1}{\sqrt{6\times10^{-3}\times C}} \Rightarrow \quad C = 0.416\mu F$$

Bridge Inverters

- The bridge inverters are current source inverters (**CSI**).
- Used in high frequency applications for induction heating and for solar cells power transfer.
- AC filters are required.



+ , − polarities correspond to the current $i_L = i_{T1\text{-}T4}$

⊞ , ⊟ polarities correspond to the current $i_L = i_{T2\text{-}T3}$

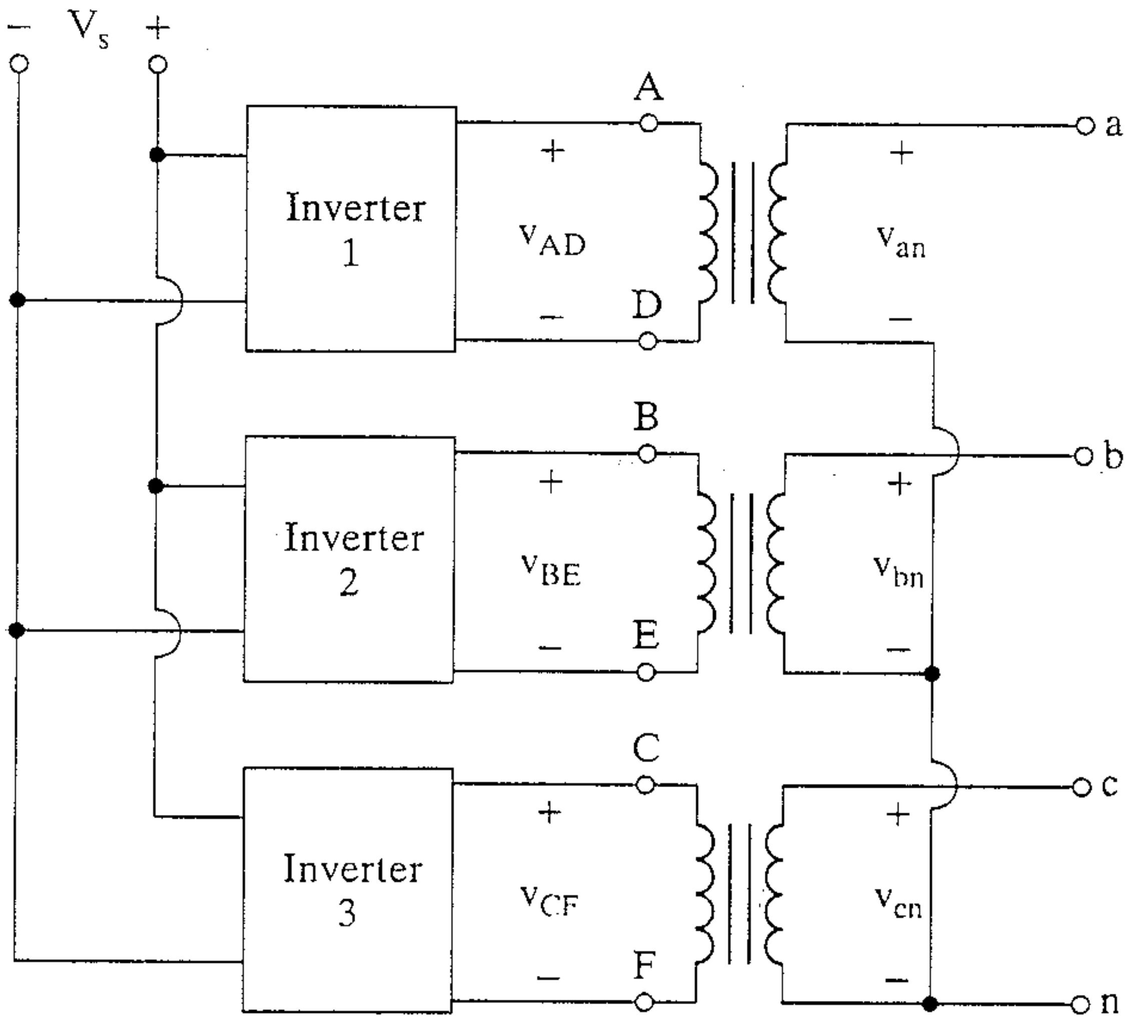
——► Charging current of the capacitor C

- - -► Direction of the load current $i_L = i_{T1\text{-}T4}$

▫——► Discharging and recharging of capacitor C

▫- - -► Direction of the load current $i_L = i_{T2\text{-}T3}$

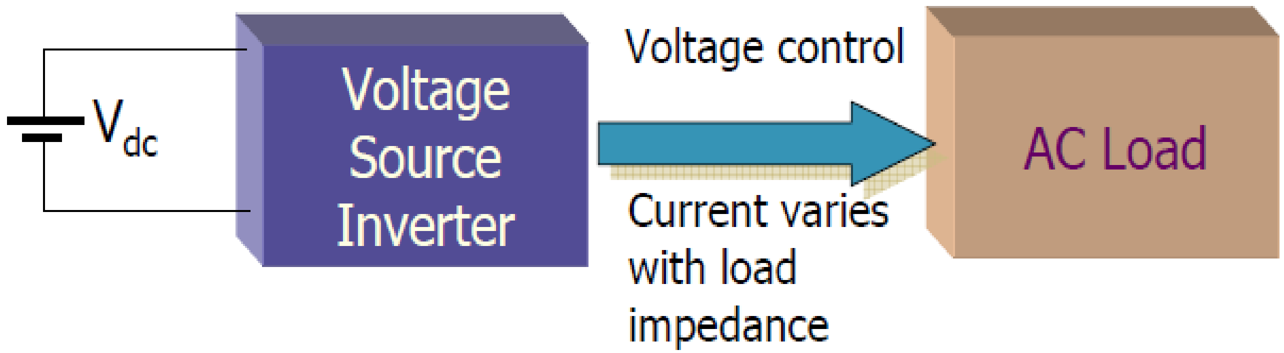
Three Phase Voltage Source Inverters (VSI)



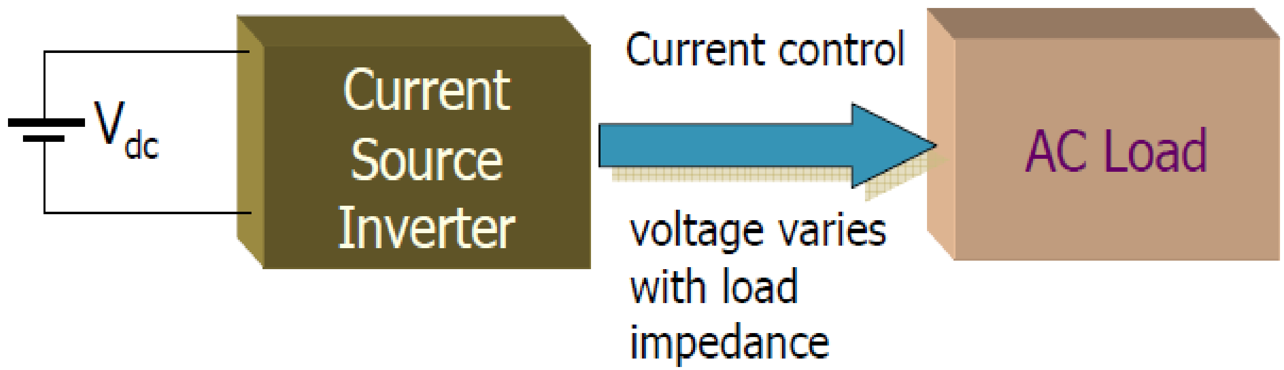
Three single-phase inverters in parallel, driven 120° apart

Inverters Control

- **Voltage Control:**



- **Current Control:**

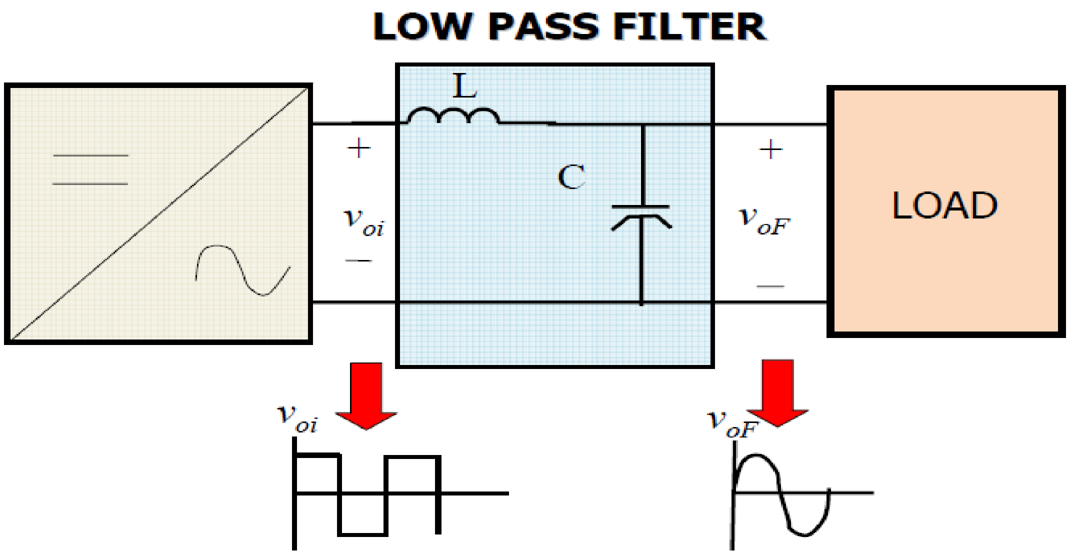


Pulse Width Modulation (PWM) for VSI

- **Objective of PWM:**
  - Control of output voltage.
  - Reduce harmonics.

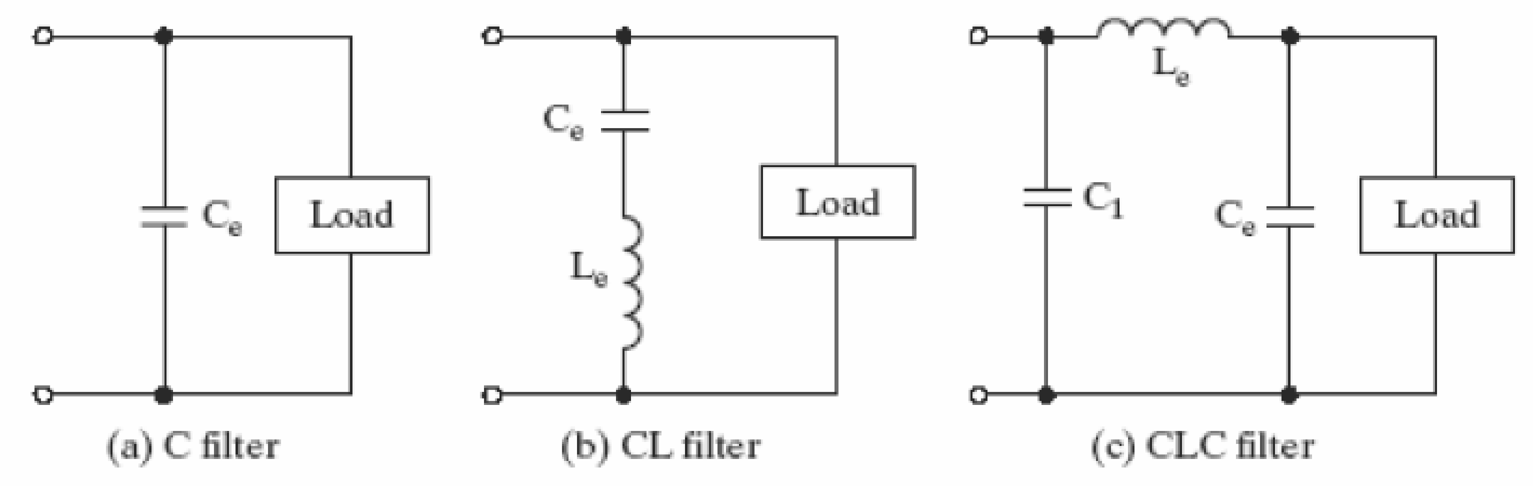
- **Disadvantages of PWM:**
  - Increase of switching losses due to high PWM frequency.
  - Reduction of available voltage.
  - EMI problems due to high order harmonics.

AC Filters for Inverters



- Output of the inverter is "chopped AC voltage with zero DC component". In some applications such as UPS, "*high purity*" sine wave output is required.
- An LC section low-pass filter is normally fitted at the inverter output to reduce the high frequency harmonics.
- In some applications such as AC motor drive, filtering is not required.

Types of AC Filters



C filter is very simple but draws more reactive power.

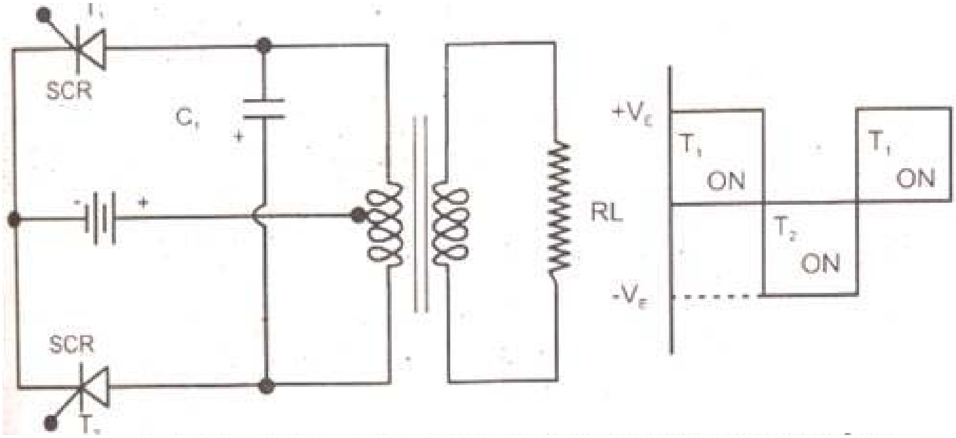
LC tuned filter can eliminates only one frequency.

CLC filter is more effective in reducing harmonics of wide bandwidth and draws less reactive power.

Problems and Solutions

Question 1 A single Phase parallel inverter has 0.68μF commutation capacitor and 500Ω resistive load. If $t_{ON}/t_{OFF} = 3$ and the centre-tap transformer has a turn ratio of n=12, calculate: 1) the maximum output frequency f_{max} , 2) find the value of L and draw the circuit diagram.

Solution:



1) $$f_{max} = \frac{n^2}{48RC} = \frac{12^2}{48 \times 500 \times 0.68 \times 10^{-6}} = 8.8\text{kHz}$$

2) $$\frac{t_{ON}}{t_{OFF}} = 3 \quad \Rightarrow t_{ON} = 3t_{OFF}$$

$$\frac{4RC}{n^2} = 3\frac{Ln^2}{R} \quad \Rightarrow \quad \frac{4 \times 500 \times 0.68 \times 10^{-6}}{12^2} = 3\frac{L \times 12^2}{500}$$

$$\therefore \quad L = 11\mu H$$

Question 2 Calculate the output frequency (f_{out}) of a series inverter in which L=10mH, C=2μf, and R=100Ω . The time period between t_{OFF} of one SCR and t_{ON} of the other SCR is 0.25ms. If the load resistance varies between 100Ω and 75Ω, find the output frequency range.

Solution:

$$f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{4L}} = \sqrt{\frac{1}{10\times10^{-3}\times2\times10^{-6}} - \frac{100^2}{4\times10\times10^{-3}}} = 500Hz$$

$$T = \frac{\pi}{f_r} = \frac{\pi}{500} = 6.28m\sec$$

$$f_{out} = \frac{1}{(T/2)+t_{Off}} = \frac{1}{(6.28\times10^{-3}/2)+0.25\times10^{-3}} = 290Hz$$

For $R = 75\Omega$ we have:

$$f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{4L}} = \sqrt{\frac{1}{10\times10^{-3}\times2\times10^{-6}} - \frac{75^2}{4\times10\times10^{-3}}} = 600Hz$$

$$T = \frac{\pi}{f_r} = \frac{\pi}{600} = 5.23m\sec$$

$$f_{out} = \frac{1}{(T/2)+t_{Off}} = \frac{1}{(5.23\times10^{-3}/2)+0.25\times10^{-3}} = 350Hz$$

$\therefore$ The frequency range is: $290Hz \le f_{out} \le 350Hz$

Question 3 **Choose the correct answer for the following Questions**

1) An inverter is an electrical device that converts ………………….. currents.

A. AC-DC B. DC-AC C. DC-DC D. AC-AC

2) The inverters performs the opposite function of the …………………..

A. regulators B. transformers C. rectifiers D. filters

3) Static inverters are used in ………………………..

A. Induction heaters B. Solar panels C. UPS D. All the above

4) Inverters are classified according to their connections as …… ………..

A. Series inverters B. Parallel inverters C. Bridge inverters
D. All the above

Chapter 9

AC-AC Converters

AC - AC Converters

- Used to convert the AC voltage to another AC voltage with waveforms of different amplitude, frequency, or phase.
- They can be single or three phase circuits.
- The AC-AC converters are classified as:
  - **AC Controllers (AC Regulators):**
    - ✓ Two SCRs or TRIACs are used under natural commutation.
    - ✓ Output voltage has variable rms value with fixed or variable frequencies.
    - ✓ Used in light-dimmers and speed control of induction motors.
  - **Cycloconverters:**
    - ✓ Converts input power at one frequency to output power at different frequency with one-stage of conversion.
    - ✓ Cycloconverters are naturally commutated and no DC link is required.
    - ✓ Maximum output frequency is a fraction of the source frequency (up to $f/3$).
    - ✓ Used in low speed ac motor drives for ranges of frequencies from 0 to 20 Hz.
    - ✓ Called also frequency changers (step-up or step-down).

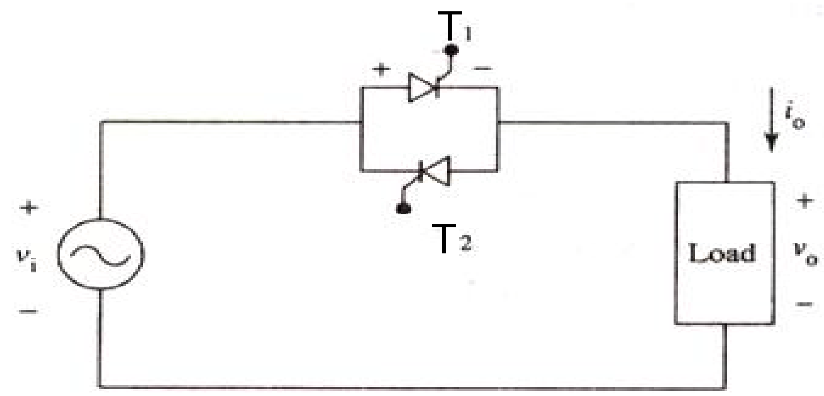
Types of AC Voltage Controllers

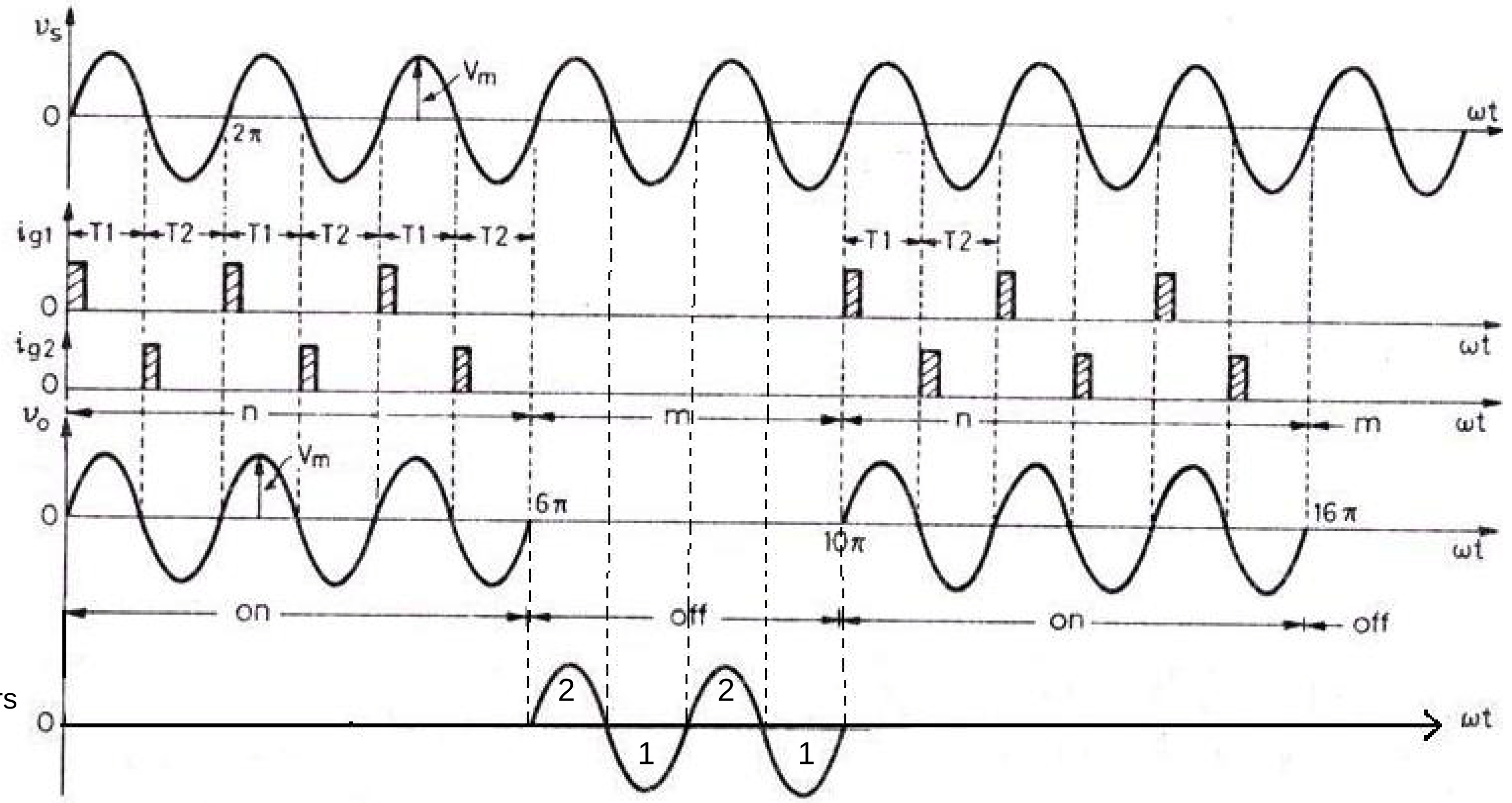
- **Integral Cycle Control (On-Off Control):**
  - ✓ Load power is controlled by connecting the source for few cycles then disconnecting for another number of cycles, periodically.
  - ✓ Suitable for systems with large time constants.
  - ✓ Average load power can be varied from 0% through 100%.
  - ✓ Used for heating loads and speed control.

- **Phase Control:**
  - ✓ The SCRs are fired once every cycle and the firing angle can be delayed from the zero crossing.
  - ✓ Only part of the voltage appears across the load providing voltage control.
  - ✓ Load current appears in pulses and its zero crossing is used to turn-off the SCRs.
  - ✓ Suitable for loads with short time constants.
  - ✓ Firing instants of the SCRs are 180° apart, each occurs in its respective half cycle.
  - ✓ By controlling the firing instant, the effective load voltage can be varied.
  - ✓ Since the two SCRs having firing pulses delayed by 180°, the AC current pulses are periodic and symmetrical without DC current component.

Integral Cycle Control

Single Phase AC voltage Control with Resistive Load





Voltage across thyristors

The ratio of ON time to the total cycle time T controls the average load power as well as the rms output voltage.

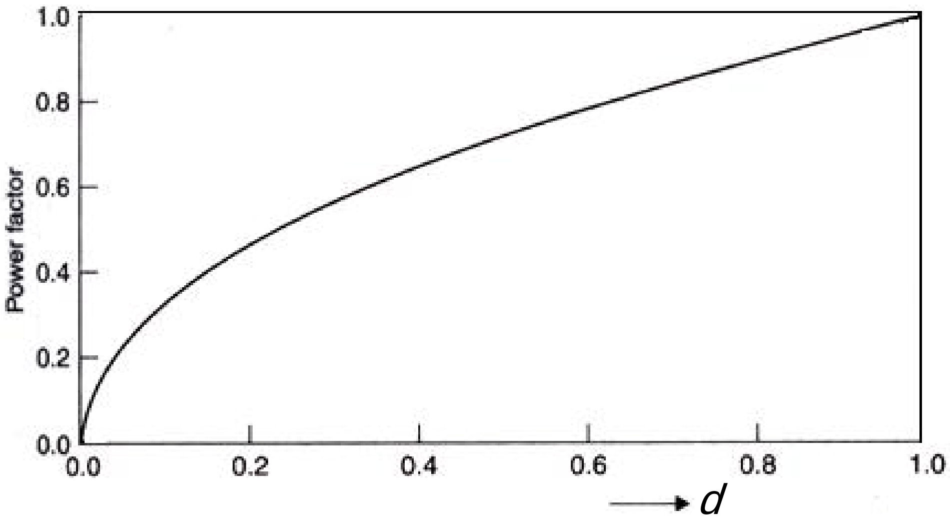
Duty cycle: $d = \frac{t_{ON}}{T}$

Average load power: $P_{O(avg)} = \frac{V_i^2 t_{ON}}{RT} = \frac{V_i^2 d}{R} = P_{O(\max)} d$

RMS output voltage: $V_{O(rms)} = \frac{V_m}{\sqrt{2}} \sqrt{d} = V_i \sqrt{d}$

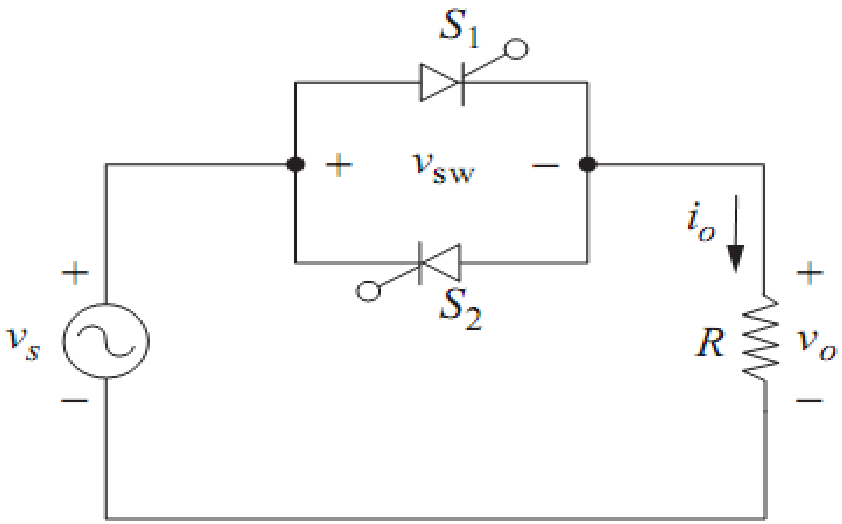
Power factor: $PF = \sqrt{d}$

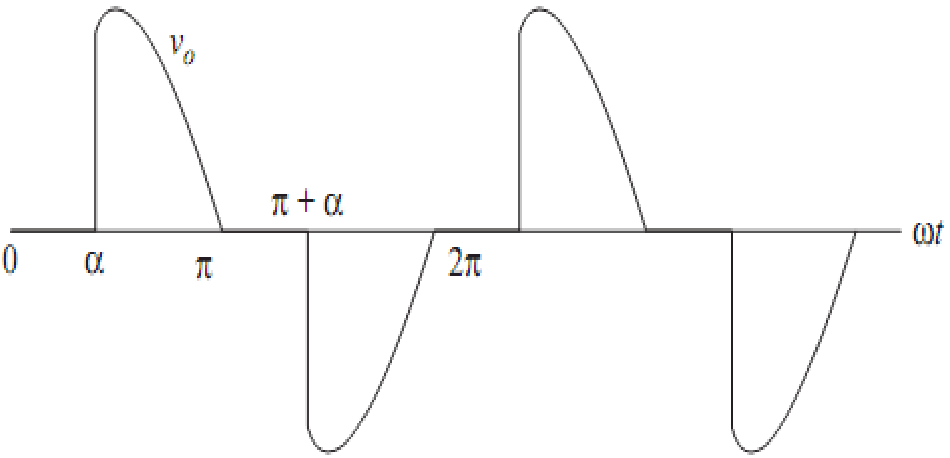
- The Source current is always in time phase with source voltage.
- However, the integral cycle circuit does not operate at unity PF.
- For part of time, the source current is not present at all and therefore is not in phase with source voltage.



Phase Control

Single Phase AC phase Control with Resistive Load





$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } \alpha < \omega t < \pi \text{ and } \alpha + \pi < \omega t < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

$$V_{o,\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$I_{o,\text{rms}} = \frac{V_{o,\text{rms}}}{R}$$

$$I_{\text{SCR,rms}} = \frac{I_{o,\text{rms}}}{\sqrt{2}}$$

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{V_{o,\text{rms}}^2 / R}{V_{s,\text{rms}}(V_{o,\text{rms}}/R)} = \frac{V_{o,\text{rms}}}{V_{s,\text{rms}}}$$

$$= \frac{\frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{(\sin 2\alpha)}{2\pi}}}{V_m/\sqrt{2}}$$

$$\text{pf} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$I_{\text{SCR,avg}} = \frac{1}{2\pi}\int_{\alpha}^{\pi} \frac{V_m \sin(\omega t)}{R}\, d(\omega t) = \frac{V_m}{2\pi R}(1 + \cos\alpha)$$

Example 1

TRIAC AC controller supplies an electrical furnace of 8Ω heating element with [240V , 50Hz] voltage supply and integral cycle control circuit that allows the current to flow for 2 cycles ON and one OFF cycle.

Calculate: 1) the average load power, and 2) the PF.

Solution:

$$d = \frac{t_{ON}}{T} = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{2}{2+1} = \frac{2}{3}$$

1) $$P_{O(avg)} = \frac{V_i^{\,2} d}{R} = \frac{240^2 (2/3)}{8} = 4800\ W$$

2) $$PF = \sqrt{d} = \sqrt{2/3} = 0.816$$

Example 2

Thyristor AC Controllers with 7Ω resistive load is connected to ac voltage source V_{IN} = 350 sin(315t). If the conduction time of each thyristor is 2.5 msec, calculate the load power .

Solution:

$$\theta_C = \omega t_C = 315 \times 2.5 \times 10^{-3} = 0.7875 rad = 45^o$$

$$\alpha = 180 - \theta_C = 180^o - 45^o = 135^o; \quad V_m = 350\ \text{Volt}$$

$$V_{O(rms)} = \frac{V_m}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} = 73.3\ V \quad \Rightarrow \quad P = \frac{\left[V_{O(rms)}\right]^2}{R} = \frac{[75.3]^2}{7} = 810\ W$$

Cycloconvertors

- **Types of Cycloconverters :**
  - Single phase to single phase cycloconvertor.
  - Three phase to three phase cycloconvertor.
  - Single phase to three phase cycloconvertor.

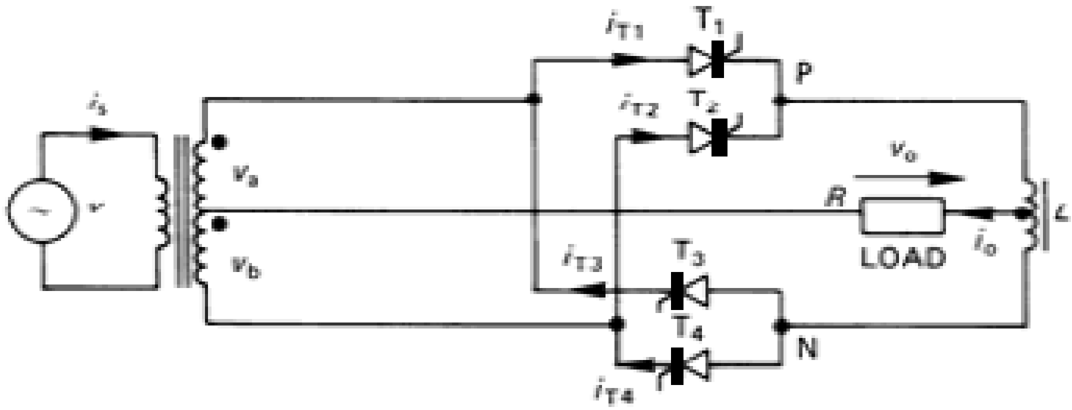
Single Phase Cycloconvertor

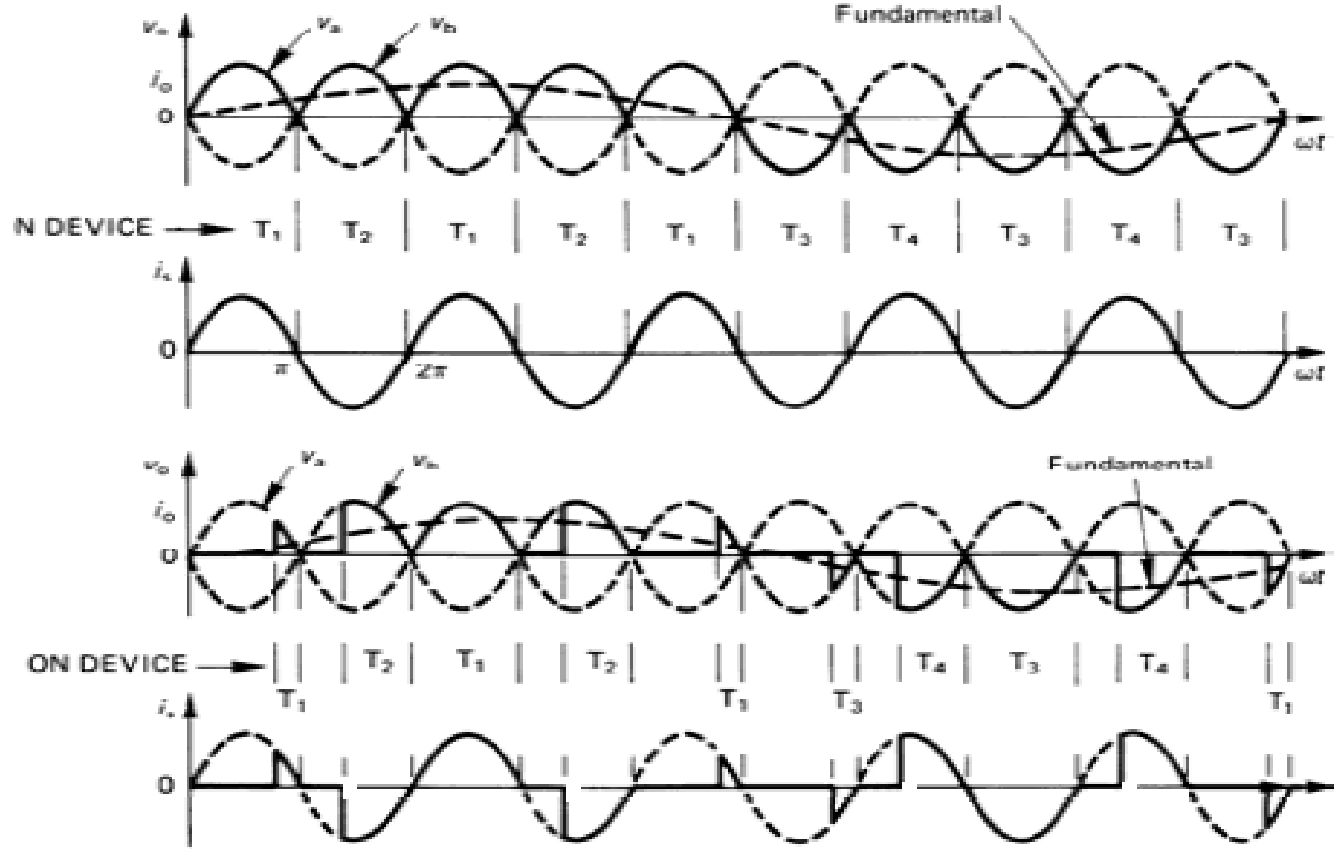
- First, two single-phase controlled converters are operated as bridge rectifiers.
- The delay angles are configured such that the output voltage of one converter is equal and opposite to that of other converter.

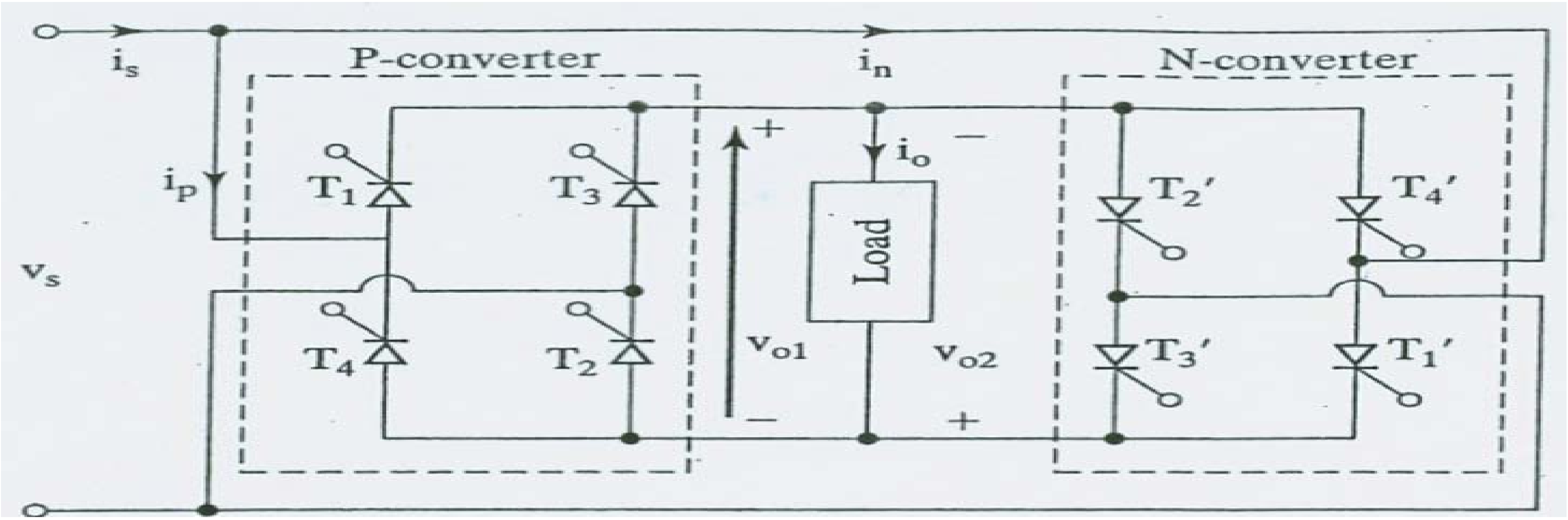
$$V_{dc} = \frac{2V_m}{\pi}\cos(\alpha) = 0.9V_{rms}\cos(\alpha)$$

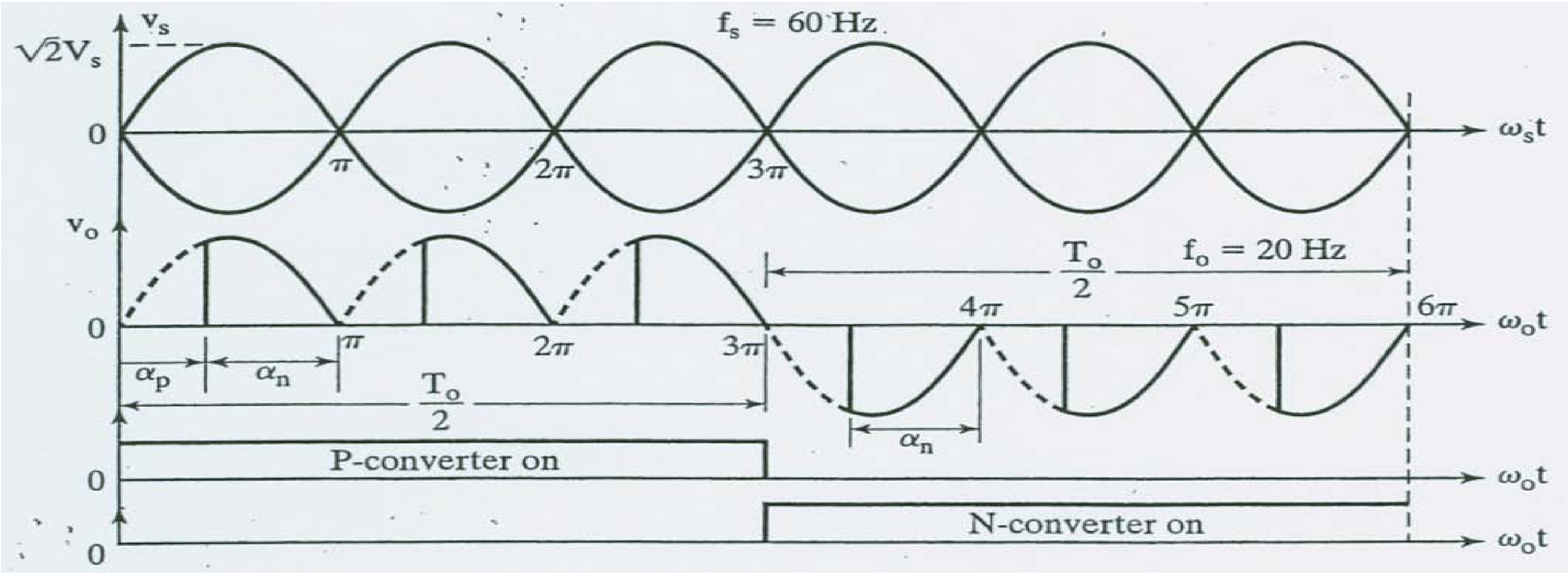
- If α_p is the delay angle of the positive converter, the delay angle of the negative converter is :

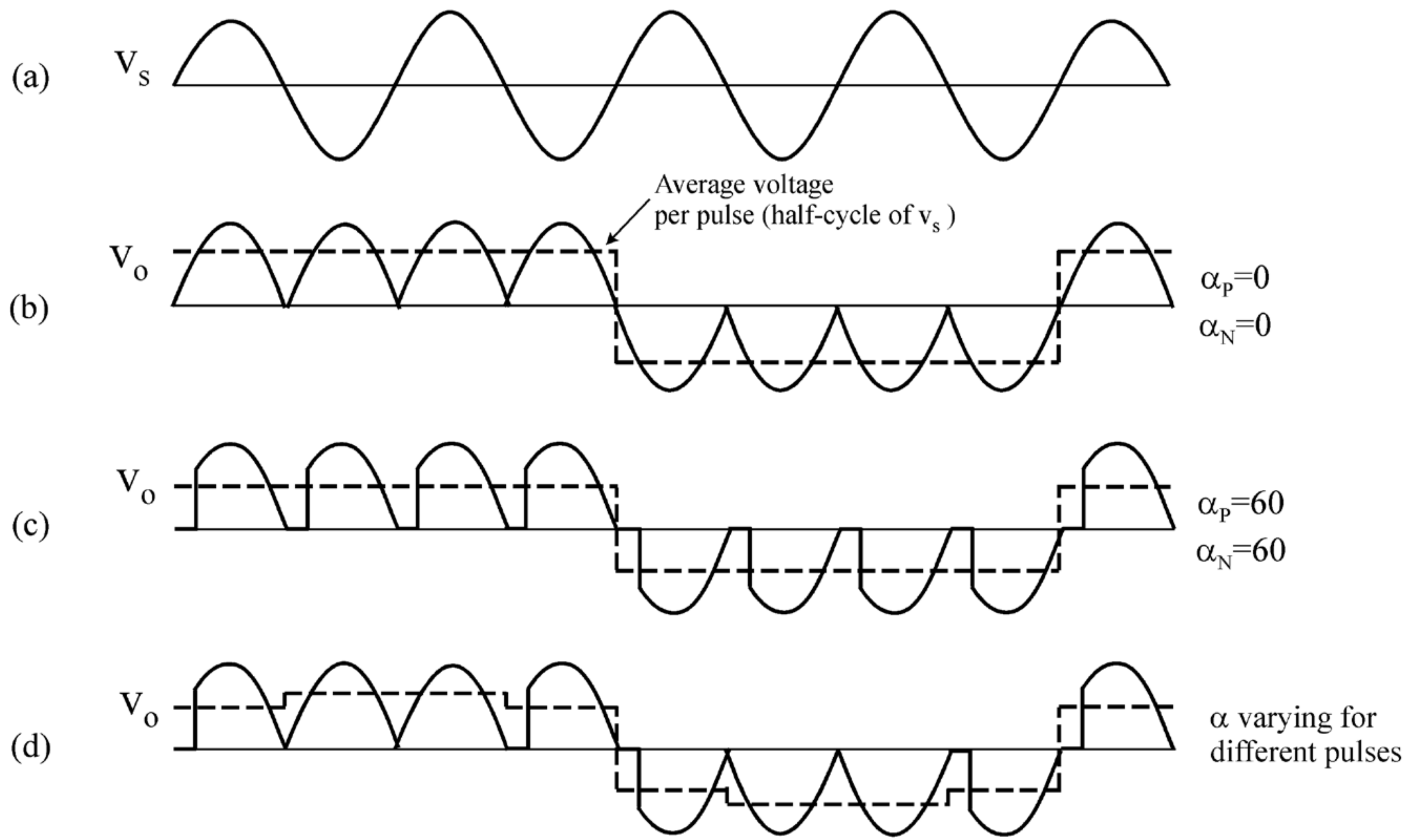
$$\alpha_n = \pi - \alpha_p$$


T1
T2
T3
T4
P
N
LOAD
R
L


Fundamental
N DEVICE
ON DEVICE
0


P-converter
N-converter
Load
T1
T3
T4
T2
T2′
T4′
T3′
T1′
i_s
i_n
i_p
i_o
v_s
v_{o1}
v_{o2}


$f_s = 60$ Hz
$f_o = 20$ Hz
$\sqrt{2}V_s$
v_s
v_o
$\omega_s t$
$\omega_o t$
α_p
α_n
$\frac{T_o}{2}$
P-converter on
N-converter on



a) input voltage **b)** output voltage for zero firing angle
c) output voltage with firing angle π/3 rad. **d)** output voltage with varying firing angle.

Problems and Solutions

Question 1 1Φ AC to AC Converter with 100Ω resistive load . If the control mode is integral cycle of 4 ON cycles from 5 total cycles and the supply voltage is [240 V, 50Hz], find 1) the average load power and 2) the supply power factor.

Solution:

$$d = \frac{t_{ON}}{T} = \frac{4}{5}$$

1) $$P_{O(avg)} = \frac{V_i^{\,2} d}{R} = \frac{240^2 \left(4/5\right)}{100} = 460.8\ W$$

2) $$PF = \sqrt{d} = \sqrt{4/5} = 0.894$$

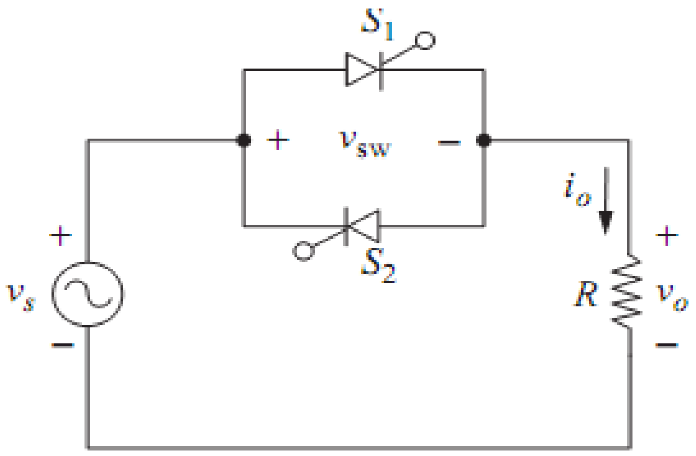
Question 2 1Φ AC voltage controller feeding a pure resistance load of rms voltage of 200V. If the rms voltage of the source is 250V, find the power factor of the controller.

Solution:

$$PF = \frac{P}{S} = \frac{V_{O(rms)} I_{O(rms)}}{V_{in(rms)} I_{O(rms)}} = \frac{V_{O(rms)}}{V_{in(rms)}} = \frac{200}{250} = 0.8$$

Question 3 The voltage controller shown has 120 Vrms, 60Hz source and the load resistance is 15Ω. Determine: a) the delay angle required to deliver 500 W to the load, b)The rms source current , c) The rms and average current in SCRs, and d) The power factor.

Solution:



a) $$P = \frac{\left[V_{O(rms)}\right]^2}{R} \Rightarrow 500 = \frac{\left[V_{O(rms)}\right]^2}{15}$$

$$V_{O(rms)} = \sqrt{500 \times 15} = 86.6\ V$$

$$V_{O(rms)} = \frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$86.6 = (120)\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} \Rightarrow \therefore\ \alpha = 1.54 rad = 88.1^o$$

b) $$I_{O(rms)} = \frac{V_{O(rms)}}{R} = \frac{86.6}{15} = 5.77\ A$$

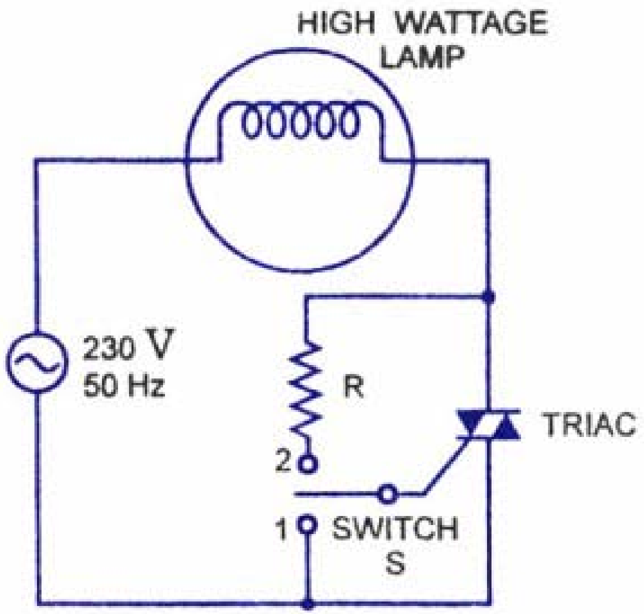
c) $$I_{SCR(rms)} = \frac{I_{O(rms)}}{\sqrt{2}} = \frac{5.77}{\sqrt{2}} = 4.08\ A$$

$$I_{SCR(avg)} = \frac{V_m}{2\pi R}\left(1 + \cos(\alpha)\right) = \frac{\sqrt{2} \times 120}{2\pi \times 15}\left(1 + \cos(88.1)\right) = 1.86\ A$$

d) $$PF = \frac{P}{S} = \frac{500}{120 \times 5.77} = 0.72$$

Question 4 For the given Triac regulator, if the rated power of the lamp is 1KW, find the approximate value of that reduces the wattage of the lamp to 500W with switch at position 2.

Solution:



$$P = \frac{\left[V_{O(rms)}\right]^2}{R} \quad ; \quad V_{O(rms)} = \frac{V_m}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$P = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R}\left(1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}\right)$$

$$500 = 1000 \times \left(1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}\right)$$

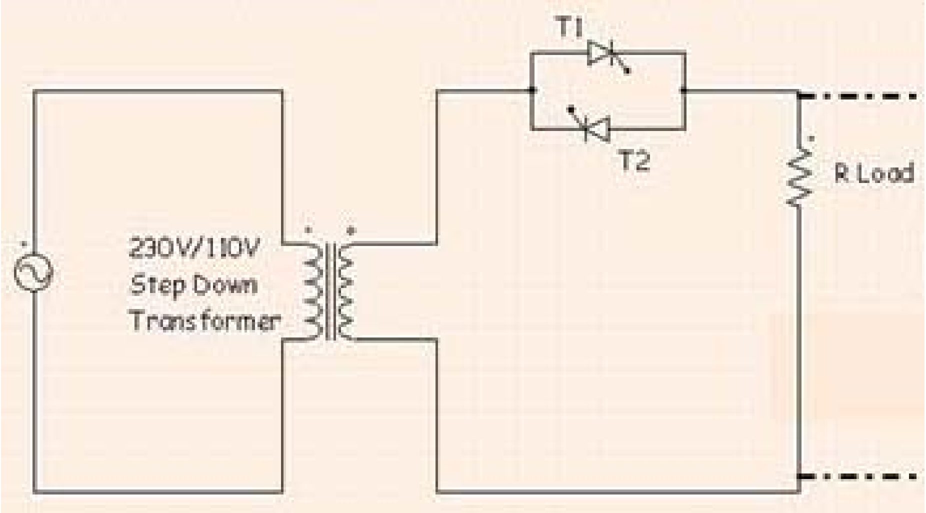
$$0.5 = 1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}$$

$$0.5\pi = \pi - \alpha + 0.5\sin(2\alpha)$$

$$\therefore \quad \alpha - 0.5\sin(2\alpha) = \pi / 2$$

By estimation, we can find that: $\alpha = \pi / 2$

Question 5 For the shown 1Φ AC controller, calculate the rms output voltage for $\alpha = 45°$ symmetrical trigger.



Solution:

$$V_{O(rms)} = \frac{V_m}{\sqrt{2}}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}}$$

$$V_{O(rms)} = 110\sqrt{1-\frac{45}{180}+\frac{\sin(2\times45)}{2\times3.14}} = 104.9\ V$$

Question 6 1Φ to 1Φ step-down cycloconvertor with the following data: [240V, 60Hz] supply, output frequency is 10Hz, the positive and negative converter groups are set to give V_{dc3} = 216V, V_{dc2} = 187V, and V_{dc1} = 108V. Find the values of firing angles.

Solution:

$$V_{dc} = 0.9V_{rms}\cos(\alpha)$$

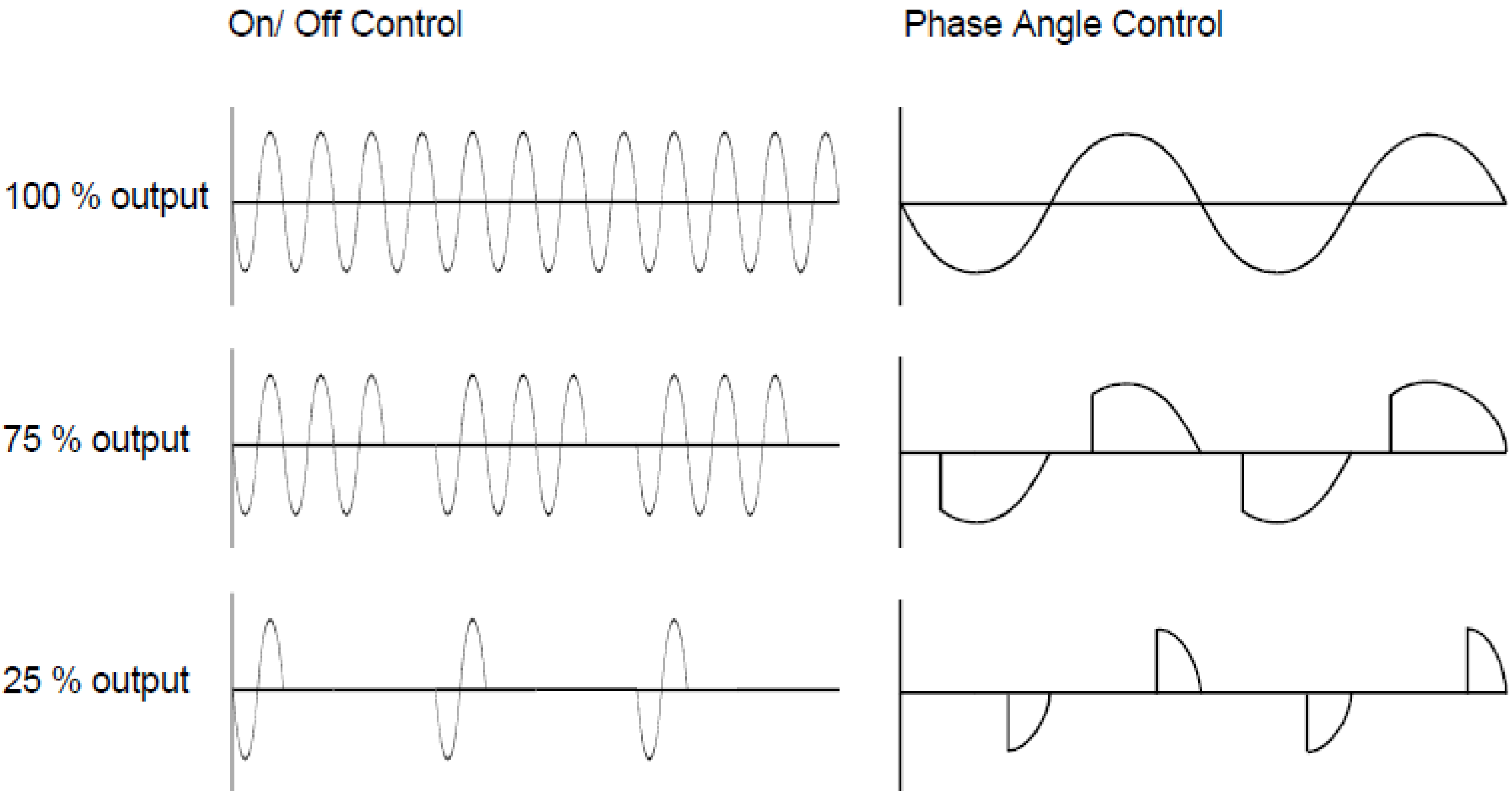
$$V_{dc3} = 0.9V_{rms}\cos(\alpha_3) \Rightarrow 216 = 0.9\times240\times\cos(\alpha_3) \Rightarrow \alpha_3 = 0^o$$

$$V_{dc2} = 0.9V_{rms}\cos(\alpha_2) \Rightarrow 187 = 0.9\times240\times\cos(\alpha_2) \Rightarrow \alpha_2 = 30^o$$

$$V_{dc1} = 0.9V_{rms}\cos(\alpha_1) \Rightarrow 108 = 0.9\times240\times\cos(\alpha_1) \Rightarrow \alpha_1 = 60^o$$

Question 7 Draw the voltage waveforms for an AC controller using On/Off and Phase control methods for the following outputs: 100%, 75% ,and 25% .

Solution:



Question 8 **Fill in the blanks with the appropriate word(s)**

1) Triac is the word derived by combining the capital letters from the words ---------------and ------------------.

2) AC Regulators are also called AC ------------------------

3) --------------- circuits are used to prevent premature triggering caused for example by voltage spikes in the AC supply or those produced by inductive loads such as motors.

4) Integral Cycle Control Method is applied to load with -------------------- element.

5) The ----------- is fabricated by integrating two thyristors in an -------- parallel connection.

Answers:

1) Triode, alternating current. 2) AC controllers. 3) Snubber. 4) heating. 5) Triac, anti

Question 9 Choose the correct answer for the following Questions

1) A cycloconverter can be

(A) Step down. (B) Step up.
(C) Step down or Step up. (D) Neither of above.

---

2) Match **List I** (Converters) with **List II** (Type of conversion) and select the correct answer :

	List I		List II
A.	Controlled Rectifier	1.	Fixed DC to variable voltage and variable frequency AC
B.	Chopper	2.	Fixed DC to variable DC
C.	Inverter	3.	Fixed AC to variable DC
D.	Cycloconverter	4.	Fixed AC to variable frequency AC

Codes :

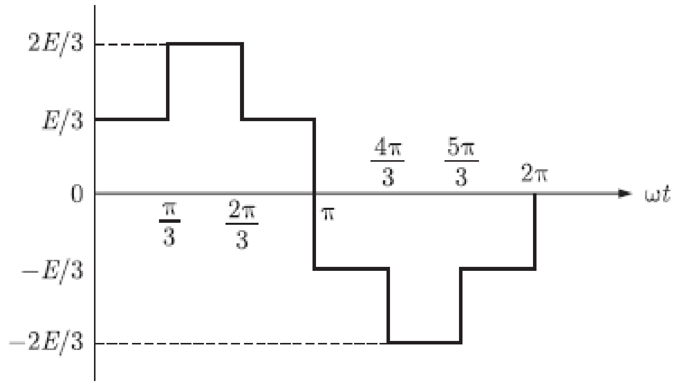
	A	B	C	D
(A)	2	3	1	4
(B)	3	2	4	1
(C)	2	3	4	1
(D)	3	2	1	4

3)

1. Single phase fully controlled ac to dc converter
2. Voltage commutated dc to dc chopper with input dc voltage E.
3. Phase voltage of a three phase inverter with 180° conduction and input dc voltage E.
4. Line voltage of a three phase inverter with 120° conduction and input dc voltage E.
5. Three-phase diode bridge rectifier.

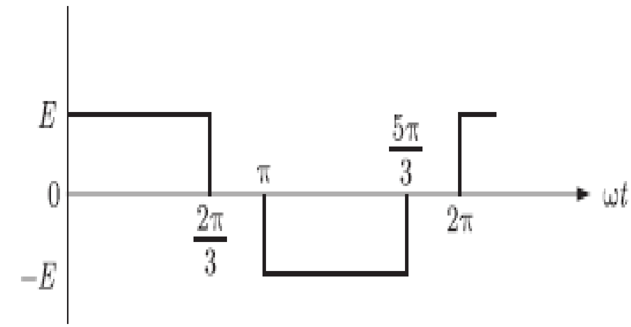
A.

3



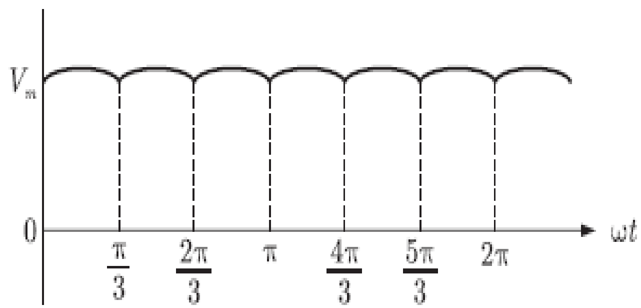
B.

4



D.

5



C.

1