Research Article

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Seismic evaluation cylindrical concrete shells

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Abstract: The reasons for concrete roof shells' apparent seismic resistance have been subject to limited research but they have been shown to be inherently resilient to earthquakes. Shells constructed of concrete exhibit high structural efficiency and can therefore be made very thin. As a result of their relatively lightweight nature, thin shell structures are implicitly resistant to earthquake forces. The shell structure is typically designed so that it performs optimally under gravity loads, which are carried mainly by membrane action over the shell surface. As earthquakes induce unexpected horizontal forces, concrete shell structures can be damaged by bending stresses. By studying 8 cm-thick concrete roof shells using parametric analysis, this research shows that small and midsized (span <30 m) thin concrete roof shells can indeed be intrinsically earthquake resistant. These structures have high geometric stiffness and low mass, which results in fundamental frequencies far higher than those of realistic seismic events. Under earthquake excitation, these characteristics result in elastic shell behavior, without exceeding the maximum concrete strength. A shallow shell exhibits greater stress in response to earthquake vibrations caused by the vertical components than by horizontal components. Further, by increasing the rise and curvature of large shells, the fundamental frequency increases and the damaging effect of vertical earthquake vibration is reduced. The aim of this study in general is to show the analysis and the effect of earthquakes on cylindrical concrete shells.

Keywords: cylindrical shells, earthquake, shell concrete, finite element method, ANSYS software

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1 Introduction

A thin concrete shell is used for a variety of purposes, including roofs for public spaces, auditoriums, and industrial facilities [1]. It is possible to span large spaces without using interior supports with thin concrete shell roofs, as they have a high strength-to-weight ratio and are highly rigid [2]. In the past decade, large span thin shell-reinforced concrete roof structures have demonstrated their ability to withstand extreme loads during natural disasters. As an example, Félix Candela's reinforced concrete roof shell structures (Mexico City, 1950–1960s) survived the 8.0 Mw earthquake that shook Mexico city in 1985 without damage [3].

Cylindrical shells have a singly curved shell, developable surface, and commonly an arc-shaped cross-section. Besides two straight edges parallel to the cylinder's axis, the surface has two curved edges perpendicular to the axis. The length-to-radius ratio of cylindrical shells can be classified as long ($L/R > 5$) or short ($L/R < 1$) and intermediate between them. Across the longitudinal edge of a short cylindrical shell, loads are transferred to transverse supports by deep beams on the shell's edge sections. In long cylindrical shells, it behaves as a large beam with a thin curved section, although there is still some arching near the crown [4].

These structures are often the result of collaboration between architects, engineers, and builders. In many cases, roof shells are built simultaneously by architects and engineers in different parts of the world. For example, consider the roof shells designed by Isler (Switzerland), Candela (Mexico), and Nervi (Italy). In terms of the number and variety of concrete shells built, the 1920s to early 1960s are considered the golden age of concrete shell construction. The number of concrete shells built and the articles published on their analysis and design methods have decreased steadily since then. This was largely caused by the difficulty of building concrete shells as well as some serious collapses [5,6].

In response to an earthquake, buildings shake for a few seconds. During this time, multiple types of seismic waves are combined to shake the building in different ways, depending on which earthquake is occurring. Due to variations in fault slippage, different rock types through which the waves pass and different geological characteristics at each location, the resultant shaking differs at each one.

Each building is different, whether in size, configuration, material, structural system, method of analysis, or age and quality of construction; each of these characteristics affects the building's response [7].

The authors are not aware of any thin concrete shell structure that sustained significant structural damage as a result of an earthquake. Due to the curved geometry of shells, they are very thin and have high structural efficiency. Thus, the forces generated by dynamic actions such as earthquakes are relatively low because they are directly proportional to the mass of the shell. It is the overall shape of a shell structure that is most important among the common design parameters, such as support conditions, material, thickness, and overall shape, that determines if a shell will have enough safety, stability, and stiffness to span a space without intermediate supports [8]. However, roof shell structures are usually shaped in a way that allows gravity loading to be applied optimally [9]. Furthermore, they often carry loads to the foundation through membrane action, thus avoiding tensile stresses caused by bending [3]. Nevertheless, earthquake-induced bending moments could cause structural damage to shells. Designing shells in earthquake-prone regions differs from typical shell design in that, instead of focusing on gravity loads, more attention needs to be paid to the strength of the shell against large bending moments [10]. In this study, three types of short, intermediate, and long cylindrical concrete shells are presented, and they were exposed to the Landers and El-Centro earthquakes. The results were compared with those of Michiels and Adriaenssens [11], noting that they studied the square type of concrete shells.

2 Methodology description

In this study, parametric analyses are used to examine the effect of shell geometry on the vibrational properties and earthquake resilience of a concrete roof shell with a different size plan. The purpose of this study is to assess the performance of singly curved shells based on response spectrum analysis of recorded spectra and to determine the key design parameters that will produce a shell structure for the same plan area that is more durable and effective in resisting stresses and deformations. The flowchart shown in Figure 1 illustrates the research methodology.

2.1 Shell geometry and material properties

Three types of concrete cylindrical shells, long, medium, and short shell cylinders, were analyzed, starting with variable plan dimensions, a thickness of 8 cm and an angle of 60° against curvature, as indicated by the first group. The thickness of the concrete shell for the second group was changed according to the fixed span of 20 m and the angle of curvature of 60°. In the last group, the curvature angle was changed for each concrete shell type with an 8 cm thickness and a 20 m span as well. Each of the above cases is simply supported (Figure 2).

The material properties used in the finite element implementation via ANSYS 21R2 software [12] are given in Table 1.

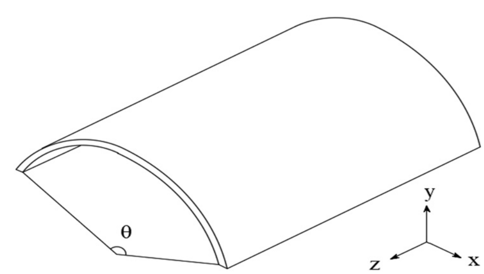

Figure 2: Cylindrical shell; perpendicular displacements in the horizontal plane are permitted. Displacements vertically and parallel to the edge are restrained.

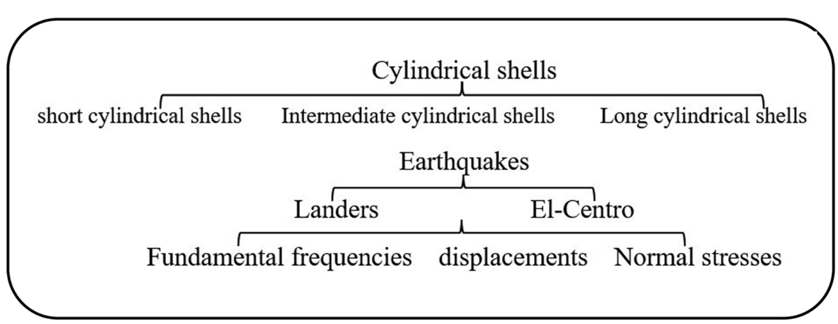

Figure 1: Flowchart illustrating the research methodology.

Table 1: Material properties used for the parametric study [11]

Compressive strength	30 MPa
Tensile strength	3 MPa
Young's modulus	21.5 GPa
Density	2,400 kg/m^3
Poisson's ratio	0.2

2.2 Numerical modeling of concrete shells

Solid elements (SOLID65) with eight nodes are used to model the concrete shell material, which includes three degrees of freedom at each point and translations in x, y, and z directions. Also, this element is capable of plastic deformation, cracking in the x, y, and z directions, until it reaches the crushed concrete [13]. In modeling concrete materials, the element-type SOLID65 provides results by calculating the nonlinear behavior of concrete shells [14].

2.3 Modal analysis and earthquake response

Besides performing a response spectrum analysis, the earthquake response of the shell is also determined indirectly by evaluating its fundamental frequencies. The eigenvalues and corresponding eigenmodes of a shell depend only on its stiffness and mass distribution, and as such, are independent of its loading. A normal mode analysis can be used to determine these values by solving

$$[\boldsymbol{K} - \lambda \boldsymbol{M}]\varnothing = 0, \qquad (1)$$

where $\boldsymbol{K}$ is the stiffness matrix of the shell structure and $\boldsymbol{M}$ is the mass matrix. The eigenvalues of the shell can be represented by vector λ and the corresponding eigenvectors by vector ϕ. The Lanczos algorithm is used to calculate n eigenvalues λ and n eigenvectors ϕ, where n is the number of degrees of freedom [15]. Based on the excitation and the mass participation factor for each eigenvalue, corresponding modal shapes contribute to the dynamic response of the system. A response spectrum analysis is used to determine the response of each shape to the seismic input spectra in addition to determining the fundamental frequencies [16]. First, the response spectrum of the 1992 7.3 Mw Landers earthquake in California, USA (Figure 3), was used as an earthquake input. Model calibration used this earthquake input as the earthquake response of another shell reported in the study of Ostovari Dailamani [17]. An additional factor that made the earthquake so relevant is that it had a particularly strong vertical component. The vertical component of the shells studied in this article is more significant than the horizontal component, which will be discussed in Section 3.2. The second spectrum is derived from the El-Centro earthquake in 1940, in Southern California (Figure 4). The reason why a response spectrum analysis is used instead of a time history analysis is that a response spectrum analysis is more computationally efficient, allowing for more parametric variations to be processed more effectively. In addition, since the behavior of the shells in this study is elastic, only the maximum response of the structure is of significant interest, while the evolution of the response over time provides little value to the research.

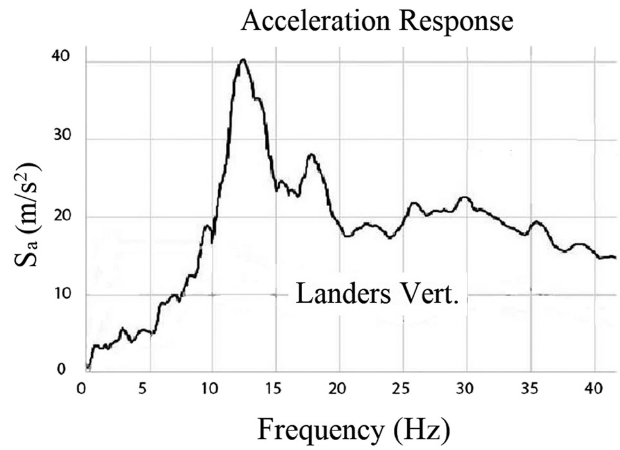

Figure 3: The acceleration response spectrum for the vertical component of the Landers earthquake.

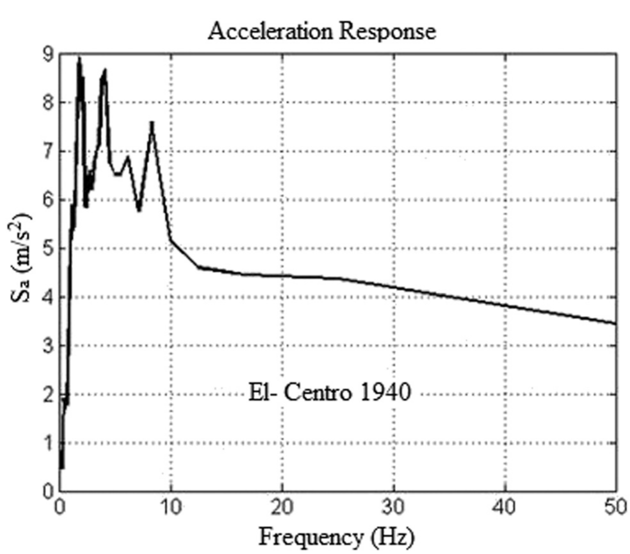

Figure 4: The acceleration response spectrum of the El-Centro 1940 earthquake in the vertical direction.

2.4 Variation in shape

Generally, structures with high fundamental frequencies are well suited to withstand both vertical and horizontal external forces. Typically, the displacement response decreases with increasing frequency (Figure 5). By examining the seismic spectrum, it is noted that in large spaces from 30 to 50 m, there is a slight change in low frequencies, which gives higher deformations. As the concrete shell thickness increases (Figure 6), the frequency increases, which naturally reduces deformation. With respect to changing the curvature angle θ (Figure 7), we notice that the bigger the angle, the higher the rigidity of the structure, and thus the decreased deformations in it. These results were adopted for the Landers and El-Centro earthquakes. There was a slight difference in the deformation values of concrete shells, with some oscillations in them that might be due to the convergence of seismic frequencies with the natural frequency of the concrete shell, leading to high deformation values.

An approach to understanding the seismic behavior of shells is to determine whether the fundamental frequencies of the analyzed shells fall within these low-frequency ranges and to determine the parametric variations of the initial shell shape for which the fundamental frequency is at the highest level. It is acceptable to increase the shell's fundamental frequencies as long as this results in the elasticity of the shell structure. Reinforced concrete shells form plastic hinges if their elastic limit is exceeded, and these hinges will reduce the shell's stiffness and therefore its fundamental frequency. Therefore, it can be assumed that the adopted elastic approach applies to thin reinforced concrete shells.

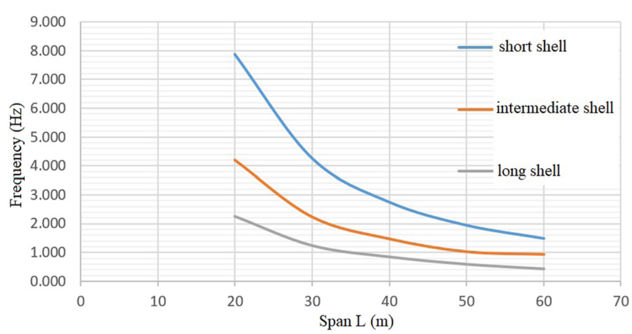

Figure 5: Evolution of frequency for reinforced concrete cylindrical shells with a constant thickness of 8 cm and variable span.

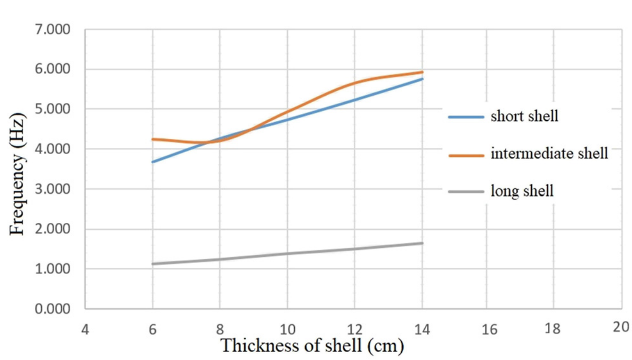

Figure 6: Evolution of frequency for reinforced concrete cylindrical shells with a constant span and variable thickness.

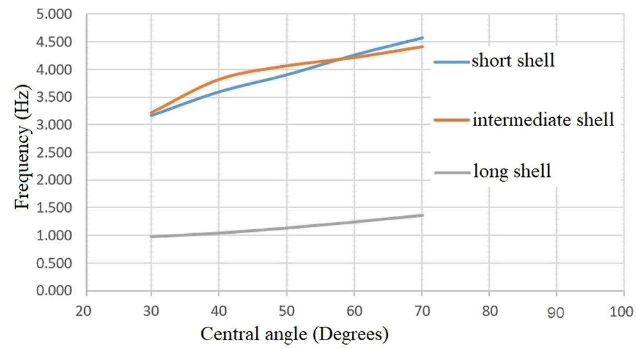

Figure 7: Evolution of frequency for reinforced concrete cylindrical shells with a constant thickness of 8 cm and variable central angle.

3 Results

3.1 Validations

The normal modes and response spectrum analyses are initially calibrated for a 20 m × 20 m, 0.08 m-thick shell with 60° curvature, as reported by Michiels and Adriaenssens [11]. During model validation, deformations due to the vertical component of the Landers earthquake were determined using mesh convergence analysis, modal analysis with modal mass participation analysis, and response spectrum analysis [16]. In Section 2.1, a parametric analysis is performed to determine a realistic plan size. It is shown that 8 cm-thick singly curved shells of this form with spans greater than 50 m will fail due to large deformations. Additionally, for shells with spans greater than 30 m, the concrete's maximum tensile strength (3 MPa) is exceeded locally. The Landers earthquake had this result, but the El-Centro earthquake had all stresses developed as less than 3 MPa.

Studies of cylindrical shells with spans less than 30 m demonstrate that their fundamental frequencies are higher than 4 Hz, not coincident with earthquake frequencies (Landers and El-Centro 1940). As a result, earthquakes should have little effect on these shells. In order to

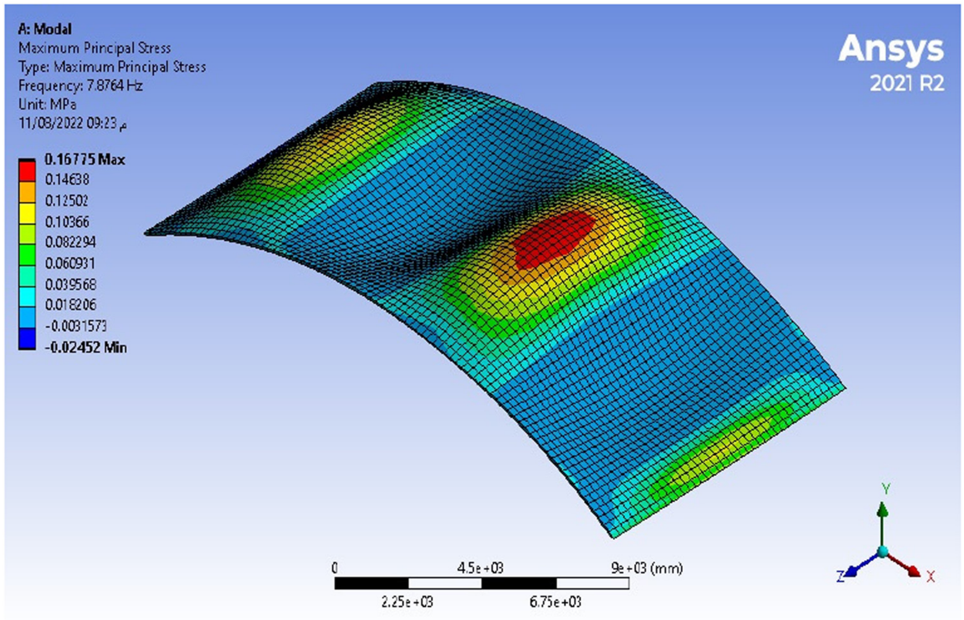

Figure 8: Maximum stress under self-weight in the short shell with a span of (20 × 10) m.

confirm this hypothesis of favorable behavior based on a high fundamental frequency, a span of (20 × 10) m was chosen for the short shell, (20 × 20) m for the intermediate shell, and (20 × 40) m for the long shell as a base model for an in-depth analysis under earthquake loading. The rise of the shell is 2.68 m (a 60° central angle), and the calculated fundamental frequencies are 7.88, 4.21, and 2.26 Hz for short, intermediate, and long shell, respectively. The maximum deformation under the self-weight condition did not exceed 0.5 mm. The deformation of this shape is lower than the span of the shell divided by 200, which is used as a criterion for maximum deflection, where u_v^{max} is the maximum vertical deflection in the shell [18]:

$$u_v^{max} \leq L/200 = 10\text{ cm}. \qquad (2)$$

The stress values throughout the shells are low under a self-weight (Figures 8–10). As a result of the self-weight of the concrete shell, maximum stress values are found at its top. Based on a conservative reinforced concrete compressive strength of 30 MPa and the corresponding tensile strength of 3 MPa, the concrete shell will not crack under a self-weight since the maximum permissible compressive and tensile stresses are never exceeded.

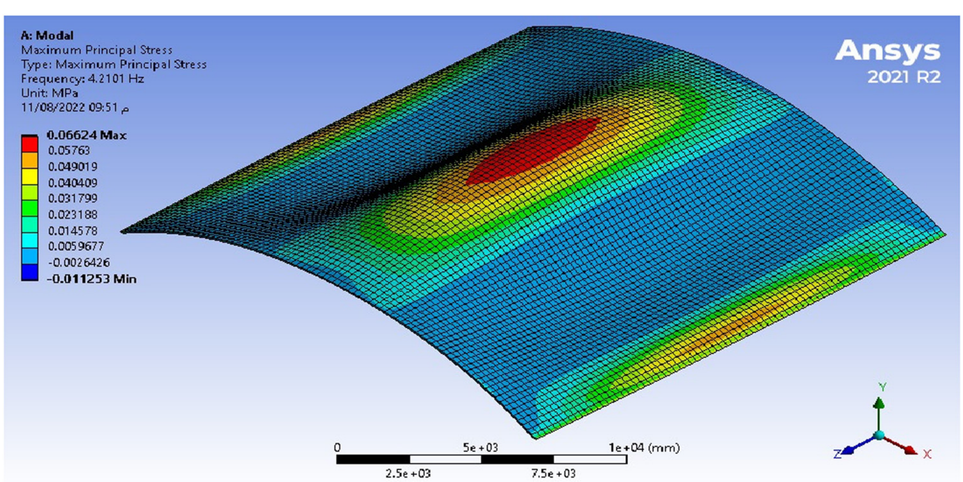

Figure 9: Maximum stress under a self-weight in the intermediate shell with a span of (20 × 20) m.

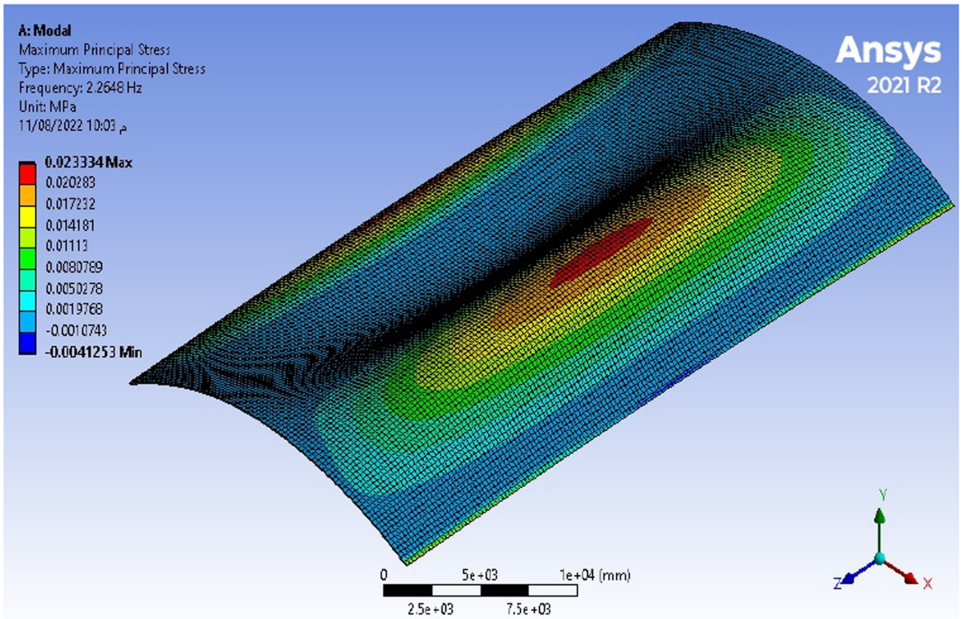

Figure 10: Maximum stress under self-weight in a long shell with a span of (20 × 40) m.

3.2 Response to the initial earthquake

There are two earthquake spectra used in the models: one for the vertical components of the Landers earthquake (Figure 3) and one for El-Centro 1940 (Figure 4). Landers' vertical component is more important than its horizontal component because the vertical component is much larger (about four times at peak response). For an earthquake, this behavior is atypical, and a large vertical component is only realistic in near-field events but is often much more destructive [19]. As compared with the Landers earthquake, the El-Centro spectrum exhibits a greater proportion of lower frequencies. The response spectra analysis indicates, however, that the deformation values of the concrete shells give greater results in the case of vertical earthquake components (for the Landers and El-Centro earthquakes and for concrete shells of short, intermediate, and long lengths, it does not exceed 3 cm), mostly (Tables 2–4). Additionally, the vertical component causes much higher maximum stresses. Since the shells under investigation are relatively shallow, this conclusion is not unexpected. As a result, they are more susceptible to vertical loading, which operates

Table 3: Total deformations and maximum normal stresses of intermediate shells

Span (m)	Max. deformation (cm)		Max. normal stress (MPa)	
	Landers	El-Centro	Landers	El-Centro
20	1.271	0.419	1.859	1.256
30	0.832	1.315	2.825	2.144
40	1.457	2.934	3.627	3.965
50	2.174	3.906	2.497	3.867
60	3.653	4.121	3.423	3.765

Table 2: Total deformations and maximum normal stresses of short shells

Span (m)	Max. deformation (cm)		Max. normal stress (MPa)	
	Landers	El-Centro	Landers	El-Centro
20	0.286	0.089	2.564	0.558
30	0.997	0.270	3.998	0.929
40	0.878	0.768	4.268	1.720
50	1.106	1.561	4.189	3.289
60	1.098	1.919	2.829	3.307

Table 4: Total deformations and maximum normal stresses of long shells

Span (m)	Max. deformation (cm)		Max. normal stress (MPa)	
	Landers	El-Centro	Landers	El-Centro
20	0.811	1.024	2.520	1.766
30	1.668	3.215	3.967	5.623
40	3.985	5.990	2.986	4.457
50	7.549	10.822	3.444	6.185
60	13.938	13.004	5.242	11.560

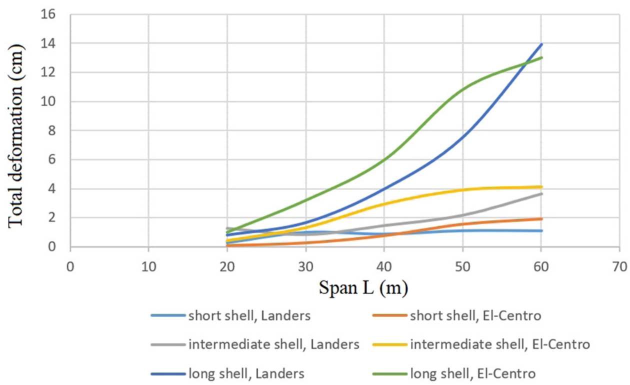

Figure 11: Total deformations with different span lengths of shells, subjected to the vertical components of Landers and El-Centro earthquakes.

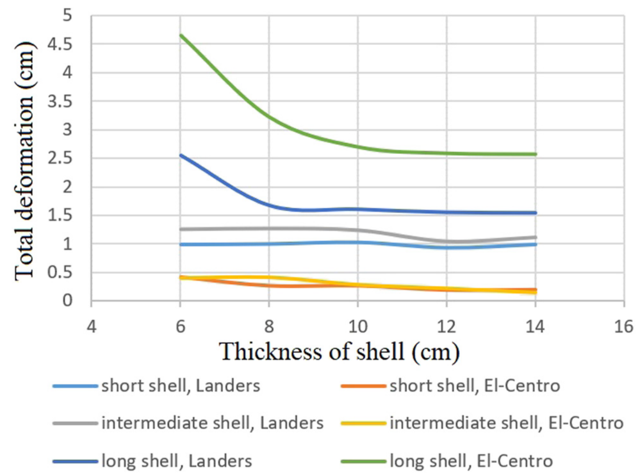

Figure 13: Total deformations with different thicknesses of shells, subjected to the vertical components of Landers and El-Centro earthquakes.

mostly out-of-plane, than to horizontal loading, which operates in-plane. For the shells, the normal stresses imposed by Landers' were 2.6–4.2, 1.4–3.6, and 2.5–3.9 MPa for short, intermediate, and long shells, respectively, and in the case of the El-Centro earthquake, the normal stresses were between 0.5–3.3, 0.3–3.9, and 1.8–11.6 MPa for short, medium, and long shells, respectively. Although the stress values in the El-Centro earthquake are mostly within the permissible range to withstand the elastic behavior, because the tensile strength of concrete (3 MPa) is not exceeded mostly, in the Landers earthquake, the maximum permissible tensile strength of concrete is exceeded, which affects the earthquake on the shells in some cases. As a result of these stresses, cracking would occur in the shell structure and the reinforcement would be activated, affecting fundamental frequencies through stiffness reduction. Additionally, by selecting a smaller span, for example 20 m, the maximum principal stresses under vertical earthquake loading would not exceed 3 MPa for both earthquakes, thus ensuring elastic behavior (Figures 11 and 12). The results were compared with the fundamental frequency; according to the research, it was 3.63 Hz and it appeared to us as 4.1 Hz. The vertical deformation was 1.2 cm, while according to our analysis it was 1.271 cm, as shown in Figure 4 and Table 3, of the results of the Landers earthquake for a cylindrical concrete shell 20 m × 20 m, where the results are very similar.

3.3 Effect of shell dimensions

The changing shape of the shells will show that the fundamental frequency and stiffness can be altered and excessive stresses resulting from vertical excitations can also be avoided.

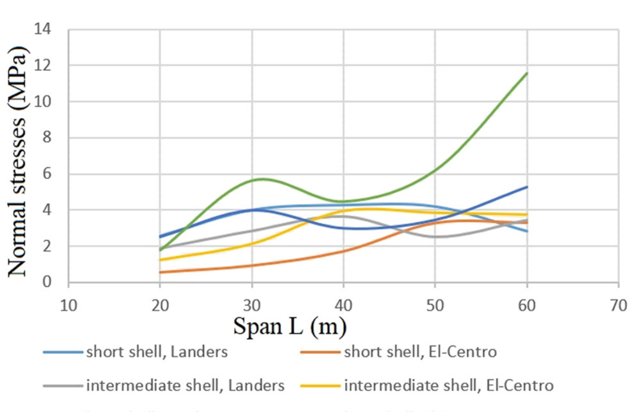

Figure 12: Maximum normal stresses with different span lengths of shells, subjected to the vertical components of Landers and El-Centro earthquakes.

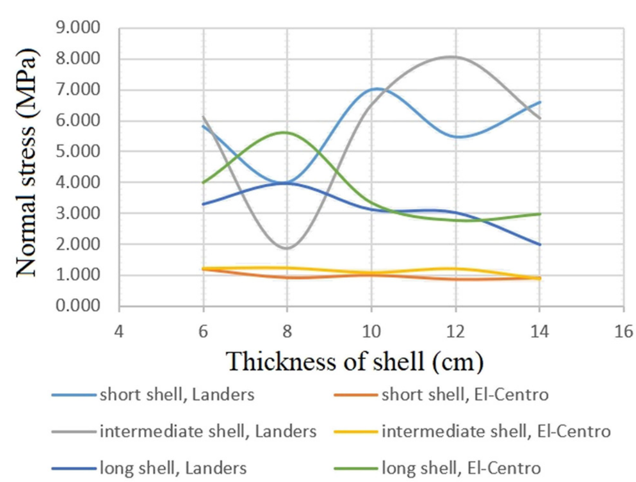

Figure 14: Maximum normal stresses with different thicknesses of shells, subjected to the vertical components of Landers and El-Centro earthquakes.

The first approach to reduce earthquake-induced deformations and stresses is by increasing the fundamental shell frequency via increasing the shell thickness and keeping the span constant (Figures 6, 13 and 14). The central angle of each shell is kept constant at 60°. The plan size is kept constant at 30 m × 15 m, 20 m × 20 m, and 30 m × 60 m for short, intermediate, and long shells, respectively, and the geometry is still based on a cylinder and all other design parameters remain unchanged.

In addition to increasing the fundamental frequency of the shell, the angle at which it is included (Figures 7, 15 and 16) can also be adjusted to reduce earthquake response. Each shell's thickness is kept uniform at 8 cm. The geometric stiffness increases with curvature, and fundamental frequencies increase almost linearly.

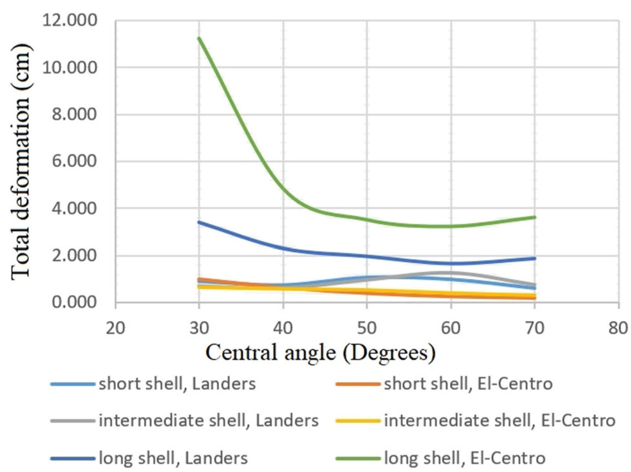

Figure 15: Total deformations with different central angles of shells, subjected to the vertical components of Landers and El-Centro earthquakes.

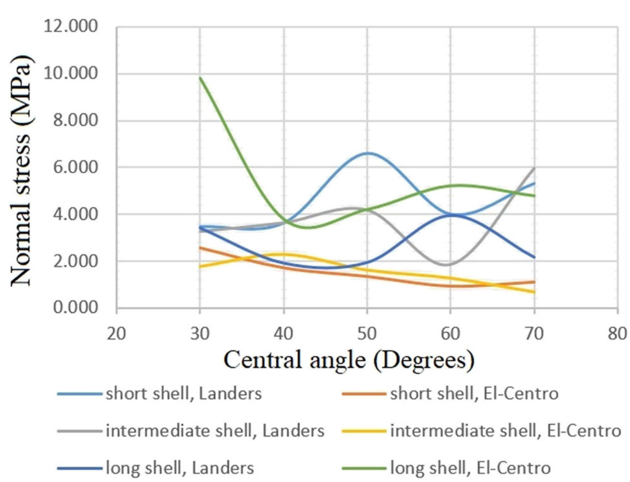

Figure 16: Maximum normal stresses with different central angles of shells, subjected to the vertical components of Landers and El-Centro earthquakes.

4 Discussion of parametric analysis

According to the response spectrum analyses, shells with a high fundamental frequency sustain lower structural damage from the studied earthquake loading because their behavior remains elastic. For an 8 cm-thick cylindrical shell, the effects of span on the fundamental frequency demonstrate that certain shells have a lower seismic risk than others due to these higher frequencies. The shapes of shells with smaller spans have higher fundamental frequencies, so their structural modes are less excited by seismic action than those with comparable forms but larger spans (Figure 5). Under horizontal and vertical earthquake excitations, 8 cm-thick shells with small to medium spans (up to 20 m) will behave entirely elastically. In addition to their high fundamental frequencies, their stiffness and mass play a role in determining their fundamental frequencies. Due to their lightweight design, yet stiffness due to their curved geometry, these shells are not at risk of damage during an earthquake.

The larger spans of reinforced concrete shells have a lower fundamental frequency; hence, they are more vulnerable to earthquake damage. The cylindrical reinforced concrete shells with spans of 20 m and thicknesses of 8 cm are relatively resistant to earthquake damage. In most earthquakes, the horizontal components are critical, making this finding important. However, these shells are at risk of damage from vertical seismic components. Interestingly, the relatively shallow shells presented in this article are more affected by vertical earthquake loading than by horizontal, in-plane earthquake loading. Insufficient rise can cause stresses induced by vertical earthquakes to exceed the concrete maximum tensile stress, resulting in cracking and a reduction in stiffness.

It is possible to increase the shell's fundamental frequency to ensure its elastic behavior under this vertical excitation. Fundamental frequency can be most effectively impacted by changing the shape of the shell. In fact, increasing the curvature of cylindrical shells reduces the stress and deformation response to vertical excitation. Frequency increases almost linearly when the central angle increases from 30 to 70°. Overall, increasing fundamental frequencies and reducing deformations of these shells are best achieved by increasing their stiffness through curvature and thickness of the shell. The fundamental frequency is less affected by changing the shell thickness.

5 Conclusions

A set of reinforced concrete roof shells with a short, square, and long plan is analyzed under earthquake loading. Using cylindrical shells with different spans, this study compares and contrasts fundamental frequencies as well as deformations and stresses for El-Centro, as well as a particularly strong earthquake with a severe vertical component (Landers).

When subjected to either the horizontal or vertical components of the considered ground motions, 8 cm-thick shells with concrete compressive strength of 30 MPa are shown to exhibit elastic behavior as the permissible compressive and tensile strengths are never exceeded in the considered cases. Furthermore, shells with a span of 30 m or more are unlikely to sustain structural damage as a result of the considered seismic actions. As a result of their high stiffness and lightweight nature, these shells exhibit superior structural behavior. Consequently, the examined earthquakes, which mostly trigger lower frequency modes, only slightly affect the shell structural modes due to these characteristics.

Additionally, it is shown that the ultimate tensile stresses in these concrete shells and a span of 20 m can be exceeded by earthquakes with vertical components, which are more common during near-field earthquakes. By increasing the rise and thickness of these shells, the fundamental frequency can be increased, thereby reducing deformations and tensile stresses in the shells, again ensuring elastic response.

Despite the fact that shape and span have a significant impact on the earthquake resistance of concrete roof shells, the proper shape might not be sufficient during certain disastrous seismic events. It is also possible for nearfield seismic events with a high-frequency content or earthquakes with a strong vertical component to cause structural damage. It is important to consider other seismic protection measures beyond shell shape in areas where such seismic events are likely to occur. Shells commonly transfer loads to the ground through a limited number of supports, so base isolation of the supports could be an effective seismic protection measure.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: Most datasets generated and analyzed in this study are included in this manuscript. The other datasets are available on reasonable request from the corresponding author with the attached information.

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