**Southern Technical University**
Basrah Technical Institute
Department of Electrical Techniques

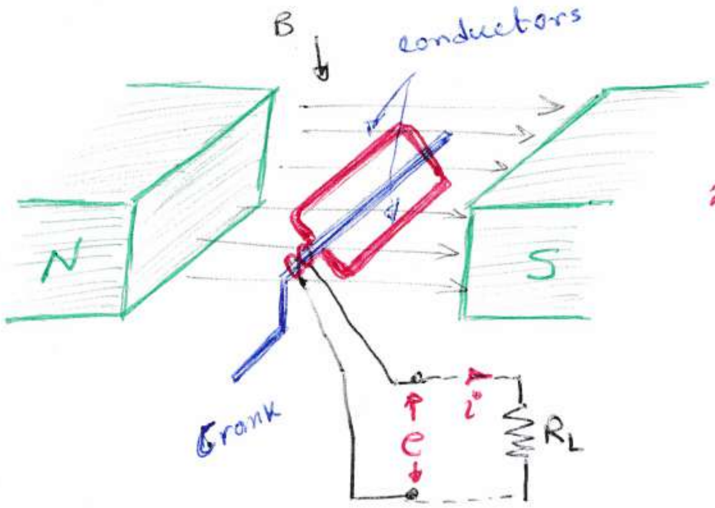
# AC Electrical Circuits

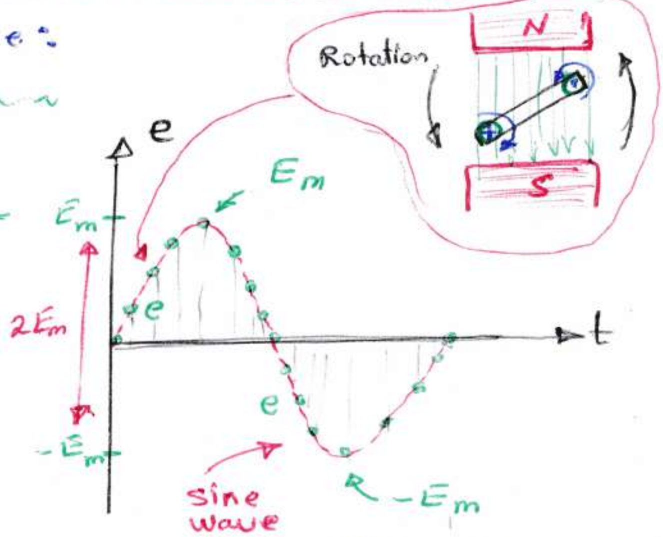
First Year

2022-2023

# Chapter (10)
## Alternating Current and Voltage

### 10-1 Generation of AC voltage :-




[ sinusoidal voltage wave form ]

* A sinusoidal voltage waveform is generated at the terminals of a conducting loop rotated in amagnetic field.
* peak o/p voltage is produced when the conductors are moving perpendicular to the field.
* Zero o/p when the conductors are moving parallel to the field.

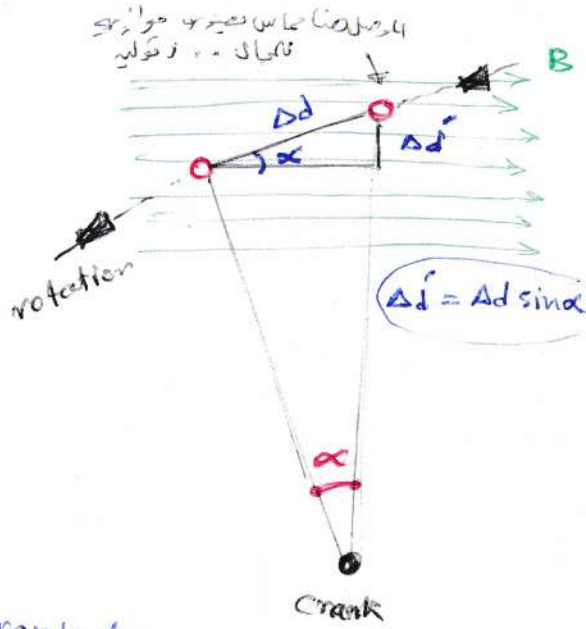
emf induced

$$e = \frac{\Delta \Phi}{\Delta t} = B \cdot \frac{\Delta A}{\Delta t}$$

area of one conductor swept in field by $\Delta t$

$$e = B \cdot l \cdot \frac{\Delta d'}{\Delta t}$$

$$\Delta d' = \Delta d \sin \alpha$$




$$e = B l \frac{\Delta d}{\Delta t} \sin \alpha$$

velocity

$$\boxed{e = B l v \sin \alpha}$$

emf induced

$$\boxed{e = E_m \sin \alpha}$$

angle of conductor with the field. (Phase).

max. emf induced. magnitude peak value

**Example**

In the-hand-cranked generator $l$ of each conductor is 25 cm , $r$ = 5cm , $B$ = 0.1 T , rpm = 100

Calculat $E_m$ , $e$ ($\alpha$ = 45°, 90°, 135°, 225°).

**solution :**

$$E_m = Blv = Bl\frac{\Delta d}{\Delta t}$$

$$\Delta d = 2\pi r * \overset{r.p.m}{100} = 2\pi * 5 \times 10^{-2} * 100 = 10\pi \ m$$

$$\Delta t = 1\,\text{minit} = 60\ \text{sec}$$

$$\therefore E_m = 0.1 * 0.25 * \frac{10\pi\ m}{60\ sec} = 26.2\ mV$$

$$e = E_m \sin\alpha$$

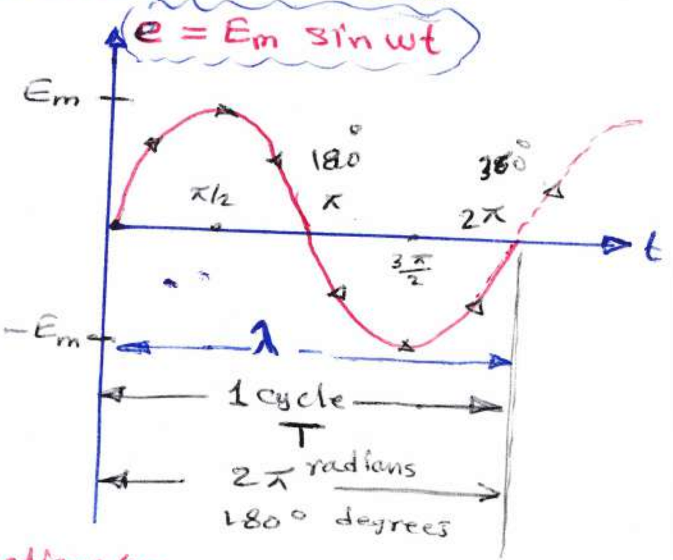
for $\alpha = 45°$ $\Rightarrow$ $e = 26.2 \sin 45 = 18.5$ mV

$\alpha = 90°$ $\Rightarrow$ $e = 26.2 \sin 90 = 26.2$ mV

$\alpha = 135°$ $\Rightarrow$ $e = 18.5$ mV

$\alpha = 225°$ $\Rightarrow$ $e = -18.5$ mV.

## 10-2 : Frequency , phase angle and wavelength.




* time of one cycle = T
* frequency $f = \frac{1}{T}$

number of cycles in 1sec

$f = 1\ Hz$ = 1cycle /sec

* Angular velocity $\omega = \frac{2\pi}{T} = \underbrace{2\pi f}$ radians/sec

for one revolution in sec

* phase angle $\alpha$ at any time $= \omega t = 2\pi f t$ radians.

velocity of light m/s

$$* \quad \text{wavelength} \quad \underset{\text{in(m)}}{\lambda} = \frac{c}{f} = c\,T$$ time period in sec.

$$c = 3 \times 10^{8} \; m/s$$

Example: An ac waveforme with frequency $f = 1.5\ KH_z$ has apeak value of $E_m = 3.3$ volt. find the time periode $T$ and the wavelength $\lambda$ and $e$ at $t_1 = 0.65\ \mu s$, $t_2 = 1.2\ ms$

solution:

$$T = \frac{1}{f} = \frac{1}{1.5 \times 10^{+3}} = 0.66 \text{ m sec}$$

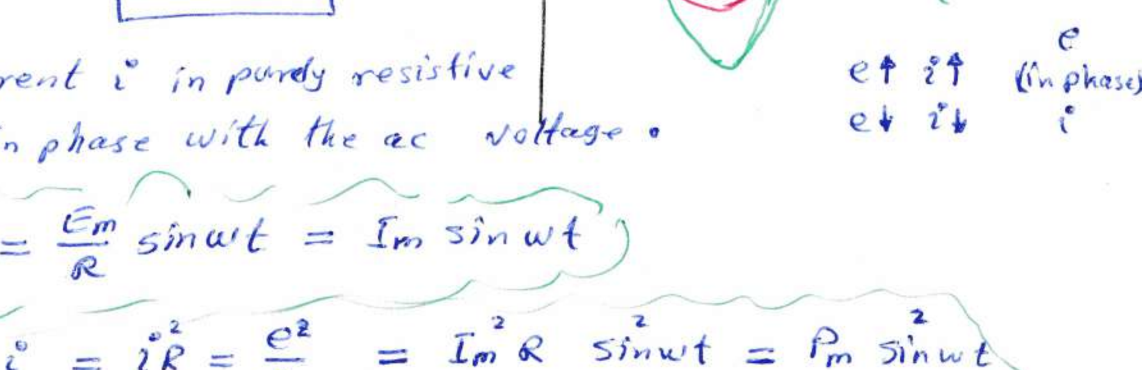
$$\lambda = \frac{c}{f} = \frac{3 \times 10^{8}}{1.5 \times 10^{3}} = 2 \times 10^{5} = 200 \text{ Km}$$

$$\omega = 2\pi f = 2\pi \times 1.5 \times 10^{3} = 3\pi \times 10^{3} \text{ rad/sec.}$$

$$e_1 = E_m \sin \omega t_1 = 3.3 \sin(3\pi \times 10^{3} * 0.65 \times 10^{-6})$$
$$= 20.2 \text{ mV}$$

$$e_2 = E_m \sin \omega t_2 = 3.3 \sin(3\pi \times 10^{3} * 1.2 \times 10^{-3})$$
$$= -3.1 \text{ volt.}$$

## 10-3: resistive load with AC supply




The Ac current $i$ in purely resistive circuit is in phase with the ac voltage.

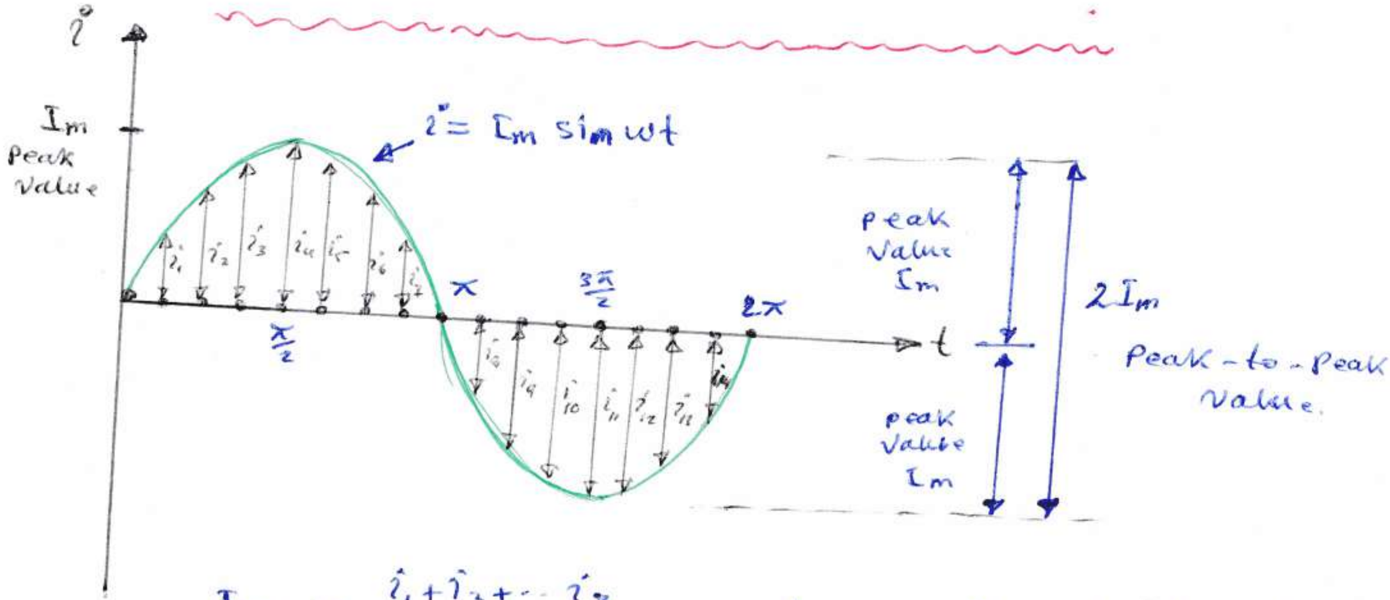
$e\uparrow \; i\uparrow$
$e\downarrow \; i\downarrow$
$e$ (in phase) $i$

$$i = \frac{e}{R} = \frac{E_m}{R} \sin \omega t = I_m \sin \omega t$$

$$p = e\,i = i^2 R = \frac{e^2}{R} = I_m^2 R \sin^2 \omega t = P_m \sin^2 \omega t$$

$$P_m = I_m^2 R = \frac{V_m^2}{R} = I_m V_m$$

## 10-4: Peak, Average and RMS Values of Sine waves




dc $\rightarrow$ $I_{av_1} = \dfrac{i_1 + i_2 + \cdots i_7}{7}$ for positive half cycle.

$$I_{av_1} = \frac{I_m}{7}\{\sin \omega t_1 + \sin \omega t_2 + \cdots + \sin \omega t_7\} \cong 0.637 I_m$$

$I_{av_2} = \dfrac{i_8 + i_9 + \cdots i_{14}}{7}$ for negative half cycle

$$\cong -0.637 I_m$$

$\therefore I_{av} = I_{av_1} + I_{av_2} = 0$ Zero. (Average value) dc

or $I_{av} = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} I_m \sin \omega t \; dt = 0$

$\therefore$ Average value of 1 cycle of sinwave $i$ or $e$ $= 0$

* Effective value (rms) (It is used in ohm's Law)

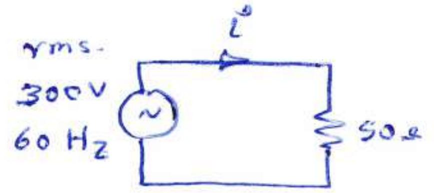
dc التيار الفعلي الذي يكافئ $\rightarrow$ $I_{rms} = \dfrac{I_m}{\sqrt{2}} = 0.707 \; I_m$

والفولتية الفعلية $\rightarrow$ $V_{rms} = \dfrac{V_m}{\sqrt{2}} = 0.707 \; V_m$

ac - power dissipation $= I_{rms}^2 R = \frac{1}{2} \cdot I_m^2 R$

$$\therefore P = \frac{1}{2} P_m$$

**Example** A 300 V sinusoidal ac supply is applied to 50 Ω resistor. Determine:




(a) $E_m$ , $I_m$
(b) rms current $I$
(c) average current for half-cycle
(d) average current for 1-complete cycle.
(e) power dissipation.
(f) instantinous power at phase angle $\pi/2$

**Solution:**

(a) $E_m = \sqrt{2}\; E = 1.414 * 300 = 424$ volt.

$$I_m = \frac{E_m}{R} = \frac{424}{50} = 8.48 A$$

(b) rms $I = \frac{I_m}{\sqrt{2}} = \frac{8.48}{\sqrt{2}} = 6\ A$ or $I = \frac{E}{R} = \frac{300}{50} = 6\ A$

(c) $I_{av\ of\ half\ cycle} = 0.637\ I_m = 0.637 * 8.48 = 5.4\ A$

(d) $I_{av}$ of 1 complete cycle $= 0$

(e) $P = I^2 * R = 6^2 * 50 = 1.8$ KW. $= \frac{1}{2} P_m = \frac{1}{2} I_m^2 R$

(f) $P = I_m^2 R \sin^2 \omega t = I_m^2 R \sin^2 \alpha$

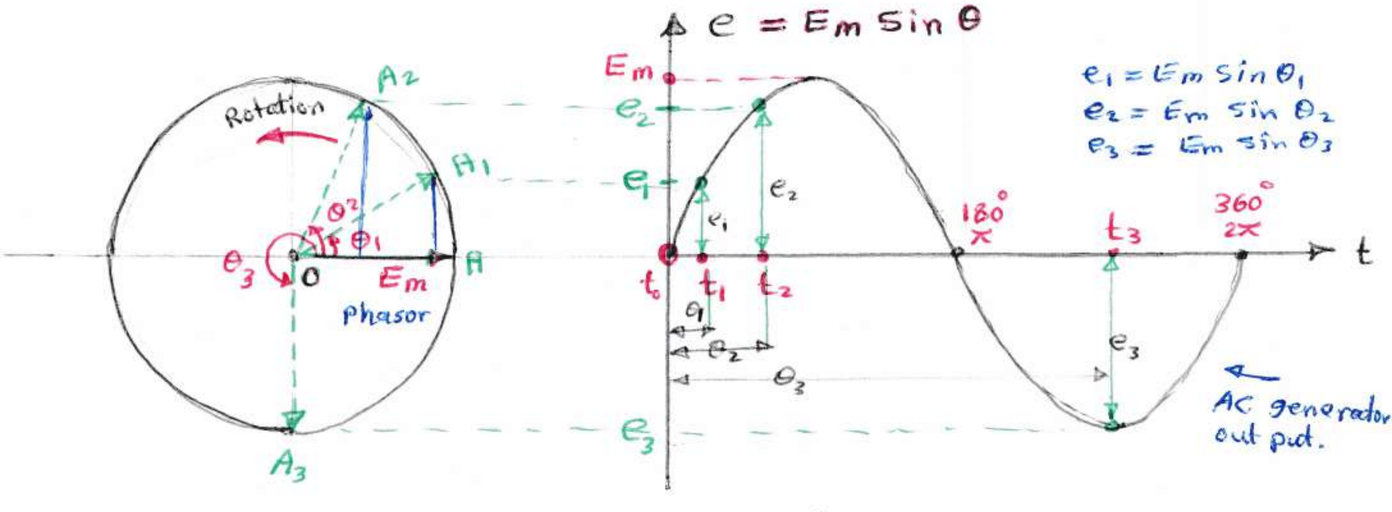
$= (8.48)^2 * 50 * \sin^2(\pi/2)$

$= 3.5$ KW.

**Note:** * with Ac voltages and currents, all quantities are assumed to be rms quantities unless otherwise indicated.

* For analysis use all quantities in rms or in peak values. (don't mix).

# Chapter (11) : Phasors and complex numbers



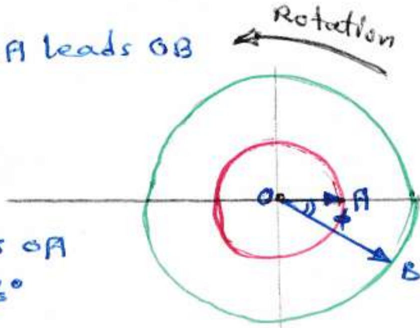

Phasor representation of a sinwave:

At any angle $\theta$ , $OA \sin\theta$ is the instantaneous value of the sine wave.

- scaler : number of unites (magnitude) , (volume, resistance, ...)
- Vectors : number of unites and direction , (velocity , forces, ..)
- phasors : vectors rotates by angular velocity $\omega$.
  ( Ac current , Ac voltage).

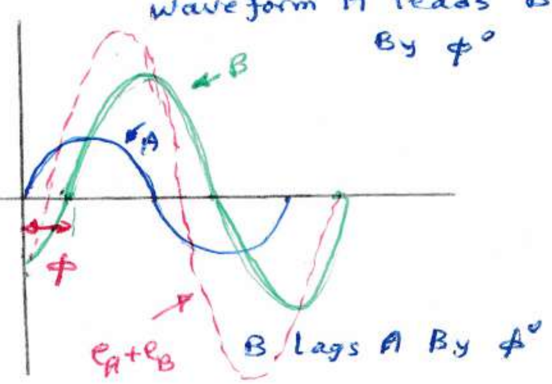
## Addition and Subtraction of phasors :

phasor OA leads OB By $\phi^\circ$

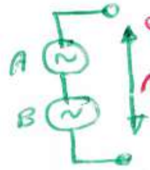



OB lags OA By $\phi^\circ$

waveform A leads B By $\phi^\circ$

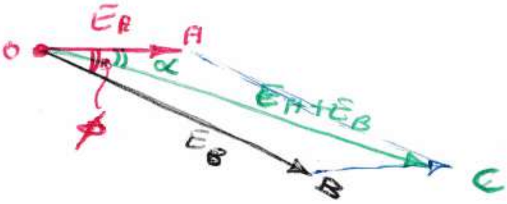



B lags A By $\phi^\circ$




$$e_A = OA \sin\theta$$

$$e_B = OB \sin(\theta - \phi)$$



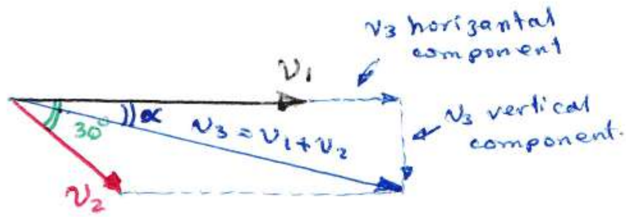
**Example**

$v_1 = 120 \sin\theta$ , $v_2 = 75 \sin(\theta - 30^\circ)$

find the resultant of (a) $v_1 + v_2$ (b) $v_1 - v_2$

**solution**

(a)




$v_3 = v_1 + v_2$

horizontal component of $v_1 = 120 \cos 0 = 120$

" " of $v_2 = 75 \cos 30^\circ = 65$

" " of $v_3 = 120 + 65 = 185$

vertical " " $v_1 = 120 \sin 0 = 0$

" " " $v_2 = -75 \sin 30 = -37.5$

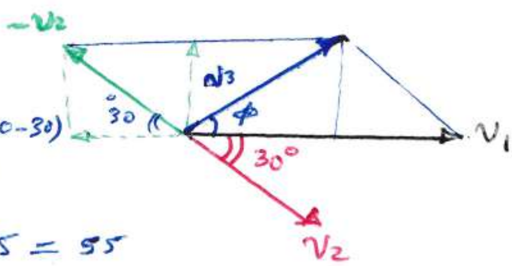
" " " $v_3 = 0 - 37.5 = -37.5$

$$v_3 = \sqrt{(185)^2 + (37.5)^2} = 188.8$$

$$\alpha = \tan^{-1} \frac{v_3 \text{ vertical componant}}{v_3 \text{ horizontal componant}} = \tan^{-1} \frac{-37.5}{185} = -11.5^\circ$$

$$\therefore v_3 = 188.8 \sin(\theta - 11.5^\circ)$$

(b) $v_3 = v_1 - v_2$




horizontal componant of $v_1 = 120$

" " $= -v_2 = 75 \cos(180-30)$

$= -65$

" " of $v_3 = 120 - 65 = 55$

vertical component of $v_1 = 120 \sin 0 = 0$

" " $-v_2 = 75 \sin(180 - 30^\circ) = 37.5$

$$v_3 = \sqrt{(55)^2 + (37.5)^2} = 66.6$$

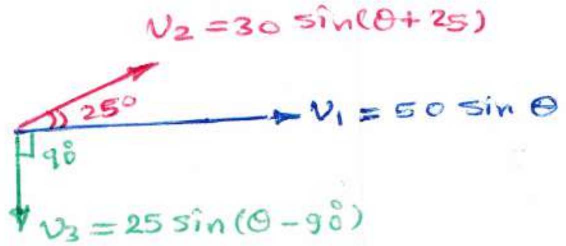
$$\phi = \tan^{-1} \frac{37.5}{55} = 34.3^\circ$$

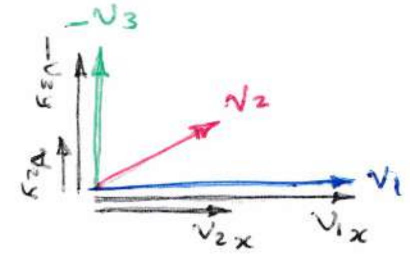
$$\therefore v_3 = 66.6 \sin(\theta + 34.3^\circ)$$

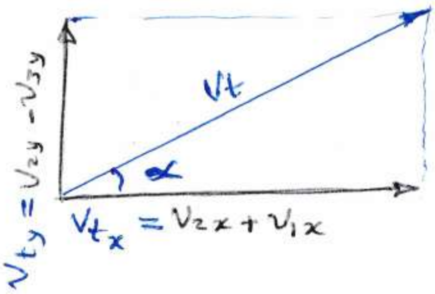
**Example** Graphically find $v_t = v_1 + v_2 - v_3$

where: $v_1 = 50 \sin\theta$ , $v_2 = 30 \sin(\theta + 25°)$

$v_3 = 25 \sin(\theta - 90°)$.

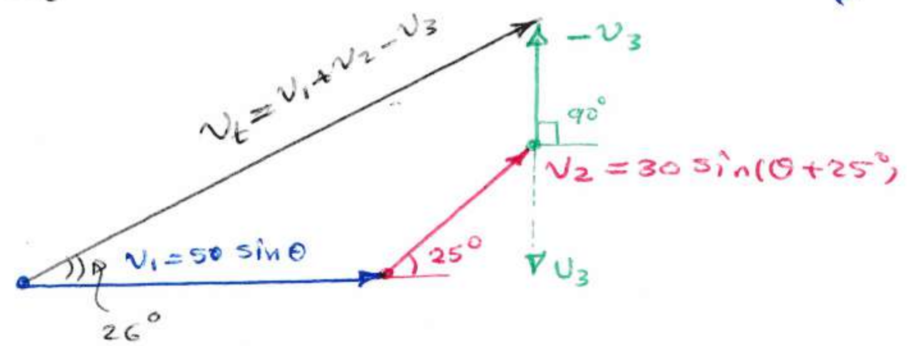



$$\alpha = \tan^{-1} \frac{V_{ty}}{V_{tx}}$$

$v_t = 86 \sin(\theta + 26°)$



# Polar and Rectangular Form, The j operator

magnitude, angle, Real part, Imaginary part

$$e = E_m \sin\theta = E_m \angle\theta = E_m[\cos(\theta) + j\sin(\theta)]$$

$$i = I_m \sin(\theta+\alpha) = I_m \angle\theta+\alpha = I_m[\cos(\theta+\alpha) + j\sin(\theta+\alpha)]$$

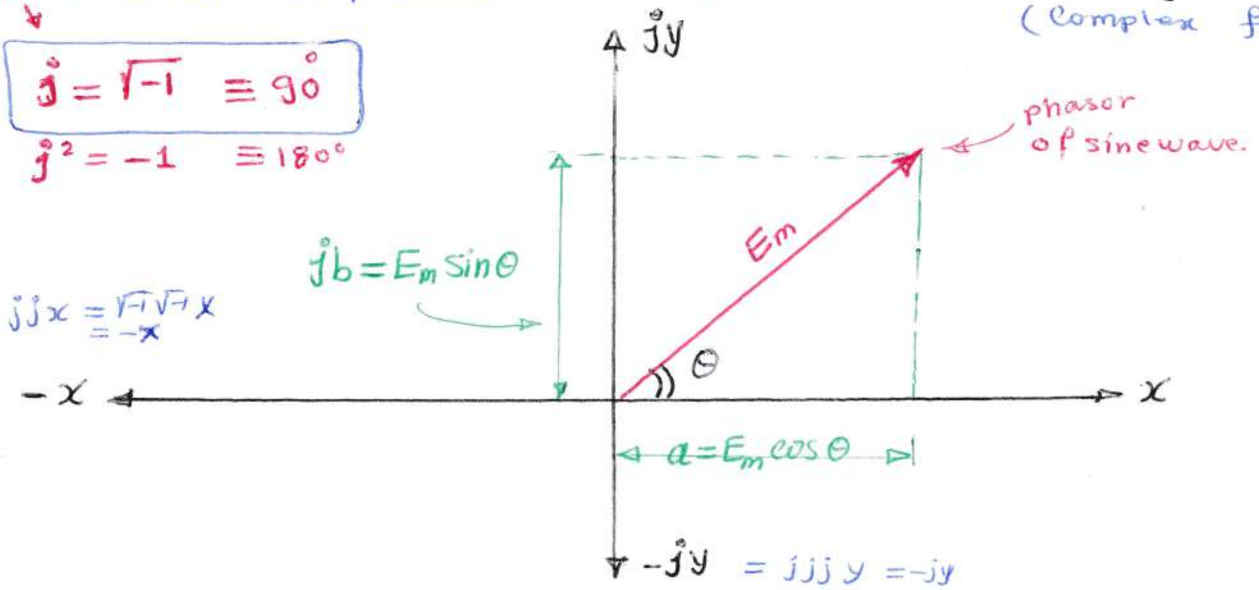
j operator

$\underbrace{\qquad\qquad}_{\text{sinusoidal quantity}}$ $\underbrace{\qquad}_{\text{polar form}}$ $\underbrace{\qquad\qquad}_{\text{Rectangular form (Complex form)}}$

imaginary number

$$\boxed{j = \sqrt{-1} \equiv 90^\circ}$$

$$j^2 = -1 \equiv 180^\circ$$




Example:

$12\sin(30)$, $40\sin(105)$, $270\sin(1.7\pi)$

① $12\sin(30) = 12\angle 30^\circ = 12\cos(30) + j\,12\sin 30$

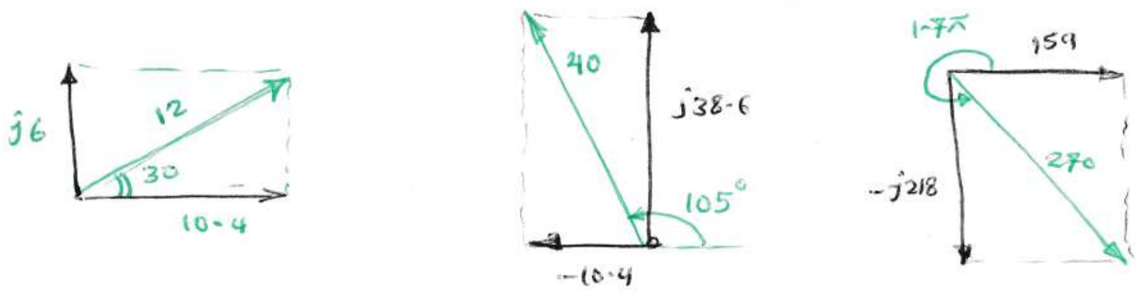
$= \underline{10.4 + j\,6}$

② $40\sin(105) = 40\angle 105^\circ = 40\cos(105) + j\sin(105) * 40$

$= \underline{-10.4 + j\,38.6}$

③ $270\sin(1.7\pi) = 270\angle 1.7\pi = 270\cos(1.7\pi) + j\,270\sin(1.7\pi)$

$= \underline{159 - j\,218}$



## * Mathematics of rectangular (complex) form and polar form

$j = \sqrt{-1}$ ; $j^2 = \sqrt{-1}\sqrt{-1} = -1$ ; $j^3 = -j$ , $j^4 = 1$

$$\frac{1}{j} = \frac{1}{j} * \frac{j}{j} = -j$$

### ① complex form

Let $f = a + jb$ , $h = c + jd$

⊛ $f + h = (a + jb) + (c + jd) = (a + c) + j(b + d)$

⊛ $f - h = (a + jb) - (c + jd) = (a - c) + j(b - d)$

⊛ $f * h = (a + jb) * (c + jd) = ac + jbc + jad - bc$
$= (ac - bd) + j(bc + ad)$

⊛ $$\frac{f}{h} = \frac{a + jb}{c + jd} = \frac{a + jb}{c + jd} * \frac{c - jd}{c - jd} = \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2}$$

conjugate of denominator

$$= \frac{ac + bd}{c^2 + d^2} + j\frac{bc - ad}{c^2 + d^2}$$

### ② polar form

Let $m = E_1 \angle \theta_1$ , $n = E_2 \angle \theta_2$

* $m * n = E_1 \angle \theta_1 * E_2 \angle \theta_2 = E_1 E_2 \angle \theta_1 + \theta_2$

* $$\frac{m}{n} = \frac{E_1 \angle \theta_1}{E_2 \angle \theta_2} = \frac{E_1}{E_2} \angle \theta_1 - \theta_2$$

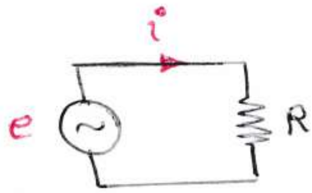
Note: * addition and subtraction are easy in complex form

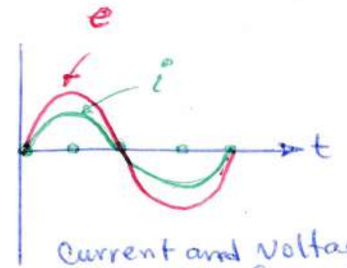
* multiplication and division are simpler in polar form

chapter (12)

# Inductance and capacitance in AC circuits

## 1) AC supply with pure resistance :

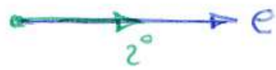



Current and Voltage Waveforms.

phasor diagram

e in phase with i

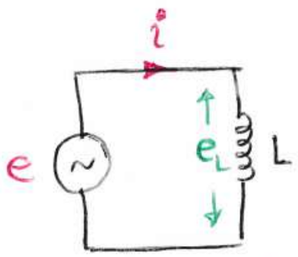


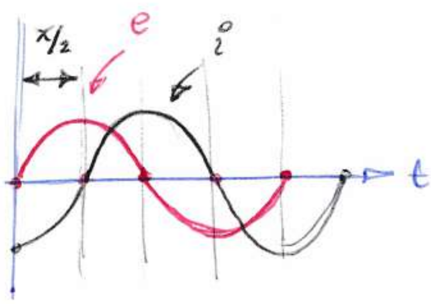

$e = E_m \sin \omega t$

$i = \frac{e}{R} = \frac{E_m}{R} \sin \omega t = I_m \sin(\omega t)$

$G = \frac{1}{R}$ siemens.

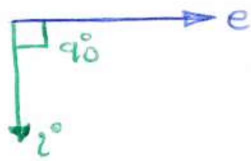
## 2) AC supply with pure inductive load :




phasor diagram

$i$ Lag $e$ by $\frac{\pi}{2}$




$e = E_m \sin \omega t$

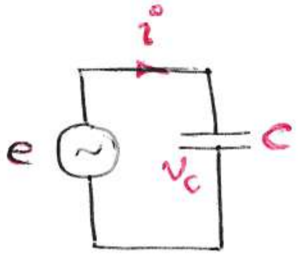
$e_L = L \frac{\Delta i}{\Delta t} = \omega L \cos(\omega t - \pi/2)$

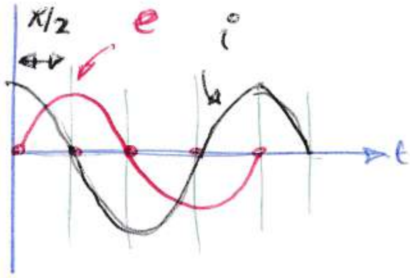
$i = I_m \sin(\omega t - \pi/2)$

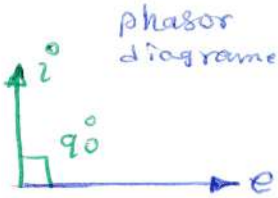
$$\boxed{X_L = \frac{E_{rms}}{I_{rms}} = \omega L = 2\pi f L}$$ $\Omega$ (inductive reactance)

$$\boxed{B_L = \frac{1}{X_L} \quad \text{(Siemens)}}$$ (Inductive susceptance)

## 3) AC supply with pure capacitive load:




i lead e by π/2

$e = E_m \sin \omega t$

$i = C \dfrac{\Delta V_C}{\Delta t} = E_m(\omega C) \cos \omega t = E_m \omega C \sin(\omega t + \pi/2)$

$i = I_m \sin(\omega t + \pi/2)$

$$\boxed{X_C = \frac{E_{rms}}{I_{rms}} = \frac{1}{\omega C} = \frac{1}{2\pi f C}} \quad \Omega$$ capacitive reactance

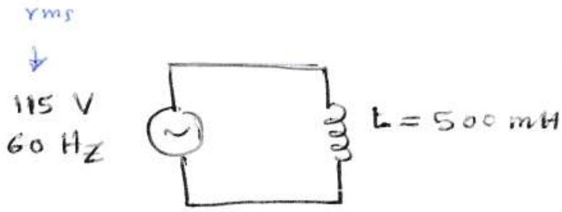
$B_C = \dfrac{1}{X_C} = 2\pi f C$ (seimens) capacitive susceptance.

Memory aid

ELI the ICE man
(ELI: voltage lead current; ICE: current lead voltage)

Example

find $X_L$, I



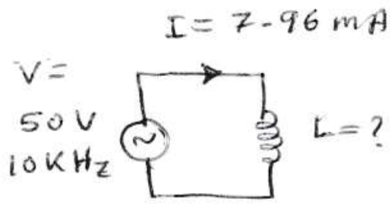

inductive reactance $X_L = 2\pi f L$

$X_L = 2\pi * 60 * 500\ mH \cong 188.5\ \Omega$

$I = \dfrac{E}{X_L} = \dfrac{115}{188.5} = 610\ mA$ (rms)

Example

find L = ?

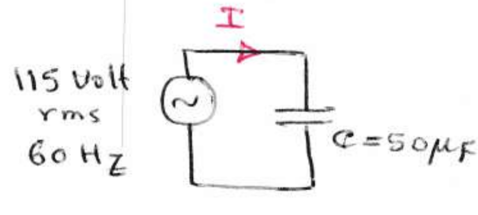



$X_L = \dfrac{E}{I} = \dfrac{50}{7.96\ mA} = 6.28\ K\Omega$

$X_L = 2\pi f L \Rightarrow L = \dfrac{X_L}{2\pi f} = 100\ mH$

**Example**

find $X_C$ and $I$



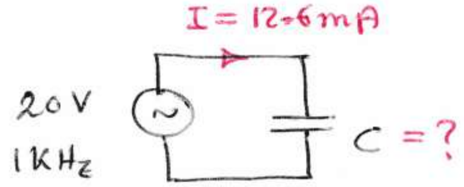

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi * 60 * 50\times10^{-6}}$$

$$= 53.1\ \Omega$$

$$I = \frac{E}{X_C} = \frac{115}{53.1} = 2.2\ A$$

**Example:**

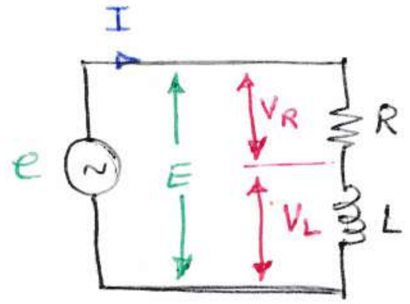
find $C$



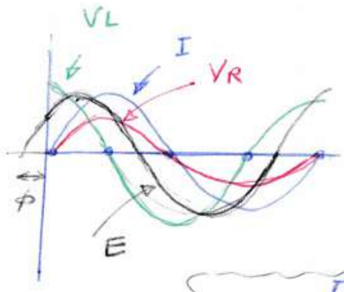

$$X_C = \frac{E}{I} = \frac{20}{12.6} = 1.59\ K\Omega$$

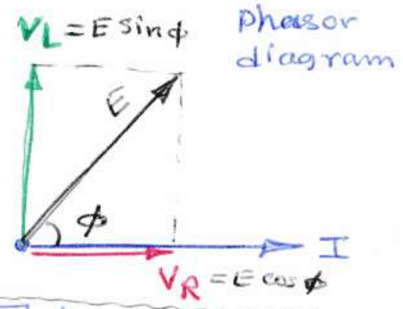
$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi * 1000 * 1590} = 0.1\ \mu F$$

## 4) Series RL in AC circuits:




Series RL circuit.



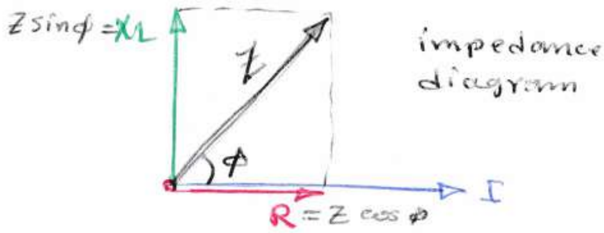

$$\boxed{E = \sqrt{V_R^2 + V_L^2}\ \angle \tan^{-1}(V_L/V_R)}$$

$$\boxed{E = V_R + j V_L} \qquad E = |E| \angle \phi$$

$$\frac{E}{I} = \frac{V_R}{I} + j\frac{V_L}{I}$$

$$\boxed{Z = R + j X_L}$$ impedance $\Omega$

$$Z = |Z| \angle \phi$$

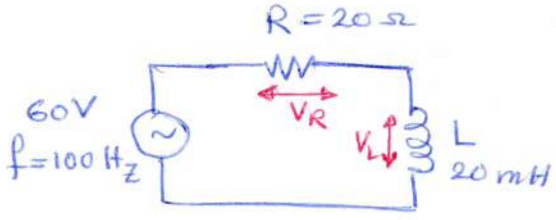



$$\boxed{Z = \sqrt{R^2 + X_L^2}\ \angle \tan^{-1}(X_L/R)}$$

$$Y = \frac{1}{Z}$$ (siemens) admittance.

## Example

Calculate the inductor voltage, resistor voltage and the phase angle of current w.r.t. supply voltage.




$$X_L = 2\pi f L = 2\pi * 100 * 20 * 10^{-3} = 12.57\ \Omega$$

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(20)^2 + (12.57)^2} = 23.6\ \Omega$$

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{12.57}{20} = 32.1^\circ$$

$$Z = R + jX_L = 20 + j\,12.57$$

$$Z = |Z|\angle\phi = 23.6\angle 32.1^\circ$$

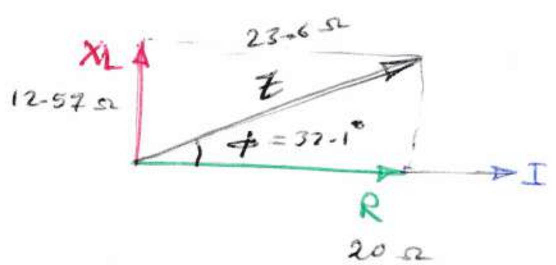
$$|I| = \frac{E}{|Z|} = \frac{60\ V}{23.6} = 2.54\ A$$

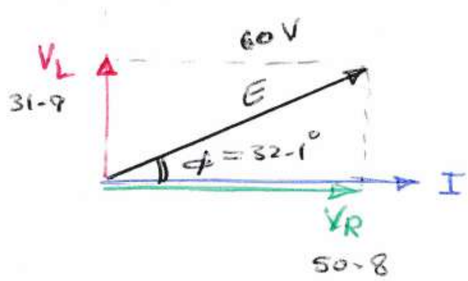
$$V_R = E\cos\phi = 60\cos(32.1^\circ) = 50.8 = I * R$$

$$V_L = E\sin\phi = 60\sin(32.1^\circ) = 31.9 = I * X_L$$

$$E = V_R + jV_L = 50.8 + j\,31.9$$

$$E = |E|\angle\phi = 60\angle 32.1^\circ$$



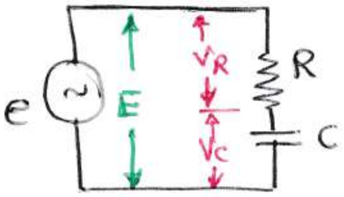

$$R = Z\cos\phi$$

$$X_L = Z\sin\phi$$

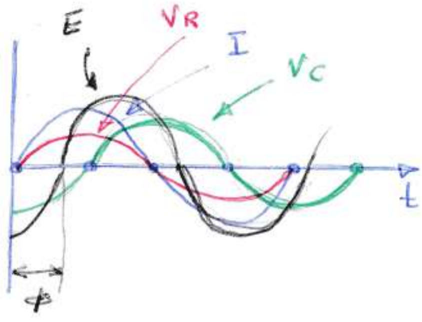
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

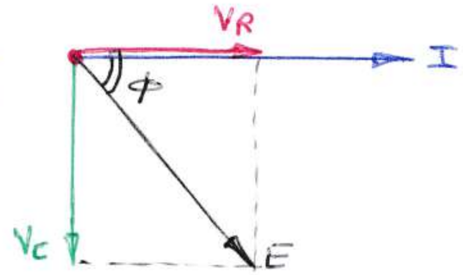
## 5) series RC in AC circuits:




series RC circuit

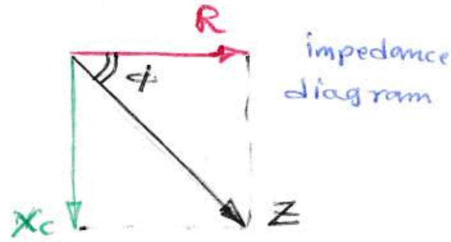



phasor diagram




$$\boxed{E = V_R - jV_C = |E| \angle \phi}$$

$$\boxed{|E| = \sqrt{V_R^2 + V_C^2} \quad , \phi = \tan^{-1}\frac{V_C}{V_R}}$$




$$\frac{E}{I} = \frac{V_R}{I} - j\frac{V_C}{I}$$

$$\boxed{Z = R - jX_C} \boxed{= |Z| \angle \phi}$$

$$\boxed{|Z| = \sqrt{R^2 + X_c^2}}$$

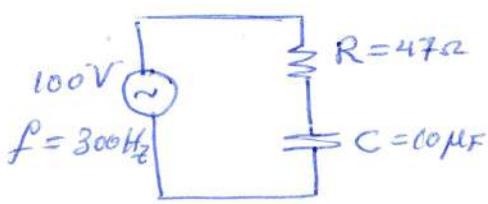
$$\boxed{\phi = \tan^{-1}\frac{X_C}{R}}$$

$V_R = E\cos\phi$ , $V_c = E\sin\phi$

$R = Z\cos\phi$ , $X_C = Z\sin\phi$

$Y = \frac{1}{Z}$ (siemens).

**Example:** calculate $V_R$, $V_C$, I, $\phi$ of current and supply voltage.




$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi * 300 * 10 * 10^{-6}} = 53\,\Omega$$

$$|Z| = \sqrt{R^2 + X_c^2} = \sqrt{(47)^2 + (53)^2} = 71\,\Omega$$

$$\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{53}{47} = 48.5^\circ$$

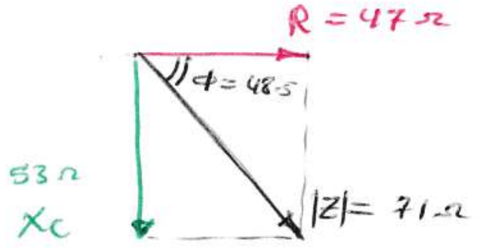
$$|I| = \frac{E}{|Z|} = \frac{100}{71} = 1.41\,A$$

Example تكملة السابقة

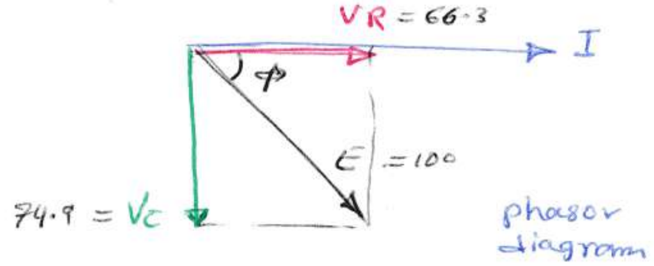
$V_R = E \cos \phi = 100 \cos(48.5) = 66.3$

OR $V_R = I R = 1.41 * 47 = 66.3$

$V_C = E \sin \phi = 100 \sin(48.5) = 74.9 = I * X_C$




impedance diagrame

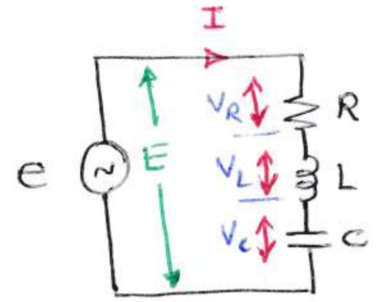


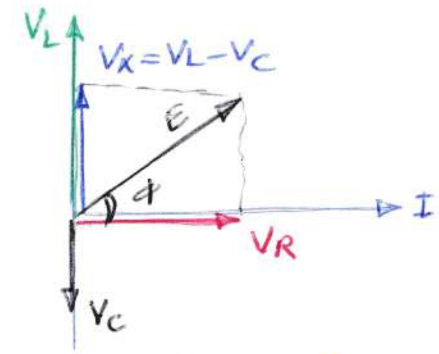

phasor diagram

Current lead E by $\phi$

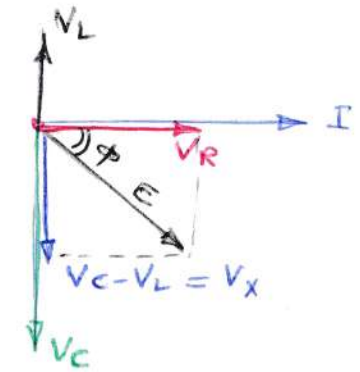
E lag I by $\phi$

## 6) Series RLC in AC circuits:




phasor diagram for $X_L > X_C$




phasor diagram for $X_L < X_C$

$$\boxed{E = V_R + j(V_L - V_C) = V_R + j V_X}$$

$$\boxed{V_X = V_L - V_C}$$

$$E = |E| \angle \phi = \sqrt{V_R^2 + V_X^2} \angle \tan^{-1} \frac{V_X}{V_R}$$

$$|E| \angle \phi = |E| \cos \phi + j |E| \sin \phi$$

$$\boxed{Z = R + j(X_L - X_C) = |Z| \angle \phi} \qquad \boxed{X_L - X_C = X_{eq}}$$

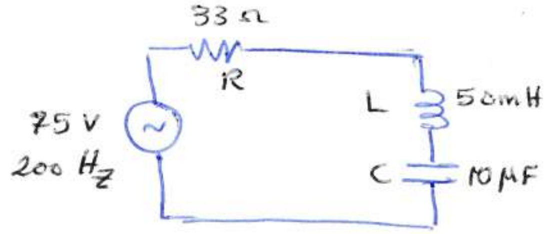
$$\boxed{|Z| = \sqrt{R^2 + X_{eq}^2}} \ , \ \boxed{\phi = \tan^{-1} \frac{X_{eq}}{R}}$$

$$Z \angle \phi = Z \cos\phi + jZ \sin\phi$$
$$= R + j X_{eq}$$


impedance diagram

$X_L > X_C$ $\qquad\qquad$ $X_C > X_L$

**Example** series RLC circuit




Calculate $I, V_R, V_L, V_C, \phi$

$$X_L = 2\pi f L = 62.8 \ \Omega$$
$$X_C = \frac{1}{2\pi f C} = 79.6 \ \Omega$$
$$Z = R + j(X_L - X_C) = 33 + j(62.8 - 79.6) = 33 - j16.8 \ \Omega$$

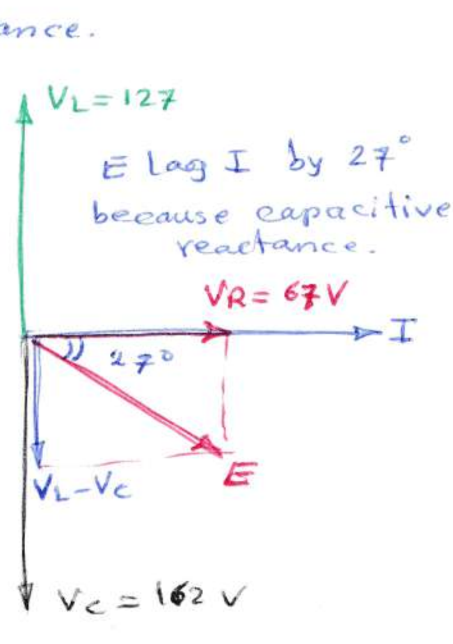
$X_{eq} = X_L - X_C = -16.8$ capacitive reactance.

$$Z = \sqrt{R^2 + X_{eq}^2} \angle \tan^{-1} \frac{X_{eq}}{R} = 37 \angle -27^\circ$$
$$I = \frac{E}{Z} = \frac{75}{37 \angle -27^\circ} = 2 \angle 27^\circ$$

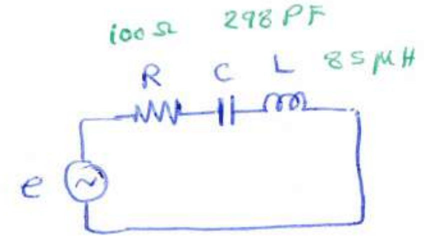
$V_R = I * R = 67$ volt.

$V_L = I * X_L = 127$ volt.

$V_C = I * X_C = 162$ volt.



## 7) Resonance in series RLC .




impedance.

$Z = R + j(X_L - X_C)$

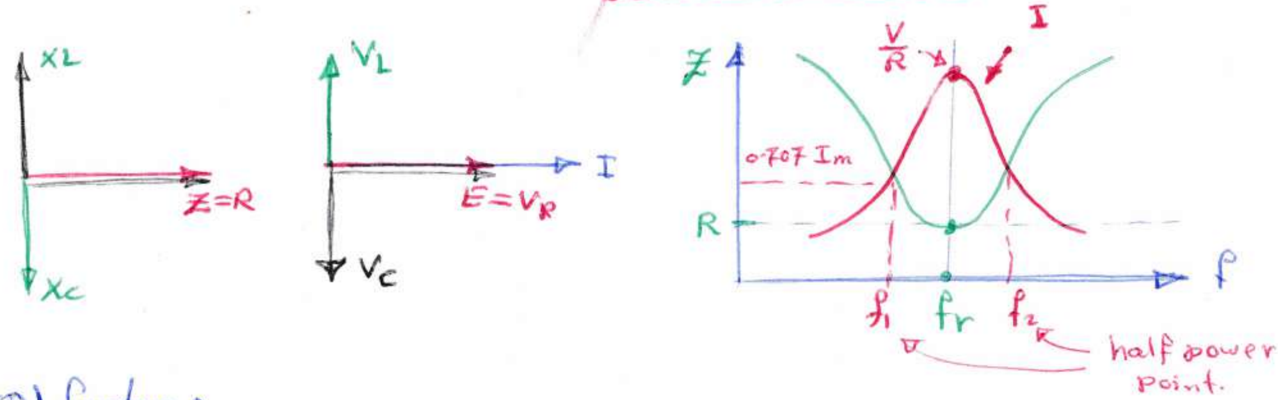
when $X_L = X_C \Rightarrow X_{eq} = 0$

$Z = R + j0 = R$ (Resistance only). minimum impedance.

$f_r$ : resonance frequency at which $X_L = X_C$

$$2\pi f_r L = \frac{1}{2\pi f_r C} \Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

$f_r = 1\,MHz$




### (Q) factor :

voltage magnification factor.

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \text{ at } f_r$$

(Example)

for LCR circuit at

$f_r = 500\,KHz$

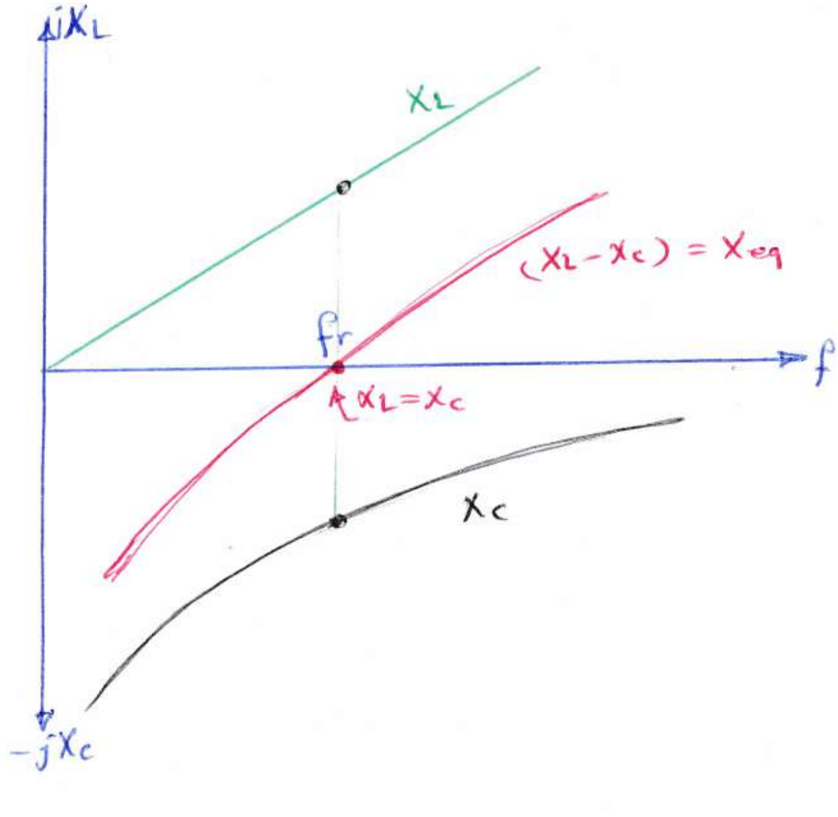
$L = 100\,\mu H$ , $C = 1\,nF$

$R = 25\,\Omega$

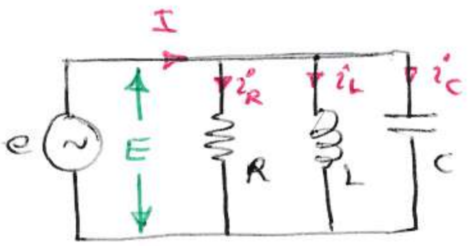
$$Q = \frac{1}{25}\sqrt{\frac{100\times10^{-6}}{1\times10^{-9}}} = 12.6$$

if $L = 200\,\mu H \Rightarrow Q \cong 25$

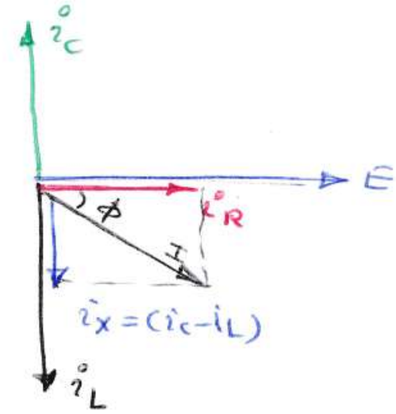
$$C = \frac{1}{4\pi^2 f_r^2 L} = 500\,pF$$



## 8) Parallel RLC circuits:




phasor diagram




$$I = i_R + j(i_C - i_L)$$

$$\boxed{I = i_R + j\, i_x} \qquad i_x = i_C - i_L$$

$$I = |I| \angle \phi = \sqrt{i_R^2 + i_x^2} \angle \tan^{-1} \frac{i_x}{i_R}$$

$$\frac{I}{E} = \frac{i_R}{E} + j\left(\frac{i_C}{E} - \frac{i_L}{E}\right)$$

$$\frac{1}{Z} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$\boxed{Y = G + j(B_C - B_L)}$$

admittance

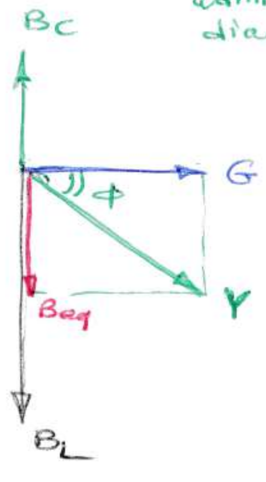
inductive susceptance

conductance

capacitive susceptance

$$\boxed{Y = G + jB_{eq}} \qquad B_{eq} = B_C - B_L$$

admittance diagram.



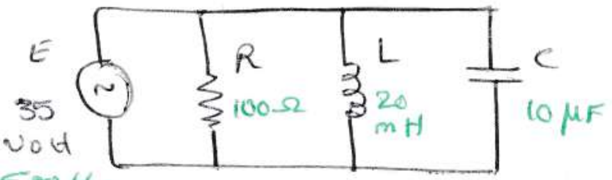

$$Y = |Y| \angle \phi$$

$$|Y| = \sqrt{G^2 + B_{eq}^2}$$

$$\phi = \tan^{-1}\left(\frac{B_{eq}}{G}\right)$$

## Example

- Calculate $i_R, i_L, i_C, I$
- determine the phase angle of $I$ with respect to $E$.
- draw the phasor and admittance diagrams.




$f = 500\,Hz$

$$X_L = 2\pi f L = 62.8\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = 31.8\,\Omega$$

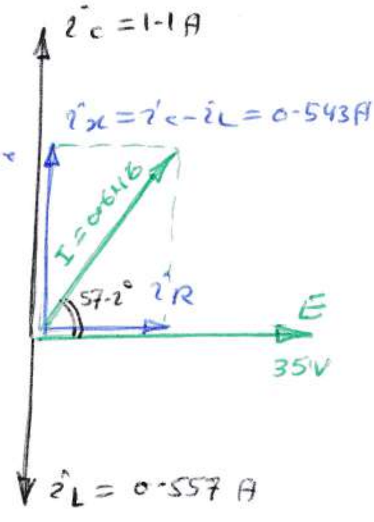
$$i_R = \frac{E}{R} = \frac{35}{100} = 350\,mA$$

$$i_L = \frac{E}{X_L} = \frac{35}{62.8} = 557\,mA$$

$$i_C = \frac{E}{X_C} = 1.1\,A$$

$$i_x = i_C - i_L = 1.1 - 0.557 = 0.543\,A$$

phasor diagram




$$I = i_R + j\,i_x = (0.35 + j\,0.543)\,A$$

$$I = \sqrt{i_R^2 + i_x^2}\ \angle \tan^{-1} i_x/i_R = \sqrt{(0.35)^2 + (0.543)^2}\ \angle \tan^{-1}\frac{0.543}{0.35}$$

$$I = 0.646\angle 57.2^\circ$$

$$G = \frac{1}{R} = 10\,ms$$

$$B_L = \frac{1}{X_L} = 15.9\,ms$$

$$B_C = \frac{1}{X_C} = 31.4\,ms$$

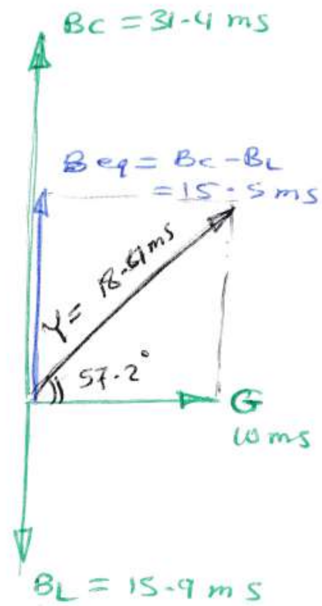
$$B_{eq} = B_C - B_L = 15.5\,ms$$

$$Y = \sqrt{G^2 + B_{eq}^2}\ \angle \tan^{-1}\frac{B_{eq}}{G_{eq}}$$

$$Y = 18.4\,ms\ \angle 57.2^\circ$$

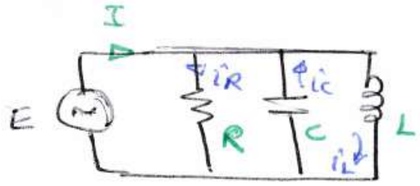
$$Z = \frac{1}{Y} = \frac{1}{18.4\,ms\ \angle 57.2^\circ}$$

$$Z = 54.3\,\Omega\ \angle -57.2^\circ$$




admittance diagram

## 9) Resonance in parallel RLC

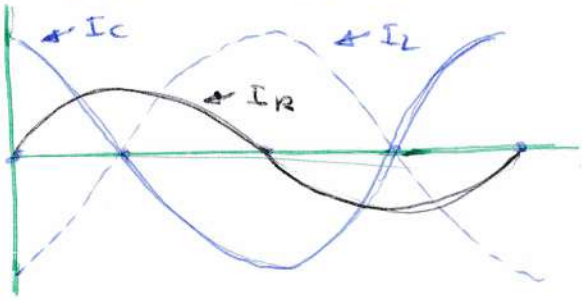


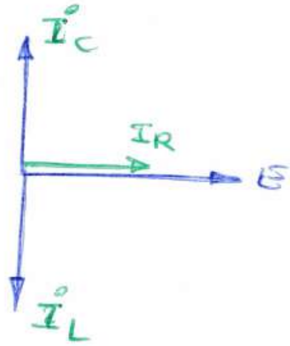

$$Y = \frac{1}{R_p} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$I = i_R + j(i_C - i_L)$$

When $i_C = i_L \Rightarrow I = i_R$ (Resonance)

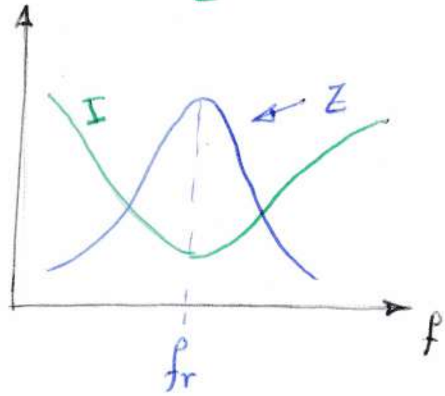
then E in phase with I




* for parallel RLC circuits we have maximume Z impedance at $f_r$. and minimum current.

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$




Example:

$L = 100\mu H$, $R = 12\ \Omega$, C Variable

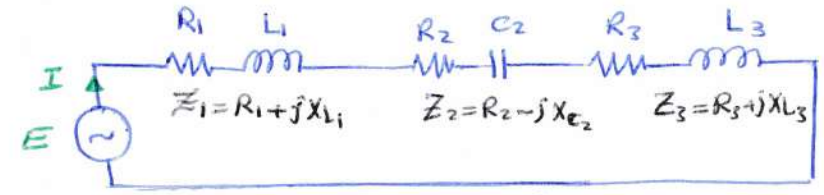
$200pF \leq C \leq 300pF$, find $f_r$ range.

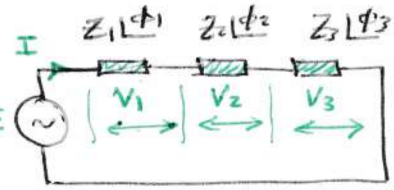
$$f_{r_{min}} = \frac{1}{2\pi\sqrt{L\,C_{max}}} = \frac{1}{2\pi\sqrt{100\times10^{-6} * 300\times10^{-12}}} = 919\ KHz$$

$$f_{r_{max}} = \frac{1}{2\pi\sqrt{L\,C_{min}}} = \frac{1}{2\pi\sqrt{100\times10^{-6} * 200\times10^{-12}}} = 1.13\ MHz$$

$$919\ KHz \leq f_r \leq 1.13\ MHz$$

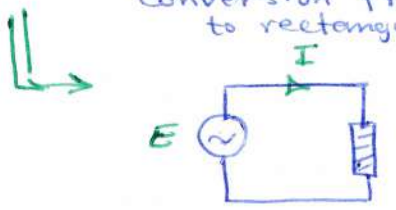
## 10) Impedances in series :




impedances in series

conversion from polar to rectangular form.




$Z\angle\phi = R \mp jX$

equivalent circuit

$R = R_1 + R_2 + R_3$ , $X = X_1 + X_2 + X_3$

### Example :

For circuit shown above $Z_1 = 70.7 \angle 45^\circ \ \Omega$
$Z_2 = 92.4 \angle 330^\circ \ \Omega$ , $Z_3 = 67 \angle 60^\circ \ \Omega$ , $E = 100$ volt.
OR $E = 100 \angle 0^\circ$ volt.

① find $I$ ② draw the phasor diagrame of $E$ & $I$.
③ find $V_1$, $V_2$, $V_3$ and draw complete phasor diagrame.

### Solution :

① $Z = Z_1 + Z_2 + Z_3$

$Z_1 = 70.7 \cos 45^\circ + j\, 70.7 \sin 45^\circ = 50 + j50 \ \Omega$

$Z_2 = 92.4 \cos 330^\circ + j\, 92.4 \sin 330^\circ = 80 - j\, 46.2 \ \Omega$

$Z_3 = 67 \cos 60^\circ + j\, 67 \sin 60^\circ = 33.5 + j\, 58 \ \Omega$

$\therefore Z = (50 + 80 + 33.5) + j\,(50 - 46.2 + 58) = \underline{163.5 + j\,61.8} \ \Omega$
$= \underline{174.8 \angle 20.7^\circ}$

inductive impedance

$I = \dfrac{E}{Z} = \dfrac{100 \angle 0^\circ}{174.8 \angle 20.7^\circ} = \underline{0.572 \angle -20.7^\circ} \ A$

I lag E by $20.7^\circ$

②

(3)

$E = 100\angle 0^\circ$ , $Z = Z_1 + Z_2 + Z_3 = 174.8^\circ \angle 20.7^\circ\ \Omega$

$$V_1 = E * \frac{Z_1}{Z_1+Z_2+Z_3} = 100\angle 0^\circ * \frac{70.7\angle 45^\circ}{174.8^\circ\angle 20.7^\circ} = 40.4\angle 24.3^\circ \text{ Vo}$$

$$V_2 = E * \frac{Z_2}{Z_1+Z_2+Z_3} = 100\angle 0^\circ * \frac{92.4\angle 330^\circ}{174.8^\circ\angle 20.7^\circ} = 52.9\angle 309.3^\circ \text{ volt}$$

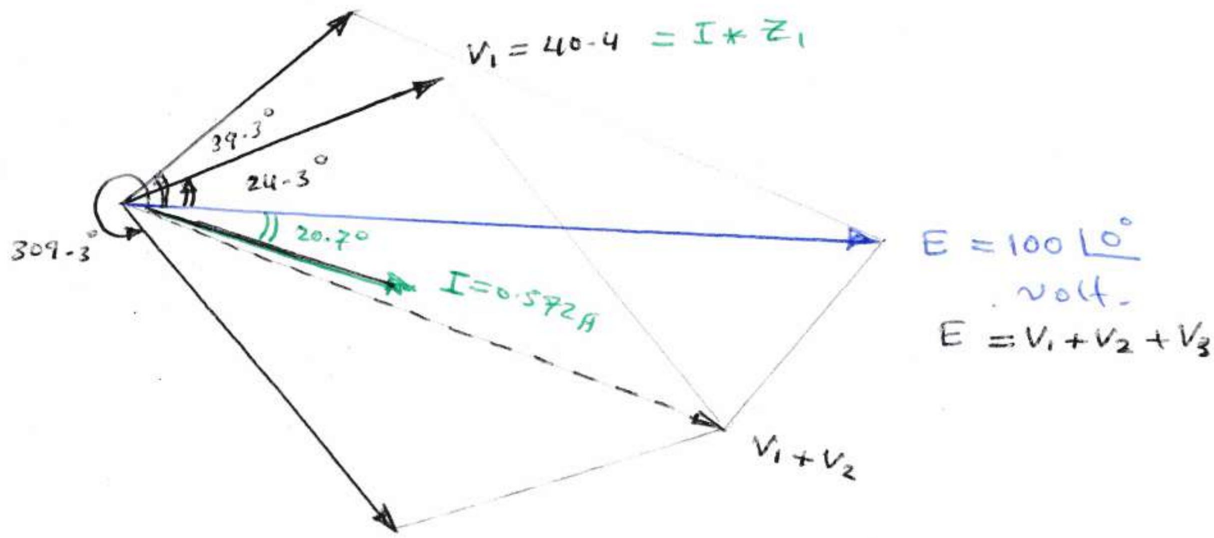
$$V_3 = E * \frac{Z_3}{Z_1+Z_2+Z_3} = 100\angle 0^\circ * \frac{67\angle 60^\circ}{174.8^\circ\angle 20.7^\circ} = 38.3\angle 39.3^\circ \text{ volt}$$

$\boxed{\text{OR}}$ $V_1 = I * Z_1 = 0.572\angle -20.7^\circ * 70.7\angle 45^\circ$

$= 40.4\angle 24.3^\circ$ volt.

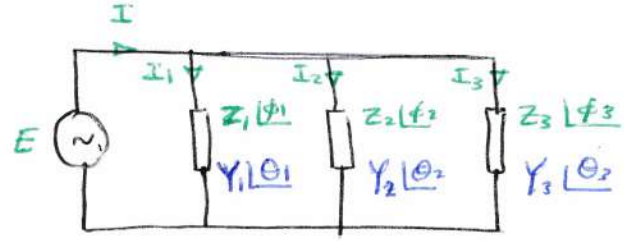
$V_2 = I * Z_2$

$V_3 = I * Z_3$



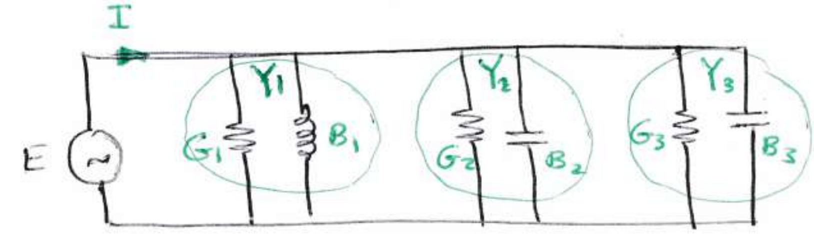

complete phasor diagrame.

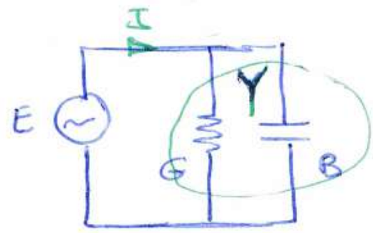
## II) Impedances in parallel




$$Y_1\angle\theta_1 = \frac{1}{Z_1\angle\phi_1} = \frac{1}{Z_1}\angle-\phi_1 = G_1 \mp jB_1$$

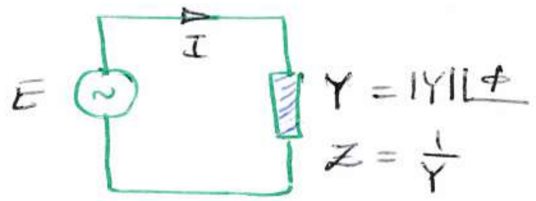
$$Y_2\angle\theta_2 = \frac{1}{Z_2}\angle-\phi_2 = G_2 \mp jB_2$$




$$Y = Y_1 + Y_2 + Y_3 = G \mp jB = |Y|\angle\phi$$

$$G = G_1 + G_2 + G_3 \quad , \quad B = B_1 + B_2 + B_3$$




equivalent circuit.

### Example

For the parallel circuit shown above if

$Z_1 = 1606\angle 51^\circ\ \Omega$ , $Z_2 = 977\angle -33^\circ\ \Omega$ , $Z_3 = 953\angle -19^\circ\ \Omega$

$E = 33$ volt. find :

① $Z$ total and $I_1$ , $I_2$ , $I_3$ , $I$

② draw the complete phasor diagrame of currents.

### solution :

① $Y_1 = \frac{1}{Z_1} = \frac{1}{1606\angle 51^\circ} = 622.7\angle -51^\circ\ \mu S = 392 - j484\ \mu S$

$Y_2 = \frac{1}{Z_2} = 1.02\angle 33^\circ\ mS = 855 + j556\ \mu S$

$Y_3 = \frac{1}{Z_3} = 1.05\angle 19^\circ\ mS = 993 + j342\ \mu S$

$$Y = Y_1 + Y_2 + Y_3 = (392 + 855 + 993)\,\mu S + j(-484 + 556 + 342)\,\mu S$$
$$= 2.24 + j\,0.414 \text{ mS} \equiv G_{eq} + j\,B_{eq}$$
$$= 2.28 \angle 10.5^\circ \text{ mS}$$

$$Z = \frac{1}{Y} = \frac{1}{2.28 \angle 10.5^\circ} = 439 \angle -10.5^\circ \ \Omega$$

$$I = EY = 33 \angle 0^\circ * 2.28 \angle 10.5^\circ \text{ mS} = 75.2 \text{ mA} \angle 10.5^\circ$$
$$= \frac{E}{Z}$$

$$I_1 = \frac{E}{Z_1} = EY_1 = 33 \angle 0^\circ * 622.7 \angle -51^\circ \ \mu S = 20.5 \angle -51^\circ \text{ mA}$$

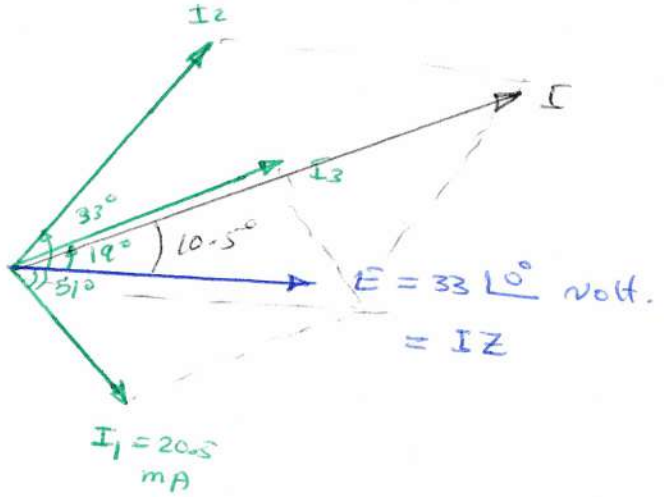
$$I_2 = \frac{E}{Z_2} = EY_2 = 33.8 \angle 33^\circ \text{ mA}$$

$$I_3 = \frac{E}{Z_3} = EY_3 = 34.6 \angle 19^\circ \text{ mA}$$

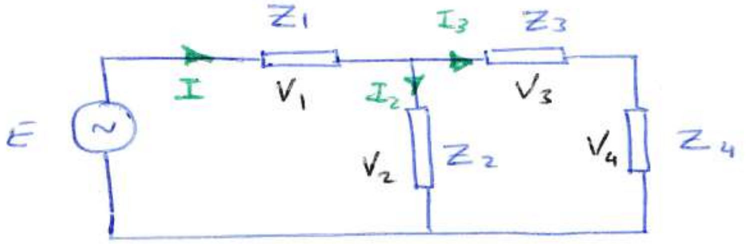
OR using current divider

$$I_1 = I\frac{Y_1}{Y}, \quad I_2 = I\frac{Y_2}{Y}, \quad I_3 = I\frac{Y_3}{Y}$$

$$I = I_1 + I_2 + I_3 \quad \text{(phasor sume).}$$



## 12) Impedances in series and parallel:




**Note**

all calculation in complex form or polar form.

$$Z_{eq} = Z_1 + \left[ Z_2 \,//\, (Z_3 + Z_4) \right]$$

$$I = \frac{E}{Z_{eq}} \quad ; \quad I_2 = I * \frac{(Z_3 + Z_4)}{Z_2 + Z_3 + Z_4}$$

$$I_3 = I * \frac{Z_2}{Z_2 + Z_3 + Z_4} \quad ; \quad I = I_2 + I_3$$

$$V_1 = I Z_1 \quad , V_2 = I_2 Z_2 \quad , V_3 = I_3 Z_3 \quad , V_4 = I_3 Z_4$$

**Example**

for the circuit shown above if $Z_1 = 560 - j620\ \Omega$, $Z_2 = 330 + j470\ \Omega$, $Z_3 = 390 + j270\ \Omega$, $Z_4 = 220 - j220\ \Omega$, $E = 30$ volt. find $I_2$

$$Z_5 = Z_3 + Z_4 = 610 + j50 = 612 \angle 4.7^\circ\ \Omega$$

$$Z_2 = 330 + j470\ \Omega = 574 \angle 54.9^\circ\ \Omega$$

$$Z_2 // Z_5 = \frac{Z_2 Z_5}{Z_2 + Z_5} = 329 \angle 30.6^\circ = 283 + j167\ \Omega$$

$$Z_{eq} = Z_1 + (Z_2 // Z_5) = (566 - j620) + (283 + j167) = 843 - j453\ \Omega$$

$$= 957 \angle -28.3^\circ$$

$$I = \frac{E}{Z_{eq}} = \frac{30 \angle 0^\circ}{957 \angle -28.3^\circ} = 31.3\,mA \angle 28.3^\circ$$

$$I_2 = I * \frac{Z_5}{Z_2 + Z_5} = 31.3\,mA \angle 28.3^\circ * \frac{612 \angle 4.7^\circ}{(330 + j470) + (610 + j50)}$$

$$= 17.8\,mA \angle 4^\circ$$

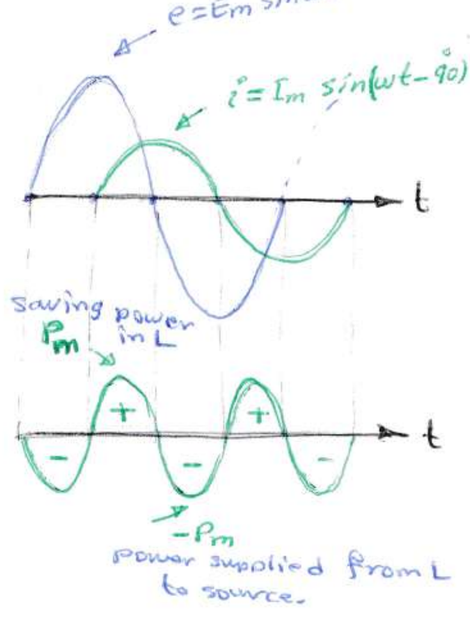
## 13) power in AC circuits

* power dissipated in aresistance:

$$\boxed{P = \underset{rms}{E}\ \underset{rms}{I} = \frac{E_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} = \frac{1}{2} E_m I_m = \frac{1}{2} P_m} \text{ watt}$$

$$\boxed{P = \frac{E^2}{R} = I^2 R = \underset{rms}{E\ I}}$$

* power in an inductance:



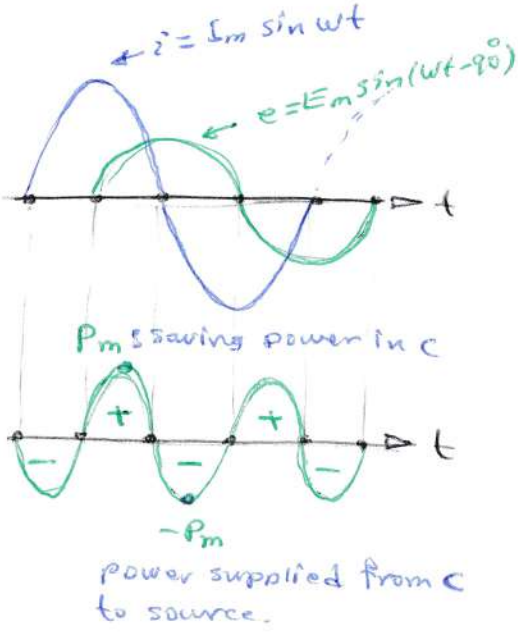

$P = e * i$

$= \underbrace{E_m I_m}_{P_m} \sin(\omega t) . \sin(\omega t - 90)$

$\therefore P \Rightarrow$ as shown $0 \to P_m \to 0 \to -P_m$

average power $= 0$

* power in a capacitance:




$P = e * i$

$= \underbrace{E_m I_m}_{P_m} \sin(\omega t) . \sin(\omega t - 90^\circ)$

$\therefore P \Rightarrow$ as shown

$0 \to P_m \to 0 \to -P_m \to$

average power is zero

## True power and reactive power :

* True power = power dissipated in Resistance.

$$= P = E\,I \quad \text{(watt)} \qquad \text{(heating resistance)}$$

$$= I^2 R = E^2/R$$

* Reactive power = power supplied to reactance $X_L$ or $X_C$

$$= Q \qquad \text{(average} = 0\text{)}.$$

$$Q_L = \text{reactive power for } L = E_L I_L = I_L^2 X_L = \frac{E_L^2}{X_L} \quad \text{(VAR)}$$

$$Q_C = \text{reactive power for } C = E_C I_C = I_C^2 X_C = \frac{E_C^2}{X_C} \quad \text{(VAR)}$$

VAR = Volt. ampear. reactive.

**Example** : calculate the power supplied when a 120 V ; 60 Hz source is connected to :

(a) 60 Ω resistor (b) L = 50 mH (c) C = 33 μF

**solution:**

(a) $$P = \frac{E^2}{R} = \frac{(120)^2}{60} = 240 \text{ watt} \quad \text{(true power)}$$

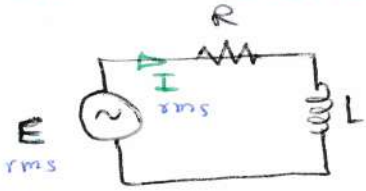
(b) $$X_L = 2\pi f L = 18.8\ \Omega$$

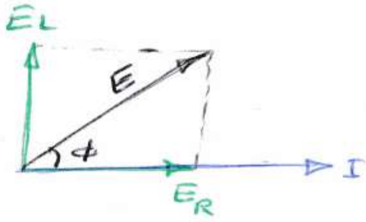
$$Q_L = \frac{E_L^2}{X_L} = \frac{(120)^2}{18.8} = 766 \text{ VAR} \quad \text{(reactive power)}$$

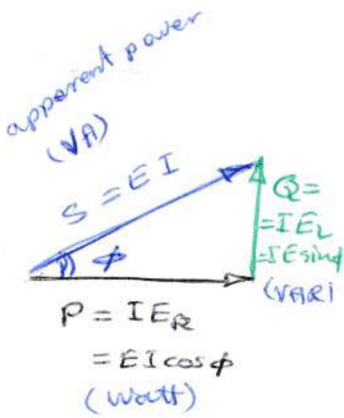
(c) $$X_C = \frac{1}{2\pi f C} = 80.4\ \Omega$$

$$Q_C = \frac{E_C^2}{X_C} = \frac{(120)^2}{80.4} = 179 \text{ VAR} \quad \text{(reactive power)}.$$

## power in RL circuit :




Power triangle

$S$ = apparent power (VA)

$S = EI$ (VA) Volt. amper

$P$ = true power = resistive power = $I * \underbrace{E\cos\phi}_{E_R}$ watt

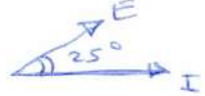
$Q$ = reactive power = $I * \underbrace{E\sin\phi}_{E_L}$ . (VAR)

$$\boxed{\cos(\phi) = \text{power factor} = \frac{\text{true power}}{\text{apparent power}} = \frac{P}{S}}$$ like effeciency

* we need to supply all ... power to load
$\cos(\phi) = 1$ (better case) , $S = P$ (no reactive power).

* for RL we have lagging power factor.

**Example :** for series RL AC circuite.
$E = 50$ volt. , $I = 100\,mA \angle 25°$ , find $S, Q, P$.
power factor.

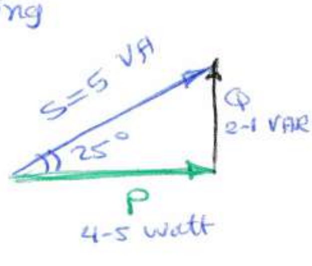



$S = EI = 50 * 100\,mA = 5$ (VA)

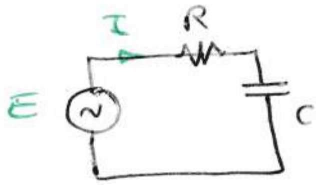
$Q = EI\sin\phi = 50 * 100\,mA * \sin(25°) = 2.1$ (VAR)

$P = EI\cos\phi = 50 * 100\,mA * \cos(25°) = 4.5$ (watt)

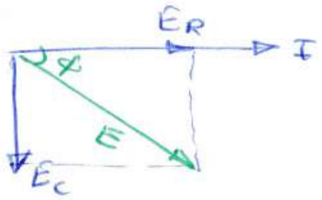
power factor = $\cos(\phi) = \cos(25°) = 0.9$ Lagging
$= 90\%$



## power in RC circuit:



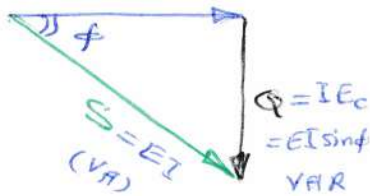

phasor diagrame.




power triangle.

$P = IE\cos\phi$

$P = IE_R$ watt




$S = EI$ volt-ampear (VA) $\Rightarrow$ apparent power.

$Q = IE_c = IE\sin(\phi)$ (VAR) $\Rightarrow$ reactive power

$P = IE_R = IE\cos(\phi)$ watt. $\Rightarrow$ true power

power factor $= \cos(\phi)$ (leading) ( I Lead E)

$=$ true power P / apparen power S

**Example:** For series RC AC circuit we have:

$R = 1.2\ K\Omega$, $C = 0.1\ \mu F$, $E = 45$ volt.

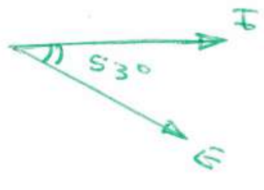
$f = 1KHz$. find P, S, Q, power factor.

**solution:**

$$X_c = \frac{1}{2\pi f_c} = 1.59\ K\Omega$$

$$|Z| = \sqrt{R^2 + X_c^2} = \sqrt{(1200)^2 + (1590)^2} = 1.99\ K\Omega$$

$$\phi = \tan^{-1}\left(\frac{X_c}{R}\right) = 53^\circ$$




$$|I| = \frac{E}{|Z|} = \frac{45}{1.99K\Omega} = 22.6\ mA$$

$S = EI = 45 * 22.6 \times 10^{-3} = 1$ VA

$Q = EI\sin(\phi) = 45 * 22.6 \times 10^{-3} * \sin(53) = 0.81$ VAR

$P = EI\cos\phi = 0.61$ watt.

$= I^2 R$

also $Q = I^2 X_c$

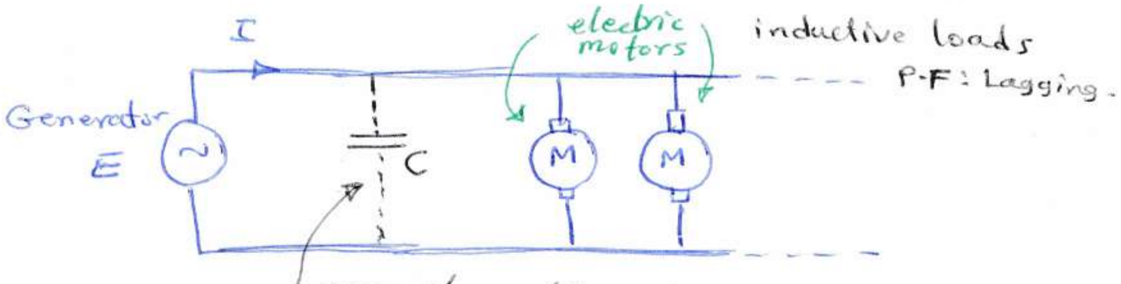
power factor $= \cos(\phi) = \cos(53) = 0.6$ Leading

60% leading.

## power factor correction:

(H-W: 21.6.1, 21.6.2 } p.587)

For good electrical power systems P.F. $\cos\phi \simeq 1$




capacitor adding for power factor correction (P.F. leading).

IF $\phi$ becomes Zero $\Rightarrow$ P.F $= \cos\phi = 1$
i.e no reactive power. (only true power).

**Example:** The current taken from a 115V, 60HZ supply is measured as 20 A with a lagging power factor of 75%. Calculate the apparent power S, true power P, reactive power Q. also determine the amount of capacitance that must be connected in parallel with the load to correct the power factors to 95% lagging.

* $P.F = \cos\phi = 75\% = 0.75$ Lag $\Rightarrow \phi = \cos^{-1}(0.75) = 41.4^\circ$

$S = EI = 115 * 20 = 2.3$ KVA

$P = EI\cos\phi = 115 * 20 * 0.75 = 1.725$ KW

$Q_L = EI\sin\phi = \underbrace{2300}_{S} * \sin(41.4^\circ) = 1.52$ KVAR

reactive power

* For $P.F = 95\%$ Lag $= 0.95$ Lag $\Rightarrow \phi = 18.2^\circ$

$P = I^2R = EI\cos\phi = 1725$ watt remains at this value.

$S = EI = \frac{P}{\cos\phi} = \frac{1725}{0.95} = 1820$ VA (new value)

$Q = EI\sin\phi = 1820 * \sin(18.2^\circ) = 568$ VAR (new value)

$Q = Q_L - Q_c \Rightarrow Q_c = Q_L - Q = 1520 - 568 = 952$ VAR

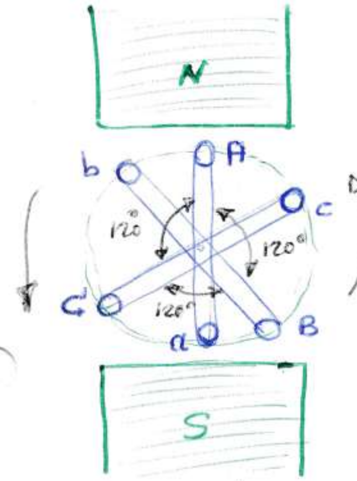
$Q_c = \frac{E^2}{X_c} \Rightarrow X_c = (115)^2/952 = 13.9\ \Omega$

$X_c = 1/2\pi f C \Rightarrow C = 191\ \mu F$

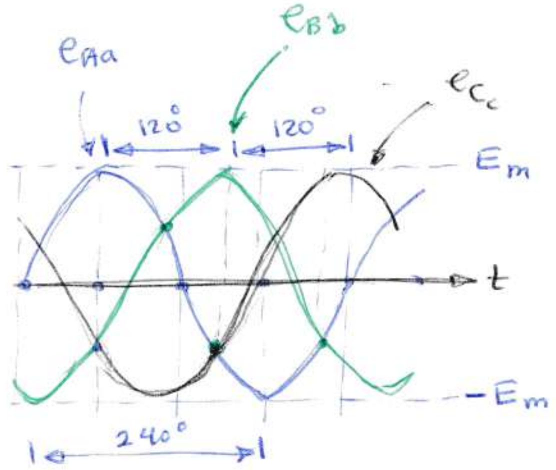
# Chapter (13)

## Three-Phase AC systems

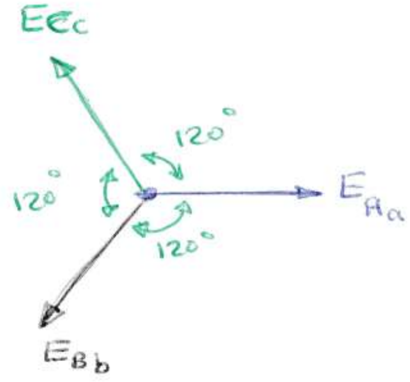
### Generation of 3 phase voltage :




3ph-Generator




Three phase waveform

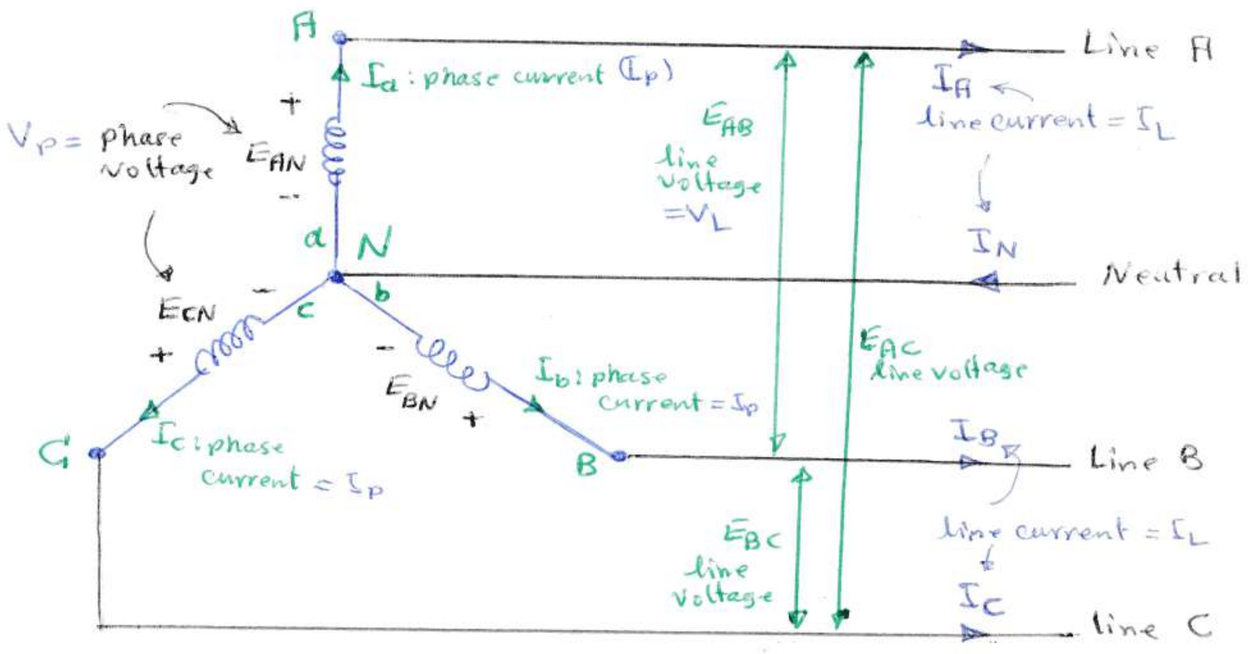



phasor diagram of voltage.

$e_{Aa} = E_m \sin \alpha$

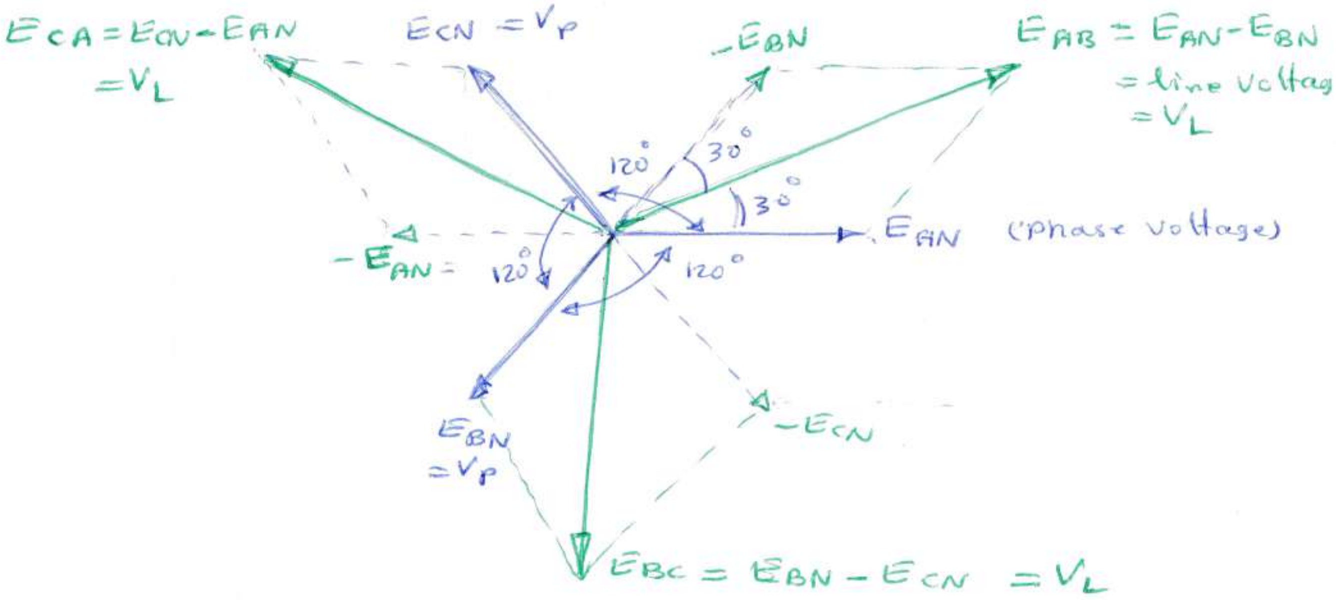
$e_{Bb} = E_m \sin(\alpha - 120°)$ ; $e_{Cc} = E_m \sin(\alpha - 240°)$

### 3-phase generator circuit and phasors in WYE Y connection




$I_A = I_a$ , $I_B = I_b$ , $I_C = I_c$ $\Rightarrow I_L = I_p$

in Y connection phase current = line current.



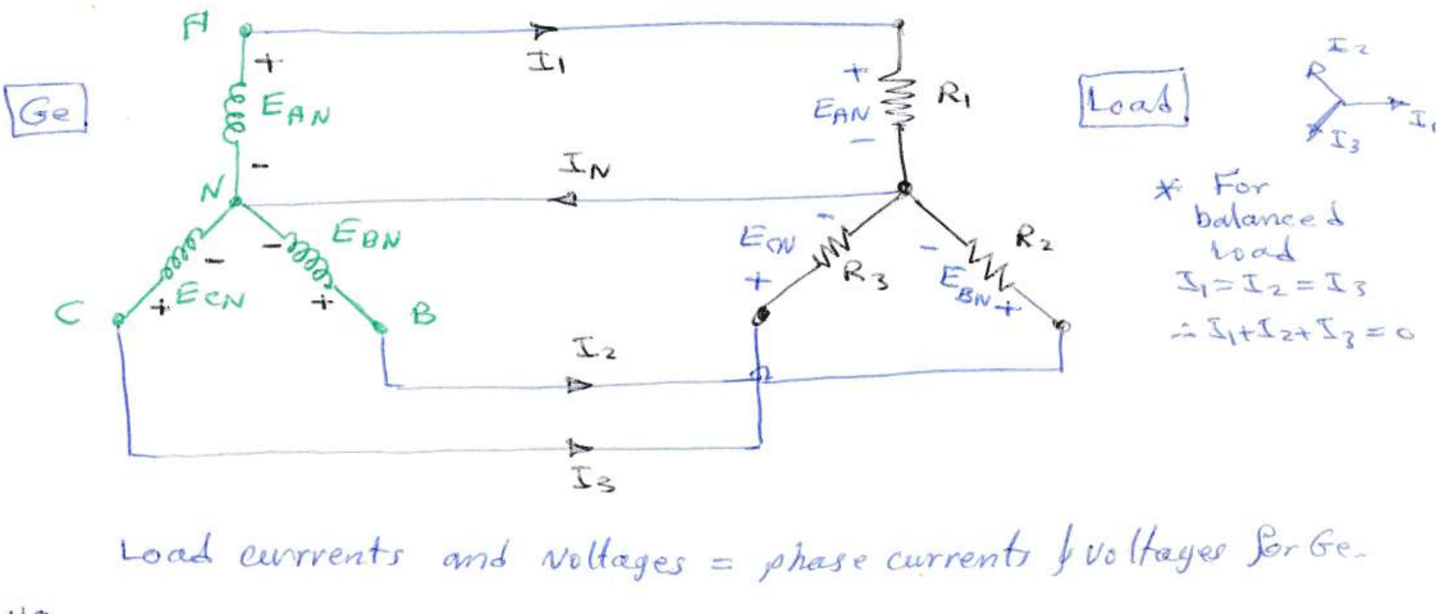

$|E_{AN}| = |E_{BN}| = |E_{CN}| = V_P$ : phase voltage.

$|E_{AB}| = |E_{BC}| = |E_{CA}| = V_L$ : line voltage.

$$E_{AB} = E_{AN}\cos(30^\circ) + (-E_{BN}\cos 30^\circ)$$
$$= V_P\cos(30) + V_P\cos(30) = 2\,V_P\cos 30^\circ$$
$$= 1.732\,V_P = \sqrt{3}\,V_P$$

$$\therefore\ V_L = \sqrt{3}\,V_P \quad \& \quad I_L = I_P$$

## Y–Y connection for generators and loads




* For balanced load
$I_1 = I_2 = I_3$
$\therefore I_1 + I_2 + I_3 = 0$

Load currents and voltages = phase currents & voltages for Ge.

**Note:** Load $R_1, R_2, R_3$ can be used $Z_1, Z_2, Z_3$ with the same analysis.

$$P_p = V_p I_p \cos(\phi) \quad \text{watt} \qquad (\text{phase power})$$

$$P = P_1 + P_2 + P_3 = 3 V_p I_p \cos\phi \quad \text{watt.}$$

$$V_L = \sqrt{3}\, V_p \quad , \quad I_L = I_p.$$

$\therefore$ $\boxed{P = \sqrt{3}\, V_L I_L \cos\phi = 3 V_p I_p \cos\phi}$ true power (Watt)

$\boxed{Q = \sqrt{3}\, V_L I_L \sin\phi = 3 V_p I_p \sin\phi}$ Reactive power (VAR)

$\boxed{S = \sqrt{3}\, V_L I_L = 3 V_p I_p}$ apparent power (VA)

$\boxed{P.F = \cos\phi = \frac{P}{S}}$

**Example:** 3-ph load resistors $R_1, R_2$ and $R_3$ have:

① equal values = 100 Ω

② 100 Ω , 200 Ω , 50 Ω

$V_p = 100$ V, find line current $I_L$, Neutral current $I_N$, line voltage $V_L$. (see figure before).

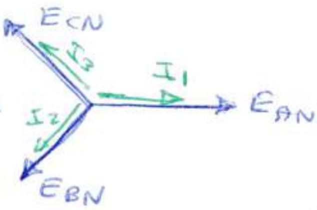
**Solution:**

① $E_{AN} = E_{BN} = E_{CN} = V_p = 100$ Volt.

$I_L = I_1 = I_2 = I_3$ (balanced load.

$I_L = V_p / R_1 = 100/100 = 1$ A

$I_N = I_1 + I_2 + I_3 = 0$ ; $V_L = \sqrt{3}\, V_p = 173.2$ Volt.




$$I_1 = \frac{E_{AN}}{R_1} = \frac{100\angle 0^\circ}{100\,\Omega} = 1\angle 0^\circ = 1 + j0 \text{ A}$$

$$I_2 = \frac{E_{BN}}{R_2} = \frac{100\angle -120^\circ}{100} = 1\angle -120^\circ = -0.5 - j0.8 \text{ A}$$

$$I_3 = \frac{E_{CN}}{R_3} = \frac{100\angle -240^\circ}{100} = 1\angle -240^\circ = -0.5 + j0.8$$

②

$$I_1 = \frac{E_{AN}}{R_1} = \frac{100\angle 0}{100} = 1\angle 0^\circ \ A = 1 + j0$$

$$I_2 = \frac{E_{BN}}{R_2} = \frac{100\angle -120^\circ}{200} = 0.5\angle -120^\circ = -0.25 - j0.433 \ A$$

$$I_3 = \frac{E_{CN}}{R_3} = \frac{100\angle -240^\circ}{50} = 2\angle -240^\circ = -1 + j1.73 \ A$$

$$I_N = I_1 + I_2 + I_3 = -0.25 + j1.3 \ A = 1.32\angle 100.9^\circ \ A$$

$V_L = \sqrt{3}\ V_P = 173.2$ volt as before.

③ if we have $Z_1 = 100\ \Omega$, $Z_2 = 100 - j40\ \Omega = 107.7\angle -21.8^\circ$
$Z_3 = 100 + j60\ \Omega = 116.6\angle 31^\circ$
find the lines currents and neutral current.

$$I_1 = \frac{E_{AN}}{Z_1} = \frac{100\angle 0^\circ \text{ volt}}{100\ \Omega} = 1\angle 0^\circ \ A = 1 + j0 \ A$$

$$I_2 = \frac{E_{BN}}{Z_2} = \frac{100\angle -120^\circ}{107.7\angle -21.8^\circ} = 0.9\angle -98^\circ = -0.133 - j0.92 \ A$$

$$I_3 = \frac{E_{CN}}{Z_3} = \frac{100\angle -240^\circ}{116.6\angle 31^\circ} = 0.86\angle -271^\circ = 0.015 + j0.858 \ A$$

$$I_N = I_1 + I_2 + I_3 = 0.882 - j0.062 \ A$$
$$= 0.884\angle -4^\circ \ A.$$

Example: find for part ③ the power dissipated in the load.

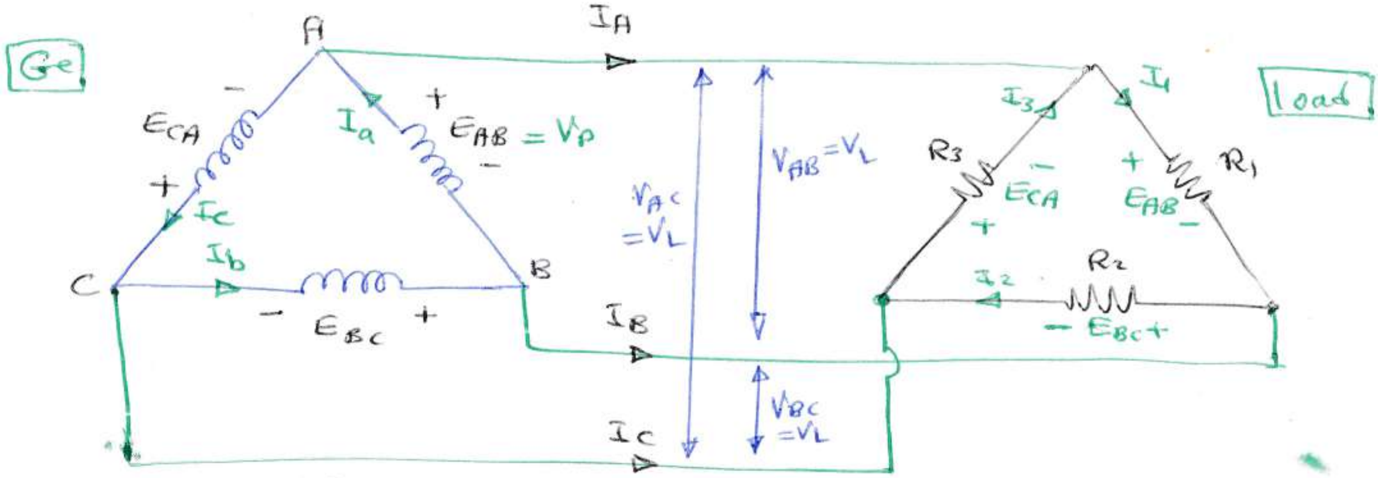
$$P_1 = V_{P_1} I_{P_1} \cos\phi_1 = 100\,V * 1A * \cos(0) = 100 \text{ watt}$$

$$P_2 = V_{P_2} I_{P_2} \cos\phi_2 = 100 * 0.9 * \cos(21.8) \simeq 86.3 \ W$$

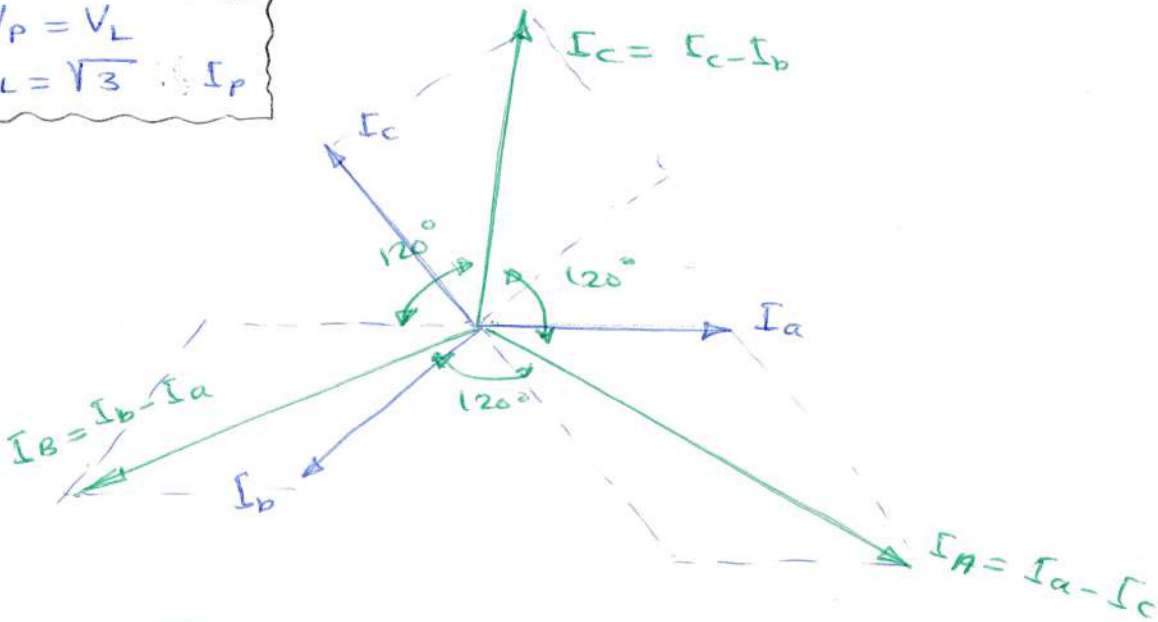
$$P_3 = V_{P_3} I_{P_3} \cos\phi_3 = 100 * 0.858 * \cos(31) = 73.5 \ W$$

$$P_T = P_1 + P_2 + P_3 = 259.8 \text{ watt.}$$

## Δ - Δ connections for generators and loads.




$$\boxed{\begin{aligned} V_P &= V_L \\ I_L &= \sqrt{3}\, I_p \end{aligned}}$$




$$P = \sqrt{3}\, V_L I_L \cos\phi$$
$$= 3 V_p I_p \cos\phi \quad \text{watt}$$

$$Q = \sqrt{3}\, V_L I_L \sin\phi \quad \text{VAR}$$
$$= 3 V_p I_p \sin\phi$$

$$S = \sqrt{3}\, V_L I_L$$
$$= 3 V_P I_P \quad \text{VA}$$