Electrical Circuits

First Year

First Course

2023/2024

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اسم المادة الدوائر الكهربائية 1	لغة التدريس الانكليزية	السنة الأولى – الفصل الأول	الساعات الأسبوعية			الوحدات
			ن	ع	م	
			2	2	4	4

الأسبوع	تفاصيل المفردات
الأول	• نظام الوحدات المستخدم في الكهرباء ووحدات القياس لكل مادة (أجزائها ومضاعفاتها) تطبيقات رياضية لتحويل القيم باستخدام الوحدات. • تعريف الوحدات الأساسية الفولتية والتيار والمقاومة • مكونات الدائرة الكهربائية • قانون اوم • العوامل المؤثرة على قيمة المقاومة • المقاومة النوعية للمادة الموصلة والعازلة.
الثاني	دوائر التيار المستمر وتشمل: • ربط المقاومات على التوالي مع أمثلة • ربط المقاومات على التوازي مع أمثلة • ربط مختلط للمقاومات مع أمثلة • الربط ألنجمي والمثلثي (Y / Δ) للمقاومات والتحويل من كل منهم إلى الآخر مع أمثلة
الثالث	تطبيقات على دوائر التوالي والتوازي والربط المختلط والربط ألنجمي والمثلثي
الرابع	• قوانين كيرشوف – تعريف قانوني كيرشوف للتيار والفولتية مع حل أسئلة • ماكسويل مع حل أمثلة
الخامس	• نظرية ثيفنن – تعريف النظرية – كيفية تطبيقها في دوائر التيار المستمر • نظرية نورتن – تعريف النظرية – كيفية تطبيقها في دوائر التيار المستمر
السادس	• تطبيقات على نظرية ثيفنن ونورتن
السابع	• نظرية التطابق – تعريف النظرية – خطوات تطبيقها في حل دوائر التيار المستمر التي تحوي على أكثر من مصدر واحد – حل أمثلة • تعريف مصدر التيار ومصدر الفولتية (موزع القدرة المستمرة) وكيفية التحويل من احدهما إلى الأخر • نظرية نقل أعظم قدرة ممكنة – تعريف النظرية واشتقاق العلاقات الخاصة بها – أمثلة تطبيقية
الثامن	الكميات المتناوبة ويشمل • تعريفها خصائص التيار المتناوب – كيفية توليد التيار المتناوب ورسم الموجة له والعلاقات الخاصة به • تعريف القيمة الفعالة (RMS) ومتوسط القيمة والعلاقات الخاصة بها لإيجاد عامل التكوين وعامل القيمة لإشكال موجية غير منتظمة مع أمثلة تطبيقية

التاسع	• الكميات المتناوبة المتجهة • تعريفها التمثيل ألطوري والاتجاهي لها • زاوية الطور وكيفية إيجادها • إيجاد محصلة الكميات المتجهة ويشمل الضرب والقسمة والجمع والطرح – مع أمثلة تطبيقية
العاشر	دراسة تأثير التيار المتناوب على • دائرة تحتوي على مقاومة فقط • دائرة تحتوي على محاثة نقية فقط • دائرة تحتوي على سعة نقية فقط • إيجاد زاوية للطور بين الفولتية والتيار لكل دائرة مع حل امثلة
الحادي عشر	تأثير التيار المتناوب على دائرة تحتوي على • مقاومة ومحاثة على التوالي • دائرة تحتوي على مقاومة ومتسعة على التوالي • دائرة تحتوي على مقاومة ومحاثة ومتسعة على التوالي • أمثلة تطبيقية
الثاني عشر	تأثير التيار المتناوب على دائرة تحتوي على • مقاومة ومحاثة على التوازي • مقاومة ومتسعة على التوازي • مقاومة ومحاثة ومتسعة على التوازي • إيجاد العلاقة بين التيار والفولتية في الحالات الثلاثة – زاوية الطور – وتعريفها وكيفية إيجادها • إيجاد الممانعة – السماحية مع أمثلة تطبيقية
الثالث عشر	استخدام التوصيف 1-7 (J-Operator) أو العامل المركب لإيجاد • الممانعة الكلية • والسماحية الكلية • والتيار والفولتية • وزاوية الطور لدوائر ربط الممانعات على التوالي وعلى التوازي مع • حل أمثلة
الرابع عشر	دوائر الرنين ويشمل • دائرة رنين التوالي • تعريف حالة الرنين وكيفية الوصول إليها • حساب التيار والفولتية والممانعة وزاوية التردد عند الرنين • إيجاد عرض الحزمة • إيجاد عامل الجودة • ورسم العلاقة بين المفاعلة الحثية والمفاعلة السعوية مع التردد • حل أمثلة

الخامس عشر	دائرة رنين التوازي • تعريفها • حساب التيار والفولتية والممانعة وزاوية الممانعة وزاوية الطور وتردد الرنين • إيجاد عرض الحزمة • ورسم العلاقات البيانية مع التردد • إيجاد عامل الجودة • حل أمثلة

References

- C. K. Alexander and M. N. O. Sadiku, fundamental of electrical circuits,3rd edition, McGraw-Hill

SYSTEMS OF UNITS

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

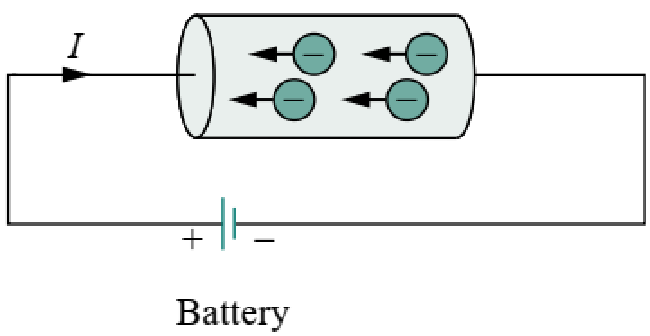
Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

CHARGE AND CURRENT

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

- The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.
- According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
- The ***law of conservation of charge*** states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

Electric current is the time rate of change of charge, measured in amperes (A).

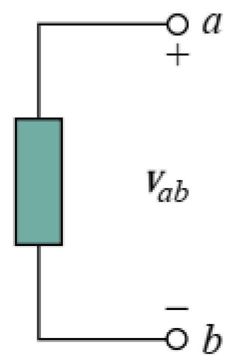


A **direct current** (dc) is a current that remains constant with time.

An **alternating current** (ac) is a current that varies sinusoidally with time.

VOLTAGE

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

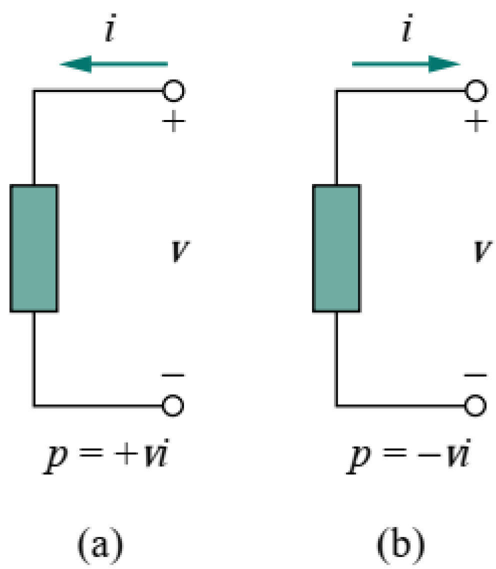


POWER AND ENERGY

Power is the time rate of expending or absorbing energy, measured in watts (W).

$$p = vi$$

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.



Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where 1 Wh = 3,600 J

The Resistance and Resistivity

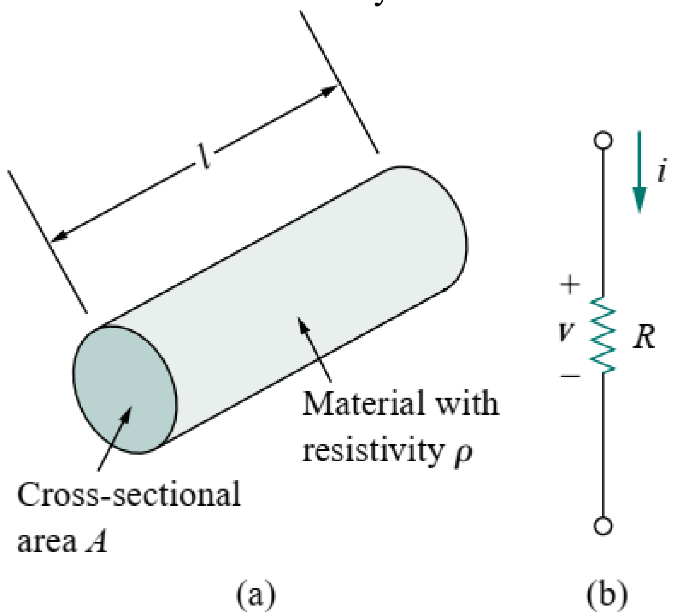
The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

- materials in general have a characteristic behavior of resisting the flow of electric charge.
- This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol *R*.

- The resistance of any material with a uniform cross-sectional area *A* depends on *A* and its length ℓ,

$$R = \rho \frac{\ell}{A}$$

where ρ is known as the resistivity of the material in ohm-meters.



Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Ex: Most homes use solid copper wire having a diameter of 1.63 mm to provide electrical distribution to outlets and light sockets. Determine the resistance of 75 meters of a solid copper wire having the above diameter.

Solution:

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.63 \times 10^{-3}\ \text{m})^2}{4} = 2.09 \times 10^{-6}\ \text{m}^2$$

$$R = \frac{\rho \ell}{A} = \frac{(1.723 \times 10^{-8}\ \Omega\text{-m})(75\ \text{m})}{2.09 \times 10^{-6}\ \text{m}^2} = 0.619\ \Omega$$

Ohm's law

> **Ohm's law** states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v \propto i$$

$$R = \frac{v}{i}$$

Ex: An electric iron draws 2 A at 120 V. Find its resistance

Solution:

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Ex: In the circuit shown, calculate the current i, and the power p.

Solution:

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6\ \text{mA}$$

$$p = vi = 30(6 \times 10^{-3}) = 180\ \text{mW}$$

or

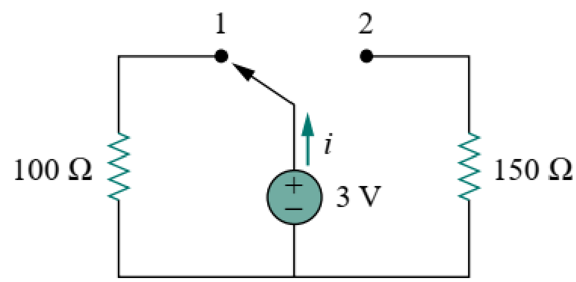
$$p = i^2 R = (6 \times 10^{-3})^2 5 \times 10^3 = 180\ \text{mW}$$

Review Questions

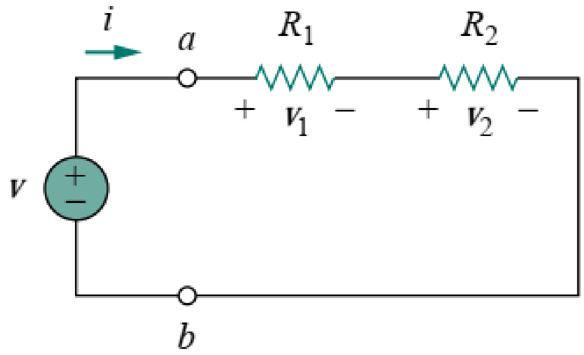
Ex: Find the resistance of a 100-m long tungsten wire which has a circular cross-section with a diameter of 0.1 mm. the resistivity of tungsten is $5.485\ 10^{8}\ \Omega.m$
Answer: 698 Ω

Ex: The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 12 Ω at 110 V?
Answer: 9.167 A.

Ex: (a) Calculate current i in Fig. below when the switch is in position 1.
(b) Find the current when the switch is in position 2



SERIES RESISTORS AND VOLTAGE DIVISION



$$v_1 = iR_1, \qquad v_2 = iR_2$$

$$-v + v_1 + v_2 = 0$$

$$v = v_1 + v_2 = i(R_1 + R_2)$$

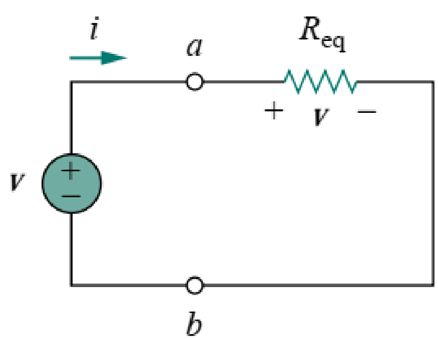
$$i = \frac{v}{R_1 + R_2}$$

$$v = iR_{eq}$$

$$R_{eq} = R_1 + R_2$$

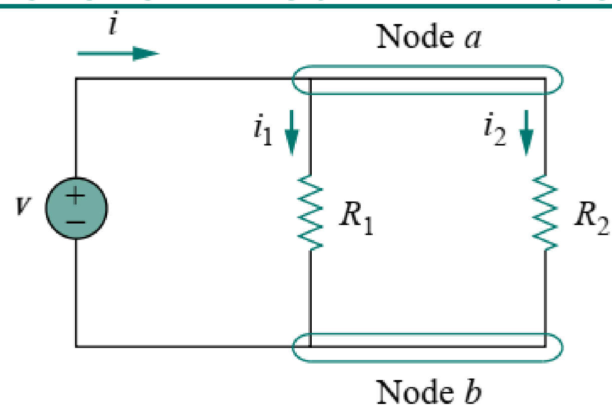
The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$



$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

PARALLEL RESISTORS AND CURRENT DIVISION



$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

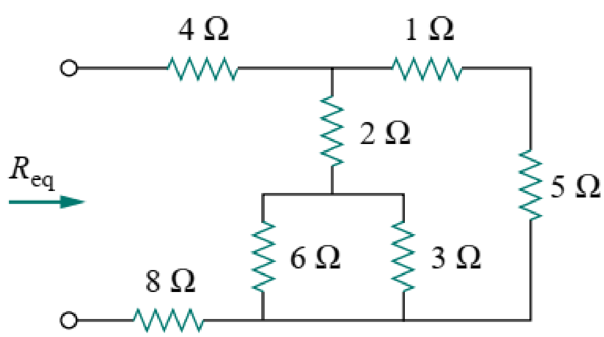
The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

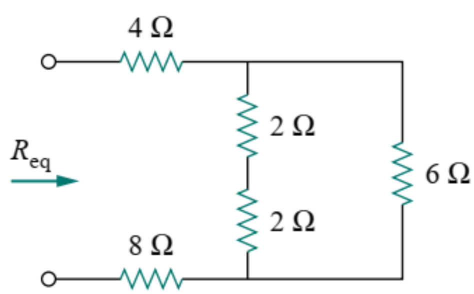
$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

Ex: Find R_{eq} for the circuit shown below.
Solution:



$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

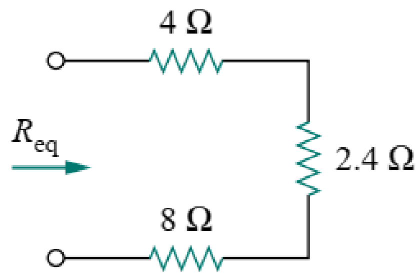
$$1\ \Omega + 5\ \Omega = 6\ \Omega$$



(a)

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

$$4\,\Omega \parallel 6\,\Omega = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

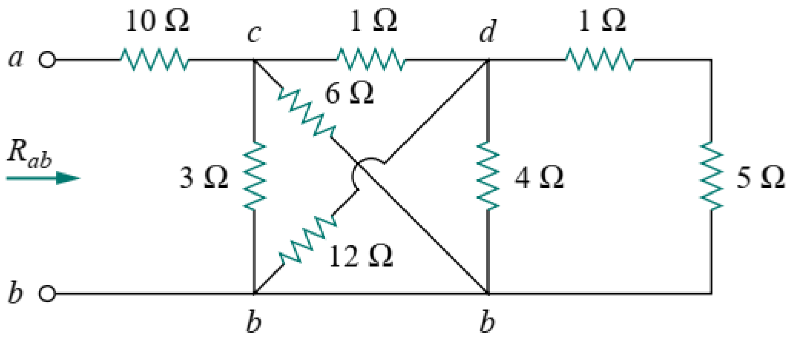


(b)

$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

Ex :Calculate the equivalent resistance $R_{ab.}$

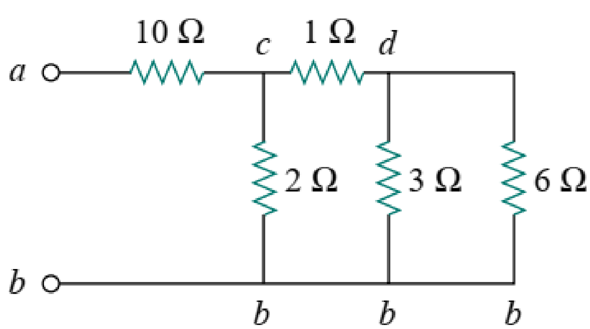
Solution:



$$3\,\Omega \parallel 6\,\Omega = \frac{3 \times 6}{3 + 6} = 2\,\Omega$$

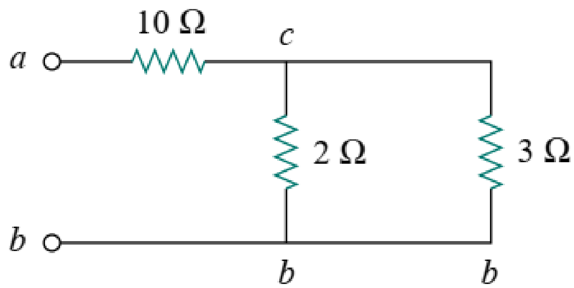
$$12\,\Omega \parallel 4\,\Omega = \frac{12 \times 4}{12 + 4} = 3\,\Omega$$

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$



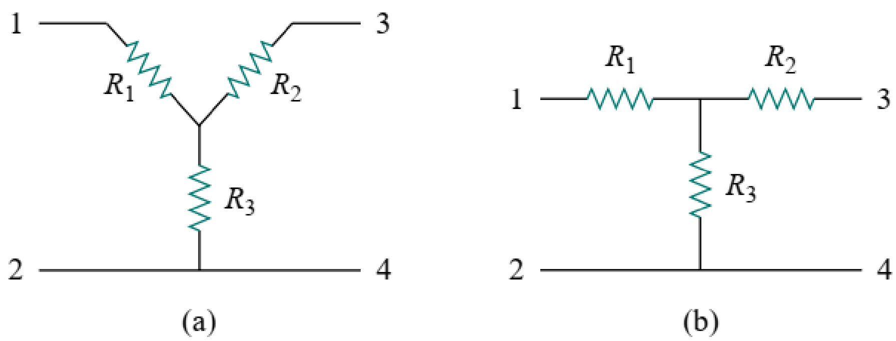
$$1\,\Omega + 2\,\Omega = 3\,\Omega.$$

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$



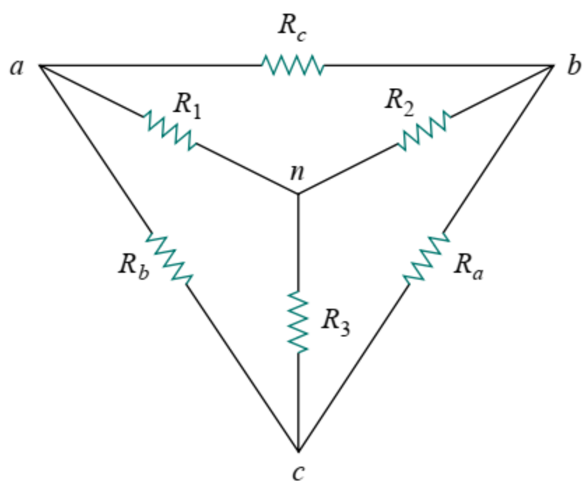
$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

WYE-DELTA TRANSFORMATIONS



Two forms of the same network: (a) Y, (b) T.

Delta to Wye Conversion



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye to Delta Conversion

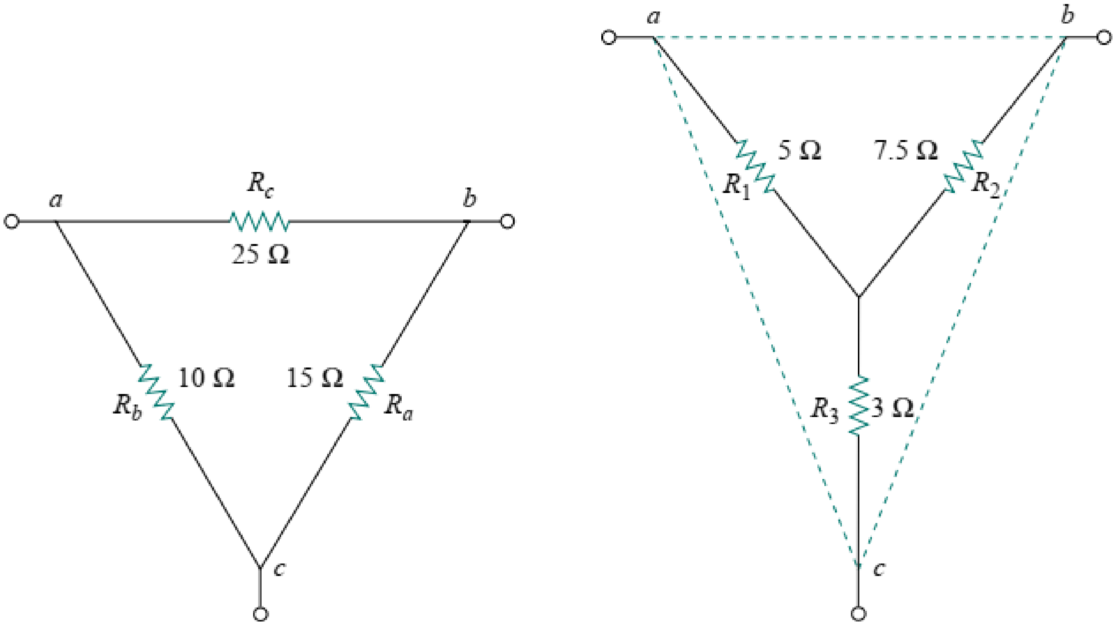
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

Ex: Convert the Δ network in Fig. below to an equivalent Y network.



(a) (b)

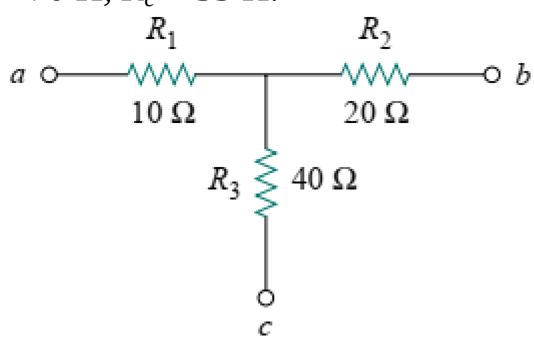
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25 + 10 + 15} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

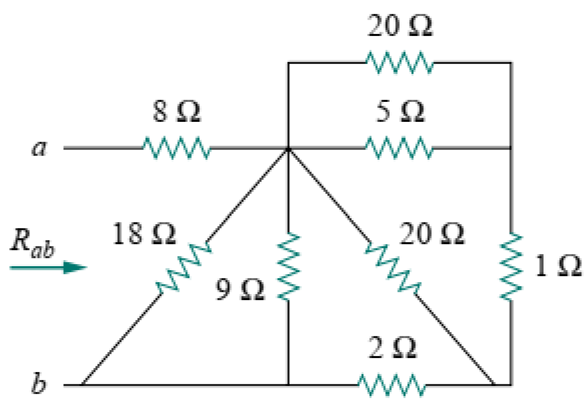
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Review Questions

1. Transform the wye network in Fig. below to a delta network.
Answer: $R_a = 140\Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.



2. Find R_{ab} for the circuit in Fig. below.
Answer: 11 Ω.

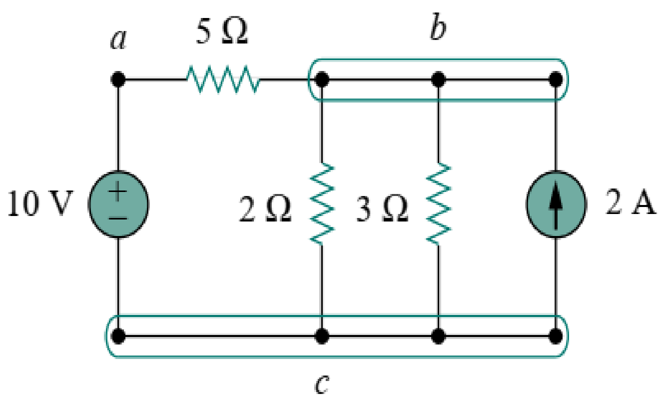


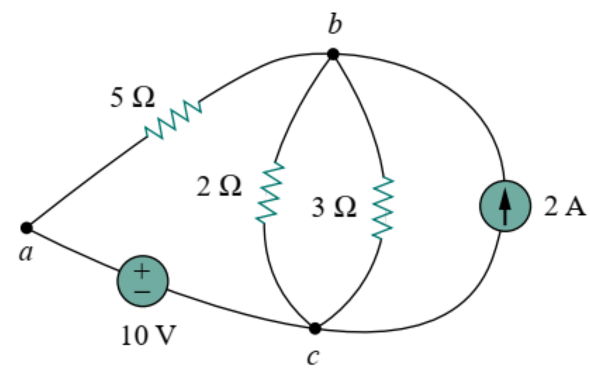
NODES, BRANCHES, AND LOOPS

A branch represents a single element such as a voltage source or a resistor.

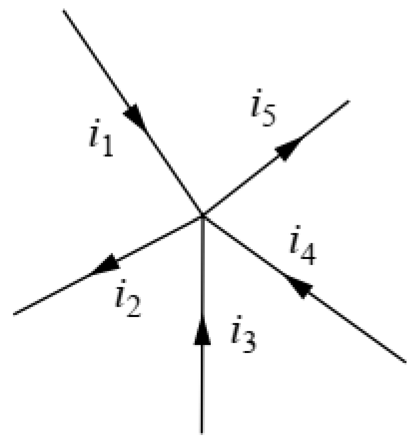
A node is the point of connection between two or more branches.

A loop is any closed path in a circuit.





KIRCHHOFF'S LAWS
Kirchhoff's current law (KCL)



Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

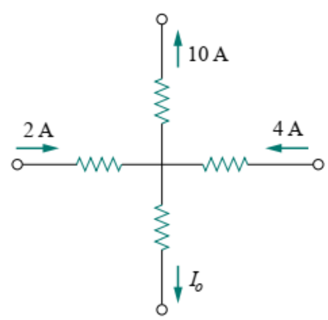
$$\sum_{n=1}^{N} i_n = 0$$

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

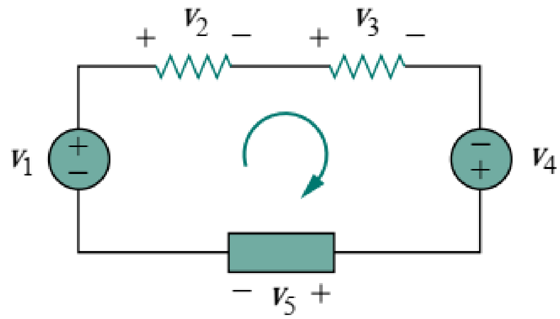
$$i_1 + i_3 + i_4 = i_2 + i_5$$

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Ex: Find the current I_o
Solution:



Kirchhoff’s voltage law (KVL)



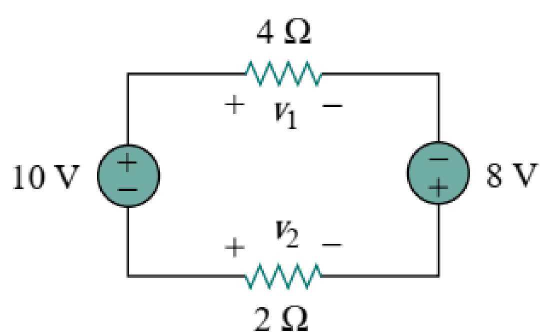
> **Kirchhoff’s voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$

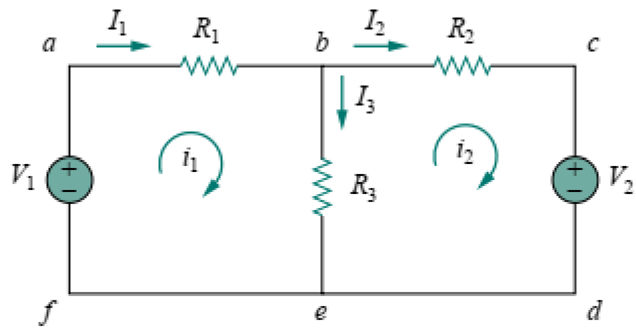
Sum of voltage drops = Sum of voltage rises

Ex: Find v_1 and v_2 in the circuit
Answer: 12 V, -6 V
Solution:



METHODS OF ANALYSIS
Maxwell's loop current analysis (Mesh Analysis)

A **mesh** is a loop which does not contain any other loops within it.



Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

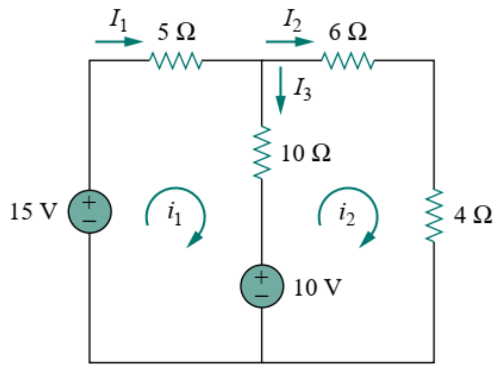
$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

Ex: For the circuit below , find the branch currents $I1$, $I2$, and $I3$ using mesh analysis.



Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{3.5.1}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \tag{3.5.2}$$

METHOD 1 Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \qquad \Longrightarrow \qquad i_2 = 1 \text{ A}$$

From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

METHOD 2 To use Cramer's rule, we cast Eqs. (3.5.1) and (3.5.2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

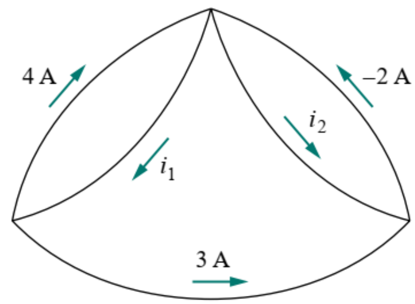
Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

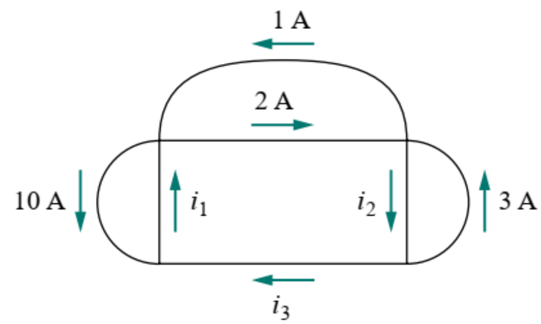
as before.

Review Questions

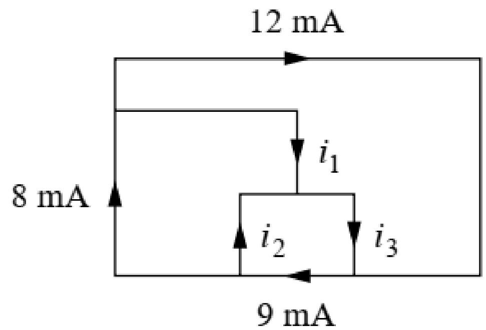
Ex: Determine *i*1 and *i*2 in the circuit



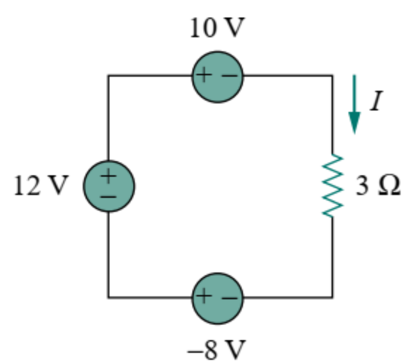
Ex: Find i1, i2, and i3 in the circuit



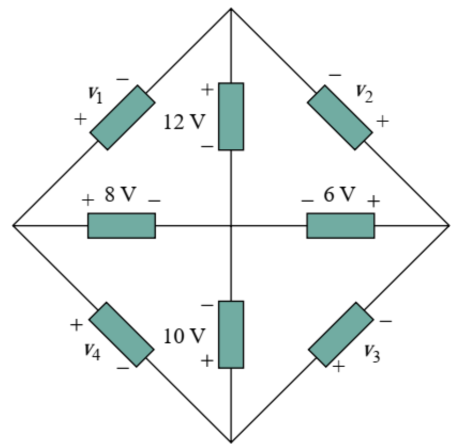
Ex: Use KCL to obtain currents i1, i2, and i3 in the circuit shown



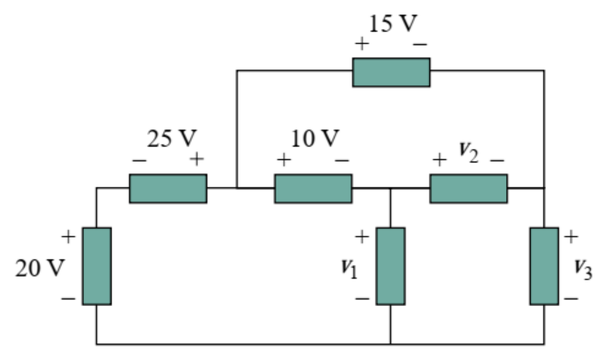
Ex : From the circuit in Fig. 2.80, find *I*, the power dissipated by the resistor, and the power supplied by each source.



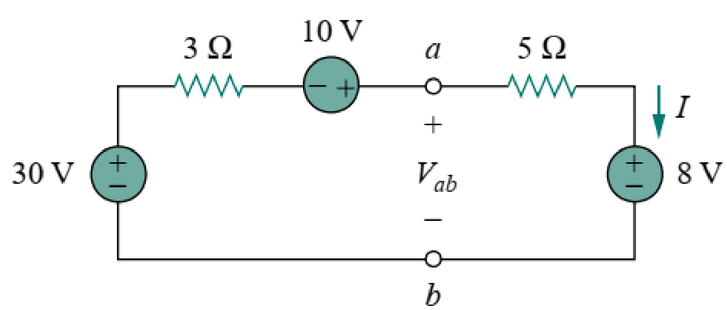
Ex: Determine *v*1 through *v*4 in the circuit



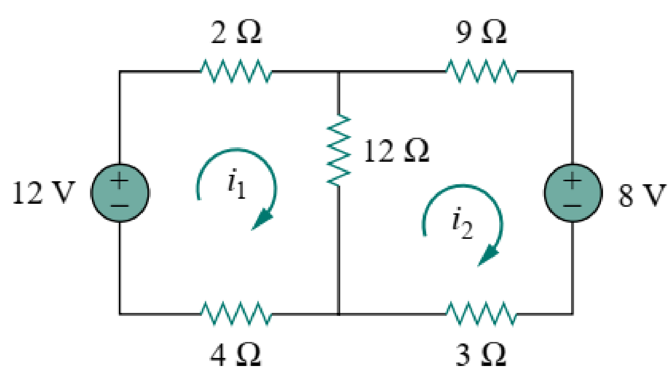
Ex: In the circuit in Fig. 2.76, obtain *v*1, *v*2, and *v*3.



Ex: Find *I* and *Vab* in the circuit



Ex: Calculate the mesh currents *i*1 and *i*2 in the circuit below.
Answer: i1 = 2/3 A, i2 = 0 A



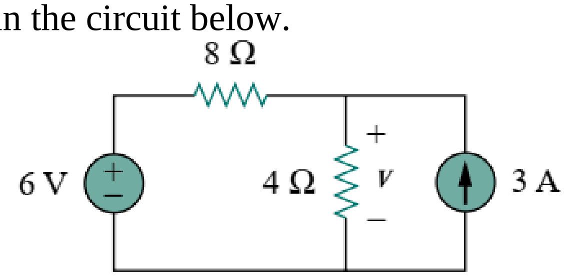
CIRCUIT THEOREMS
SUPERPOSITION

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Steps to Apply Superposition Principle:

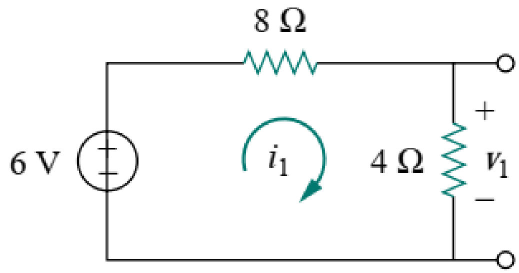
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Ex: Use the superposition theorem to find *v* in the circuit below.



Solution:
Since there are two sources, let v = v1 + v2 where v1 and v2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively.

To obtain *v*1, we set the current source to zero



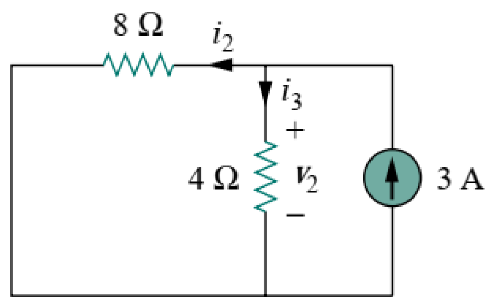
(a)

$$12i_1 - 6 = 0 \quad \Longrightarrow \quad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

we set the voltage source to zero, Using current division,



(b)

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Ex: Using the superposition theorem, find *vo* in the circuit below.
Answer: 12 V.